

Electronic properties of a biased graphene bilayer

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(Dated: April 29, 2010)

We study, within the tight-binding approximation, the electronic properties of a graphene bilayer in the presence of an external electric field applied perpendicular to the system – *biased bilayer*. The effect of the perpendicular electric field is included through a parallel plate capacitor model, with screening correction at the Hartree level. The full tight-binding description is compared with its 4-band and 2-band continuum approximations, and the 4-band model is shown to be always a suitable approximation for the conditions realized in experiments. The model is applied to real biased bilayer devices, either made out of SiC or exfoliated graphene, and good agreement with experimental results is found, indicating that the model is capturing the key ingredients, and that a finite gap is effectively being controlled externally. Analysis of experimental results regarding the electrical noise and cyclotron resonance further suggests that the model can be seen as a good starting point to understand the electronic properties of graphene bilayer. Also, we study the effect of electron-hole asymmetry terms, as the second-nearest-neighbor hopping energies t' (in-plane) and γ_4 (inter-layer), and the on-site energy Δ .

PACS numbers: 73.20.At, 73.21.Ac, 73.43.-f, 81.05.Uw

I. INTRODUCTION

The double layer graphene system – the so-called bilayer graphene (BLG) – is now a subject of considerable interest due to its unusual properties,¹⁻⁴ dissimilar in large extent to those of the single layer graphene (SLG).⁵ The integer quantum Hall effect (QHE) is a paradigmatic case; characterized by the absence of a plateau at the Dirac point,⁶ and thus still anomalous, it is associated with massive Dirac fermions and two zero energy modes.⁷

One of the most remarkable properties of BLG is the ability to open a gap in the spectrum by electric field effect – biased BLG.⁷ This has been shown both experimentally and theoretically, providing the first semiconductor with externally tunable gap.⁸⁻¹⁷ In the absence of external perpendicular electric field – unbiased BLG – the system is characterized by four bands, two of them touching each other parabolically at zero energy, and giving rise to the massive Dirac fermions mentioned above, and other two separated by an energy $\pm t_{\perp}$. Hence, an unbiased BLG is a two-dimensional zero-gap semiconductor.^{6,7,18} At the neutrality point the conductivity shows a minimum of the order of the conductance quantum,^{6,18-25} a property shared with SLG.²⁶ This prevents standard device applications where the presence of a finite gap producing high on-off current ratios is of paramount importance. The fact that a simple perpendicular electric field is enough to open a gap, and even more remarkable, to control its size, clearly demonstrates the potential of this system for carbon-based electronics.^{27,28}

The biased BLG reveals interesting properties on its own. The gap has shown to be robust in the presence of disorder,²⁹⁻³¹ induced either by impurities or dilution, but is completely absent in rotated (non *AB*-stacked) bilayers, where the SLG linear dispersion is recovered.^{32,33} The band structure near the gap shows a "Mexican-hat" like behavior, with a low doping Fermi surface which is a ring.¹¹ Such a topologically nontrivial Fermi surface leads to an enhancement of electron-electron interactions, and to a ferromagnetic instability at low enough density of carriers.^{34,35} In the presence of a perpendicular magnetic field, the biased BLG shows cyclotron mass renormalization and an extra plateau at zero Hall conductivity, signaling the presence of a sizable gap at the neutrality point.^{10,12,36} Gaps can also be induced in stacks with more than two layers as long as the stacking order is of the rhombohedral-type,^{11,14} although screening effects may become important in doped systems with increasing number of layers.¹⁵ Recently, a ferromagnetic proximity effect was proposed as a different mechanism which can also open a gap in the spectrum of the BLG, leading to a sizable magnetoresistive effect.³⁷ Strain applied to the biased BLG has also shown to produce further gap modulation.³⁸

In this paper the electronic properties of a biased BLG are studied within a full tight-binding model, which enables the analysis of the whole bandwidth, validating previous results obtained using low-energy effective models. The screening of the applied perpendicular electric field is obtained within a self-consistent Hartree approach, and a comparison with experiments is provided. The effect of the bias in the cyclotron mass and cyclotron resonance is

addressed, and the results are shown to agree well with experimental measurements.

The paper is organized as follows: in Sec. II the lattice structure of BLG and the tight-binding Hamiltonian are presented; bulk electronic properties are discussed in Sec III, with particular emphasis on the screening correction; the effect of a perpendicular magnetic field is studied in Sec. IV; Sec. V contains our conclusions. We have also included three appendices: Appendix A provides details on the calculation of the density asymmetry between layers for a finite bias; in Appendix B we give the analytical expression for the biased BLG density of states, valid over the entire energy spectrum; analytical expressions for the cyclotron mass obtained within the full tight-binding model are given in Appendix C.

II. MODEL

Here we consider only *AB*-Bernal stacking, where the top layer has its *A* sublattice on top of sublattice *B* of the bottom layer. We use indices 1 and 2 to label the top and bottom layer, respectively. The unit cell of a bilayer has twice the number of atoms of a single layer. The basis vectors may be written as $\mathbf{a}_1 = a \hat{e}_x$ and $\mathbf{a}_2 = a(\hat{e}_x - \sqrt{3}\hat{e}_y)/2$, where $a = 2.46 \text{ \AA}$.

In the tight-binding approximation, the in-plane hopping energy, t , and the inter-layer hopping energy, t_\perp , define the most relevant energy scales. The simplest tight-binding Hamiltonian describing non-interacting π -electrons in BLG reads:

$$H_{TB} = \sum_{i=1}^2 H_i + t_\perp \sum_{\mathbf{R}, \sigma} [a_{1,\sigma}^\dagger(\mathbf{R}) b_{2,\sigma}(\mathbf{R}) + \text{h.c.}] + H_V, \quad (1)$$

with the SLG Hamiltonian

$$H_i = -t \sum_{\mathbf{R}, \sigma} [a_{i,\sigma}^\dagger(\mathbf{R}) b_{i,\sigma}(\mathbf{R}) + a_{i,\sigma}^\dagger(\mathbf{R}) b_{i,\sigma}(\mathbf{R} - \mathbf{a}_1) + a_{i,\sigma}^\dagger(\mathbf{R}) b_{i,\sigma}(\mathbf{R} - \mathbf{a}_2) + \text{h.c.}], \quad (2)$$

where $a_{i,\sigma}(\mathbf{R})$ [$b_{i,\sigma}(\mathbf{R})$] is the annihilation operator for electrons at position \mathbf{R} in sublattice *Ai* (*Bi*), $i = 1, 2$, and spin σ . The in-plane hopping t can be inferred from the Fermi velocity in graphene $v_F = t a \hbar^{-1} \sqrt{3}/2 \approx 10^6 \text{ ms}^{-1}$,³⁹ yielding $t \approx 3.1 \text{ eV}$, in good agreement with what is found experimentally for graphite.⁴⁰ This value also agrees with a recent Raman scattering study of the electronic structure of BLG.⁴¹ As regards the inter-layer hopping t_\perp , angle-resolved photoemission spectroscopy (ARPES) measurements in epitaxial BLG give $t_\perp \approx 0.43 \text{ eV}$,⁸ and Raman scattering for BLG obtained by micro-mechanical cleavage of graphite yields $t_\perp \approx 0.30 \text{ eV}$.⁴¹ The experimental value for bulk graphite is $t_\perp \approx 0.39 \text{ eV}$,⁴² which means that for practical purposes we can always assume $t_\perp/t \sim 0.1 \ll 1$. This values for t and t_\perp compare fairly well with what is obtained from first-principles calculations for graphite⁴³

$\gamma_3/t^{41,48}$	$\gamma_4/t^{41,48,49}$	Δ/t^{48-50}	t'/t^{51}
0.03 – 0.1	0.04 – 0.07	0.005 – 0.008	~ 0.04

Table II: Approximate parameter values as obtained in recent experiments (except for t' quoted from DFT calculations).

using the well established Slonczewski-Weiss-McClure (SWM) parametrization model^{44,45} to fit the bands near the Fermi energy. The SWM model assumes extra parameters that can also be incorporated in a tight-binding model for BLG. Namely, the inter-layer second-NN hoppings γ_3 and γ_4 , where γ_3 connects different sublattices (*B1*–*A2*) and γ_4 equal sublattices (*A1*–*A2* and *B1*–*B2*). Additionally, there is an on-site energy Δ reflecting the inequivalence between sublattices *A1*, *B2* and *B1*, *A2* – the former project exactly on top of each other while the latter lay on the hexagon center of the other layer. The consequences of these extra terms for the band structure obtained from Eq. (1) are well known: γ_3 induces trigonal warping and both γ_4 and Δ give rise to electron-hole asymmetry.^{7,46,47} The in-plane second-NN hopping energy t' is not considered in the usual tight-binding parametrization of the SWM model. Nevertheless, this term can have important consequences since it breaks particle-hole symmetry but does not modify the Dirac spectrum. Typical values are given in Table II as obtained in recent experiments, except for t' quoted from density functional theory (DFT) calculations.

We are interested in the properties of BLG in the presence of a perpendicular electric field – the biased BLG. The effect of the induced energy difference between layers, parametrized by V , may be accounted for by adding H_V to Eq. (1), with H_V given by

$$H_V = \frac{V}{2} \sum_{\mathbf{R}, \sigma} [n_{A1}(\mathbf{R}) + n_{B1}(\mathbf{R}) - n_{A2}(\mathbf{R}) - n_{B2}(\mathbf{R})], \quad (3)$$

where $n_{Ai}(\mathbf{R})$ and $n_{Bi}(\mathbf{R})$ are number operators.

III. BULK ELECTRONIC PROPERTIES

Introducing the Fourier components $a_{i,\sigma,\mathbf{k}}$ and $b_{i,\sigma,\mathbf{k}}$ of operators $a_{i,\sigma}(\mathbf{R})$ and $b_{i,\sigma}(\mathbf{R})$, respectively, with the layer index $i = 1, 2$, we can rewrite Eq. (1) as $H = \sum_{\mathbf{k}, \sigma} \psi_{\sigma,\mathbf{k}}^\dagger H_{\mathbf{k}} \psi_{\sigma,\mathbf{k}}$, where $\psi_{\sigma,\mathbf{k}}^\dagger = [a_{1,\sigma,\mathbf{k}}^\dagger, b_{1,\sigma,\mathbf{k}}^\dagger, a_{2,\sigma,\mathbf{k}}^\dagger, b_{2,\sigma,\mathbf{k}}^\dagger]$ is a four component spinor, and $H_{\mathbf{k}}$ is given by

$$H_{\mathbf{k}} = \begin{pmatrix} V/2 & -ts_{\mathbf{k}} & 0 & -t_\perp \\ -ts_{\mathbf{k}}^* & V/2 & 0 & 0 \\ 0 & 0 & -V/2 & -ts_{\mathbf{k}} \\ -t_\perp & 0 & -ts_{\mathbf{k}}^* & -V/2 \end{pmatrix}. \quad (4)$$

The factor $s_{\mathbf{k}} = 1 + e^{i\mathbf{k}\cdot\mathbf{a}_1} + e^{i\mathbf{k}\cdot\mathbf{a}_2}$ determines the matrix elements for the SLG Hamiltonian in reciprocal space

($t_{\perp} = 0, V = 0$), from which the SLG dispersion is obtained, $\epsilon_{\mathbf{k}} = \pm t|s_{\mathbf{k}}|$. The resultant conduction (+) and valence (-) bands touch each other in a conical way at the corners of the first Brillouin zone (BZ), the K and K' points.⁵ This touching occurs at zero energy, the Fermi energy for undoped graphene. The 4-band continuum approximation for Eq. (4), valid at energy scales $E \ll t$, may be obtained by introducing the small wave vector \mathbf{q} which measures the difference between \mathbf{k} and the corners of the BZ. Linearizing the factor $s_{\mathbf{k}}$ around the K points Eq. (4) reads

$$H_K = \begin{pmatrix} V/2 & v_F p e^{-i\varphi_{\mathbf{p}}} & 0 & -t_{\perp} \\ v_F p e^{i\varphi_{\mathbf{p}}} & V/2 & 0 & 0 \\ 0 & 0 & -V/2 & v_F p e^{-i\varphi_{\mathbf{p}}} \\ -t_{\perp} & 0 & v_F p e^{i\varphi_{\mathbf{p}}} & -V/2 \end{pmatrix}, \quad (5)$$

where $\mathbf{p} = \hbar\mathbf{q}$ and $\varphi_{\mathbf{p}} = \tan^{-1}(p_y/p_x)$. Around the K' points Eq. (5) with complex conjugate matrix elements defines $H_{K'}$.^{7,52}

Equation (5) can be further simplified if one assumes $v_F p, V \ll t_{\perp}$. By eliminating high energy states perturbatively we can write a two-band effective Hamiltonian describing low-energy states whose electronic amplitude is mostly localized on $B1$ and $A2$ sites. Near the K points the resulting Hamiltonian may be written as

$$H_{eff} = - \begin{pmatrix} -V/2 & e^{i2\varphi_{\mathbf{p}}} v_F^2 p^2 / t_{\perp} \\ e^{-i2\varphi_{\mathbf{p}}} v_F^2 p^2 / t_{\perp} & V/2 \end{pmatrix}, \quad (6)$$

whereas the complex conjugate matrix elements should be taken for a low-energy description around the K' points. The two-component wave functions have the form $\Phi = (\phi_{B1}, \phi_{A2})$.^{7,52,53}

In the following we discuss the electronic structure resulting from the tight-binding Hamiltonian (4), and comment on the approximations given above by Eqs. (5) and (6).

A. Electronic structure

Let us briefly discuss the electronic structure of the biased BLG using the full tight-binding Hamiltonian given by Eq. (1). The spectrum of Eq. (1) for $V \neq 0$ reads:

$$E_{\mathbf{k}}^{\pm\pm}(V) = \pm \sqrt{\epsilon_{\mathbf{k}}^2 + \frac{t_{\perp}^2}{2} + \frac{V^2}{4} \pm \sqrt{t_{\perp}^4/4 + (t_{\perp}^2 + V^2)\epsilon_{\mathbf{k}}^2}}. \quad (7)$$

As can be seen from Eq. (7), the $V = 0$ gapless system turns into a semiconductor with a gap controlled by V . Moreover, the two bands close to zero energy are deformed near the corners of the BZ,⁵ so that the minimum of $|E_{\mathbf{k}}^{\pm\pm}(V)|$ no longer occurs at these corners. As a consequence, the low doping Fermi surface is completely different from the $V = 0$ case, with its shape controlled by V .

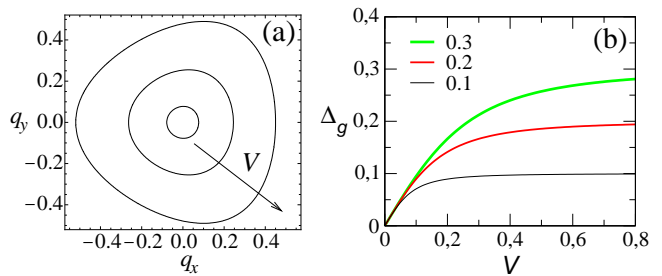


Figure 1: (Color online) (a) Solution of Eq. (8) for $V = t_{\perp}/2, 2t_{\perp}, 4t_{\perp}$. (b) Δ_g vs V for various t_{\perp} values. Energy is given in units of t and momentum in units of a^{-1} .

It can be readily shown that the minimum of sub-band $E_{\mathbf{k}}^{+-}(V)$ [or equivalently, the maximum of $E_{\mathbf{k}}^{-+}(V)$] occurs for all \mathbf{k} 's satisfying

$$\epsilon_{\mathbf{k}}^2 = \alpha(V, t_{\perp}), \quad (8)$$

with $\alpha(V, t_{\perp}) = (V^4/4 + t_{\perp}^2 V^2/2)/(V^2 + t_{\perp}^2)$ – note that $\partial E_{\mathbf{k}}^{\pm\pm}/\partial \epsilon_{\mathbf{k}} = 0$ at the desired extrema. Equation (8) has solutions for $\sqrt{\alpha} \leq 3t$ ($3t$ is half of the single layer bandwidth). When $\sqrt{\alpha} > 3t$ the minimum of $E_{\mathbf{k}}^{+-}(V)$ occurs at the Γ point. Figure 1(a) shows the solution of Eq. (8) around the K point for $V = t_{\perp}/2, 2t_{\perp}, 4t_{\perp}$ (around the K' point the figure is rotated by $\pi/3$). At low doping the Fermi sea acquires a line shape given by the solution of Eq. (8), the line width being determined by the doping level. As can be seen in Fig. 1(a), when $V < t_{\perp}$ the Fermi sea approaches a ring, the Fermi ring, centered at the BZ corners.^{11,34} As V is increased there is an apparent trigonal distortion showing up, which originates from the single layer dispersion in Eq. (8).

The existence of a Fermi ring is easily understood using the continuum version of Eq. (7), i.e., the eigenvalues of Eq. (5). This amounts to substituting the single layer dispersion in Eq. (7) by $v_F p$, which immediately implies cylindrical symmetry around K and K' . If we further assume that $v_F p \ll V \ll t_{\perp}$ holds, Eq. (7) is then well approximated by the “Mexican hat” dispersion,¹¹

$$E^{\pm\pm}(V) \approx \pm \frac{V}{2} \mp \frac{V v_F^2 p^2}{t_{\perp}^2} \pm \frac{v_F^4 p^4}{t_{\perp}^2 V}, \quad (9)$$

which explains the Fermi ring. If, instead, we have $V < v_F p \ll t_{\perp}$, we can approximate Eq. (7) by

$$E^{\pm\pm}(V) \approx \pm \sqrt{V^2/4 + v_F^4 p^4/t_{\perp}^2}, \quad (10)$$

which corresponds exactly to the eigenvalues of the effective two-band Hamiltonian in Eq. (6). Note that no continuum approximation can produce the trigonal distortion shown in Fig. 1(a).

The gap between conduction and valence bands, Δ_g , is twice the minimum value of $E_{\mathbf{k}}^{+-}(V)$ due to electron-hole

symmetry, and is given by,

$$\Delta_g = \begin{cases} \sqrt{t_\perp^2 V^2 / (t_\perp^2 + V^2)} & V \leq V_c \\ 2t \sqrt{9 + \frac{t_\perp^2}{2t^2} + \frac{V^2}{4t^2} - \sqrt{\frac{t_\perp^4}{4t^4} + 9 \frac{t_\perp^2 + V^2}{t^2}}} & V > V_c \end{cases}, \quad (11)$$

where $V_c = [18t^2 - t_\perp^2 + (18t^4 + t_\perp^4)^{1/2}]^{1/2} \simeq 6t$, the approximation being valid for $t_\perp \ll t$. From Eq. (11) it can be seen that for both $V \ll t_\perp$ and $V \gg t$ one finds $\Delta_g \sim V$. However, there is a region for $t_\perp \lesssim V \lesssim 6t$ where the gap shows a plateau $\Delta_g \sim t_\perp$, as depicted in Fig. 1(b). The plateau ends when $V \simeq 6t$ (not shown).

B. Screening of the external field

So far we have considered V , i.e. the electrostatic energy difference between layers felt by a single electron, as a band parameter that controls the gap. However, the parameter V can be related with the perpendicular electric field applied to BLG, avoiding the introduction of an extra free parameter in the present theory.

Let us call $\mathbf{E} = E\hat{z}$ the perpendicular electric field felt by electrons in BLG. The corresponding electrostatic energy $U(z)$ for an electron of charge $-e$ is related to the electric field as $eE = \partial U(z)/\partial z$, and thus V is given by

$$V = U(z_1) - U(z_2) = eEd, \quad (12)$$

where z_1 and z_2 are the positions of layer 1 and 2, respectively, and $d \equiv z_1 - z_2 = 3.4 \text{ \AA}$ is the inter-layer distance. Given the experimental conditions, the value of E can be calculated under a few assumptions, as detailed in the following.

1. External field in real systems

If we assume the electric field E in Eq. (12) to be due exclusively to the external electric field applied to BLG, $E = E_{ext}$, all we need in order to know V is the value of E_{ext} ,

$$V = eE_{ext}d. \quad (13)$$

The experimental realization of a biased BLG has been achieved in epitaxial BLG through chemical doping^{8,54} and in back gated exfoliated BLG.^{9,10} In either case the value of E_{ext} can be extracted assuming a simple parallel plate capacitor model.

In the case of exfoliated BLG, devices are prepared by micromechanical cleavage of graphite on top of an oxidized silicon wafer (300 nm of SiO₂), as shown in the left panel of Fig. 2(a). A back gate voltage V_g applied between the sample and the Si wafer induces charge carriers due to the electric field effect, resulting in carrier densities $n_g = \beta V_g$ relatively to half-filling ($n_g > 0$ for electrons and $n_g < 0$ for holes). The geometry of the resulting capacitor determines the coefficient β . In

particular, the electric field inside the oxidized layer is $E_{ox} = en_g/(\varepsilon_{\text{SiO}_2}\varepsilon_0)$, where $\varepsilon_{\text{SiO}_2}$ and ε_0 are the permittivities of SiO₂ and free space, respectively. This implies a gate voltage $V_g = en_g t/(\varepsilon_{\text{SiO}_2}\varepsilon_0)$, from which we obtain the coefficient $\beta = \varepsilon_{\text{SiO}_2}\varepsilon_0/(et)$. For a SiO₂ thickness $t = 300 \text{ nm}$ and a dielectric constant $\varepsilon_{\text{SiO}_2} = 3.9$ we obtain $\beta \simeq 7.2 \times 10^{10} \text{ cm}^{-2}/\text{V}$, which is in agreement with the values found experimentally.^{6,39,55} In order to control independently the gap value and the Fermi level, in Ref. 10 the devices have been chemically doped by deposition of NH₃ on top of the upper layer, which adsorbed on graphene and effectively acted as a top gate providing a fixed electron density n_0 .⁵⁶ Charge conservation then implies a total density n in BLG given by $n = n_g + n_0$, or in terms of the applied gate voltage,

$$n = \beta V_g + n_0. \quad (14)$$

In Fig. 2(b) the charge density in BLG is shown as a function of V_g . The symbols are the experimental result obtained from Hall effect measurements,¹⁰ and the line is a linear fit with Eq. (14). The fit provides n_0 , which for this particular experimental realization is $n_0 \simeq 1.8 \times 10^{12} \text{ cm}^{-2}$, and validates the parallel plate capacitor model applied to the back gate, since the fitted $\beta \simeq 7.2 \times 10^{10} \text{ cm}^{-2}/\text{V}$ is in excellent agreement with the theoretical value. Extending the parallel plate capacitor model to include the effect of dopants, the external field E_{ext} is the result of charged surfaces placed above and below BLG. The accumulation or depletion layer in the Si wafer contributes with an electric field $E_b = en_g/(2\varepsilon_r\varepsilon_0)$, while dopants above BLG effectively provide the second charged surface with electric field $E_t = -n_0/(2\varepsilon_r\varepsilon_0)$. A relative dielectric constant ε_r different from unity may be attributed to the presence of SiO₂ below and vacuum on top, which gives $\varepsilon_r \approx (\varepsilon_{\text{SiO}_2} + 1)/2 \approx 2.5$, a value that can be slightly different due to adsorption of water molecules.^{56,57} Adding the two contributions, $E_{ext} = E_b + E_t$, and making use of the charge conservation relation, we arrive at an electrostatic energy difference V [Eq. (13)] that depends linearly on the BLG density,

$$V = \left(\frac{n}{n_0} - 2 \right) \frac{e^2 n_0 d}{2\varepsilon_r\varepsilon_0}. \quad (15)$$

In treating the dopants as a homogeneous charged layer we ignore possible lattice distortion induced by adsorbed molecules, as well as the electric field due to the NH₃ electric dipole, which may contribute to the gap in the spectrum. However, it has been shown recently⁵⁸ that for NH₃ these effects counteract, giving rise to a much smaller gap than other dopant molecules,⁵⁹ as for instance NH₂ and CH₃. For the biased BLG realized in Ref. 9, independence of Fermi level and carrier density was achieved with a real top gate, which makes the parallel plate capacitor model a suitable approximation in that case.

In the case of epitaxial BLG, devices are grown on SiC by thermal decomposition of the Si-face.⁶⁰ The substrate

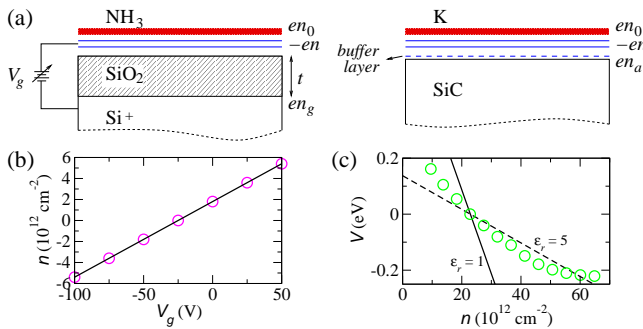


Figure 2: (Color online) (a) Biased BLG devices. (b) n vs V_g for the left device shown in (a): experimental data is shown as symbols;¹⁰ the line is a linear fit with Eq. (14). (c) V vs n for the right BLG device shown in (a): symbols are experimental data from Ref. 8; the lines are the result of Eq. (16).

is fixed (SiC), and graphene behavior develops for carbon layers above the buffer layer,^{61–64} as schematically shown in the right panel of Fig. 2(a).⁶⁵ Due to charge transfer from substrate to film, the as-prepared BLG devices appear electron doped with density n_a . First-principles calculations indicate that such doping is coming from interface states that develop between the buffer layer and the Si-terminated substrate.^{61,62} [Scanning tunneling microscopy (STM) measurements corroborate the presence of interface states.^{64,66–68}] From the point of view of our theoretical approach, we may interpret these interface states as an effective depletion layer that provides the external electric field necessary to make the system a biased BLG. In Ref. 8 the BLG density n was varied by doping the system with potassium (K) on top of the upper layer [see Fig. 2(a)], which originates an additional charged layer contributing to the external electric field. Applying the same parallel plate capacitor model as before, we get an electrostatic energy difference that can be written as

$$V = \left(2 - \frac{n}{n_a}\right) \frac{e^2 n_a d}{2\epsilon_r \epsilon_0}. \quad (16)$$

Following a similar reasoning to the case of exfoliated graphene on top of SiO₂, we would write $\epsilon_r \approx (\epsilon_{\text{SiC}} + 1)/2 \approx 5$. However, this value neglects that interface states (the effective bottom plate capacitor) occur above the SiC substrate, close to the graphene system, and thus $\epsilon_r \approx 1$ should be more appropriate. In Fig. 2(c) we compare Eq. (16) with experimental results for V obtained by fitting ARPES measurements from Ref. 8. For this particular biased BLG realization, the as-prepared carrier density was $n_a \approx 10^{13} \text{ cm}^{-2}$. From Eq. (16), this n_a value implies a zero V , i.e., zero electric field and therefore zero gap, for the bilayer density $n^{\text{th}} \approx 2 \times 10^{13} \text{ cm}^{-2}$. Experimentally, a zero gap was found around $n^{\text{exp}} \approx 2.3 \times 10^{13} \text{ cm}^{-2}$. Given the simplicity of the theory, it can be said that n^{th} and n^{exp} are in good agreement. However, the agreement is only good at $V \sim 0$, since the

measured V is not a linear function of n , as Eq. (16) implies. In what follows we analyze in detail the effect of screening and how it modifies Eqs. (15) and (16).

2. Screening correction

In deriving Eqs. (15) and (16) we assumed that the electric field E in the BLG region was exactly the external one, E_{ext} . There is, however, an obvious additional contribution: the external electric field polarizes the BLG, inducing some charge asymmetry between the two graphene layers, which in turn give rise to an internal electric field, E_{int} , that screens the external one.

To estimate E_{int} we can again apply a parallel plate capacitor model. The internal electric field due to the charge asymmetry between planes may thus be written as

$$E_{\text{int}} = \frac{e\Delta n}{2\epsilon_r \epsilon_0}, \quad (17)$$

where $-e\Delta n$ is the induced charge imbalance between layers, which can be estimated through the weight of the wave functions in each layer,

$$\Delta n = n_1 - n_2 = \frac{2}{N_c A_{\square}} \sum_{j,l=\pm} \sum_{\mathbf{k}}' (|\varphi_{A1,\mathbf{k}}^{jl}|^2 + |\varphi_{B1,\mathbf{k}}^{jl}|^2 - |\varphi_{A2,\mathbf{k}}^{jl}|^2 - |\varphi_{B2,\mathbf{k}}^{jl}|^2), \quad (18)$$

where the factor 2 comes from spin degeneracy, N_c is the number of unit cells and $A_{\square} = a^2\sqrt{3}/2$ is the unit cell area, jl is a band label, and the prime sum runs over all occupied \mathbf{k} 's in the first BZ. The amplitudes $\varphi_{Ai,\mathbf{k}}^{jl}$ and $\varphi_{Bi,\mathbf{k}}^{jl}$, with $i = 1, 2$, are determined by diagonalization of Eq. (4), enabling Δn to be written as

$$\Delta n = \frac{2}{N_c A_{\square}} \sum_{j,l=\pm} \sum_{\mathbf{k}}' \frac{(\epsilon_{\mathbf{k}}^2 + \mathcal{K}_{\mathbf{k},-}^{jl})(\epsilon_{\mathbf{k}}^2 - \mathcal{K}_{\mathbf{k},+}^{jl})^2 - (\epsilon_{\mathbf{k}}^2 + \mathcal{K}_{\mathbf{k},+}^{jl})t_{\perp}^2 \mathcal{K}_{\mathbf{k},-}^{jl}}{(\epsilon_{\mathbf{k}}^2 + \mathcal{K}_{\mathbf{k},-}^{jl})(\epsilon_{\mathbf{k}}^2 - \mathcal{K}_{\mathbf{k},+}^{jl})^2 + (\epsilon_{\mathbf{k}}^2 + \mathcal{K}_{\mathbf{k},+}^{jl})t_{\perp}^2 \mathcal{K}_{\mathbf{k},-}^{jl}}, \quad (19)$$

where $\epsilon_{\mathbf{k}}$ is the SLG dispersion, $\mathcal{K}_{\mathbf{k},\pm}^{jl} = (V/2 \pm E_{\mathbf{k}}^{jl})^2$ with $E_{\mathbf{k}}^{jl}$ given by Eq. (7). Taking the limit $N_c \rightarrow \infty$, it is possible to write Eq. (19) as an energy integral weighted by the density of states of SLG, as described in Appendix A. What is important to note is that in order to calculate Δn we must specify V , which in turn depend Δn through Eq. (17). Thus, a self-consistent procedure must be followed. In particular, for the two experimental realizations of biased BLG discussed in Sec. III B 1, the self-consistent equation that determines V reads: in the case of exfoliated BLG,¹⁰

$$V = \left[\frac{n}{n_0} - 2 + \frac{\Delta n(n, V)}{n_0} \right] \frac{e^2 n_0 d}{2\epsilon_r \epsilon_0}; \quad (20)$$

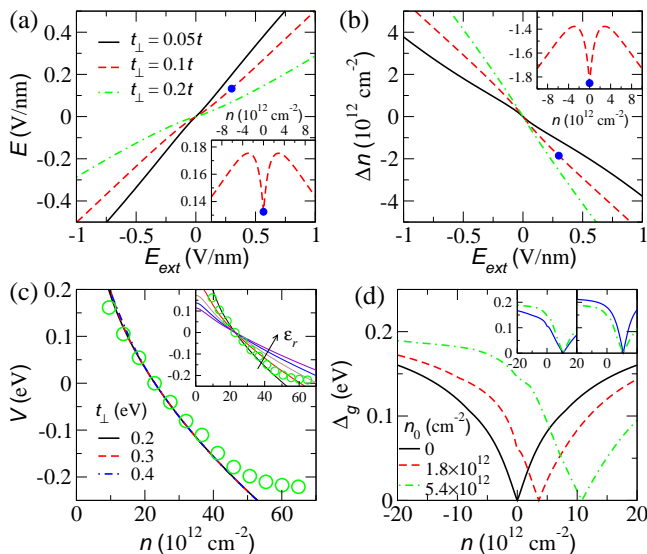


Figure 3: (Color online) (a)-(b) Respectively, screened electric field and charge imbalance vs E_{ext} at half-filling; the insets show the effect of changing n at fixed $E_{ext} = 0.3$ V/nm, signaled by the (blue) dot in main panels. (c) V vs n for the BLG device shown in the right panel of Fig. 2(a): symbols are experimental data from Ref. 8; lines are the result of Eq. (21) for $\epsilon_r = 1$; the effect of changing $\epsilon_r = 1 - 5$ is shown in the inset. (d) Gap vs n for the BLG device shown in the left panel of Fig. 2(a) with $t_{\perp} \simeq 0.22$ eV and $\epsilon_r = 1$; the left inset compares the $n_0 = 5.4 \times 10^{12}$ cm $^{-2}$ result for $\epsilon_r = 1$ (green dashed-dotted) with $\epsilon_r = 2$ (blue full line); the right inset shows the $n_0 = 5.4 \times 10^{12}$ cm $^{-2}$ result for the screened V given by Eq. (20) (dashed-dotted line) and for the unscreened V given by Eq. (15) (full line). We used as in-plane hopping $t \simeq 3$ eV.

in the case of epitaxial BLG,⁸

$$V = \left[2 - \frac{n}{n_a} + \frac{\Delta n(n, V)}{n_a} \right] \frac{e^2 n_a d}{2\epsilon_r \epsilon_0}. \quad (21)$$

The self-consistent electric field $E = E_{ext} + E_{int}$ at the BLG region, with E_{int} given by Eq. (17) for $\epsilon_r = 1$, is shown at half-filling as a function of E_{ext} in Fig. 3(a). The screened E is approximately a linear function of E_{ext} , with a constant of proportionality that depends on the specific value of t_{\perp} . Increasing t_{\perp} leads to an increased screening, which can be understood as due to an increased charge imbalance between layers, as shown in Fig. 3(b). The highly non-linear effect of inducing a finite carrier density ($n \neq 0$) can be seen in the insets of Fig. 3(a) and 3(b), for $t_{\perp} = 0.1t$ and $E_{ext} = 0.3$ V/nm.

As a validation test to the present self-consistent treatment, we compare Eq. (21) with experimental results for V obtained by fitting ARPES measurements from Ref. 8, as mentioned in Sec. III B 1. The result is shown in Fig. 3(c). Clearly, the self-consistent V given by Eq. (21) for $\epsilon_r = 1$ is a much better approximation than the unscreened result of Eq. (16) [see Fig. 2(c)]. The best fit is obtained for $\epsilon_r \sim 1 - 2$, as can be seen in the inset

of Fig. 3(c). The value $\epsilon_r \approx (\epsilon_{\text{SiC}} + 1)/2 \approx 5$ is too high, possibly because the bottom capacitor plate is, indeed, due to interface states,⁶⁹ and therefore is not buried inside the SiC substrate.^{61,62,66} Note, however, that the dielectric constant ϵ_r may effectively be tuned externally, as recently shown in SLG by adding a water overlayer in ultra-high vacuum.⁷⁰ In Fig. 3(d) we show the gap Δ_g as a function of carrier density n for the biased BLG device shown in the left panel of Fig. 2(a), with realistic values of chemical doping n_0 .¹⁰ The gap is given by Eq. (11), with $t_{\perp} \simeq 0.22$ eV¹⁰ and V obtained by solving self-consistently Eq. (20) for $\epsilon_r = 1$. Note that for $E_{ext} = 0$ we always have $E_{int} = 0$ (the charge imbalance must be externally induced), and therefore we also have $V = 0$ and $\Delta_g = 0$. For this particular biased BLG device the present model predicts $E_{ext} = 0$ for $n = 2n_0$, which explains the asymmetry for Δ_g vs n shown in Fig. 3(d).

The most important characteristic of such devices, from the point of view of applications, is the maximum size of the gap which could be induced. The maximum Δ_g occurs when V_g reaches its maximum, which occurs just before the breakdown of SiO₂. The breakdown field for SiO₂ is $\gtrsim 1$ V/nm, meaning that V_g values as high as 300 V are possible for the device shown in the left panel of Fig. 2(a). From Eq. (14) we see that $V_g \simeq \pm 300$ V implies $n - n_0 \simeq \pm 22 \times 10^{12}$ cm $^{-2}$, and therefore Fig. 3(d) nearly spans the interval of possible densities. It is apparent, specially for $n_0 = 5.4 \times 10^{12}$ cm $^{-2}$, that when the maximum allowed densities are reached the gap seems to be approaching a saturation limit. This saturation is easily identified with the plateau shown in Fig. 1(b) for Δ_g vs V , occurring for $V \gtrsim t_{\perp}$. We may then conclude that such devices enable the entire range of allowed gaps (up to t_{\perp}) to be accessed — as has been shown in very recent experiments.^{16,17} The effect of using a different dielectric constant ($\epsilon_r = 2$) is shown as a full line in the left inset of Fig. 3(d), and the result for the unscreened case in the right inset, both for $n_0 = 5.4 \times 10^{12}$ cm $^{-2}$. The former makes the gap slightly smaller, and the latter slightly larger, but the main conclusions remain.

3. Screening in continuum models

The self consistent Hartree approach considered in the previous section has been applied to the full tight-binding Hamiltonian given in Eq. (1). Here we compare the results for the potential difference V and gap Δ_g when the screening correction is used within the continuum approximation, either for the 4-band model of Eq. (5) or for the 2-band model of Eq. (6). This self consistent Hartree approach in the continuum has been followed in Refs. 12,27.

In the case of the 4-band model, Δn is still given by Eq. (19) with the substitutions $\epsilon_{\mathbf{k}} \rightarrow v_{\text{FP}}$ and $\frac{2}{N_c A_{\text{O}}} \sum_{j,l=\pm} \sum_{\mathbf{k}}' \rightarrow \frac{2}{\pi \hbar^2} \sum_{j,l=\pm}' \int_{p_1}^{p_2} dp p$, where the prime on the right hand summation means sum over to-

tal or partially occupied bands. Depending on the band in question and the value of the Fermi energy E_F , the limits of integration are $p_1, p_2 = \{0, p^\pm, \Lambda\}$, where

$$v_F p^\pm = \sqrt{E_F^2 + V^2/4 \pm \sqrt{E_F^2(V^2 + t_\perp^2) - t_\perp^2 V^2/4}}, \quad (22)$$

and Λ is a BZ cutoff that can be chosen such that $\frac{4\pi}{\hbar^2} \int_0^\Lambda dp p = \frac{4\pi^2}{A_O} \Leftrightarrow \Lambda = \hbar\sqrt{\pi/A_O}$. As regards the gap Δ_g , in the 4-band model it is still given by Eq. (11).

For the 2-band model case, the charge imbalance can be written as an integral in momentum space of the function $|\phi_{B1}|^2 - |\phi_{A2}|^2 = \pm V/(V^2 + 4v_F^4 p^4/t_\perp^2)^{1/2}$, where $\Phi = (\phi_{B1}, \phi_{A2})$ is the two component wave function obtained by diagonalizing Eq. (6). The \pm signs stand for the contribution of valence and conduction bands, respectively. In particular, at half-filling the charge imbalance is given by

$$\Delta n_{1/2} \simeq -\frac{t_\perp V}{2\pi v_F^2 \hbar^2} \ln(2t_\perp/|V| + \sqrt{4t_\perp^2/V^2 + 1}), \quad (23)$$

where we have included a factor of 4 to account for both spin and valley degeneracies. The BZ cutoff Λ has been chosen such that $v_F \Lambda = t_\perp$.¹² Since in the 2-band model it is assumed that $V \ll t_\perp$ holds we can write $\Delta_{1/2} \approx -t_\perp V/(2\pi v_F^2 \hbar^2) \ln(4t_\perp/|V|)$, which, from Eq. (17), leads to the logarithmic divergence of the screening ratio at small external electric field, $E_{ext}/E \sim -\ln E$, as mentioned in Ref. 13. For a general filling n the charge imbalance reads

$$\Delta n \approx \frac{t_\perp V}{2\pi v_F^2 \hbar^2} \ln\left(\frac{v_F^2 \hbar^2 \pi |n|}{2t_\perp^2} + \sqrt{\frac{v_F^4 \hbar^4 \pi^2 n^2}{4t_\perp^4} + 1}\right), \quad (24)$$

where the charge density is given in terms of the Fermi wave vector as $n = \pm p_F^2/(\pi \hbar^2)$. Inserting Eq. (24) into Eq. (20) or (21) we get the expression for V in the 2-band approximation, which is exactly the gap in the 2-band model, $\Delta_g = |V|$.

In Fig. 4(a) the obtained electrostatic energy difference between planes V is shown for the three different approaches discussed above. The full (black) lines stand for the full tight-binding result, with V given by Eq. (20) and the charge imbalance Δn by Eq. (19). The result obtained in the 4-band approximation is shown as dashed (red) lines. It can hardly be distinguished from the full tight-binding result, even when the chemical doping n_0 is as high as $5.4 \times 10^{12} \text{ cm}^{-2}$ (see figure caption). In fact, the only prerequisite for the continuum 4-band approximation [Eq. (5)] to hold is that $|E_F| \ll t$, which is always realized for the available BLG devices. As regards the 2-band approximation model, we show as dotted (blue) lines the self-consistent result for V , obtained from Δn as in Eq. (24). Clearly, it is only when both the bilayer density n and the chemical doping n_0 are small enough for the relation $|E_F|, V \ll t_\perp$ to hold that the 2-band model is a good approximation (see inset). The same conclusions apply to the behavior of the gap Δ_g as a function

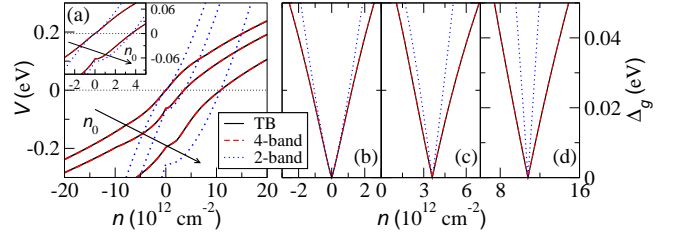


Figure 4: (Color online) (a) Screened V vs n for the BLG system shown in the left panel of Fig. 2(a) computed within three different approaches (see text): full tight-binding (TB), 4-band approximation, and 2-band approximation. Three different chemical dopings have been considered, $n_0 = \{0, 1.8, 5.4\} \times 10^{12} \text{ cm}^{-2}$. The inset shows a zoom around $V = 0$ for $n_0 = \{0, 1.8\} \times 10^{12} \text{ cm}^{-2}$. (b)-(c) Screened gap vs n obtained using V shown in (a), respectively for $n_0 = \{0, 1.8, 5.4\} \times 10^{12} \text{ cm}^{-2}$. Parameters: $t \simeq 3 \text{ eV}$, $t_\perp \simeq 0.22 \text{ eV}$, and $\varepsilon_r = 1$.

of carrier density n , which is shown in panels 4(b)-(d) for $n_0 = \{0, 1.8, 5.4\} \times 10^{12} \text{ cm}^{-2}$, respectively. The failure of the 2-band model in the presence of interactions was also observed in Hartree calculations of the electron compressibility.⁷¹

4. Electron-hole asymmetry

As we have seen in Secs. III B 1 and III B 2, the two biased BLG devices shown in Fig. 2(a) have zero gap when the carrier density is twice the system's chemical doping. The closing of the gap at a finite density induces an electron-hole asymmetric behavior in the system, where obvious examples are the gap Δ_g and the electrostatic energy difference between layers V , as shown in Figs. 2(d) and 4(a). An experimental confirmation for this electron-hole asymmetric behavior comes from measurements of the cyclotron mass in the biased BLG device shown in the left panel of Fig. 2(a)¹⁰ (discussed in more detail in Sec. IV A). However, real electron-hole asymmetry can also be present in BLG due to extra hopping terms, as mentioned in Sec. II. Here we study how Δ_g and V are effected by the electron-hole symmetry breaking terms t' , γ_4 , and Δ , taking into account the screening correction.

Inclusion of in-plane second-NN hopping t' leads to a generalized version of Eq. (4), which can be written as $H_{\mathbf{k},t'} = H_{\mathbf{k}} - (\epsilon_{\mathbf{k}}^2 t'/t - 3t')\mathbf{1}$, where $H_{\mathbf{k}}$ is given by Eq. (4), $\epsilon_{\mathbf{k}}$ is the SLG dispersion, and $\mathbf{1}$ is the 4×4 identity matrix. The generalized the BLG dispersion, either biased or unbiased, is given by the $t' = 0$ result added by $-\epsilon_{\mathbf{k}}^2 t'/t + 3t'$, which clearly breaks electron-hole symmetry. Note that a finite t' has no influence on the wavefunctions' amplitude. Therefore, the integrand in Eq. (18) – the definition of the charge carrier imbalance between layers Δn – is independent of t' . We have found numerically, using a 4-band continuum model, that neither the screened V nor the gap Δ_g are affected by t' , although

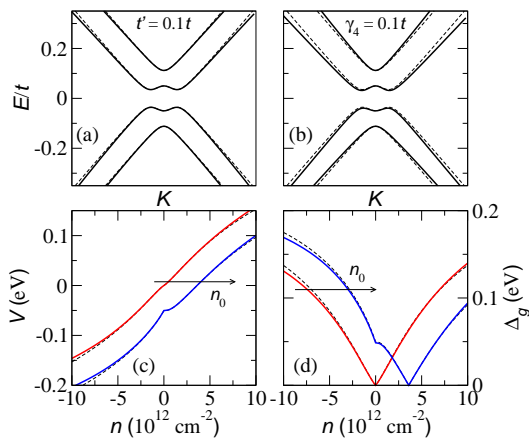


Figure 5: (Color online) (a)-(b) Band structure around K for the biased BLG with $t' = 0.1t$ and $\gamma_4 = 0.1t$, respectively, for $V = t_\perp = 0.1t$. Dashed lines: $t' = \gamma_4 = 0$. (c)-(d) Respectively, V vs n and Δ_g vs n for the BLG device shown in the left panel of Fig. 2(a), modeled with a finite γ_4 . Parameters: $t \simeq 3$ eV, $t_\perp = 0.1t$, $\gamma_4 = 0.1t$, $\varepsilon_r = 1$, and $n_0 = \{0, 1.8\} \times 10^{12} \text{ cm}^{-2}$. Dashed lines: $t' = \gamma_4 = 0$.

the gap becomes indirect for finite t' . This means that the structure of occupied \mathbf{k} 's is insensitive to t' , and thus Δn in Eq. (18) is fully t' independent, at least as long as $E_F \ll t$. Even though the presence of t' can lead to the suppression of the Mexican hat in the valence band, this only happens for $|V| < t_\perp^2 t' \sim 10^{-3}t$. For such a small $|V|$ value the Mexican hat plays an irrelevant role. The band structure around the K point for $t' = 0.1t$ (solid line) and $t' = 0$ (dashed line) can be seen in Fig. 5(a) for typical parameter values.

Now we turn to the effect of the inter-layer second-NN hopping γ_4 . The generalized version of Eq. (4) for finite γ_4 , which we call $H_{\mathbf{k},\gamma_4}$, can be obtained by replacing the null entries $(A1, A2)$ and $(B1, B2)$ by $\gamma_4 s_{\mathbf{k}}^*$ and $(A2, A1)$ and $(B2, B1)$ by $\gamma_4 s_{\mathbf{k}}$. The associated eigenproblem admits an analytic treatment at low energies and small biases $v_F p, V \ll t_\perp$,⁵² but as has been seen previously $V \sim t_\perp$ is possible in real systems. Therefore, we analyze the problem numerically using a 4-band continuum approximation. The matrix Hamiltonian $H_{\mathbf{k},\gamma_4}$ may then be written as $H_{K,\gamma_4} = M^\dagger \tilde{H}_{K,\gamma_4} M$ near the K points, with $M = \text{diag}[1, e^{i\varphi_{\mathbf{p}}}, e^{-i\varphi_{\mathbf{p}}}, 1]$, and \tilde{H}_{K,γ_4} obtained from Eq. (5) with $\varphi_{\mathbf{p}} = 0$ and the null entries $(A1, A2)$, $(B1, B2)$, $(A2, A1)$, and $(B2, B1)$ replaced by $-v_4 p$, where $v_4 = \gamma_4 a \hbar^{-1} \sqrt{3}/2 \lesssim 10^5 \text{ ms}^{-1}$. The canonical transformation defined by M clearly shows that the problem still has cylindrical symmetry in the continuum approximation. Around the K' points we have $H_{K',\gamma_4} = M \tilde{H}_{K,\gamma_4} M^\dagger$. The obtained band structure for $\gamma_4 = 0.1t$ (solid lines) and $\gamma_4 = 0$ (dashed lines) is shown in Fig. 5(b) for typical parameter values. Note that, even though the gap becomes indirect for $\gamma_4 \neq 0$, we still have $E_{p=0} = \{\pm V/2, \pm \sqrt{t_\perp^2 + V^2}/4\}$ as in the $\gamma_4 = 0$ case. The screened electrostatic energy difference between lay-

ers V for the biased BLG device shown in the left panel of Fig. 2(a) is shown as a function of the carrier density in Fig. 5(c). The result for V has been obtained by solving Eq. (20) with carrier imbalance Δn given by the continuum version of Eq. (18), with wavefunctions obtained numerically through \tilde{H}_{K,γ_4} for $\gamma_4 = 0.1t$ (see figure caption for other parameter values). The corresponding screened gap Δ_g is shown in panel 5(d). The $\gamma_4 = 0$ result is also shown as a dashed line for both V and Δ_g . The effect of γ_4 may clearly be considered small, even for such a large value as $\gamma_4 \simeq 0.3$ eV. However, electronic properties which are particularly sensitive to the changes of the Fermi surface (like, for instance, the cyclotron mass), may, in principle, be measurably affected by γ_4 . We will come back to this point in Sec. IV A.

As regards the on-site energy Δ , since it is smaller than γ_4 (see Sec. II) we consider their simultaneous effect. The additional term in the Hamiltonian adds to the matrix $H_{\mathbf{k},\gamma_4}$ the contribution $\text{diag}[\Delta, 0, 0, \Delta]$, and therefore the 4-band continuum approximation for finite γ_4 and Δ may be written as $\tilde{H}_{K,\gamma_4,\Delta} = \tilde{H}_{K,\gamma_4} + \text{diag}[\Delta, 0, 0, \Delta]$, where we use the same transformation M introduced above. Similarly to γ_4 , the effect of Δ is negligible in both V and Δ_g .

C. DOS and LDOS

Insight into the electronic properties of biased (and unbiased) BLG can also be achieved by studying the density of states (DOS) and the local DOS (LDOS) of the system. In particular, the LDOS can be accessed through scanning tunneling microscopy/spectroscopy measurements,⁷² providing a useful way to validate theoretical models. On the other hand, the knowledge of the DOS turns out to be very useful for practical purposes, as it provides a way to relate the Fermi energy E_F and the carrier density n in the system: $|n| = \int_0^{|E_F|} dE \rho_2(E)$, where $\rho_2(E)$ stands for the BLG DOS.

We have computed the analytical expression for the DOS of BLG, valid over the entire energy spectrum and for zero and finite bias. The expression is given in Appendix B. As regards the LDOS, the results have been obtained using the recursive Green's function method.⁷³ The DOS and LDOS of unbiased BLG has been obtained previously within the effective mass approximation in Ref. 74. The effect of disorder on the DOS and LDOS of BLG, both biased and unbiased, has also been studied recently.^{18,29-31,74,75}

The DOS (full line) and LDOS (dashed and dash-dotted lines) for the biased BLG is shown in Fig. 6(a)-(b) for $V = 0.05t$. The asymmetry between the four sublattices is evident, in particular between sites $B1$ and $A2$, and $A1$ and $B2$, which are equivalent in the unbiased system. Note that close to the gap edges the states corresponding to positive energies have a larger amplitude at $B1$ sites, while those corresponding to negative energies have a larger amplitude at $A2$ sites. This behavior agrees

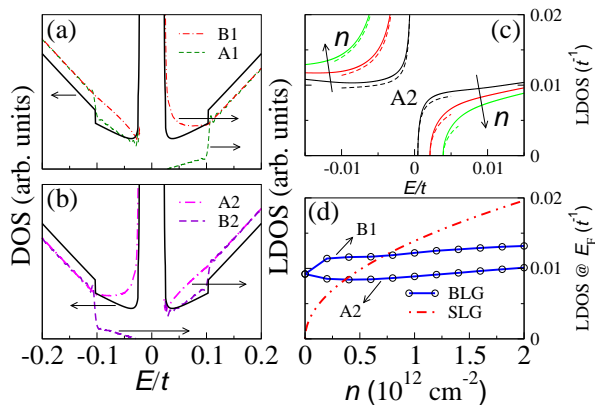


Figure 6: (Color online) (a)-(b) LDOS of BLG at A1/B1 and A2/B2 sites, respectively, for $V = 0.05t$ and $t_{\perp} = 0.1t$. The total DOS is shown as a full line. (c) LDOS at A2 sites for $n \simeq \{0.2, 0.8, 1.4\} \times 10^{12} \text{ cm}^{-2}$ and $t_{\perp} = 0.1t$. Full lines for numerical results and dashed lines for Eq. (25). (d) LDOS at E_F vs n for BLG and SLG.

with the observation that $B1$ and $A2$ are the low energy active sites (the basis for the 2-band model), and it also reflects our choice of electrostatic energies in Eq. (3): $+V/2$ in layer 1 and $-V/2$ in layer 2. The asymmetry between $B1$ and $A2$ sites can be understood with the 2-band continuum model, valid for $v_F p, V \ll t_{\perp}$. Defining the LDOS as $\rho_{B1/A2}(E) = \frac{1}{N_c} \sum_{\mathbf{k}} |\phi_{B1/A2, \mathbf{k}}|^2 \delta(E - E_{\mathbf{k}})$, where $\Phi_{\mathbf{k}} = (\phi_{B1, \mathbf{k}}, \phi_{A2, \mathbf{k}})$ is the two component wave function obtained by diagonalizing Eq. (6), we can readily arrive at the following expressions,

$$\rho_{B1/A2}(E) = \frac{1}{2\sqrt{3}\pi} \frac{t_{\perp}}{t^2} \text{sgn}(E) \frac{E \pm V/2}{\sqrt{E^2 - V^2/4}}. \quad (25)$$

The asymmetric behavior is apparent, with $\rho_{B1}(E)$ diverging for $E \rightarrow V/2^+$ and $\rho_{A2}(E)$ for $E \rightarrow -V/2^-$. The result for $\rho_{A2}(E)$ is shown in Fig. 6(c) for $V \simeq \{0.87, 4.23, 7.87\} \times 10^{-3}t$ and $t_{\perp} = 0.1t$. Within the screening corrected parallel plate capacitor model discussed in Sec. III B [Eq. (20)], these V values correspond to carrier densities $n \simeq \{0.2, 0.8, 1.4\} \times 10^{12} \text{ cm}^{-2}$, respectively, where we have used $t \simeq 3.1 \text{ eV}$, $n_0 = 0$, and $\varepsilon_r = 1$. The full lines are the recursive Green's function method⁷³ results and dashed lines are the results of Eq. (25). As expected, the closer to the gap edges the better the agreement between the two approaches.

A strong suppression of electrical noise in BLG has been reported recently by Lin and Avouris.⁷⁶ In devices made from exfoliated BLG on top of SiO_2 , the current fluctuations are thought to originate from the fluctuating trapped charges in the oxide. Therefore, the more effective the impurity charge screening in the system the lower the electrical noise. The lower noise in BLG than in SLG may then be attributed to the low energy finite DOS in the former. However, it has also been reported in Ref. 76 that while increasing the carrier density in SLG leads to lower noise, as expected due to more effective impurity

screening, it results in higher noise in BLG. Insight into this behavior is achieved by analyzing the LDOS at the Fermi level E_F in a biased BLG, as charging the system through the back gate V_g leads to a finite perpendicular electric field. In Fig. 6(e) we show the biased BLG LDOS at E_F for $B1$ and $A2$ sites as a function of carrier density n in the system. For a given n , the electrostatic energy difference V is evaluated self-consistently through Eq. (20), with $n_0 = 0$ and $\varepsilon_r = 1$, and E_F is obtained by integrating over the DOS. Additionally, we use $t \simeq 3 \text{ eV}$ and $t_{\perp} = 0.1t$. We have chosen densities in the range $n \in [0 - 2] \times 10^{12} \text{ cm}^{-2}$, which corresponds to back gate voltages $V_g \in [0 - 27] \text{ eV}$ through Eq. (14), similar to the experimental range in Ref. 76. The main observation to be made as regards the results of Fig. 6(e) is that for the low energy active sublattices $B1$ and $A2$ the LDOS at E_F remains approximately constant with increasing electron density, as opposed to the $\sim \sqrt{n}$ dependence found in SLG. This is an indication that impurity screening may not be increasing with carrier density in the biased BLG system, which may be contributing to enhance electrical noise.

IV. MAGNETIC FIELD EFFECTS

In the biased BLG system, as a consequence of the gapped band structure discussed in Sec. III, a perpendicular magnetic field is expected to induce distinct features in electronic properties. In this section we focus on the cyclotron mass (semi-classical approach) and on the cyclotron resonance (quantum regime) comparing the theory with experimental results.

A. Cyclotron mass

In the semi-classical approximation the cyclotron effective mass m_c is given by

$$m_c = \frac{\hbar^2}{2\pi} \left. \frac{\partial A(E)}{\partial E} \right|_{E=E_F}, \quad (26)$$

where $A(E)$ is the k -space area enclosed by the orbit of energy E , and the derivative is evaluated at the Fermi energy E_F .⁷⁷⁻⁷⁹ It can be accessed experimentally through the Shubnikov-de Haas effect, providing a direct probe to the Fermi surface. In the case of exfoliated graphene, either SLG or (un)biased BLG, the Fermi energy can be varied by tuning the back gate voltage, and therefore a significant portion of the whole band structure may be unveiled. In particular for the biased BLG, the presence of a finite gap can be checked and the model developed in Sec. III tested.

1. Comparison with experiment

General expressions for m_c obtained for the full tight-binding bands in Eq. (7), valid for the relevant parameter range $V \lesssim t_\perp \ll t$ and restricted to $E_F < t$, are given in Appendix C. In Fig. 7(a) we compare the theory results for the cyclotron mass with experimental measurements¹⁰ on the biased BLG system shown in the left panel of Fig. 2(a). We have only considered m_c associated with low energy bands $E_{\mathbf{k}}^{\pm-}$ [see Eq. (7)], since $E_{\mathbf{k}}^{\pm+}$ are inactive for the experimentally available carrier densities. The dashed lines stand for the unscreened result, where V is given by Eq. (15), and the solid lines are the screened result, with V given by Eq. (20). The inter-layer coupling t_\perp has been taken as an adjustable parameter, keeping all other fixed: $t \simeq 3$ eV, $\varepsilon_r = 1$, and $n_0 = 1.8 \times 10^{12} \text{ cm}^{-2}$. The value of t_\perp could then be chosen so that theory and experiment gave the same m_c for $n = 2n_0 \approx 3.6 \times 10^{12} \text{ cm}^{-2}$. As discussed in Sec. III B 2, at this particular density the gap closes, meaning that the theoretical value becomes independent of the screening assumptions. We found $t_\perp \approx 0.22$ eV, in good agreement with values found in the literature. The theoretical dependence $m_c(n)$ agrees well with the experimental data for the case of electron doping. Also, as seen in Fig. 7(a), the screened result provides a somewhat better fit than the unscreened model, especially at low electron densities. This fact, along with the good agreement found for the electrostatic energy difference data of Ref. 8 [see Fig. 3(c)], allows us to conclude that for doping of the same sign from both sides of bilayer graphene, the gap is well described by the screened approach. In the hole doping region in Fig. 7(a), the Hartree approach underestimates the value of m_c whereas the simple unscreened result overestimates it. This can be attributed to the fact that the Hartree theory used here is reliable only if the gap is small compared to t_\perp . In the experimental realization of Ref. 10, $n_0 > 0$ and, therefore, the theory works well for a wide range of electron doping $n > 0$, whereas even a modest overall hole doping $n < 0$ corresponds to a significant electrostatic difference between the two graphene layers. In this case, the unscreened theory overestimates the gap whereas the Hartree calculation underestimates it.

In Fig. 7(b) we compare our best fit to the cyclotron mass (full line) with results obtained for different parameter values. The dashed-dotted lines stand for m_c obtained with $\varepsilon_r = 2$ in Eq. (20). As can be seen clearly, the $n > 0$ result is not substantially affected, while for $n < 0$ the theory description of m_c worsens. This is due to the reduction of the gap when ε_r is increased [see left inset in Fig. 3(d)]. The dashed lines in Fig. 7(b) are obtained with $n_0 = 0$, where the zero gap occurs at the neutrality point. The dotted lines are the result for $E_{ext} = 0 = V$, i.e., zero gap at every density value. Note that these two results, $n_0 = 0$ and $V = 0$, show an electron-hole symmetric m_c , contradicting the experimental result. It may then be said that the electron-hole asymmetry observed

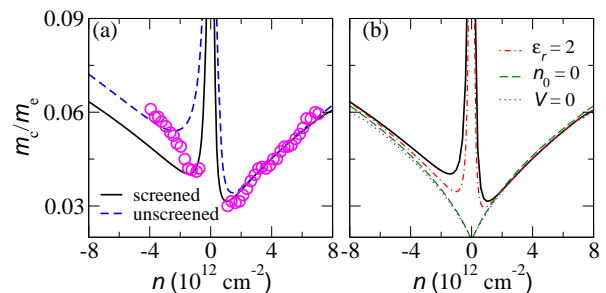


Figure 7: (Color online) Cyclotron mass vs n , normalized to the free electron mass, m_e . (a) Solid lines are the result of the self-consistent procedure and the dashed lines correspond to the unscreened case; $t \simeq 3$ eV, $t_\perp \simeq 0.22$ eV, $\varepsilon_r = 1$, and $n_0 = 1.8 \times 10^{12} \text{ cm}^{-2}$. Circles are experimental data from Ref. 10. (b) The screened result in (a) is compared with the result for $\varepsilon_r = 2$, the case without chemical doping ($n_0 = 0$), and the case where the external field is zero ($V = 0$).

in m_c is a clear indication of the presence of a finite gap in the spectrum. It will be shown in Sec. IV A 3 that, if we ignore the gap, this electron-hole asymmetry cannot be described by taken into account t' , γ_4 or Δ .

2. Cyclotron mass in continuum models

Here we compare our results for the cyclotron mass, which has been obtained with expressions shown in Appendix C, with the results of continuum models.

Within the 4-band continuum model given by Eq. (5), where the dispersion is just the full tight-binding result [Eq. (7)] with the substitution $\epsilon_{\mathbf{k}} \rightarrow v_F p$, we can easily derive the following analytical expression for m_c ,

$$m_c = \frac{E_F}{v_F^2} \left[1 + \frac{V^2 + t_\perp^2}{2\sqrt{E_F^2(V^2 + t_\perp^2) - t_\perp^2 V^2/4}} \right]. \quad (27)$$

In Fig. 8(a) the dashed line is the result of Eq. (27), where V has been computed self-consistently using Eq. (20) and the 4-band continuum approximation discussed in Sec. III B 3. As expected, the agreement with the full tight-binding result (shown as a full line) is excellent for the considered densities. Note that there is an extra solution given by $\tilde{m}_c v_F^2 = E_F [1 - (V^2 + t_\perp^2)/\sqrt{4E_F^2(V^2 + t_\perp^2) - t_\perp^2 V^2}]$, valid when $|E_F| < V/2$ or $|E_F| > \sqrt{V^2/4 + t_\perp^2}$, which corresponds to the extra orbit appearing when E_F falls in the Mexican-hat region, or above the bottom of high energy bands. We can estimate the densities for which these two regions start playing a role: using $V \sim 0.1t_\perp \sim 0.01t$ in the Mexican hat region (valid for $n_0 \lesssim 2 \times 10^{12} \text{ cm}^{-2}$) we get $n \lesssim 10^{11} \text{ cm}^{-2}$; setting $V \sim t_\perp \sim 0.1t$ around the bottom of high energy bands we get $n \gtrsim 10^{13} \text{ cm}^{-2}$. These two density values are outside the range of experimentally realized densities [see Fig. 7(a)].

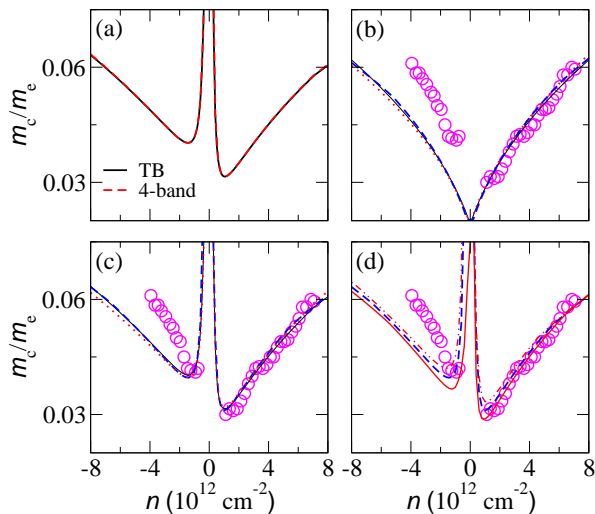


Figure 8: (Color online) Cyclotron mass vs n , normalized to the free electron mass, m_e . (a) Comparison between full tight-binding (TB) and 4-band approximation for $t_{\perp} \simeq 0.22$ eV and $n_0 = 1.8 \times 10^{12}$ cm $^{-2}$. (b)-(c) Effect of finite t' and γ_4 for $n_0 = 0$ and $n_0 = 1.8 \times 10^{12}$ cm $^{-2}$, respectively: dotted line is for $t' \simeq 0.3$ eV and $t_{\perp} \simeq 0.22$ eV; dashed line is for $\gamma_4 \simeq 0.12$ eV and $t_{\perp} \simeq 0.19$ eV; full thin line is for $t' = \gamma_4 = 0$ and $t_{\perp} = 0.22$ eV. (d) Effect of Δ for $\gamma_4 \simeq 0.12$ eV and $t_{\perp} \simeq 0.19$ eV: full line for $\Delta \simeq 0.03$ eV; dotted-dashed line for $\Delta \simeq -0.03$ eV; dashed line for $\Delta = 0$. Circles are experimental data from Ref. 10. We have used $t \simeq 3$ eV and $\varepsilon_r = 1$.

3. Effect of electron-hole asymmetry

In Sec. III B 4 the effect of electron-hole symmetry breaking parameters – namely, t' , γ_4 , and Δ – has been studied regarding the self-consistent description of the gap. Here we extend the analysis to the cyclotron mass, restricting ourselves to the biased BLG device shown in the left panel of Fig. 2(a). Results have been obtained within the 4-band model. As all cases have cylindrical symmetry around K and K' , the cyclotron mass may be written as $m_c = p_F / (\partial E_F / \partial p_F)$.

In Fig. 8(b) we show the m_c result for finite t' (dotted red line) and finite γ_4 (dashed blue line), keeping $n_0 = 0$ (absence of electron-hole asymmetry due to chemical doping). The thin full line is the result obtained for $t' = \gamma_4 = 0$ in Sec. IV A 1, and circles are experimental data from Ref. 10. The $n > 0$ region, where the smaller gaps are realized experimentally, is still well fitted if we choose $t_{\perp} \simeq 0.22$ eV with $t' \simeq 0.3$ eV or $t_{\perp} \simeq 0.19$ eV with $\gamma_4 = 0.12$ eV (we use $t \simeq 3$ eV). However, it is clear that neither of these results can account for the electron-hole asymmetry observed experimentally. In fact, a closer look reveals that the m_c for finite t' have the opposite trend, being smaller than the $t' = 0$ result for $n < 0$ and larger for $n > 0$, as would be expected by inspection of the energy bands in Fig. 5(a). Such an opposite trend should also be seen for finite γ_4 , although the effect is not as large as expected from the considerable

distortion of the energy bands shown in Fig. 5(b). This attenuation can be understood as the result of fixing the carrier density n and not the Fermi energy E_F : changing γ_4 (or t') for a given n leads to a different E_F , and the new E_F is such that it counteracts the expected effect of γ_4 (or t') in m_c . Fig. 8(c) shows the same as 8(b) for $n_0 = 1.8 \times 10^{12}$ cm $^{-2}$. The effect of the on-site energy Δ is shown in Fig. 8(d) for fixed $\gamma_4 \simeq 0.12$ eV, $t_{\perp} \simeq 0.19$ eV and $n_0 = 1.8 \times 10^{12}$ cm $^{-2}$. The result for $\Delta = 0$ (dashed line) is shown along with the result for $\Delta \simeq 0.03$ eV (full line) and $\Delta \simeq -0.03$ eV (dotted-dashed line). It is clear that the effect of t' , γ_4 and Δ on the cyclotron mass can be neglected.

B. Cyclotron resonance

The effect of a perpendicular magnetic field can be studied within the continuum approximation through minimal coupling $\mathbf{p} \rightarrow \mathbf{p} - e\mathbf{A}$.⁷ The case of biased BLG has been studied both within the 4-band [Eq. (5)] and 2-band [Eq. (6)] continuum models in Refs. 11,12,36,80. Here we use the same approach to study the cyclotron resonance (i.e. the Landau level transition energies) with the extra ingredient that the parameter V depends on the filling factor, as discussed in Sec. III B.

In the 4-band model standard manipulations^{7,11,36,81} lead to the unbiased BLG Landau level spectrum

$$E_n^{\pm} = \pm \sqrt{(1+2n)\frac{\gamma^2}{2} + \frac{t_{\perp}^2}{2} \pm \sqrt{(\gamma^2 + t_{\perp}^2)^2/4 + n\gamma^2 t_{\perp}^2}}, \quad (28)$$

where $\gamma = \sqrt{2}v_F\hbar/l_B$, with $l_B = \sqrt{\hbar/|e|B}$ for the magnetic length. Non-zero ($n \geq 1$) eigenenergies are fourfold degenerate due to valley and spin degeneracy, while zero energy Landau levels have eightfold degeneracy, since there are two zero energy Landau states ($n = -1, 0$) per valley per spin. The 2-band model result $E_n^{\pm} \approx \pm \gamma^2 t_{\perp}^{-1} \sqrt{n(n+1)}$ is easily recovered from Eq. (28) for $\gamma \ll t_{\perp}$, being valid for magnetic fields up to $B \approx 1$ T.⁷

The Landau level transition energies in BLG have been recently obtained through cyclotron resonance measurements.⁸² The data was found to deviate from what would be expected through Eq. (28), especially for larger filling factors. It should be noted, however, that in order to keep a constant filling factor and vary the magnetic field, as done in Ref. 82, the back gate voltage V_g has to be tuned to compensate for the variation of Landau level degeneracy. As we have seen previously, tuning V_g is equivalent to change V – the electrostatic energy difference between layers – which means that Eq. (28) is no longer valid, as recently shown within the 4-band continuum model.³⁶ To have an estimate for the effect of the back gate voltage on the Landau level spacing we have computed Landau level energy differences taking into account the variation of V with carrier density n . We have used the unscreened result given by Eq. (15), with $n_0 = 0$

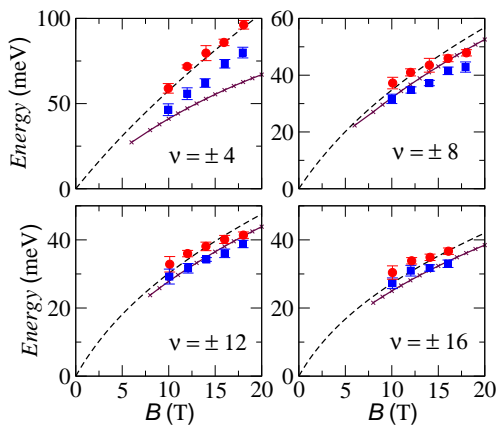


Figure 9: (Color online) Landau level transition energies vs magnetic field for the given filling factors. The dashed line is the unbiased BLG result [Eq. (28)] and the line with crosses is the biased BLG result (see text). We used $t = 3.5$ eV and $t_{\perp} = 0.1t$. Filled symbols are experimental data from Ref. 82: circles for electrons and squares for holes.

and $\epsilon_r = 1$. Within this approximation we can easily write V in terms of the filling factor ν and magnetic field B as $V = \nu B e^2 d / (2\epsilon_0 \phi_0) \approx 7.4 \times 10^{-4} \nu B$, with B in Tesla in the last step. Thus, for fixed filling factor, V varies linearly with B . Note that the comparison between this unscreened treatment of the biased BLG and the unbiased result in Eq. (28) should give lower and upper limits for the effect of the perpendicular external field in the cyclotron frequency. In Fig. 9 we show the obtained Landau transition energies vs magnetic field for the given filling factors. The dashed lines represent the unbiased BLG result, as given by Eq. (28). The lines with crosses are the results for the unscreened biased BLG, and filled symbols are experimental data from Ref. 82: circles for $\nu > 0$ and squares for $\nu < 0$. We have used $t = 3.5$ eV and $t_{\perp} = 0.1t$, consistent with Ref. 82. As can be seen from Fig. 9, the back gate induced electric field gives rise to sizable effects already for magnetic fields and filling factors realized in experiments. Except at $\nu = \pm 8$, the result of Eq. (28) for the unbiased BLG and the unscreened biased BLG result effectively provide upper and lower limits to the experimental data. The observed electron-hole asymmetry could then be interpreted as due to an asymmetry in V vs n : larger V , and therefore larger gap, for $n < 0$; smaller V and gap for $n > 0$, which would make the result more close to the unbiased case. It should be noted that in such a case we would expect the neutrality point to occur for $V_g < 0$, as is the case of the NH_3 doped BLG studied before. For the system reported in Ref. 82, however, the opposite seems to be happening, as indicated by the Hall resistivity. A neutrality point for $V_g > 0$ is, in fact, the more usual effect of H_2O molecules adsorbed on graphene samples.⁵⁶ As a final remark regarding the results presented in Fig. 9, we note that the experimental data trend, which makes Eq. (28) a poor fit at $|\nu| \geq 8$, is still not accounted for in the biased BLG result. An

alternative approach is the inclusion of the screening correction, which should go beyond Eq. (19) including the magnetic field effect. It has been reported recently that Dirac liquid renormalization may also be contributing to the observed trend.⁸³

V. CONCLUSIONS

We have studied the electronic behavior of bilayer graphene in the presence of a perpendicular electric field – *biased bilayer* – using the minimal tight-binding model that describes the system. The effect of the perpendicular electric field has been included through a parallel plate capacitor model, with screening correction at the Hartree level. We have compared the full tight-binding description with its 4-band and 2-band continuum approximations, and found that the 4-band model is always a suitable approximation for the conditions realized in experiments. Also, we have studied the effect of electron-hole asymmetry terms and found that they have only a small effect on the electronic properties addressed here. The model has been applied to real biased bilayer devices, either made out of SiC ⁸ or exfoliated graphene.^{9,10} The good agreement with experimental results – namely, for the electrostatic energy difference between layers obtained through ARPES⁸ and for the Shubnikov-de Haas cyclotron mass¹⁰ – clearly indicates that the model is capturing the key ingredients, and that a finite gap is effectively being controlled externally. Analysis of recent experimental results regarding the electrical noise⁷⁶ and cyclotron resonance⁸² further suggests that the model can be seen as a good starting point to understand the electronic properties of graphene bilayer.

Acknowledgments

E.V.C., N.M.R.P., and J.M.B.L.S. acknowledge financial support from POCI 2010 via project PTDC/FIS/64404/2006. A.H.C.N. acknowledges the partial support of the U.S. Department of Energy under grant No. DE-FG02-08ER46512.

Appendix A: Asymmetry between layers

In order to write Eq. (19) as an energy integral, we start by introducing the SLG density of states per spin per unit cell defined for the conduction band as

$$\rho(\epsilon) = \frac{1}{N_c} \sum_{\mathbf{k}} \delta(\epsilon - t|s_{\mathbf{k}}|), \quad (\text{A1})$$

with $s_{\mathbf{k}}$ as in Eq. (4). The momentum sum in Eq. (A1) can be written as an integral by letting $N_c \rightarrow \infty$. The integral can be performed and written in terms of complete elliptic integrals of the first kind.⁵

With the definition of $\rho(\epsilon)$ in Eq. (A1) the charge imbalance between layers in Eq. (19) can be written as $\Delta n = \Delta n_{1/2} + \Delta \tilde{n}$, where the charge imbalance at half-filling $\Delta n_{1/2}$ is given by

$$\Delta n_{1/2} = \frac{2}{A_{\text{O}}} \sum_{l=\pm} \int_0^{3t} d\epsilon \rho(\epsilon) \mathcal{I}_{-l}(\epsilon), \quad (\text{A2})$$

and the fluctuation $\Delta \tilde{n}$ with respect to the half-filled case is given by

$$\Delta \tilde{n} = \frac{2}{A_{\text{O}}} \begin{cases} \sum_{l=\pm} \int_{\epsilon_1}^{\epsilon_2} d\epsilon \rho(\epsilon) \mathcal{I}_{+l}(\epsilon), & n > 0 \\ -\sum_{l=\pm} \int_{\epsilon_1}^{\epsilon_2} d\epsilon \rho(\epsilon) \mathcal{I}_{-l}(\epsilon), & n < 0 \end{cases}, \quad (\text{A3})$$

where n is the carrier density with respect to half-filling. The integral kernel in Eqs. (A2) and (A3) reads

$$\mathcal{I}_{jl}(\epsilon) = \frac{[\epsilon^2 + \mathcal{K}_{-}^{jl}(\epsilon)](\epsilon^2 - \mathcal{K}_{+}^{jl})^2 - (\epsilon^2 + \mathcal{K}_{+}^{jl})t_{\perp}^2 \mathcal{K}_{-}^{jl}(\epsilon)}{[\epsilon^2 + \mathcal{K}_{-}^{jl}(\epsilon)][\epsilon^2 - \mathcal{K}_{+}^{jl}(\epsilon)]^2 + [\epsilon^2 + \mathcal{K}_{+}^{jl}(\epsilon)]t_{\perp}^2 \mathcal{K}_{-}^{jl}(\epsilon)}, \quad (\text{A4})$$

where $\mathcal{K}_{\pm}^{jl}(\epsilon) = [V/2 \pm E^{jl}(\epsilon)]^2$, with $E^{jl}(\epsilon)$ given by Eq. (7) with the substitution $\epsilon_{\mathbf{k}} \rightarrow \epsilon$. The limits of integration in Eq. (A3) depend on the band label l and E_{F} as follows: with $l = -$ we have $\epsilon_1 = \epsilon^{-}$ and $\epsilon_2 = \epsilon^{+}$ for $E_{\text{F}}^2 < V^2/4$, while for $E_{\text{F}}^2 > V^2/4$ we have $\epsilon_1 = 0$ and $\epsilon_2 = \epsilon^{+}$; with $l = +$ we only have contribution for $E_{\text{F}}^2 > t_{\perp}^2 + V^2/4$, and the limits are $\epsilon_1 = 0$ and $\epsilon_2 = \epsilon^{-}$. We use the notation $\epsilon^{\pm} = [E_{\text{F}}^2 + V^2/4 \pm \sqrt{E_{\text{F}}^2(V^2 + t_{\perp}^2) - t_{\perp}^2 V^2/4}]^{\frac{1}{2}}$.

Appendix B: Bilayer DOS

The DOS per unit cell of BLG, either biased or unbiased, is defined as

$$\rho_2(E) = \frac{2}{N_{\text{c}}} \sum_{\mathbf{k}} [\delta(E - E_{\mathbf{k}}^{\pm-}) + \delta(E - E_{\mathbf{k}}^{\pm+})], \quad (\text{B1})$$

where $E_{\mathbf{k}}^{\pm\pm}$ is given by Eq. (7). Equation (B1) can be written as a sum of two contributions, $\rho_2(E) = \sum_{l=\pm} \rho_2^l(E)$, where the label $l = \pm$ stands for contributions coming from bands $E_{\mathbf{k}}^{\pm l}$. The analytical expressions for each contribution are

$$\rho_2^{-}(E) = \frac{4}{t^2 \pi^2} \begin{cases} \psi^{--}(E) \frac{\chi^{-}(E)}{\sqrt{F[\chi^{-}(E)/t]}} \mathbf{K} \left(\frac{4\chi^{-}(E)/t}{F[\chi^{-}(E)/t]} \right), & \begin{cases} \Delta_g/2 < |E| < V/2 \wedge \alpha \leq t^2 \\ \vee \\ E^{+-}(t) < |E| < V/2 \wedge \alpha > t^2 \end{cases} \\ + \\ \psi^{-+}(E) \frac{\chi^{+}(E)}{\sqrt{F[\chi^{+}(E)/t]}} \mathbf{K} \left(\frac{4\chi^{+}(E)/t}{F[\chi^{+}(E)/t]} \right), & \Delta_g/2 < |E| < E^{+-}(t) \wedge \alpha < t^2 \\ \psi^{--}(E) \frac{\chi^{-}(E)}{\sqrt{4\chi^{-}(E)/t}} \mathbf{K} \left(\frac{F[\chi^{-}(E)/t]}{4\chi^{-}(E)/t} \right), & \Delta_g/2 < |E| < E^{+-}(t) \wedge \alpha > t^2 \\ + \\ \psi^{-+}(E) \frac{\chi^{+}(E)}{\sqrt{4\chi^{+}(E)/t}} \mathbf{K} \left(\frac{F[\chi^{+}(E)/t]}{4\chi^{+}(E)/t} \right), & \begin{cases} E^{+-}(t) < |E| < E^{+-}(3t) \wedge \alpha < t^2 \\ \vee \\ \Delta_g/2 < |E| < E^{+-}(3t) \wedge t^2 \leq \alpha < 9t^2 \end{cases} \end{cases} \quad (\text{B2})$$

and

$$\rho_2^{+}(E) = \frac{4}{t^2 \pi^2} \begin{cases} \psi^{+-}(E) \frac{\chi^{-}(E)}{\sqrt{F[\chi^{-}(E)/t]}} \mathbf{K} \left(\frac{4\chi^{-}(E)/t}{F[\chi^{-}(E)/t]} \right), & \sqrt{t_{\perp}^2 + V^2/4} < |E| < E^{++}(t) \\ \psi^{+-}(E) \frac{\chi^{-}(E)}{\sqrt{4\chi^{-}(E)/t}} \mathbf{K} \left(\frac{F[\chi^{-}(E)/t]}{4\chi^{-}(E)/t} \right), & E^{++}(t) < |E| < E^{++}(3t) \end{cases}, \quad (\text{B3})$$

with $\psi^{\pm l}(E)$ given by

$$\psi^{\pm l}(E) = \frac{\sqrt{t_{\perp}^4/4 + (t_{\perp}^2 + V^2)\chi^l(E)^2} \sqrt{\chi^l(E)^2 + t_{\perp}^2/2 + V^2/4 \pm \sqrt{t_{\perp}^4/4 + (t_{\perp}^2 + V^2)\chi^l(E)^2}}}{\chi^l(E) \left| \sqrt{t_{\perp}^4/4 + (t_{\perp}^2 + V^2)\chi^l(E)^2} \pm (t_{\perp}^2 + V^2)/2 \right|} \quad (\text{B4})$$

and $\chi^{\pm}(E)$ as in the right-hand side of Eq. (22) with $E_{\text{F}} \rightarrow E$. We use $F(x) = (1+x)^2 - (x^2-1)^2/4$ and

$\mathbf{K}(m)$ for the complete elliptic integral of the first kind,

and $E^{\pm\pm}(x)$ is given by Eq. (7) with the substitution $\epsilon_{\mathbf{k}} \rightarrow x$ and $\alpha = (V^4/4 + t_{\perp}^2 V^2/2)/(V^2 + t_{\perp}^2)$.

Appendix C: Cyclotron mass

Based on the full tight-binding band structure $E_{\mathbf{k}}^{\pm\pm}$ given in Eq. (7), it is possible to derive general expressions for the cyclotron mass in Eq. (26). The key observation is that the area of a closed orbit at the Fermi level $A(E_F)$ may be written as $A(E_F) \propto \sum'_{\mathbf{k}} \Delta_{\mathbf{k}}$, where the prime means summation over all \mathbf{k} 's inside the orbit, and $\Delta_{\mathbf{k}} = (2\pi)^2/(N_c A_O)$ is the area per k -point in

the first BZ. The cyclotron mass may then be written as $m_c \propto \partial_{E_F} \sum_i \int_{\mathcal{E}_i}^{E_F} dE \sum_{\mathbf{k}} \delta(E - E_{\mathbf{k}}^{\mu\nu}) \Theta(\epsilon^{\pm} - \epsilon_{\mathbf{k}})$, where $\epsilon_{\mathbf{k}}$ is the SLG dispersion and ϵ^{\pm} is given by Eq. (7). The integration limits \mathcal{E}_i and the choice between the two possibilities ϵ^{\pm} depend on the particular band and on the position of the Fermi level. Skipping the details of the derivation, what is worth noting is that, due to the sum of delta functions in the previous expression for m_c , the result has a mathematical structure similar to the derived expressions for the DOS of BLG (see Appendix B). The cyclotron effective mass of the biased BLG for the relevant parameter range $V \lesssim t_{\perp} \ll t$ and $|E_F| \lesssim t$ is then given by

$$m_c(E_F) = \frac{\hbar^2}{A_O t^2} \frac{2}{\pi} \begin{cases} -\psi^{--}(E_F) \frac{\chi^-(E_F)}{\sqrt{F[\chi^-(E_F)/t]}} \mathbf{K} \left(\frac{4\chi^-(E_F)/t}{F[\chi^-(E_F)/t]} \right), & \Delta_g/2 < |E_F| < V/2 \\ \psi^{-+}(E_F) \frac{\chi^+(E_F)}{\sqrt{F[\chi^+(E_F)/t]}} \mathbf{K} \left(\frac{4\chi^+(E_F)/t}{F[\chi^+(E_F)/t]} \right), & \Delta_g/2 < |E_F| \lesssim t \\ \psi^{+-}(E_F) \frac{\chi^-(E_F)}{\sqrt{F[\chi^-(E_F)/t]}} \mathbf{K} \left(\frac{4\chi^-(E_F)/t}{F[\chi^-(E_F)/t]} \right), & \sqrt{t_{\perp}^2 + V^2/4} < |E_F| \lesssim t \end{cases}. \quad (\text{C1})$$

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