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# Elementary Differential Geometry

Second Edition

TECHNISCHE INFORMATIONSBIBLIOTHEK UNIVERSITÄTSBIBLIOTHEK HANNOVER



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