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# Electron scattering on microscopic corrugations in graphene

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We discuss various scattering mechanisms for Dirac fermions in single-layer graphene. It is shown that scattering on a short-range potential (e.g. due to neutral impurities) is mostly irrelevant for electronic quality of graphene, which is likely to be controlled by charged impurities and ripples (microscopic corrugations of a graphene sheet). The latter are an inherent feature of graphene due to its two-dimensional nature and can also be an important factor in defining the electron mean-free path. We show that certain types of ripples create a long-range scattering potential, similar to Coulomb scatterers, and result in charge-carrier mobility practically independent of carrier concentration, in agreement with experimental observations.

**Keywords:** graphene; electron transport; scattering mechanisms; two-dimensional crystals

## 1. Introduction

The experimental discovery of graphene, the first example of strictly two-dimensional crystals (Novoselov *et al.* 2004, 2005*a*), has led to the development of a new active research area (for review, see Geim & Novoselov 2007; Katsnelson 2007), which started to expand particularly rapidly after it was demonstrated that graphene had a relativistic-like electronic spectrum with massless charge carriers (Novoselov *et al.* 2005*b*; Zhang *et al.* 2005). One of the most striking features of graphene is its very large electron mean-free path  $l$ , which makes graphene an enticing material for novel electronics applications (Geim & Novoselov 2007). Already in the first experiments, room temperature ballistic transport at distances up to 1  $\mu\text{m}$  was demonstrated (Novoselov *et al.* 2004). Charge-carrier mobilities currently reach 15 000  $\text{cm}^2 \text{V}^{-1} \text{s}^{-1}$  or more in single-layer graphene but further improvement of its electronic quality is desirable from the point of view of both potential applications and the search for new transport phenomena (such as, for example, the fractional quantum Hall effect and other interaction phenomena that are believed to be suppressed by impurity scattering). Therefore, it is not

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surprising that mechanisms of electron scattering in graphene have been a subject of intensive theoretical investigation (see Ando 2006; Katsnelson 2006; McCann *et al.* 2006; Nomura & MacDonald 2006; Ostrovsky *et al.* 2006; Peres *et al.* 2006; Hwang *et al.* 2007 and references therein). In particular, it has been shown that scattering by defects with a short-range potential having a typical radius  $R \approx a$  does not significantly change graphene's resistivity  $\rho$ , resulting in a rather small additional contribution  $\delta\rho \approx (h/4e^2)n_i a^2$ , where  $a$  is the interatomic distance and  $n_i$  is the impurity concentration (Ando 2006; Nomura & MacDonald 2006; Katsnelson & Novoselov 2007). This scattering contribution can be neglected for any feasible concentration of such defects. On the other hand, graphene's resistivity was experimentally found to be inversely proportional to concentration  $n$  of charge carriers, which means that their mobility  $\mu$  is almost independent of  $n$  (Novoselov *et al.* 2004, 2005*b*; Zhang *et al.* 2005; Geim & Novoselov 2007). To explain this non-trivial observation, scattering on a long-range Coulomb potential due to charged impurities was invoked (Ando 2006; Nomura & MacDonald 2006; Hwang *et al.* 2007), and it is commonly believed that Coulomb scatterers limit charge-carrier mobility in the existing graphene samples.

In this paper we point out that in addition to charged impurities there is another source of static disorder that can lead to concentration-independent mobility  $\mu$  but so far has not been taken into account. This disorder is microscopic corrugations of a graphene sheet, which are unavoidable because strictly two-dimensional crystals are extremely soft and flexible and, in fact, on the verge of structural instability (Peierls 1934, 1935; Landau 1937; Mermin 1968; Landau & Lifshitz 1980; Nelson *et al.* 2004). The fact that graphene is actually not flat was revealed by transmission electron microscopy (TEM) where suspended graphene membranes were found to exhibit pronounced corrugations with a dominant lateral size of several nanometres and a height of up to 1 nm (Meyer *et al.* 2007*a,b*). Somewhat larger corrugations were earlier observed for graphene films lying on a SiO<sub>2</sub> substrate which is attributed to the inevitable wrinkling that any thin fabric is likely to exhibit after its deposition onto a rough surface (Morozov *et al.* 2006). Nanometre-sized ripples similar to those found in TEM were also reported in scanning-probe microscopy studies of graphene (Ishigami *et al.* 2007; Stolyarova *et al.* 2007). In the latter case, ripples were explained by intrinsic roughness of thermally grown SiO<sub>2</sub> (Ishigami *et al.* 2007).

Whatever the origin of graphene's non-flatness, it must act as an additional source of scattering. In this work we investigate this scattering mechanism and estimate its contribution to electron transport in graphene. We show that if ripples are originally created by thermal fluctuations and then become quenched (and static) during deposition on a substrate, this non-flatness should result in sufficient scattering to explain both graphene's submicrometre mean-free path and mobility independent of concentration  $n$ .

## 2. Scattering of Dirac fermions by microscopic defects with radially symmetric potential

To appreciate specifics of scattering mechanisms for massless Dirac fermions, as compared with conventional two-dimensional electron systems (Ando *et al.* 1982), it is instructive to consider first the case of a small concentration of defects

$n_i \ll 1/a^2$  with a radial scattering potential (Katsnelson & Novoselov 2007). Resistivity  $\rho$  due to static impurities can generally be written as (Shon & Ando 1998; Ziman 2001)

$$\rho = \frac{2}{e^2 v_F^2 N(E_F)} \frac{1}{\tau(\mathbf{k}_F)}, \quad \frac{1}{\tau(\mathbf{k}_F)} = n_{\text{imp}} v_F \int_0^{2\pi} d\varphi \frac{d\sigma(\varphi)}{d\varphi} (1 - \cos \varphi), \quad (2.1)$$

where  $v_F$  and  $\mathbf{k}_F$  are the Fermi velocity and wavevector (the Fermi wavevector is given by  $\mathbf{k}_F = (\pi n)^{1/2}$ , where  $n$  is the two-dimensional carrier concentration), respectively;  $\tau$  is the mean-free path time;  $\sigma(\varphi)$  is the angle-dependent scattering cross section; and  $N(E_F) = 2\mathbf{k}_F / \pi \hbar v_F$  is the density of states at the Fermi level (the latter expression takes into account the double-spin and double-valley degeneracy of charge carriers in graphene). The product  $N(E_F) \cdot v_F$  is proportional to  $\mathbf{k}_F$  for both Dirac fermions and conventional two-dimensional electrons and, therefore, any difference in transport properties of the two systems can only come through differences in their  $\sigma(\varphi)$ . Note that equation (2.1) is based on the semi-classical Boltzmann equation but its applicability for Dirac fermions has been proven by Auslender & Katsnelson (2007) who showed that, away from the Dirac point, inter-band (electron-hole) scattering processes resulted in negligible corrections to conductivity.

To determine the scattering cross section, one needs to solve the two-dimensional Dirac equation, which for the case of massless particles and radially symmetric scattering potential  $V(r)$  takes the form

$$\left. \begin{aligned} \frac{dg_l(r)}{dr} - \frac{l}{r} g_l(r) - \frac{i}{\hbar v_F} (E - V(r)) f_l(r) &= 0, \\ \frac{df_l(r)}{dr} + \frac{l+1}{r} f_l(r) - \frac{i}{\hbar v_F} (E - V(r)) g_l(r) &= 0, \end{aligned} \right\} \quad (2.2)$$

where  $l=0, \pm 1, \dots$  is the angular-momentum quantum number and  $g(r)e^{il\varphi}$  and  $f(r)e^{i(l+1)\varphi}$  are components of the Dirac pseudospinor (note that the spinor index in graphene labels its two crystal sublattices rather than directions of the real spin). To be specific, we will further consider the case of a finite concentration of electrons,  $E \equiv \hbar v_F \mathbf{k} > 0$ .

Modifying the standard scattering theory (Newton 1966) for our two-dimensional case, one should try the solution of equation (2.2) outside a region of the action of the scattering potential in the form

$$g_l(r) = A \left[ J_l(\mathbf{k}r) + t_l H_l^{(1)}(\mathbf{k}r) \right] \quad \text{and} \quad f_l(r) = iA \left[ J_{l+1}(\mathbf{k}r) + t_l H_{l+1}^{(1)}(\mathbf{k}r) \right], \quad (2.3)$$

where the terms proportional to Bessel (Hankel) functions describe incident (scattered) waves. As a result, we obtain

$$\frac{d\sigma}{d\varphi} = \frac{2}{\pi \mathbf{k}} \left| \sum_{l=-\infty}^{\infty} t_l e^{il\varphi} \right|^2. \quad (2.4)$$

The Dirac equation for massless particles (2.2) has an important symmetry with respect to replacements  $f \leftrightarrow g$ ,  $l \leftrightarrow -l-1$ , which means  $t_l = t_{-l-1}$ . Accordingly, equation (2.4) can be rewritten as

$$\frac{d\sigma}{d\varphi} = \frac{8}{\pi \mathbf{k}} \left| \sum_{l=0}^{\infty} t_l \cos[(l+1/2)\varphi] \right|^2. \quad (2.5)$$

Note that for  $\varphi = \pi$  the cross section is exactly zero, which means that backscattering is absent rigorously. This property was previously noted for the case of ultrarelativistic particles in three dimensions (Berestetskii *et al.* 1971). More recently, it was shown that the absence of backscattering for Dirac fermions could be responsible for their high mobility in one-dimensional channels such as carbon nanotubes (Ando *et al.* 1998), for anomalous transport through p–n graphene junctions (Cheianov & Fal’ko 2006*a*; Katsnelson *et al.* 2006) and, also, for Friedel oscillations around charged impurities in graphene (Cheianov & Fal’ko 2006*b*).

For the case of a finite-range potential with a radius  $R$  much smaller than electron wavelength  $kR \ll 1$  (examples are neutral impurities and atomic-scale roughness of a substrate), all terms in equation (2.5) except for the  $s$  scattering ( $l=0$ ) can be neglected. Then, for a random scattering potential, we generally obtain  $t_0(\mathbf{k}) \propto kR$  which yields the above mentioned estimate  $\delta\rho \approx (h/4e^2)n_{\text{imp}}R^2$ . So little scattering in graphene, as compared with conventional two-dimensional systems, is a direct consequence of its Dirac-like spectrum and is largely responsible for ballistic transport even in samples unprotected from the environment (Geim & Novoselov 2007). For intuitive understanding of this effect, one may recall that light does not note obstacles with sizes much smaller than its wavelength (Born & Wolf 1986), whereas massless Dirac electrons have exactly the same dispersion relation as photons.

Similar to light scattering, the above description breaks down in the case of resonant scattering. If the Fermi energy of Dirac fermions accidentally coincides with an energy level of an impurity, there is much stronger scattering, which in the limit of long wavelengths can be described by  $t_0(\mathbf{k}) \propto 1/\ln(kR)$  and leads to excess resistivity (Katsnelson & Novoselov 2007)

$$\delta\rho \approx \frac{h}{4e^2} \frac{n_{\text{imp}}}{n \ln^2(k_{\text{F}}R)}. \quad (2.6)$$

This means that resonant scattering could be one of the factors limiting  $\mu$  in graphene (importantly, it has the same  $n$ -dependence as observed experimentally). In fact, electronic structure calculations (Wehling *et al.* 2007) have shown that certain types of adsorbed molecules (e.g.  $\text{N}_2\text{O}_4$ ) have resonant, quasi-localized levels near zero energy. It is interesting to note that equation (2.6) is also valid for the case of conventional (non-relativistic) two-dimensional systems (Adhikari 1986). However, in the latter case, resonant scattering is not incidental, like for Dirac fermions, but the essential feature that limits electron mobility because even a weak random potential in conventional two-dimensional systems leads to resonances near the band edge (Landau & Lifshitz 1977).

The situation is qualitatively different for charged impurities that have a long-range scattering potential. For the unscreened Coulomb potential, scattering phases for massless Dirac fermions are energy independent in the limit of low energies, which leads to resistivity inversely proportional to  $n$  (Nomura & MacDonald 2006). If screening is taken into account, the scattering cross section does not change significantly except for a numerical coefficient (Ando 2006; Nomura & MacDonald 2006). Further analysis shows that for the case  $\beta = (Ze^2/\hbar v_{\text{F}}\epsilon) > 1/2$  (where  $Z$  is the dimensionless impurity charge and  $\epsilon$  the dielectric constant), in which the Dirac equation allows the electron to fall down at

the Coulomb centre (Berestetskii *et al.* 1971), vacuum polarization effects diminish the initial supercritical value of  $\beta$  down to a critical value of one-half (Shytov *et al.* 2007). Nevertheless, the concentration dependence of resistivity should again remain qualitatively the same as for the unscreened Coulomb potential.

### 3. Scattering by a generic random potential: cross-check with perturbation theory

As shown above, in the case of a small concentration of scattering centres their contribution to resistivity can be calculated without any assumptions about the strength of the potential. In this section, we use the perturbation theory to check the applicability of the phase-scattering approach and justify it.

Let us consider a generic perturbation of the form

$$\hat{V}_{\mathbf{k}\mathbf{k}'} = V_{\mathbf{k}\mathbf{k}'}^{(0)} + \boldsymbol{\sigma} \mathbf{V}_{\mathbf{k}\mathbf{k}'}, \quad (3.1)$$

where  $\mathbf{k}$ ,  $\mathbf{k}'$  are the electron wavevectors and the Pauli matrices act on the pseudospin (sublattice) indices. This equation takes into account both scalar (electrostatic) and vector (pseudomagnetic) potentials created by defects (see also McCann *et al.* 2006). Assuming that potential  $V$  is small in comparison with the bandwidth and repeating the standard derivation of the Boltzmann equation in the Born approximation (Shon & Ando 1998), we find

$$\frac{1}{\tau} = \frac{4\pi}{N(E_F)} \sum_{\mathbf{k}\mathbf{k}'} \delta(E_{\mathbf{k}} - E_F) \delta(E_{\mathbf{k}'} - E_F) (\cos \phi_{\mathbf{k}} - \cos \phi_{\mathbf{k}'})^2 |W_{\mathbf{k}\mathbf{k}'}|^2, \quad (3.2)$$

where  $\phi_{\mathbf{k}}$  is the polar angle of the wavevector  $\mathbf{k}$  and  $E_{\mathbf{k}} = \hbar v_F \mathbf{k}$  is the electron energy and

$$W_{\mathbf{k}\mathbf{k}'} = V_{\mathbf{k}\mathbf{k}'}^{(0)} \frac{1 + \exp[i(\phi_{\mathbf{k}} - \phi_{\mathbf{k}'})]}{2} + \frac{1}{2} \left[ \left( V_{\mathbf{k}\mathbf{k}'}^{(x)} - i V_{\mathbf{k}\mathbf{k}'}^{(y)} \right) \exp(i\phi_{\mathbf{k}}) + \left( V_{\mathbf{k}\mathbf{k}'}^{(x)} + i V_{\mathbf{k}\mathbf{k}'}^{(y)} \right) \exp(-i\phi_{\mathbf{k}'}) \right]. \quad (3.3)$$

Equation (3.2) corresponds to the solution of the Boltzmann equation by a variational principle (Ziman 2001) or, equivalently, using the Mori formula (1965) for resistivity, in which only intra-band matrix elements of the current operator are taken into account. For the case of a small concentration of either short-range or Coulomb scatterers, one can check by direct calculations that equation (3.2) gives the same concentration dependence of resistivity as the phase-scattering approach described in §2.

### 4. Scattering by ripples

A local curvature of a graphene sheet changes interatomic distances and angles between chemical bonds and can be described by the following nonlinear term in the deformation tensor (Nelson *et al.* 2004):

$$\bar{u}_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} + \frac{\partial h}{\partial x_i} \frac{\partial h}{\partial x_j} \right), \quad (4.1)$$

where  $u_i$  are the components of in-plane atomic displacements and  $h$  is the displacements normal to a graphene sheet. This curvature modifies the hopping integrals  $\gamma$  as

$$\gamma = \gamma_0 + \left( \frac{\partial \gamma}{\partial \bar{u}_{ij}} \right)_0 \bar{u}_{ij}. \quad (4.2)$$

The change in nearest-neighbour hopping parameters is equivalent to the appearance of a gauge field (Morozov *et al.* 2006) described by a ‘vector potential’

$$V^{(x)} = \frac{1}{2} (2\gamma_1 - \gamma_2 - \gamma_3), \quad V^{(y)} = \frac{1}{2} (\gamma_2 - \gamma_3), \quad (4.3)$$

where the indices 1, 2 and 3 label the nearest neighbours that correspond to translational vectors  $(-a/\sqrt{3}, 0)$ ,  $(a/2\sqrt{3}, -a/2)$  and  $(a/2\sqrt{3}, a/2)$ , respectively. Changes in the next-nearest-neighbour hopping also lead to an electrostatic potential  $V^{(0)}$  that fluctuates in a randomly rippled graphene sheet (Kim Neto & Castro 2007). However, as follows from equations (3.2) and (3.3) both vector and electrostatic potentials contribute to  $\rho$  in a similar manner and, for brevity, we will further discuss only the effect of vector potential (4.3). This potential is equivalent to a random sign-changing ‘magnetic field’ that was previously shown to cause additional resistivity in conventional two-dimensional electron systems (Geim *et al.* 1994) and suppression of weak localization in graphene (Morozov *et al.* 2006).

For a rough estimate, equation (3.2) can be rewritten in the simplified form

$$\frac{1}{\tau} \approx \frac{2\pi}{\hbar} N(E_F) \langle \mathbf{V}_q \mathbf{V}_{-q} \rangle_{q \approx k_F}. \quad (4.4)$$

According to equations (4.1)–(4.3), the vector potential is proportional to in-plane deformations and, thus, quadratic in derivatives  $\partial h/\partial x$ ,  $\partial h/\partial y$ . This leads to the following expression:

$$\langle \mathbf{V}_q \mathbf{V}_{-q} \rangle \approx \left( \frac{\hbar v_F}{a} \right)^2 \sum_{\mathbf{q}_1 \mathbf{q}_2} \langle h_{\mathbf{q}-\mathbf{q}_1} h_{\mathbf{q}_1} h_{-\mathbf{q}+\mathbf{q}_2} h_{-\mathbf{q}_2} \rangle [(\mathbf{q}-\mathbf{q}_1) \cdot \mathbf{q}_1][(\mathbf{q}-\mathbf{q}_2) \cdot \mathbf{q}_2]. \quad (4.5)$$

To describe scattering on ripples for the most general case, let us assume that their height-correlation function grows with increasing distance  $r$  as  $\langle [h(\mathbf{r}) - h(0)]^2 \rangle \propto r^{2H}$ , where the exponent  $H$  characterizes the fractal dimension of ripples (Ishigami *et al.* 2007). This expression immediately gives us the scaling behaviour of the Fourier transform correlation function  $\langle |h_{\mathbf{q}}|^2 \rangle \propto q^{-2(H+1)}$  as well as the  $q$  dependence of  $\langle \mathbf{V}_q \mathbf{V}_{-q} \rangle$  that is defined by the convolution of two functions  $q^{-2H}$ . The latter result corresponds to the decoupling of the four- $h$  correlation function in equation (4.5) using the Wick theorem that is rigorous if fluctuations are Gaussian but can also be used for a qualitative estimate in other cases.

As a result, for  $2H < 1$ , the correlation function has a finite limit at  $q=0$

$$\langle \mathbf{V}_q \mathbf{V}_{-q} \rangle_{q=0} \approx \left( \frac{\hbar v_F}{a} \right)^2 \frac{z^4}{R^2}, \quad (4.6)$$

where  $z$  and  $R$  are the characteristic height and radius of ripples, respectively.

This leads to excess resistivity

$$\delta\rho \approx \frac{h}{4e^2} \frac{z^4}{R^2 a^2}. \quad (4.7)$$

For  $2H > 1$ , the resistivity is proportional to  $n^{1-2H}$  and, for  $2H = 1$ , to  $\ln^2(\mathbf{k}_F a)$ .

Below we argue that ripples with  $2H = 2$  can naturally occur in graphene owing to the way it is prepared. Indeed, single-layer samples studied in transport experiments are currently obtained by micromechanical cleavage, a process in which individual graphene sheets detach from bulk graphite and then attach to a  $\text{SiO}_2$  substrate (Novoselov *et al.* 2005a). It is sensible to assume that during the deposition process there is a transient state in which a detached sheet behaves as a free-standing membrane and, accordingly, exhibits dynamic out-of-plane fluctuations induced by the room temperature environment (e.g. Abedpour *et al.* 2007; Fasolino *et al.* 2007). As graphene touches the substrate, van der Waals forces instantly pin the rippled configuration (at least, partially), and any further reconstruction should be suppressed because this would require local movements along the substrate that strongly binds the atomically thin sheet (one might imagine the deposition process as a microscopic equivalent of placing a cling film on top of a table). Therefore, the corrugations originally induced by temperature are expected to become static and, also, persist to lower temperatures without notable changes.

To estimate scattering on such ripples, we note that, in the simple harmonic approximation, the average potential energy per individual bending mode  $E_q = \kappa q^4 \langle |h_q|^2 \rangle / 2$  is equal to  $k_B T / 2$  ( $\kappa \approx 1$  eV is the bending stiffness of graphene), which yields

$$\langle |h_q|^2 \rangle = \frac{k_B T}{\kappa q^4}. \quad (4.8)$$

Numerical simulations using a realistic atomic interaction potential in graphene directly confirm this result for the case of bending fluctuations with a typical length scale smaller than  $l^* \approx 7$ –10 nm (Fasolino *et al.* 2007). The linear  $n$  dependence of conductivity is experimentally observed for doping levels such that  $\mathbf{k}_F l^* > 1$ , which justifies the use of the harmonic approximation here. We then use equation (4.8) for the pair correlation function and find the ripple resistivity as

$$\rho_r \approx \frac{h}{4e^2} \frac{(k_B T / \kappa a)^2}{n} \mathcal{A}, \quad (4.9)$$

where the factor  $\mathcal{A}$  is of the order of unity for  $\mathbf{k}_F l^* \cong 1$  and weakly, as  $\ln^2(\mathbf{k}_F l^*)$ , depends on  $n$  for  $\mathbf{k}_F l^* \gg 1$ . The above equation shows that thermally created ripples lead to charge-carrier mobility  $\mu$  practically independent of  $n$ , in agreement with experiments. Importantly, equation (4.9) also yields  $\mu$  of the same order of magnitude as observed experimentally. One can interpret  $(k_B T_q / \kappa a)^2 \approx 10^{12} \text{ cm}^{-2}$  as an effective concentration of static defects (ripples) induced at the quench temperature  $T_q$  of 300 K. We emphasize that for a weak disorder, that is, in the Born approximation, the above formalism can be used to describe electron scattering by both static (quenched) and dynamic ripples, assuming that, first, they are classical scatterers and, second, their energy at relevant  $q$  is smaller than the energy of scattered Dirac fermions  $\hbar q v_F$  (Ziman 2001). The former condition means that  $\kappa \mathbf{k}_F^2 a^2 \ll k_B T_q$  and is satisfied in the existing experiments; the latter holds if  $q \ll \hbar v_F / \kappa$  and is even less restrictive.



For completeness, we also consider small doping levels such that  $k_F l^* \leq 1$ , in which the harmonic approximation can no longer be used to calculate the correlation functions. The reason for this is that thermal fluctuations with small  $q$  are extremely soft, which can lead to a crumpling instability, that is, the amplitude of fluctuations normal to a graphene sheet grows linearly with increasing its size. According to a general theory of flexible membranes (Nelson & Peliti 1987; Radzihovsky & Le Doussal 1992; Nelson *et al.* 2004), an anharmonic coupling between bending and stretching modes partially suppresses the growth of such fluctuations so that

$$\langle |h_q|^2 \rangle \approx \frac{1}{q^4} \left( \frac{q}{q_0} \right)^\eta, \quad (4.10)$$

where  $q_0 \approx \sqrt{B/\kappa} \approx 1/a$  is a typical cut-off vector at interatomic distances;  $B$  is the two-dimensional bulk modulus; and  $\eta \approx 0.8$  is the bending stiffness exponent. This should result in a concentration dependence weaker than  $\rho \propto 1/n$ . It should be noted, however, that for graphene this general theory probably does not give a complete description at smallest scales, where peculiarities of the carbon–carbon bond become essential and, for example, lead to the appearance of ripples with preferential sizes of approximately 5–7 nm (Fasolino *et al.* 2007). Finally, we note that, in principle, strong enough ripples can also create mid-gap (resonant) states, if the pseudomagnetic flux per ripple is larger than the flux quantum (Guinea *et al.* 2007). According to equation (2.6), such prominent ripples should be very efficient scatterers. This effect cannot be described by our perturbation theory and requires further investigation.

As for comparison with experiment, the situation currently remains unclear. On one hand, according to recent scanning probe microscopy studies of graphene on a SiO<sub>2</sub> substrate (Ishigami *et al.* 2007), the main contribution to the height fluctuation function is due to the substrate roughness, and  $2H \approx 1$ . This type of bending fluctuations cannot explain the experimental behaviour of graphene's resistivity that exhibits the  $\rho \propto 1/n$  dependence. Importantly, ripples with  $2H \approx 1$  result only in a small value of  $\delta\rho$ , which is a direct consequence of their relatively short-range character, as discussed in the earlier sections. On the other hand, TEM studies of suspended membranes show that a typical height of ripples grows approximately linear with their radius (Meyer *et al.* 2007a), which is consistent with the long-range scattering exponent  $2H=2$ . Furthermore, for graphene lying on a substrate, ripples of different origins can simultaneously be present. Then, it is possible that structural characteristics of a graphene sheet could be dominated by substrate effects, whereas thermally created ripples with their stronger, long-range scattering potential would still produce the dominant contribution into resistivity. Further experiments are required to clarify scattering mechanisms in graphene.

In conclusion, we have discussed possible mechanisms that limit charge-carrier mobility in graphene. Although charged impurities are currently considered as the most plausible source of scattering, there is an alternative explanation. We show that intrinsic corrugations of a graphene sheet create a long-range scattering potential and lead to significant resistivity that could explain the existing experimental data.

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