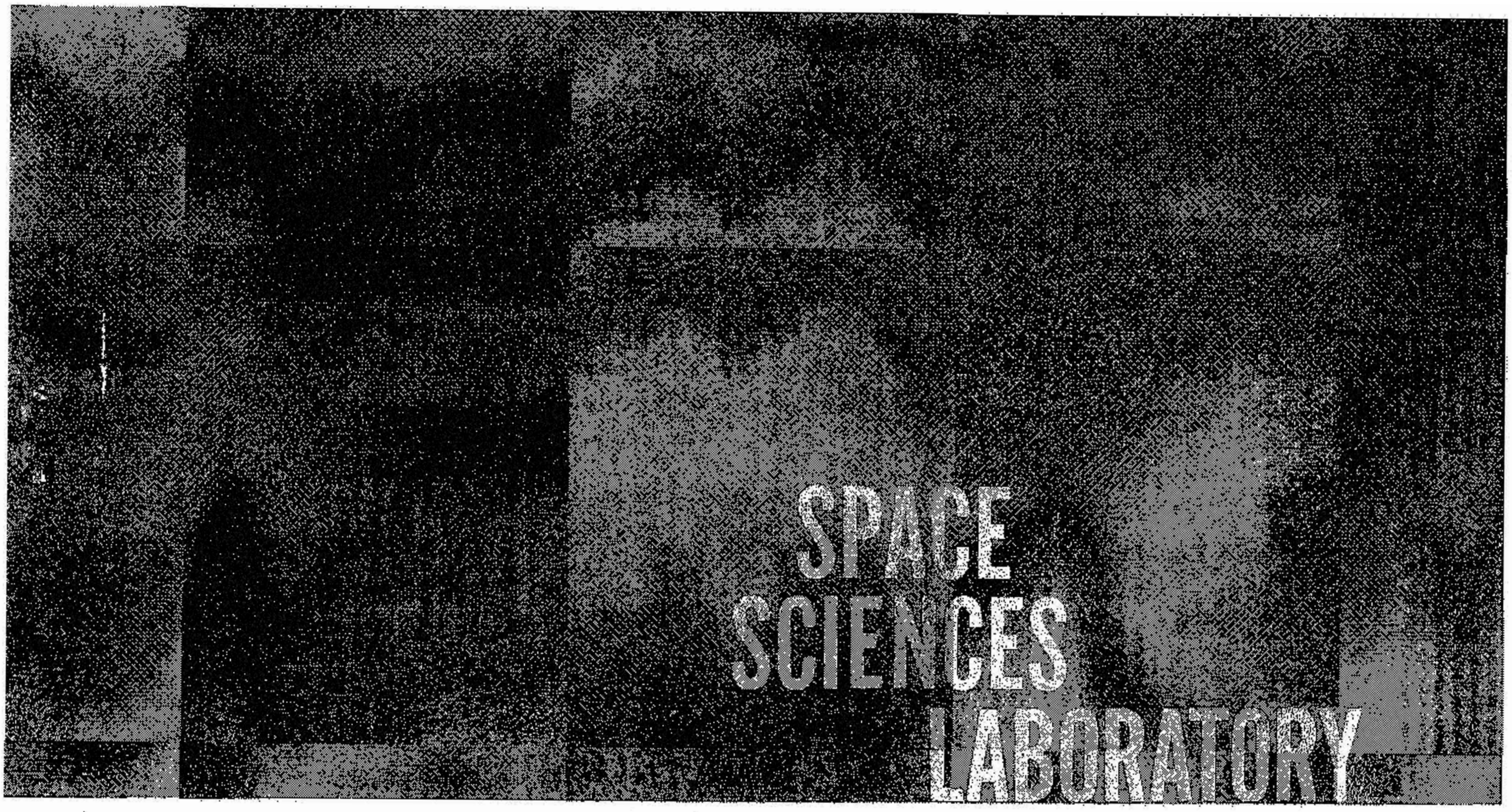

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Technical Report on
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Electron Precipitation Pulsations

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ABSTRACT

When high frequency wave turbulence is present, low frequency micropulsations can strongly modulate the high frequency wave amplitudes, leading to finite amplitude pulsations in the loss rate of energetic electrons from the magnetosphere, micropulsation amplitude is small. An extremely idealized model suggests that the precipitation modulation depends exponentially on the micropulsation amplitude, when the micropulsation period is less than the electron precipitation lifetime.

I. Introduction

It has been popular to relate the loss of charged particles from the magnetosphere to the atmosphere to pitch angle diffusion driven by microscopic plasma turbulence. (Brice, 1963; Andronov and Trakhtengehrts, 1964; Kennel and Petschek, 1966; Cornwall, 1966; Roberts, 1968, 1969; Kennel, 1969.) Those theories have generally described temporally quasi-steady precipitation. However, observed electron precipitation, which is well documented, is rarely even quasi-steady, exhibiting a variety of temporal features on millisecond to perhaps thousand second time scales (Anderson, 1968). Less is known about the temporal structure of proton precipitation. In this paper we attempt a plausible explanation for some of the electron precipitation modulations which is within the framework of pitch angle diffusion theories. In so doing, we propose a new, but simple-minded, nonlinear interaction between wave modes which is stronger than those conventionally discussed in the plasma turbulence literature.

The shortest millisecond to second time scale fluctuations in the electron precipitation rate may well be due to ionospheric effects near the detector (Lampton, 1967; Perkins, 1968). The similarity between the spatial morphology and structure of x-ray microbursts and whistler mode chorus emissions, pointed out by Russell, et al. (1969), suggests that 1 second structured precipitation is related to the structure in the basic turbulent spectrum responsible for precipitation. The mechanism driving precipitation pulsations of periods longer than a few seconds need not be localized near the earth, and may not depend strongly on the structure of the turbulent spectrum, since electrons bounce many times through the equatorial plane in a pulsation period, during which time the chorus structure has changed. Hopefully, long period pulsations can be explained by modulations of averaged pitch angle diffusion processes, which are localized to the equatorial plane.

Quasi-periodic electron precipitation pulsations have a wide range of time scales. Various pulsations with periods from 5 to 300 seconds have been observed by Anger, et al. (1963), Evans (1963), Barcus, et al. (1965), Barcus, et al. (1968), Parks, et al. (1968a), and reviewed by Brown (1966) and Anderson (1968). Riometer and balloon x-ray techniques typically measure precipitation pulsations of electrons with greater than 30 keV energy. However, auroral light also is quasi-periodically modulated, suggested that precipitation fluxes of lower energy (1-10 keV) electrons pulsate also (Belon, et al., 1969). The > 30 keV measurements indicate that precipitation pulsations only occur on an already enhanced precipitation background (Parks et al., 1968a), a fact of crucial importance. In the pulsations, the peak-to-valley precipitation intensity ratios are typically 1.5-3, so that pulsations are not small perturbations upon the background precipitation rate, another important fact. Finally, the peaks tend to have a harder energy spectrum than the valleys.

Using ground-based magnetometers, McPherron, et al. (1968) found that at least those precipitation pulsations with periods in the 5-40 second range are accompanied by magnetic micropulsations with approximately the same period as the precipitation pulsations. It is tempting to suggest that many low frequency precipitation pulsations are caused by micropulsations. (However, Maehlum and O'Brien (1968) have proposed a different mechanism for 50-200 second period precipitation pulsations.) Without good micropulsation information, it is impossible to formulate a precise theory of precipitation pulsations. Therefore, we can at most outline arguments which indicate that small amplitude micropulsations could produce large amplitude precipitation pulsations.

There are several intuitive difficulties with the hypothesized micropulsation source for precipitation pulsations. First, 5-300 second micropulsations, acting alone, conserve the electrons' first and second adiabatic invariants. Secondly, it is unlikely that micropulsation magnetic amplitudes be comparable with the main magnetic field in space. Furthermore, Parks et al. (1968b) have observed low frequency modulations of the energetic electron distributions in space, but of small amplitude; any associated micropulsation field would then have to be small. Thus, micropulsations appear neither fast enough nor strong enough to case precipitation pulsations with 2:1 peak-to-valley intensity ratios. However, it is crucial that precipitation pulsations occur only during active precipitation events when the general precipitation levels are enhanced. In the context of whistler turbulence theory, this means that before micropulsations can significantly affect precipitation, enhanced energetic electron injection must increase the trapped Van Allen electron fluxes above the critical flux for whistler instability. Then, the micropulsations can affect the precipitation rate indirectly, by modifying the whistler instability.

Consider the quasi-steady enhanced electron precipitation state which apparently exists before the onset of electron pulsation activity. The electrons are in pitch angle diffusion equilibrium with (for example) whistler turbulence, with injection of new electrons balance by pitch angle diffusion to the loss cone and subsequent precipitation loss. This equilibrium, a delicate balance between sources and sinks of particles and waves, keeps the growth rate of whistlers near but slightly above marginal stability. However, virtually any small external perturbation of the electrons resonant with whistlers, caused, for instance, by micropulsations, changes the whistler growth rate. With the whistlers already near marginal stability, a linear change in the growth rate leads to an exponential change in the whistler amplitude and therefore in pitch angle diffusion and precipitation rate. Conversely, when the whistlers are below marginal stability, there is no enhanced precipitation background, and micropulsations would not markedly affect precipitation. Thus, we wish to describe a nonlinear interaction, through the resonant particle distribution, between low and high frequency waves which is exponentially large in the low frequency micropulsation amplitude. This exponential nonlinearity should dominate the algebraic nonlinearities almost exclusively discussed heretofore in the plasma literature. By means of this interaction, the electron's first adiabatic invariant is violated, though the micropulsation alone would conserve it, and large amplitude precipitation pulsations are possible even when the micropulsation amplitude is small.

In Section 2, we review a simplified model of the electron pitch angle diffusion equilibrium; here our understanding, though limited, is reasonably secure conceptually. In Section 3, we estimate the change in whistler growth rate due to an idealized micropulsation perturbation of the electron distribution, in which the micropulsation increases the magnetic field strength seen by resonant electrons. Other perturbations would give differences in detail, but hopefully not in qualitative behavior. (One complication, which we can handle to a limited extent, is that the modulated whistler turbulence itself, through enhanced pitch angle diffusion, tends to counteract the micropulsation-induced distortion of the whistler-resonant electron distribution, thereby reducing the net change in the whistler growth rate.) These crude estimates of the growth rate modulation permit similar crude estimates of the change in whistler amplitude, from which we determine the electron precipitation pulsation rate, in Section 4. Despite the extreme assumptions made, we arrive at several physical predictions. In particular, the envelope of precipitation modulation depends strongly on the ratio of wave period to average pitch angle diffusion time and upon whether or not the peaks and valleys correspond to the strong or weak pitch angle diffusion rates.

While some electrons may be precipitated by whistlers some of the time, not all electrons can be precipitated by whistlers all the time. While we have phrased the present theory in terms of whistler turbulence theory, our discussion is sufficiently loose that other pitch angle diffusion theories could be straightforwardly modified to include the interaction with low frequency waves.

2. Steady Precipitation State

2.1) Introduction

Here we review a grossly simplified picture of a steady state pitch angle diffusion process, originally discussed by Kennel and Petschek (1966), hereinafter called KP. The same simplifications will be used in describing precipitation pulsations. We pay especial attention, in 2.3, to the distinction between strong and weak pitch angle diffusion; since the precipitation rate is relatively insensitive to the diffusion coefficient in strong diffusion, precipitation pulsations will be weak in this limit.

2.2) Diffusion Equations

Since whistler waves ought to encounter the greatest number of resonant electrons near the equatorial plane, and since they may be damped elsewhere (Kennel and Thorne, 1967), the turbulent waves are likely to be most intense near the equator. We assume that all wave particle interactions occur in a quasi-uniform region at the equator. Since the electrons must bounce many times through the turbulent region for the diffusion picture to hold, our conclusions to follow are limited to time scales longer than a bounce period. Since whistlers propagating parallel to the local magnetic field grow faster than oblique whistlers, we assume that parallel waves characterize the spectrum at the equator. Here, only electrons which satisfy the cyclotron resonance condition, $v_{\parallel} = \omega - \Omega_{-}/K$, interact resonantly with the turbulent distribution. ω is the wave frequency, K the wave number, Ω_{-} the electron cyclotron frequency, and $v_{\parallel}$ the component of electron velocity parallel to the magnetic field. A discussion of the effects of oblique whistlers is contained in Kennel and Petschek (1969).

When the parallel electron velocity $v_\parallel$ is sufficiently large, cyclotron resonance interactions produce approximately pure pitch angle diffusion. A steady state electron distribution balances pitch angle diffusion to the loss cone and subsequent loss to the atmosphere with an injection source S. If v is the particle speed and $\alpha = \cos^{-1} v_\parallel/v$, the pitch angle, the electron distribution function $f(\alpha,v)$ obeys

$$\frac{1}{\sin\alpha}\frac{\partial}{\partial\alpha}\left(D(\alpha)\sin\alpha\frac{\partial f}{\partial\alpha}\right) = S(\alpha,v) \tag{2.1}$$

outside the loss cone. $D(\alpha)$ is the pitch angle diffusion coefficient, to be estimated shortly. Let the equatorial loss cone be at α_o. Inside the loss cone, $\alpha < \alpha_o$, we neglect the source, but include an atmospheric sink:

$$\frac{1}{\alpha}\frac{\partial}{\partial\alpha}\left(D(\alpha)\,\alpha\frac{\partial f}{\partial\alpha}\right) - \frac{f}{T_B} = 0 \tag{2.2}$$

where the time scale for loss is the quarter bounce time T_B. Since averaged over many bounce periods, f is even in $v_\parallel$, similar equations apply in the two hemispheres $0 \leq \alpha \leq \pi/2$, $\pi/2 \leq \alpha \leq \pi$.

The cyclotron resonance condition specifies only the parallel energy of a resonant electron. Therefore, a given wave interacts with electrons of different total energy. If the resonant region is slightly inhomogeneous spatially, a wave can interact with electrons of different parallel energies. Finally, a given electron can interact with different components of the wave spectrum at different places. It is reasonable, therefore, that as far as the particles are concerned, the effective diffusion coefficient behaves as though the waves were quasi-uniformly distributed in K. Henceforth, we will neglect structure in the whistler spectrum. For a relatively smooth wave spectrum,

$$D(\alpha) \simeq \frac{e^2 B'^2}{m^2 c^2} \times (\Delta K v \cos\alpha)^{-1} \equiv \frac{D_o}{\cos\alpha} \tag{2.3}$$

where e is the electronic charge in esu, m the electron mass, and c the speed of light. ΔK is the effective widths of the wave spectrum, and B' is the wave amplitude integrated over the spectrum.

Equations (2.1) and (2.2) have been solved by KP for the special case that the injection source is concentrated at flat pitches, $S(\alpha,v) = S\delta(\alpha-\pi/2)$. When $\alpha > \alpha_o$,

$$f = \frac{S}{D_o} \{\log(\sin\alpha/\sin\alpha_o) + h(\alpha_o)\} \tag{2.4}$$

where $h(\alpha)$ is the solution within the loss cone

$$h(\alpha) = \sqrt{\frac{D_o T_B}{\alpha_o^2}} \; \frac{I_o\ (\alpha/\sqrt{D_o T_B})}{I_2\ (\alpha_o/\sqrt{D_o T_B})} \tag{2.5}$$

where I_o, I_2 are Bessel functions of imaginary argument.

The diffusion coefficient involves the wave energy. In principle therefore we should account for growth and decay following the individual ray paths of all components of the turbulent spectrum. However, the wave energy losses probably all occur after the waves have convected out of the region of growth. Thus a schematic equation describing the balance of growth and loss is

$$\frac{\partial}{\partial t} B'^2 = 2(\gamma-\nu)\ B'^2 \tag{2.6}$$

where γ is a "typical" growth rate (defined below), and ν is a "typical" loss rate, which is the order of the group velocity divided by the length of the growth region. Again, we have averaged over all structure in the spectral distribution.

γ , the "typical" whistler growth rate, is roughly

$$\gamma \approx \Omega\eta A \tag{2.7}$$

where η and A are defined in KP. Physically, η is roughly the fraction of the total electron distribution near resonance with whistlers. A is a measure of the resonant electron pitch angle anisotropy. For a two temperature anisotropic Maxwellian, $A = T_\perp - T_\parallel / T_\parallel$.

In steady state, $\partial B'^2/\partial t$ must be zero. To maintain a steady state, waves must e-fold a few times in crossing the equatorial plane to make up for losses elsewhere; if $\gamma < \nu$, there is no equatorial whistler growth, $D = 0$, and the source increases the trapped flux (η) unopposed by precipitation until $\gamma = \nu$. If $\gamma >> \nu$, waves e-fold rapidly, rapid precipitation decreases η , and therefore γ , until γ approaches ν . Primarily the number of trapped electrons adjusts until $\gamma \approx \nu$, since the anisotropy is fixed by the boundary conditions of the diffusion problem.

Since (2.3) indicates that $D(\alpha)$ depends dominantly upon B'^2 , (2.6) also describes the time dependence of the diffusion coefficient, whereupon,

$$\frac{\partial \nu}{\partial t} \approx 2(\gamma - \nu)\, D \qquad (2.8)$$

2.3) Limits of Weak and Strong Diffusion

The electron lifetime T_L is the total number of electrons outside the loss cone divided by the diffusion flux into the loss cone. Using (2.4) and (2.5),

$$T_L = \frac{\int_{\alpha_o}^{\pi/2} d\alpha \sin\alpha\, f(\alpha)}{D_o \tan\alpha \left.\frac{\partial f}{\partial \alpha}\right|_{\alpha=\alpha_o}} \approx \frac{h(\alpha_o) + \log(2/e_{\alpha_o})}{D_o} \qquad (2.9)$$

T_L may also be computed by taking the directional flux within the loss cone, integrating over the loss cone, dividing by T_B to give the loss rate, and normalizing to the total trapped flux. From (2.4) and (2.5), we compute the ratio of average directional flux within the loss cone J_p to the averaged directional trapped flux, J_T, by first computing the omnidirectional fluxes and dividing by the solid angle ratio $\alpha_o^2/2$. Thus,

$$\frac{J_p(>v_o)}{J_T(>v_o)} = \frac{\int_{v_o}^{\infty} v^3 dv \frac{<S(v)>}{D_o} \int_0^{\alpha_o} \alpha' \, d\alpha' \, h(\alpha')}{\int_{v_o}^{\infty} v^3 dv \frac{<S(v)>}{D_o} \int_{\alpha_o}^{\pi/2} d\alpha' \sin\alpha' \left| h(\alpha_o) + \log\left(\frac{\sin\alpha'}{\sin\alpha_o}\right) \right|} \times \frac{2}{\alpha_o^2} \tag{2.10}$$

or

$$\frac{J_p}{J_T} \approx \frac{2D_oT_B}{\alpha_o^2} [h(\alpha_o) + \log(2/e_{\alpha_o})] \equiv \frac{T_M}{T_L} \tag{2.11}$$

where T_L is defined in (2.9) and $T_M = 2T_B/\alpha_o^2$, the minimum lifetime, to be discussed shortly.

In the weak diffusion limit, defined by $\alpha_o^2/D_oT_B << 1$, an electron, once having diffused to the loss cone, is lost on the next bounce to the atmosphere. When $\alpha_o^2/D_oT_B << 1$, $h(\alpha_o) << 1$ and T_L is inversely proportional to the diffusion coefficient, $T_L \approx 3/D_o$. Since increases in the whistler amplitude decrease the electron lifetime, the whistler amplitude can adjust so that precipitation balances injection. Since $J_p/J_T << 1$, the fluxes in and near the loss cone are anisotropic.

When $\alpha_o^2/D_oT_B \approx 1$, a particle can random walk across the loss cone before it reaches the atmosphere. Not all the particles in the equatorial plane loss cone will then actually be precipitated. When $\alpha_o^2/D_oT_B >> 1$, nearly all loss cone particles remain trapped. In this limit, the steady diffusion problem approaches one without sinks, for which the only steady solution is an isotropic pitch angle distribution. Since the electrons then are equally probably distributed at all pitch angles, the lifetime is the quarter-bounce time divided by the probability that the particle be in the loss cone (which is just the ratio of the solid angle outside the loss cone to inside), or $T_M = 2T_B/\alpha_o^2$. This is clearly the smallest possible lifetime; at $L = 6$, for ≈ 40 keV electrons, it is roughly 200 seconds. Once the strong diffusion limit is approached, large increases in the diffusion coefficient diminish the precipitation lifetime only slightly. Since $J_p/J_T \to 1$, the fluxes near the loss cone approach isotropy as the diffusion coefficient increases.

2.4) Summary

In the steady precipitation state preceding a precipitation pulsation event, the sources and sinks of particles and waves are delicately balanced. Pitch angle diffusion to the loss cone removes particles at the rate they are injected, at the same time adjusting the electron pitch angle anisotropy and intensity so that the average wave growth rate balances the average wave loss rate.

In describing this steady state, we arrived at several idealizations to be helpful for the time-dependent discussions to follow. Namely, we shall neglect all structure in the wave distribution and pitch angle diffusion coefficient, assume that the "typical" growth rate (2.7) adequately describes the averaged whistler spectrum, and neglect all details of the propagation and loss of whistler energy, lumping all uncertainty into the effective convective loss rate ν .

3. Modulation of Turbulent Whistler Distribution

A wave exerts a periodic force which distorts the particle's velocity distributions. In particular, a micropulsation will modulate the distribution of electrons resonant with whistlers. Over part of the micropulsation phase, the whistler growth rate should increase. Ordinarily, this small increase is not significant; however, when whistler turbulence is already present, small increases in the already positive growth rate acting on an already enhanced wave distribution can produce significant increases in wave energy density. Whistler ray paths will also be modified by the micropulsation, altering thereby the effective loss rate of whistler energy. However, it is difficult to visualize cases in which growth and loss do not become unbalanced; thus turbulent levels and electron pitch angle scattering rates should be modulated.

Without observations of micropulsation polarizations, the low frequency wave forces are ill-defined. As a qualitative illustration, we consider model micropulsation perturbations which modify the magnetic field strength felt by the resonant electrons. For example, the micropulsation could vary the magnetic field strength, or transport resonant electrons and waves into an increasing magnetic field, or both. In this model, high frequency whistlers find themselves in a time varying magnetic field $B(t)$,

$$B(t) = B_o(1 + b \sin \omega_o t) \tag{3.1}$$

where ω_o is the micropulsation frequency, and $b \ll 1$.

We now attempt to relate temporal changes in the typical whistler growth rate (2.7),

$$\frac{d \ln \gamma}{dt} \simeq \frac{\partial \ln B}{\partial t} + \frac{\partial \ln A}{\partial t} + \frac{\partial \ln \eta}{\partial t} \quad , \tag{3.2}$$

to B(t) . The growth rate should be modulated simply because the electron cyclotron frequency, the basic scale frequency for whistler processes, changes with time. Secondly, the pitch angle anisotropy might be modified. Thirdly, because $|B|$ changes, the total electron density and the resonant electron density should also change. However, η , their ratio, should not change as much from this effect. On the other hand, if the loss rate of high energy electrons is modulated due to changes in the turbulence level, η will decrease. Thus, enhanced turbulence acts like collisions to relax the perturbation in the distribution function. However, it cannot entirely counteract the basic perturbation. Many other effects cannot be treated within our approximations. For example, changing $|B|$ alters the energies of electrons resonant with whistlers; perhaps destabilizing some waves and damping others preferentially.

In the absence of whistler turbulence, micropulsations would conserve at least the first and second electron adiabatic invariants. First invariant conservations implies that $T_\perp$ is proportional to B . Assuming that $T_\parallel$ does not vary strongly, we may estimate

$$\frac{d \ln A}{dt} \approx - \frac{T_\perp}{T_\perp - T_\parallel} \frac{\partial \ln B}{\partial t} = \frac{1 + A}{A} \frac{d \ln B}{dt} \quad . \tag{3.3}$$

Since the equilibrium $A \approx \frac{1}{6}$ (KP, 1966), A is roughly proportional to $\ln B$. The modulation of the anisotropy when whistler turbulence is present will in fact be smaller than in its absence, since pitch angle diffusion relaxes anisotropy changes.

In steady state, precipitation balances injection, and η is constant. However, increases in the turbulent whistler level (due to micropulsations) unaccompanied by increases in the injection level, might diminish η through enhanced precipitation. However, if the fractional decrease of η in one wave period is small, this effect will not completely counteract increases in the growth rate. Let T_L^* be the electron lifetime averaged over a micropulsation period; when $\omega_o T_L^* >> 1$, η will remain roughly constant over a wave period. The observation that the peak to valley precipitation rate ratio is two, implies $T_L^* \sim T_L/2$, so that comparison of the micropulsation period with the equilibrium electron lifetime establishes the validity of this small relaxation limit. When $\omega_o T_L^* >> 1$, we may drop $\partial \ln \eta/\partial t$; substituting (3.3) into (3.2) implies $\frac{\partial \ln \gamma}{\partial t} \simeq \frac{1}{A}\frac{\partial \ln B}{\partial t}$. Integrating this equation, using (3.1), and dropping terms of $O(b^2)$,

$$\frac{\gamma(t)}{\nu_o} = 1 + \frac{b}{A}\sin \omega_o t \tag{3.4}$$

where we have assumed the equilibrium growth rate γ_o equals the equilibrium loss rate ν_o.

When $\omega_o T_L^* \simeq 1$ a full diffusion treatment is needed to find $\gamma(t)$. When $\omega_o T_L^* << 1$ the micropulsation slowly modulates the equilibrium electron distribution. For a rough estimate here, we use the equilibrium formula for η (derived in KP) to find $\frac{\partial \ln \eta}{\partial t} = -\frac{\partial \ln D}{\partial t}$. Substituting this into (3.2), and integrating,

$$\frac{\gamma(t)}{\nu_o} = \left[\frac{B(t)}{B_o}\right]^{1/A} \frac{D_o}{D(t)} \quad . \tag{3.5}$$

Here, time increases in the diffusion coefficient clearly reduce the modulation of $\gamma(t)$.

In addition to the above relaxation of the growth rate due to enhanced precipitation, there might be other changes in γ due to "ripples" in the pitch angle distribution of scale $\Delta\alpha \simeq \sqrt{\omega_o/D}$ arising from nonmonotonic pitch angle solutions to the full time dependent diffusion equation.

These higher modes should have a faster time scale than ω_o , and their precise details should vary spatially on scale length of the electron gyro-radius. However, since the observations of precipitation modulation integrate over many electron gyro-radii, the "ripples" may be smoothed.

Time Dependence of Diffusion Coefficient

We now solve for $D(t)$ using (2.8) with a time dependent growth rate. Given the crude approximations leading to (3.4) and (3.5) it is wise to attach significance only to the qualitative behavior obtained in several limiting cases.

First, we consider the small relaxation limit, where $\omega_o >> D$. Substituting (3.4) into (2.8), and integrating, assuming $D = D_o$ at $t = 0$

$$D|t| = D_o \exp\left[\frac{2\nu_o b}{|\omega_o| A} (1 - \cos \omega_o t) \right] . \tag{3.6}$$

In the small relaxation limit, the modulation of D is exponentially large in b . This case gives the largest precipitation modulation. Since the increment of whistler energy created in the micropulsation compression phase equals that reabsorbed in the rarefaction phase, $D(t)$ is always greater than D_o . Relaxation will gradually decrease the growth rate and damp the diffusion coefficient modulations on time scales of $1/D$. Presumably, as more and more particles are lost, more and more waves will sink below marginal stability, and the whistler spectrum will shrink. Should the micropulsation also act as a particle injection source, relaxation could conceivably be counteracted to some extent. Finally, in strong pitch angle diffusion, the small relaxation limit ought to apply for all frequencies, since the enhanced loss of electrons will be small.

Now we turn to the quasi-static limit where $D \lesssim \omega_o$. Since a particle can diffuse to the loss cone in a wave period, (2.8) must be solved including the changes in D due to particle losses. Substituting (3.5) into (2.8), dropping terms of order b^2 , we find

$$D(t) = D_o + \frac{2\nu_o D_o}{1 + (2\gamma_o/\omega_o)^2} \left[\frac{2\nu_o}{\omega_o^2} \sin \omega_{LF} t - \frac{1 - \cos \omega_o t}{\omega_o} \right] \frac{b}{A} \tag{3.7}$$

If $\omega_o/\nu_o << 1$, then

$$D(t) = D_o \left(1 + \frac{b}{A} \sin \omega_o t\right) \quad . \tag{3.8}$$

Here the modulation of the diffusion coefficient is only linearly proportional to the micropulsation amplitude, and the whistler amplitude responds adiabatically to the micropulsation modulation.

In summary, the whistler growth rate should be modulated by a low frequency micropulsation because the resonant electron distribution is modified. Over part of the micropulsation phase, the growth rate should increase. When the wave period is much less than the electron lifetime, enhanced precipitation does not remove many particles in a wave period. Here the growth rate is linearly proportional to the micropulsation amplitude. The whistler amplitude is exponentially proportional to the micropulsation amplitude. When the wave period is much longer than the lifetime, the growth rate modulation is much weaker, since the perturbation is more nearly counteracted by enhanced diffusion, and the whistler amplitude depends only linearly upon that of the micropulsation.

4. Precipitation Modulation

The small loss cone appropriate to many geophysical situations permits a simple expression for precipitation modulation, found without solving the full time dependent diffusion equations. If $\alpha_o^2 \ll D/\omega_o$, the pitch angle profiles in and near the loss cone approach the steady state profile, since particles can random walk across the loss cone in one wave period. Thus the ratio of precipitated to trapped fluxes, observed near the earth at the mirror points, will be given roughly by (2.12), with D a prescribed function of time. This steady state solution is valid only near the loss cone. In addition, we can only describe the ratio of trapped to precipitated fluxes since the absolute flux levels can be affected by variations in the effective source strength of particles to small pitches by time dependent ripples of scale angle $\sqrt{D/\omega_o}$ in the distribution function at large pitches.

Assuming that the time dependence of D dominates all others we find

$$\frac{J_p(t)}{J_T(t)} \simeq \frac{T_{min}}{T_L(t)} \tag{4.1}$$

where $T_L(t)$ is the effective electron lifetime assuming that diffusion continues steadily at the instantaneous rate $D(t)$.

We may now distinguish four separate precipitation modulation envelopes, depending upon the background diffusion rate and the micropulsation amplitude. Either the background diffusion rate is weak or strong; similarly, the maximum diffusion rate can be weak or strong, implying the four combinations listed in Table 1.

TABLE 1

Case Number	Minimum Diffusion Rate	Maximum Diffusion Rate
1.)	Weak	Weak
2.)	Weak	Strong
3.)	Near Strong	Near - Strong
4.)	Near Strong	Strong

The weak-weak case can be calculated easily, since the maximum diffusion rate is still weak, so that $T_L(t) \sim 1/D\,(t)$. Substituting the zero relaxation approximation expression for $D\,(t)$, (3.6), into (4.1) we find

$$\frac{J_p(t)}{J_T(t)} = \frac{T_{min}}{T_L(0)} \exp\left[\frac{2\nu_o\, b}{|\omega_o|\, A} (1 - \cos\omega_o t)\right] \tag{4.2}$$

where $T_L(0)$ is the lifetime at $t = 0$. Similarly, the near strong-weak and near strong-strong cases can also be computed, since the minimum diffusion rate is strong.

$$\frac{J_p(t)}{J_T(t)} = 1 - \frac{\alpha_o^{\,2}}{D_o T_B} \exp\left[-\frac{2\nu_o\, b}{|\omega_o|\, A} (1 - \cos\omega_o t)\right] \tag{4.3}$$

where $\alpha_o^{\,2}/D_o T_B \ll 1$. When $\frac{4\nu_o}{A\,|\omega_o|}\, b \ll 1$, there are oscillations about the background state $J_p/J_t = 1 - \alpha_o^{\,2}/D\,T_B$. When $\frac{4\nu_o}{A|\omega_o|}\, b \gg 1$,

$J_p/J_T \approx 1$ for significant portions of the micropulsation period. Here the peaks of the oscillation are "clipped." The weak-strong case is mixed, with the valleys behaving like (4.2), and the peaks like (4.3), with clipped profiles. A sketch of these four cases is shown in figure 1.

In the quasi-static limit of eq. (3.8), the precipitation profile should vary more or less linearly, corresponding to slow, relatively small amplitude swells of the background precipitation rate.

Assuming that any time dependent ripples in the pitch angle distribution spatially average out and, in the zero relaxation limit, that the trapped flux is approximately constant over one micropulsation period, (4.2) describes the magnitude of the precipitation flux in the weak-weak case. We can then renormalize (4.2) to the background precipitation rate, $J_p(0)$, and taking the logarithm of both sides we have

$$\log\left(\frac{J_p(t)}{J_p(0)}\right) = \frac{2\nu_o b}{|\omega_o| A}(1 - \cos \omega_o t) \quad . \tag{4.4}$$

Log $J_p(t)$ sinusoidally varies with a positive definite time dependence, $1 - \cos \omega_o t$. This behavior qualitatively agrees with observations by Parks et al. (1968).

We now estimate b by taking a typical peak to valley ratio $J_p(\pi)/J_p(0) \sim 2$, a background whistler growth rate $\nu_o \sim 1$ rad/sec , $A \sim 1/6$, and a micropulsation wave period of 10 seconds, to find $b \sim .02$. Thus the physical statement that when the whistler noise and electron distribution are in diffusion equilibrium, a small distortion of the electron distribution can produce large changes in the whistler amplitude and precipitation rate appears self-consistent. A direct comparison with observations is difficult. For example, an absolutely reliable ground based magnetometer measurement of micropulsation amplitude would not yield the amplitude in space, because for instance, attenuation in the ionosphere and at the ionosphere-neutral atmosphere interface is difficult to estimate.

Moreover, the mode structure along the lines of force is not known, so that the amplitude at the top of the ionosphere cannot be directly related to that in the equatorial plane where whistler modulation occurs. Similarly a satellite measurement of micropulsations in space would not necessarily relate to precipitation pulsations unless correlated modulations of the VLF spectrum were also found.

5. Summary

We summarize our physical arguments, comparing them with observations where possible. Our arguments are far from rigorous, and suggest only that large precipitation pulsations could be caused by small micropulsations.

(1) At disturbed times, enhanced injection increases the trapped electron fluxes above the critical flux for precipitation instabilities. We have considered whistler instability as an example.

(2) A diffusive equilibrium is established, in which whistlers scatter electrons into the atmospheric loss cones at the rate they are injected, and the whistler growth rate from equatorial resonant electron interactions balances wave losses from convective propagation.

(3) More than one instability is likely to occur at disturbed times. For instance, in addition to the high frequency whistler turbulence driving particle precipitation, low frequency micropulsations are also enhanced at disturbed times (Coroniti et al., 1968).

(4) The low frequency oscillations perturb the diffusion equilibrium by distorting the resonant electron distribution and modifying the whistler ray paths. When the frequencies differ greatly, as for micropulsations and whistler turbulence, this interaction is physically visualizable. The micropulsation adiabatically modulates the velocity distribution of those electrons resonant with whistlers. This velocity space distortion changes the whistler growth rate, which depends upon the resonant electron distribution. On the other hand, the increased whistler turbulence which follows from the increases in the growth rate acts like collisions to relax the micropulsation modulation.

However, this relaxation can never completely counteract the micropulsation perturbation, so the whistler growth rate should always be modulated at least somewhat by micropulsations. Only when the whistlers are already unstable can small changes in their growth rate lead to large changes in their amplitude. Thus precipitation pulsations should occur only superposed on an already enhanced precipitation background, in agreement with observations by Parks et al. (1968a).

(5) If the micropulsation period is much shorter than the precipitation lifetime, only a small fraction of the electrons can be precipitated in a wave period, and the unrelaxed growth rate modulation is large in this limit. The modulation of the whistler amplitude is exponentially large in the micropulsation amplitude. Since the minimum lifetime is a few hundred seconds on auroral lines of force, large whistler modulations ought to be found only in the 3 - $\sim$ 300 second period range.

(6) If the micropulsation period is comparable with electron lifetime, the modulations of the whistler intensity should be only linearly proportional to the micropulsation amplitude.

(7) Pulsations of the whistler amplitudes in space produces electron precipitation pulsations. McPherron et al. (1968) have correlated precipitation pulsations with Pc2 and Pc3 micropulsations.

(8) Unlike the whistler amplitude modulations, precipitation pulsations should have a variety of temporal envelopes depending upon the strength of the pitch angle diffusion rate, and upon the micropulsation frequency. The largest pulsations should occur when the minimum pitch angle diffusion rate is weak, and when the micropulsation period is much shorter than the electron lifetime. Even here, however, there are two distinct cases. Either the maximum diffusion rate is weak, in which

case the precipitation is exponentially modulated with a smooth profile, or the maximum diffusion rate is strong, in which case the precipitation profile is clipped. During the micropulsation compression phase the precipitation rate exponentiates to its strong diffusion maximum, whereupon it stays constant, until the micropulsation decompression reduces the diffusion coefficient sufficiently that the precipitation rate falls exponentially below strong diffusion to its background weak diffusion rate. This cycle repeats. When pitch angle diffusion is always strong, increases in the diffusion coefficient do not increase the precipitation rate. Here, relaxation of the micropulsation perturbation should be small for all micropulsation frequencies, and whistler amplitude modulations could be large even at low frequencies. However, these modulations would not appear in the precipitation. Finally, when the micropulsation period and weak diffusion lifetime are comparable, precipitation pulsations should be weak.

(9) Several features of the pulsation envelopes described by Parks et al. (1968a) may be consistent with the ideas presented here, in particular, the sinusoidal time dependence of the logarithm of the observed x-ray fluxes. There is a suggestion of a clipped precipitation pulsation in their figure 4.

(10) Since low energy particles have larger fluxes than high energy particles, they are more likely to be strongly unstable and in strong diffusion. In this case, precipitation modulation would be most pronounced in higher energy channels. At the very highest energy channels the particles are not sufficiently numerous to be unstable in the first place. The high energy particles precipitation might still be weakly

modulated due to parasitic effects discussed by Kennel and Petschek (1969). These energy considerations are summarized in figure 2.

(11) Experimental correlations between enhanced whistler (or other high frequency) noise and micropulsations are needed to test these ideas. Helliwell (1965) has described quasi-periodic VLF emissions with periods similar to those of micropulsations and precipitation pulsations, and has already suggested that hydromagnetic modulation of the basic VLF source might be responsible.

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Figure Captions

Fig. 1. Possible Precipitation Pulsation Envelopes.

The logarithm of the ratio of peak to valley precipitation fluxes, $\log J_p/J_v$, has a time envelope which depends upon the strength of the pitch angle diffusion rate and upon the strength of the micropulsation perturbation. The four cases, summarized in Table 1, are sketched here.

Fig. 2. Schematic Energy Dependence of Pulsation Amplitudes.

If low energy fluxes are on strong diffusion, they will have weak pulsations. Intermediate energy electrons (here chosen arbitrarily to lie between 10 and 10^2 keV) could be unstable, but on weak diffusion; these will have strong pulsations. Finally, high energy electrons could have weak pulsations due to parasitic effects (Kennel and Petschek, 1969).

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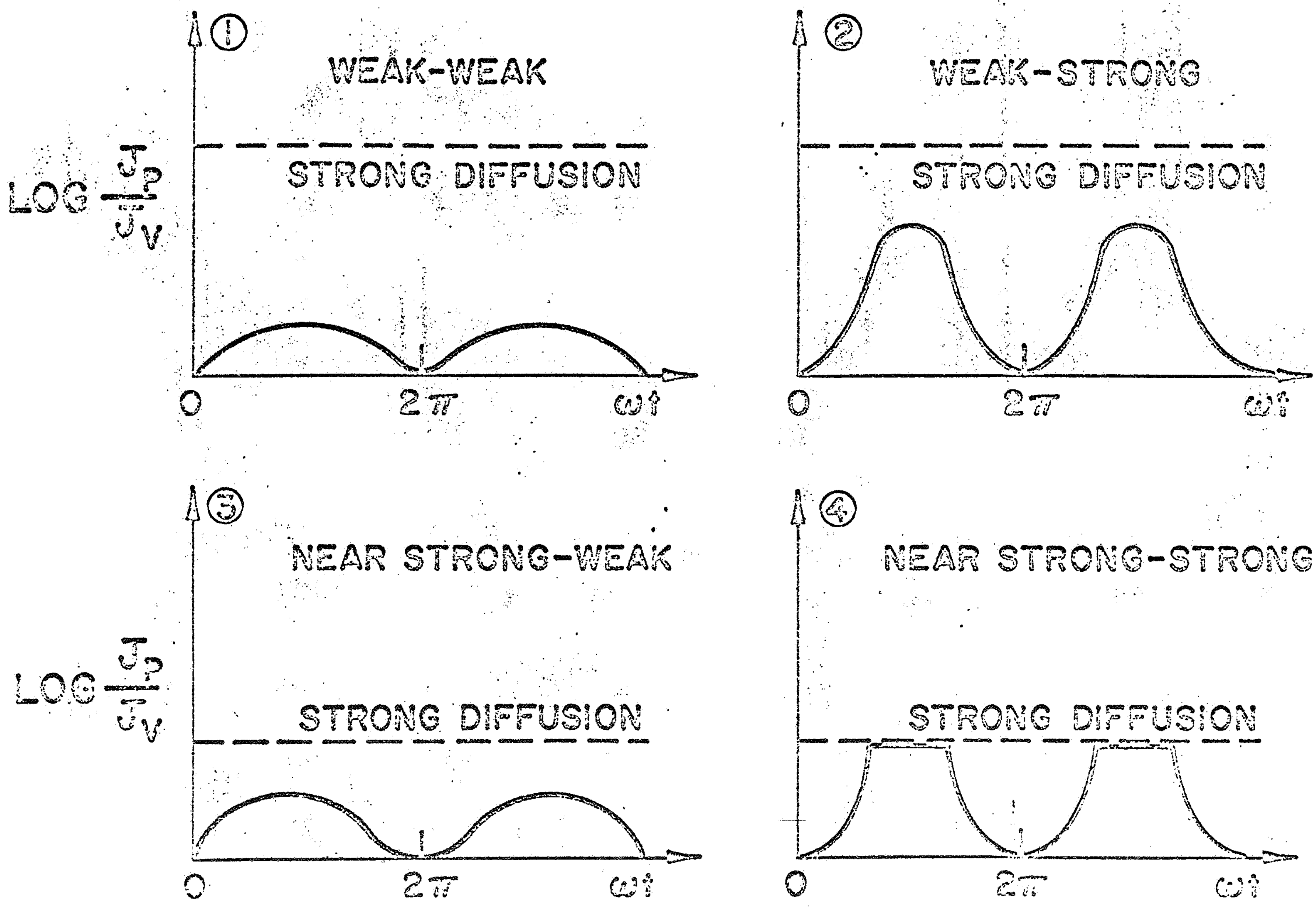
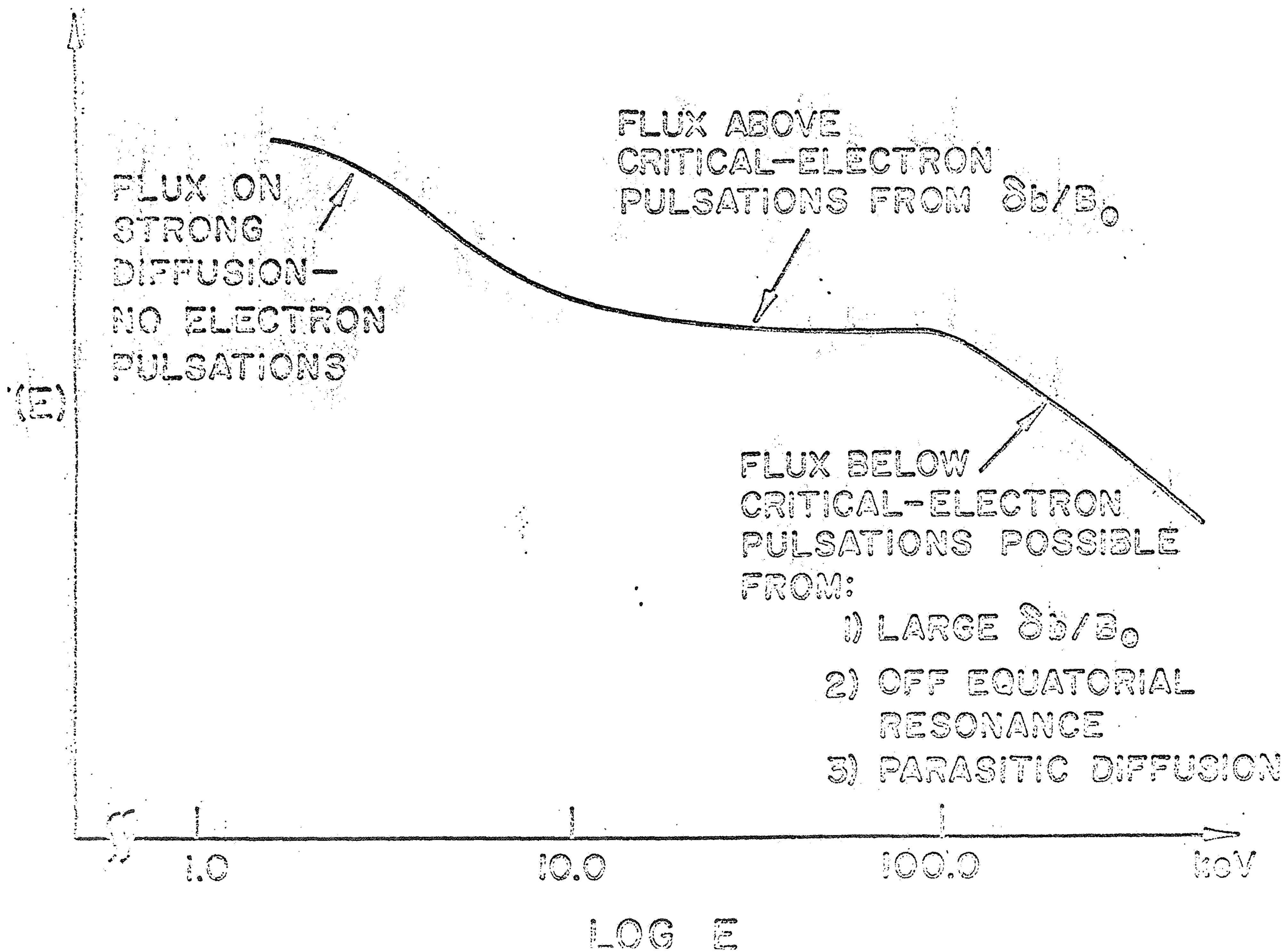
POSSIBLE ELECTRON PULSATIONS




FLUX ON STRONG DIFFUSION— NO ELECTRON PULSATIONS
FLUX ABOVE CRITICAL-ELECTRON PULSATIONS FROM δb/B₀
FLUX BELOW CRITICAL-ELECTRON PULSATIONS POSSIBLE FROM:
1) LARGE δb/B₀
2) OFF EQUATORIAL RESONANCE
3) PARASITIC DIFFUSION
(E)
1.0
10.0
100.0
keV
LOG E