

Elements of a Cognitive Model of Physics Problem Solving: Epistemic Games

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Although much is known about the differences between expert and novice problem solvers, knowledge of those differences typically does not provide enough detail to help instructors understand why some students seem to learn physics while solving problems and others do not. A critical issue is how students access the knowledge they have in the context of solving a particular problem. In this paper, we discuss our observations of students solving physics problems in authentic situations in an algebra-based physics class at the University of Maryland. We find that when these students are working together and interacting effectively, they often use a limited set of locally coherent resources for blocks of time of a few minutes or more. This coherence appears to provide the student with guidance as to what knowledge and procedures to access and what to ignore. Often, this leads to the students failing to apply relevant knowledge they later show they possess. In this paper, we outline a theoretical phenomenology for describing these local coherences and identify six organizational structures that we refer to as epistemic games. The hypothesis that students tend to function within the narrow confines of a fairly limited set of games provides a good description of our observations. We demonstrate how students use these games in two case studies and discuss the implications for instruction.

1. Introduction

Students learning physics at the college level often have considerable difficulty with problem solving despite the fact that problem solving is an integral part of most physics classes.¹ Instructors may assume that these difficulties arise from a lack of mathematical skills, but little evidence has been presented to determine if this is the case. As part of a project to reform introductory algebra-based physics,² we have collected extensive data of students learning physics and solving physics problems in a variety of environments. This data includes some familiar but remarkable student behavior, such as

- failing to use their personal knowledge in favor of misinterpretations of authority-based knowledge when reasoning in a formal context,
- using incorrect qualitative arguments to rebut a qualitative argument even when they know the correct formal argument.

These behaviors are often quite robust, with students dramatically ignoring — appearing not even to hear — explicit suggestions from an instructor speaking to them directly. As a result, these behaviors look like what one might crudely describe as “misconceptions of expectations” about how to solve problems.

In order to make sense of this data, we propose a useful way of analyzing students’ problem solving behavior in terms of locally coherent goal-oriented activities that we refer to as *epistemic games*. These games both guide and limit what knowledge students think is appropriate to apply at a given time. Identifying these games provides a way of parsing students’ tacit expectations about how to approach solving physics problems.

We work in the context of a theoretical model that allows us to describe the cognitive processes that students use — correctly and incorrectly — in the context of solving physics problems. We build on and extend ideas developed by

diSessa, Sherin, and Minstrell^{3,4,5,6,7}, and by Collins and Ferguson.⁸ Our theoretical approach fits into the more general theoretical framework we refer to as *the resource model*.^{9,10,11,12} In this broad model of student thinking, knowledge elements combine dynamically in associative structures activated by control structures in response to inputs from each other and from the environment.

Our theoretical framework offers researchers and educators a vocabulary (an ontological classification of cognitive structures) and grammar (a description of the relationship between the cognitive structures) to describe students’ problem solving (and in particular, their understanding and use of mathematics) in the context of physics.¹³ Viewing student activity through the lens created by this framework can help researchers and educators understand how teacher-student interactions can more effectively help students develop their own problem solving skills.

In the next section, we give a brief overview of our theoretical framework. In section three, we describe a cognitive model for local coherence in problem solving in physics: epistemic games. In section four, we describe the setting of the study: the student population, the modified instructional environment, and the methodology used to collect and analyze our data. In section five, we use our theoretical model to analyze two student problem-solving sessions. In the final section, we discuss some instructional implications, and present some conclusions. Much of the work described here is taken from the dissertation of Jonathan Tuminaro and more detail can be found there.¹⁴

2. The Theoretical Framework

Constructivism — the idea that a student constructs new knowledge based largely on what that student already knows — is the dominant paradigm in modern educational theories.¹⁵ The teacher’s role in the constructivist paradigm is to create environments that help students undertake this construction accurately and effectively. In order to do this, it

helps the teacher to know (i) the content and structure of the students' existing knowledge and (ii) how the students use this knowledge to construct new knowledge. There has been considerable direct observational research on the difficulties students have with various items of physics content;^{16,17} but to understand how students organize, access, and use their existing knowledge, we need a finer-grained understanding of how students think and respond. We need to know not just that students construct their new knowledge based on what they know, we need some understanding of how students construct new knowledge. To develop such an understanding, we need to know at least some of the basic elements of fundamental cognitive activities and how they are organized.

Previous Research

Research on students' naïve knowledge and on expert/novice differences in problem solving are two topics that are particularly relevant to the current study. In this subsection, we give a brief review of these two areas of research.

Students' Naïve Knowledge

The fact that students bring prior naïve knowledge into a physics class has been well documented in the research literature.^{17,18} The level of abstraction at which the students' naïve knowledge is described, however, varies considerably. Some researchers describe student knowledge that does not align well with the scientific knowledge we are trying to teach as "misconceptions," "alternative conceptions," or "naïve theories." These researchers assume that students have internally consistent models of how aspects of the world work.^{18,19,20,21} Others describe the knowledge of beginning students in physics as fragmented and spontaneous.^{4,8} Both of these approaches contain elements of the truth. Sometimes student knowledge is fragmented, other times it appears coherent. In particular, expert knowledge is often highly coherent. In order to understand the novice-to-expert transition we must have a model that can bridge these two cognitive states. The resource model allows us to do this.

Expert/Novice Differences in Problem Solving

Researchers have studied problem solving in different contexts: problem solving associated with games such as chess,²² problem solving in mathematics,^{23,24,25} and mathematical problem solving in the context of physics.^{26,27,28,29,30,31,32} There is agreement that there are substantial differences between experts and novices; experts have more knowledge and organize it better. But most attempts to model the differences at a finer scale have focused on creating computer models that would solve problems effectively. Sometimes these models are algorithmic;³³ sometimes they are based on heuristics extracted from expert informants.³⁴ While these approaches can produce computer software that can carry out some tasks that human experts do, it is not at all clear that they correctly model how a human being learns and functions. (A good summary of the successes and limitations of this approach is given in d'Andrade.³⁵) Other ap-

proaches are phenomenological, using expert heuristics to design learning environments that improve students' problem-solving skills.^{36,37} Neither approach helps us understand how students make the transition from novice to expert.

The Resource Model

If our goal is to teach a human being effectively, it is appropriate to build a theoretical model based on our knowledge of the functioning of that system (a human being) and not some other (a computer).³⁸ In order to describe student behaviors, we want to create a model that is sufficiently coarse-grained that it allows us to describe observed behaviors and sufficiently fine-grained that it gives us insight into the mechanisms responsible for those behaviors. The *resource model*^{10,11,12,13} provides such a structure. It is based on a combination of three kinds of scholarship about the functioning human: neuroscience, cognitive science, and behavioral science. It permits us to begin to create a finer-grained understanding of student behavior that can bridge the alternative- and fragmented-conception models and can help us develop a more detailed understanding of the novice-to-expert transition.

Researchers in neuroscience, cognitive science, and behavioral science attempt to model human thought at a variety of grain sizes. Much has been learned in all these areas (though one has to be cautious in applying research results at a fine-grained level from neuroscience or cognitive science to real-world situations), but there is still much to be learned, and there is much that is still uncertain about what we know about human thinking. Nevertheless, we can model reasonably safely when the basic structural elements of neuroscience, the well-documented mechanisms of cognitive science, and the observations of real-world (ecological)³⁹ behavior of real human beings acting in authentic situations all agree and support each other.

Note that in building the resource model we are not attempting to create a fundamental theory of human behavior. Rather, we are developing a theoretical framework or superstructure¹¹ within which plausible phenomenological models can be created that can help us understand what we see in our classrooms and that are also consistent with what is known about the fundamental mechanisms and operation of the brain.

The Basis of Cognition

A consistent model of cognition that is beginning to emerge from neuroscience and cognitive science is synthesized and documented in many books.^{40,41,42,43} In this model, cognitive elements of knowledge and memory are represented by networks of connected neurons. When someone recalls or uses the knowledge represented by a particular network, the neurons of the network are *activated* (increase their firing rate).⁴⁴ Particular knowledge elements tend to be multi-modal (i.e., to involve activation and interpretation of multiple sensory and interpretive structures) and involve neurons in many parts of the brain.⁴⁵ Cognitive networks arise from the building of associations among neurons

through synapse growth.⁴⁶ The association of neurons can vary in strength and increases with repeated associational activations.^{47,48}

Neural connections can be excitatory or inhibitory.⁴⁷ This creates the possibility of *executive processes* that result in the selective activation of some networks and the suppression of others.⁴⁹ Modern fMRI studies and neurophysiological studies with patients who have brain lesions suggest that the prefrontal cortex is a primary site of a large number of control structures (though they are expected to occur in other parts of the brain as well).^{50,51,52}

The critical elements of this model are the basic elements of knowledge stored in long-term memory, the way those elements are linked, and the way those linked structures are activated in different circumstances. We use the term *knowledge element* to describe any knowledge stored in long-term memory that activates as a single unit or chunk. We include both declarative and procedural knowledge. We refer to the linking patterns of association as *knowledge structures* and to the executive function that determines when those structures are activated as *control structures*. We broadly refer to all the elements of this model as *resources*.

This model only provides a framework. In order to develop a practical phenomenology, we need to identify various robust patterns of association of knowledge elements – i.e., specific knowledge structures, and demonstrate the value of recognizing these structures.

Basic Knowledge Elements: Compilation

A network corresponding to an element of knowledge becomes robust through practice and experience. For example, one can quickly and easily identify the combination of sensations associated with holding a cup of hot coffee. We effortlessly combine the perception of the pixels (activation of rods and cones) on our retina with the touch, smell, and taste of the coffee into a perception of what appears to be a single object. Neuroscientists call this *binding*, but we prefer to describe it as *compilation*.⁵³ Compilation of knowledge elements in associated knowledge structures results in new knowledge elements. They are seen as irreducible by the individual and can be used as a single chunk in working memory.⁵⁴ The instructional implications of compilation are discussed elsewhere.⁵⁵

Note that a knowledge element may have a structure and that for some purposes it might be useful to decompose it into finer-grained knowledge elements even when the user sees it as irreducible.⁵⁶ This is like considering molecules consisting of atoms. For some tightly bound molecules in some situations (e.g., molecules in a gas in kinetic theory) it suffices to consider the molecule as a single functioning unit without substructure. In other circumstances (e.g., situations in which chemical reactions occur or for more weakly bound molecules), it is essential to keep the molecule's structure in terms of atoms in mind.

Patterns of Association: Knowledge Structures

Because cognitive networks are extended and because neurons have large numbers of synapses with other neurons, an individual neuron may be a part of multiple mutually linked knowledge structures. As a result, activation of one network may result in the associated activation of other networks. Patterns of association develop, linking different resources in different situations. Learning occurs as the result of the growth of new synapses that result in changing the topology of existing networks.⁵⁷ The patterns of association individuals develop may help or hinder them in solving physics problems.¹³

In this paper, we propose that a useful way to analyze some of the common associational structures in student approaches to physics problem solving is to describe them in terms of locally coherent, goal-oriented activities. We choose to call these epistemic games because of their similarity to the structures proposed by Collins and Ferguson.⁹ Note that the activities that Collins and Ferguson describe are normative — activities carried out by experts to solve problems. We extend their idea to one that is ethnographic — descriptive of observed student behavior. (Note that some other researchers have also extended their use of the term in this way.^{58,59})

Some Specific Resources:

Reasoning Primitives and Intuitive Mathematics

A variety of specific relevant resources are available to most students studying physics. These include both knowledge about the physical world and knowledge about mathematics, both intuitive and formal.

Students use a form of intuitive knowledge about physical phenomena and processes that they have learned in their everyday life experiences to make sense of the physical world.⁶⁰ DiSessa⁴ proposes that students develop an intuitive sense of physical mechanism from everyday experience. This intuitive sense of physical mechanism arises from the activation and interaction of multiple cognitive resources that diSessa refers to as *phenomenological primitives* (p-prims).

The name, *phenomenological primitives*, is used to convey several key aspects of these cognitive structures. The word “phenomenological” reflects the idea that these resources are abstracted from everyday phenomena. For example, *closer is stronger* could be abstracted from the phenomenon that the closer one is to a fire, the warmer it feels. The word “primitive” reflects the idea that these resources are “irreducible and undetectable” to the user – they are often used as if they were self-explanatory. For example, asked why it is warmer closer to a fire, a student using *closer is stronger* may respond, “it just is.”⁶¹

Because of his focus on the irreducibility of p-prims with respect to the user, diSessa identifies p-prims at differing levels of abstraction: for example, *force as mover* and *abstract balancing*. *Force as mover* involves the very specific concept of an object moving under the influence of a force; whereas *abstract balancing* involves the general notion that two unspecified influences can be in a state of equilibrium. Because of the specific nature of p-prims like *force as*

mover, diSessa proposes that there are thousands of p-prims corresponding to the myriad of physical experiences one may have in this complex world.

To reduce the extremely large number of p-prims and to group cognitive structures at their different levels of abstraction, we follow Redish¹¹ and abstract from p-prims the notion of intuitive pieces of knowledge called *reasoning primitives*. Reasoning primitives are abstractions of everyday experiences that involve generalizations of classes of objects and influences. In this view a p-prim like *force as mover* results from mapping an abstract reasoning primitive like *agent causes effect* into a specific situation that involves forces and motion. The specific agent, in this case, is a force and the effect it causes is movement. When a reasoning primitive is mapped into a specific situation, we refer to it as a *facet* of that reasoning primitive.⁸ *Agent causes effect* could also be mapped into *force as spinner*, another p-prim identified by diSessa.⁴ This shows how the notion of reasoning primitives reduces the total number of resources necessary to describe students' previous knowledge about physical phenomena (compared to p-prims). In addition, *agent causes effect* and *abstract balancing* both reflect relationships between abstract influences and, therefore, exist at the same level of abstraction.

Another reason to consider the reasoning primitives underlying facets is to understand process components that may be addressable by instruction. If a student is using an appropriate reasoning primitive but has mapped it inappropriately, it may be simple to help the student change the mapping. This more fine-grained theoretical model activates different instructional responses than if one considers a particular p-prim to be an irreducible and robust "alternate conception."

Students can also activate a variety of resources from their intuitive mathematics knowledge including: intuitive sense of number,⁶² counting, ordering, a variety of grounding metaphors,⁶³ symbolic forms, and interpretive devices.⁷ Since these mathematical resources do not play a critical role in the examples used here, however, we save their discussion for another publication. (See ref. 15.)

3. Epistemic Games

Students have a wealth of previous knowledge and ideas that they bring to bear when solving physics problems. To understand and talk about what students are doing, we need a description of the way they organize their resources.

To determine some of these organizational structures, we analyzed 11 hours of video data drawn from about 60 hours of videotapes of groups of students solving homework problems in a reformed algebra-based physics class. The context of the reforms and the methodology of the data collection and analysis are described in section 4. In this section, we describe six locally-coherent organizational control structures that we saw students using in these tapes. Examples of these games drawn from two case studies taken from the video data are given in section 5.

One of the most interesting characteristics of the student behaviors we observed was their local coherence. Over a period of a few minutes to half an hour, we saw students reasoning using a limited set of associated resources.

We can best describe these behaviors by adapting the idea of *epistemic game* (or, *e-game*, for short) introduced by Collins and Ferguson.⁹ Collins and Ferguson define an epistemic game as a complex "set of rules and strategies that guide inquiry." They introduce the idea of epistemic games to describe expert scientific inquiry across disciplines. Students in introductory physics courses are far from experts, so using scientists' approaches to inquiry as a norm by which to describe students' inquiry would not be appropriate. For this reason, we generalize the idea of epistemic games to be descriptive rather than normative. We define an *epistemic game* to be

*a coherent activity that uses particular kinds of knowledge and processes associated with that knowledge to create knowledge or solve a problem.*¹¹

The activities are 'epistemic' in the sense that students engage in these activities as a means of constructing new knowledge. We use the word 'game' in a very real sense; a particular game (like checkers or chess) is a coherent activity that has ontology components that identify the "things" of the game (players, pieces, and a playing board) and a structure (a beginning and an end, moves, rules) that makes it distinguishable from other activities. Similarly, an e-game has ontological components (concepts, principles, equations) and a structure (starting and ending states, allowed moves, rules). The simplest epistemic game identified by Collins and Ferguson is a familiar one: *list making*. Every list is implicitly an answer to a question – it builds knowledge to satisfy some goal. Some examples are: "What do I need from the grocery store?"; "What are the fundamental forces of nature?"; and, "What are the constituents of all matter?"

Note that the idea of a "game" here — a locally coherent set of behavioral rules for achieving a particular goal — is very general. Some of the behavioral science literature (especially in the opposite extremes of popularizations⁶⁴ and mathematical economics⁶⁵) has used the term game in this way. We focus here on epistemic games — games engaged in for the purpose of creating knowledge.

Ontology of Epistemic Games

Epistemic games have two ontological components: a knowledge base and an epistemic form. An e-game is *not* simply a structure of a set of associated knowledge; it is an activation of a pattern of activities that can be associated with a collection of resources. The collection of resources that an individual draws on while playing a particular e-game constitutes the *knowledge base*. For example, to answer a question like, "What are the fundamental forces of nature?" one needs to have some requisite knowledge to list the forces.

The *epistemic form* is a target structure, often an external representation that helps guide the inquiry during an epistemic game. For example, the epistemic form in the list

making game is the list itself. The list is an external representation that cues particular resources and guides the progression of the inquiry. In some of the games we describe below, the epistemic form could be a written out series of steps, or the derivation of an equation or result.

Structure of Epistemic Games

The structural components of epistemic games include the entry and ending conditions of the game and the moves.

The entry and ending conditions specify the beginning and the ending of the game. As we mentioned above, one may enter into the list making game as a means to answer a question. When solving physics problems, students' expectations about physics problems determine the entry and ending conditions. These expectations can depend on real-time categorizations of physics problems and/or on preconceived notions about the nature of problem solving in physics. Research by Hinsley and Hayes⁶⁶ indicates that students can quickly categorize large classes of physics problems very shortly after reading the statement of the problem. In fact, these categorizations can be made after reading the first sentence. (Often these categorizations can be made after reading the first sentence.) The students' ability to very quickly categorize physics problems may stem from their experience with and expectations about physics problem solving. These expectations and categorizations of physics problems affect which epistemic game the students (perhaps tacitly) choose to play. In addition, students' preconceived epistemological stances about problem solving in physics can affect their expectations. If students believe that problem solving in physics involves rote memorization of physics equations, that can affect the strategy they employ (i.e. which e-game they choose to play) and what they believe an answer in physics is (i.e. how they know they are done playing a particular game).⁶⁷

	Ontological Components	Structural Components	
<i>Knowledge Base</i>	Cognitive resources associated with the game.	<i>Entry and ending conditions</i>	Conditions for when to begin and end playing a particular game.
<i>Epistemic Form</i>	Target structure that guides inquiry.	<i>Moves</i>	Activities that occur during the course of an e-game.

Table 1: The ontological and structural components of epistemic games

The moves in an e-game are the steps/procedures that occur in the game. In the list-making game the moves may be to add a new item, combine two (or more) items, substitute an item, split an item, and remove an item. As we will see, a

critical element of an epistemic game is that playing the game specifies a certain set of moves. What is particularly important about this is not just the moves that are included in the game, but also the moves that are excluded.

Table 1 summarizes the ontological and structural components of epistemic games.

Epistemic Games Students Play in Introductory, Algebra-Based Physics

In this section we discuss some of the epistemic games that account for the different problem-solving strategies seen in our data. We identify six epistemic games that include most of the different problem-solving behaviors we have seen. (See

Table 2.) We do not claim that this list spans all possible problem-solving approaches that could be employed during problem solving in physics and we do not claim to have identified all possible moves within each game. If we had examined a different population of students or a different domain, it is possible that the list of epistemic games would be different, though we expect some of the games identified here to have broad applicability. We present these as examples of the type of structure we are proposing. In the next section, we present two case studies showing how analyzing student behavior in terms of these games helps make sense of what they do and do not do in the context of solving a specific problem.

Each of these games is described in more detail below. For each epistemic game we give a brief introduction and discuss its ontology and structure. Note that some of the games have common moves and one game may look like a subset of another. We identify them as distinct games because they have different ending conditions; students playing different games decide they are "done" when different conditions are met. Section 5 gives an example of students playing each of these games.

<i>List of epistemic games</i>
Mapping Meaning to Mathematics
Mapping Mathematics to Meaning
Physical Mechanism Game
Pictorial Analysis
Recursive Plug-and-Chug
Transliteration to Mathematics

Table 2. List of epistemic games identified in our data set

Mapping Meaning to Mathematics: The most intellectually complex epistemic game that we identify is *Mapping Meaning to Mathematics*. In this game, students begin from a conceptual understanding of the physical situation described in the problem statement, and then progress to a quantitative solution. We identify five basic moves (see Figure (1)): (1) develop a story about the physical situation, (2) translate quantities in the physical story to mathematical entities, (3) relate the mathematical entities in accordance with the physi-

cal story, (4) manipulate symbols, and (5) evaluate and interpret the story.

The knowledge base for this game (as with all the games we identify) comes from the set of physics and mathematics resources. In general, however, different resources can be activated during the different moves of the game. During the development of the conceptual story (move 1), reasoning primitives are most often activated. That is, students often rely on their own conceptual understanding to generate this story – not on fundamental physics principles. Translating the conceptual story into mathematical entities (move 2) is difficult for most of the students in our population. Intuitive mathematics knowledge, symbolic forms, and interpretive devices may be activated during this move. Relating the mathematical entities to the physical story (move 3), again is difficult for students in our population, and depends on intuitive mathematics knowledge, symbolic forms, and interpretive devices. Once the physics equations are written, the symbolic manipulations (move 4) often are carried out without a hitch. This is probably because most of our students have had ample practice manipulating symbols. The evaluation of the story (move 5) can occur in many different ways. For example, students may check the solution with a worked example (or solution in the back of the book), students may check their quantitative answer with their conceptual story, or students may check their solution against an iconic example. Note that these evaluations do not necessarily correspond to an expert evaluation or what a teacher would want to see. A superficial similarity to previously seen results may suffice for the student to decide that the end condition of the game has been met.

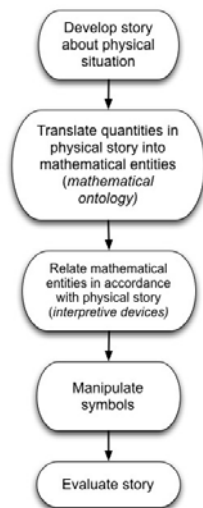


Fig. (1): Schematic diagram of some moves in the epistemic game Mapping Meaning to Mathematics

The epistemic form for *Mapping Meaning to Mathematics* is typically the collection of mathematical expressions that the students generate during moves (2) and (3). These expressions lead the direction of the inquiry. Note, however, that the epistemic form is not the entire story in this game. The interpretation (story) that goes with the series of mathe-

tical expressions generated, may or may not be explicitly expressed, depending on the instructions for giving a written output and the students' sense of how much "explanation" they are required to provide.

Mapping Mathematics to Meaning: The second most intellectually complex epistemic game that we identify is *Mapping Mathematics to Meaning*. In this game, students develop a conceptual story corresponding to a particular physics equation. The ontological components of *Mapping Mathematics to Meaning* are the same as those in *Mapping Meaning to Mathematics*. In particular, both games involve the same kind of knowledge base (mathematical resources) and the same epistemic form (physics equations). However, the particular resources and physics equations that are used in each game can vary from problem to problem.

In addition, the structural components of the two games are different. In *Mapping Meaning to Mathematics*, students begin with a conceptual story and then translate it into mathematical expressions. In contrast, in *Mapping Mathematics to Meaning* students begin with a physics equation and then develop a conceptual story.⁶⁸ The structural differences between these two games make them distinguishable from each other.

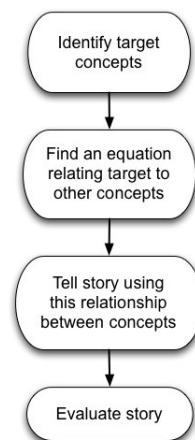


Fig. (2): Schematic diagram of some moves in the epistemic game Mapping Mathematics to Meaning.

We identify four moves in *Mapping Mathematics to Meaning* (see figure (2)): (1) identify target concepts, (2) find an equation relating the target concepts to other concepts, (3) tell a story using this relationship between concepts, and (4) evaluate story.

Physical Mechanism Game. In the *Physical Mechanism Game* students attempt to construct a physically coherent and descriptive story based on their intuitive sense of physical mechanism. The knowledge base for this game consists of reasoning primitives. In this game students do not make explicit reference to physics principles or equations.

The ontology of the *Physical Mechanism Game* is different than in *Mapping Meaning to Mathematics* and *Mapping Mathematics to Meaning*. The epistemic form in the latter two games explicitly involves physics equations. In contrast, the epistemic form in the *Physical Mechanism Game* does

not. Although the epistemic form is necessarily different, the same set of resources (intuitive mathematics knowledge, reasoning primitives, symbolic forms, and interpretive devices) may be active in this game as in the previous games.

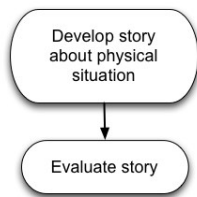


Fig.(3): Schematic diagram of some moves in the epistemic game Physical Mechanism

The structure of the *Physical Mechanism Game* is similar to the first move in *Mapping Meaning to Mathematics* – both involve the development of a conceptual story. However, we can distinguish the two because the *Physical Mechanism Game* represents a separate, coherent unit of student activities; it has a different endstate. In *Mapping Meaning to Mathematics*, after move (1) students go on to move (2), then move (3), etc. After (1) creating and (2) evaluating the conceptual story developed in the *Physical Mechanism Game* (see figure (3)) students decide they are done. The activities that follow this game do not cohere with the conceptual story – in direct contrast with the activities that follow move (1) in *Mapping Meaning to Mathematics*.

Pictorial Analysis Game. In the *Pictorial Analysis Game*, students generate an external spatial representation that specifies the relationship between influences in a problem statement. For example, students who make a schematic drawing of a physical situation, a free-body diagram, or a circuit diagram are all playing the *Pictorial Analysis Game*.

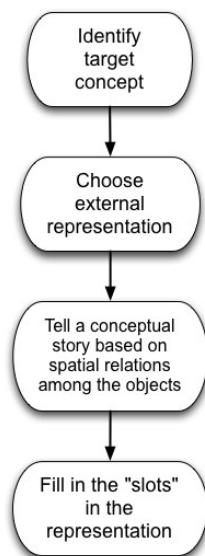


Fig. (4): Schematic diagram of some moves in the epistemic game Pictorial Analysis

In this game, as with all the games previously discussed, the knowledge base consists of all the resources listed above

plus some representational translation resources that we do not discuss here. The epistemic form in this game is the distinguishing characteristic. The epistemic form is a schematic or diagram that the students generate. For example, if the students draw a circuit diagram during their inquiry, then that diagram serves as an epistemic form that guides their inquiry. In the same way, a schematic drawing and/or free-body diagram could each serve as a target structure that guides inquiry.

The moves in this game are largely determined by the particular external representation that the students choose. For example, if the students choose to draw a free-body diagram, then one move is to determine the forces that act upon the object in question; whereas, if the students choose to draw a circuit diagram, then one move is to identify the elements (e.g. resistors, capacitors, batteries, etc.). Despite differences that may arise based on the particular external representation chosen, there are three moves that are common to all instantiations of the Pictorial Analysis Game (see figure (4)): (1) determine the target concept, (2) choose an external representation based on the spatial relation between the objects, and (3) fill in the slots in this representation. An example of students who choose to draw a free-body diagram while playing the Pictorial Analysis Game is given in our first case study in section 5.⁶⁹

Recursive Plug-and-Chug. In the *Recursive Plug-and-Chug Game* students plug quantities into physics equations and churn out numeric answers, without conceptually understanding the physical implications of their calculations.

Students do not generally draw on their intuitive knowledge base while playing this game. Instead, they simply identify quantities and plug them into an equation. Consequently, students playing this game rely only on their syntactic understanding of physics symbols, without attempting to understand these symbols conceptually. That is, other cognitive resources (such as intuitive mathematics knowledge, reasoning primitives, symbolic forms, and interpretive devices) are usually not active during this game.

The epistemic form in *Recursive Plug-and-Chug* is similar or even identical to that in *Mapping Meaning to Mathematics* and *Mapping Mathematics to Meaning*. Each game has physics equations as part of the epistemic form, but the resources that are active (*i.e.* knowledge base) are different. The rules and strategies that are employed in *Recursive Plug-and-Chug* differ from those in *Mapping Meaning to Mathematics* and *Mapping Mathematics to Meaning* – even though the epistemic form may be the same. A distinguishing feature of *Recursive Plug-and-Chug* is the resources that are not activated during this game.

In *Recursive Plug-and-Chug*, the students first identify a target quantity. This is similar to the first move in *Mapping Mathematics to Meaning*, but it differs in that here the students only identify the quantity and its corresponding symbol – they do not attempt to understand conceptually what the quantity represents physically as in *Mapping Mathematics to*

Meaning. Second, the students identify an equation that relates the target quantity to other quantities, but they do not attempt to create a story that justifies the use of that equation. Third, the students identify which quantities are known and which quantities are unknown. If the target quantity is the only unknown, then they can proceed to calculate the answer. However, if there are additional unknowns, then they must choose a sub-goal and start this process over. Herein lies the ‘recursive’ nature inherent in this game. Figure (5) shows a schematic depiction of the moves in this game.

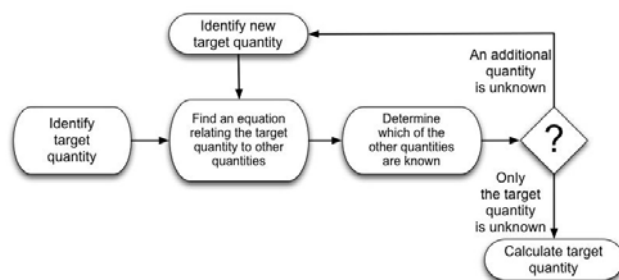


Fig. (5): Schematic diagram of some moves in the epistemic game Recursive Plug-and-Chug

Transliteration to Mathematics. Research on problem solving indicates that students often use worked examples to develop solutions to novel problems.^{70,71} *Transliteration to Mathematics* is an epistemic game in which students use worked examples to generate a solution without developing a conceptual understanding of the worked example. “Transliterate” means “to represent (letters or words) in the corresponding characters of another alphabet.”⁷² In the *Transliteration to Mathematics* game students map the quantities from a target problem directly into the solution pattern of an example problem.

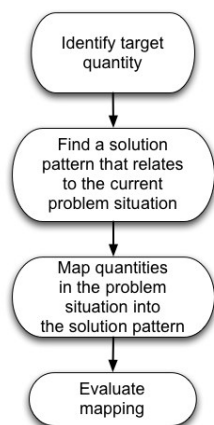


Fig. (6): Schematic diagram of some moves in the epistemic game Transliteration to Mathematics

Because students use the symbolism in this game without conceptual meaning, usually only resources associated with the syntactic structure of equations are active during this game. The solution pattern of the target example serves as

the epistemic form for the *Transliteration to Mathematics* game.

The moves in this game are as follows: (1) identify a target quantity, (2) find a solution pattern that relates to the current problem situation, (3) map quantities in the current problem situation into that solution pattern, and (4) evaluate the mapping (see figure (6)). Many students find moves (2) and (3) very tricky. Many times students may find a solution pattern that they think relates to the current problem, when in fact it does not.

4. The Setting of the Study and Methodology

This study was done as a part of a project carried out at the University of Maryland³ to determine whether an introductory physics course could serve as a venue to help biology students learn to see science as a coherent process and way of thinking, rather than as a collection of independent facts; and whether this could be achieved within the context of a traditional large-lecture class without a substantial increase in instructional resources. The project adopted reforms that were well-documented to produce conceptual gains and adapted them to create a coherent package that produced epistemological and metacognitive gains. We were able to accomplish this without sacrificing the conceptual gains associated with these reforms.⁷³

Data on the student responses to the modified environment were collected in a variety of ways in order to provide triangulation on the learning process of individual students and evaluations of the overall class results. The learning environments were constructed to encourage students to learn in group discussions taking place both in and out of the classroom. Hundreds of hours of these group discussions were recorded on video and provide the data for this project. In addition, all student homework, quizzes, and exams were scanned before grading. Finally, we gave pre-post conceptual (FCI⁷⁴ and FMCE⁷⁵) and epistemological attitude surveys (modified MPEX⁷⁶).

Student Population

The students in this study were enrolled in an introductory, algebra-based physics course. They were approximately 60% female; more than 70% were juniors and seniors, about 50% were biological science majors, and about 40% were pre-meds. (There were some year-to-year fluctuations in these numbers.) A particularly interesting statistic for this study is that more than 95% of the students had successfully completed two semesters of calculus, yet they chose to enroll in an algebra-based introductory physics course despite the availability of a calculus-based alternative. Data were collected in 10 semester-long classes over a four-year period from a total of more than 1000 students.

Structure of the modified course

The course had four major structural components. The homework, the lecture, the discussion, and the laboratory were all modified to be non-traditional in some fashion. In addition, we attempted to make all parts of the course coher-

ent with each other. We believe that the overall epistemological orientation of the class was responsible, at least in part, for the students' willingness to spend long periods working together on individual problems and for some of the behaviors we observed, such as discussing the physics qualitatively before starting to write equations. We describe here the details of the reforms that are directly relevant to the data presented. Brief descriptions of the other reforms are given in the auxiliary appendix (available online at http://linktobedetermined/aux_appendix.pdf).

Homework problems: Problems were regularly assigned and graded. The problems assigned were not traditional end-of-chapter textbook exercises. Instead, they included a mix of challenging activities including representation translation problems, context-based reasoning problems, ranking tasks, estimation problems, and essay questions with epistemological content. (For more on these types of problems see Chapter 4 of Redish's *Teaching Physics*.⁷⁷)

The instructor (Redish) expected that each problem would take the students about an hour to complete, and he communicated this expectation to the class. In accordance with his expectation, the instructor only assigned about five problems each week. (The specific problems we discuss here are given in the Appendix.) Because these problems were assigned as homework and graded, our observations of students working on these problems gave us an authentic look at how students actually behave in real-world problem-solving situations — as opposed to watching them solve problems artificially posed to them in an interview environment.

Coherence: An important characteristic of the reformed class was the attempt to make the various parts epistemologically oriented and mutually supportive. The instructor and the teaching assistants frequently cross-referenced among homework, lectures, tutorials, and laboratories. Exam questions drew from and mixed information that the students had worked on in each of the class components.

The Course Center

Since the traditional discussion sections were converted to tutorials, the students did not have time to discuss the problems on the homework set with a teaching assistant (TA) during these periods. To close this gap, a room was set up, called the *course center*, where students could gather to work on the homework problems together. The data reported on here comes from videotaped sessions of students working on homework problems in the course center.

A TA or instructor was available in the course center approximately twenty hours per week. The TA or instructor was present to offer assistance but not to explicitly solve the problems for the students, as is often the case in many traditional recitation sessions. The relevant features of this room were its architecture, the white boards, and the audio-video set up.

Architecture: Many students expect recitation sessions in which a teaching assistant stands at the front of the room and solves problems, while the students frantically copy down

the solutions. The architecture of the course center was altered to modify this expectation by removing the front of the room. All the chairs with desk arms were removed, and they were replaced with stools and five long workbenches. (See figure (7) for a schematic lay out.) This seating arrangement did not direct the attention of the students to any one location in the room — as is the case in all lecture halls in which the seating is arranged to face the 'front,' directing attention to the lecturer. The natural focus of attention of a student seated at one of these worktables is the work area in front of them and the students seated across from them.

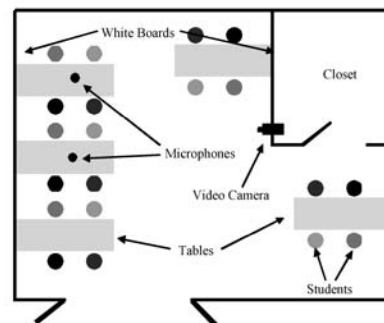


Fig. (7): Top-view of the lay-out of the course center.

Whiteboards: As a second alteration to the course center, whiteboards were mounted on the walls and the students were provided with dry erase markers. The reason for this was threefold. First, the location of the whiteboards made them difficult to reach for the TAs but easy for the students — an architectural feature that encourages the students to go to the whiteboards and discourages the TAs from solving the problem for the student or “lecturing” at the whiteboards. Second, the whiteboards facilitate group problem solving. Research on expert and novice problem solving has shown that external representations are a helpful and sometimes necessary tool in the problem-solving process.^{25,32} The whiteboards offered the students a medium to share their external representations with each other. Third, the whiteboards helped with our research agenda. The students’ shared representations on the whiteboards are visible to the video camera.

Audio-video set-up: The course center was equipped with a digital video camera. Microphones were mounted in the middle of two tables to ensure quality audio reception. The video camera was mounted about seven feet above the floor on the wall of the closet across from the tables that were equipped with microphones. The elevation of the camera had three advantages:

- Students and staff members walking by the closet did not block the camera;
- Students sitting closer to the camera did not block our view of students who sat closer to the wall; and
- We had a clear view of what the students wrote on the whiteboards.

Students were encouraged to work at the two tables that could be recorded. At the beginning of a session, the camera was pointed at an occupied table and that microphone connected to the camera. Most students were willing to work at these tables and we have strong evidence that they quickly forgot they were being recorded.

Methodology

The data for this study comes from about 60 hours of videotaped sessions of groups of students solving homework problems in the course center. Sixty hours of video is too much to be analyzed in detail, so we selected promising episodes from the full data set. We looked for episodes rich in articulated student thinking and reasoning, and ones that contained some discussion of mathematical issues (qualitative or quantitative). These selection criteria reduced the data set to about 11 hours of video that were analyzed in detail.

These 11 hours of video were transcribed and analyzed. The games were determined by a semi-phenomenographic approach.⁷⁸ We identified video segments that appeared to contain students carrying out coherent and consistent activities (whether correct or not). The authors viewed these segments multiple times and identified plausible goals, moves, and exclusions. Hypotheses for specific games were proposed. During weekly meetings of the research team (the authors plus other members of the University of Maryland Physics Education Research Group), the transcription and coding of the episodes were scrutinized and the descriptions of the proposed e-games refined. Finally, two different coders independently analyzed a sample episode in terms of epistemic games, with an inter-rater reliability of 80%. After discussion, the two codings were in complete agreement. The process resulted in the identification of the 6 games described in section 3.

5. Two Case Studies

We now present two case studies that demonstrate how an analysis in terms of resources and e-games can help make sense of student problem-solving behavior; in particular, why students often do not use what seems to the instructor to be the appropriate resources in a given context. The full transcripts of these episodes are included in the on-line appendix (http://linktobedetermined/aux_appendix.pdf).

Case 1: Building Knowledge Using E-Games

The episode for this case study involves three female students working on an electrostatics problem, which we refer to as the three-charge problem (Appendix, problem #1).⁷⁹ This episode occurs in the second week of the second semester of a two-semester introductory, algebra-based physics course. All the students in the group had been in the reformed course the first semester and were familiar with its innovative features. In particular, they were familiar with the interaction style between students and teaching assistants in the course center and with the type of homework problems that were assigned in this course. Most importantly, they

were cognizant of the fact that the instructor expected the students to spend about an hour on each homework problem – during which time they were expected to generate solutions to the questions that “made sense to them.”

An “instructor’s” solution to the three-charge problem involves straightforward balancing of forces and the use of Coulomb’s Law. The parenthetical comment in the problem states there is “no net electrostatic force” acting on charge q_3 . Symbolically, this can be written as $\vec{F}_{q_2 \rightarrow q_3} + \vec{F}_{q_1 \rightarrow q_3} = 0$. Manipulating this equation, and defining the positive \hat{i} direction to be to the right, yields:

$$\begin{aligned} \vec{F}_{q_2 \rightarrow q_3} &= -\vec{F}_{q_1 \rightarrow q_3} \\ \frac{kq_2q_3}{d^2} \hat{i} &= -\frac{kq_1q_3}{(2d)^2} \hat{i} \end{aligned} \quad (1)$$

Canceling similar terms on both sides of the equation and setting $q_2 = Q$ yields the result: $q_1 = -4Q$.

There are several inferences and steps involved in generating this solution. However, in spite of the multiple steps involved, most experienced physics teachers solve this problem in less than one minute. Some can “see” the answer in a conversational beat and give the correct answer immediately.

The most interesting aspect about the students’ approach is that it takes so long. The students work for nearly 60 minutes before arriving at a solution – two orders of magnitude longer than the typical teacher! Why does it take so long? The typical teacher has a broader mathematical knowledge base (*i.e.* a larger collection of compiled mathematical resources) and richer collection of problem-solving strategies (*i.e.* an assortment of epistemic games for solving problems in physics) than most students. For the typical teacher, the problem statement immediately cues the appropriate epistemic game and tightly compiled resources; whereas, the students’ mathematical resources do not exist in compiled form. The difference in the teacher and the students’ knowledge structure could account for the difference in the speed of the problem solution and demonstrates the power and effectiveness of cognitive compilation.⁵⁶

The students do not follow a straightforward approach to solving this problem. However, these students’ various problem-solving approaches are readily understood in terms of epistemic games. We identify five different epistemic games that are played during this problem-solving session: *Physical Mechanism*, *Pictorial Analysis*, *Mapping Mathematics to Meaning*, *Transliteration to Mathematics*, and *Mapping Meaning to Mathematics*. We divide the discussion into segments corresponding to different e-games and refer to these segments as “strips.” The names in the transcripts of the strips are gender indicative pseudonyms.

Playing the Physical Mechanism Game

The students’ initial attempt to solve this problem follows a less formal path than the instructor’s solution outlined above. Throughout this strip the students draw on intuitive reasoning primitives to explain and support their conclu-

sions. The students do not activate any formal mathematics or physics principles to support their claims. The reasoning consists almost entirely of facets. This first strip occurs about 7 minutes into the problem-solving process.

Darlene: I'm thinking that the charge q_1 must have it's...negative Q .

Alisa: We thought it would be twice as much, because it can't repel q_2 , because they're fixed. But, it's repelling in such a way that it's keeping q_3 there.

Bonnie: Yeah. It has to—

Darlene: Wait say that.

Alisa: Like— q_2 is— q_2 is pushing this way, or attracting—whichever. There's a certain force between two Q , or q_2 that's attracting.

Darlene: q_3 .

Alisa: But at the same time you have q_1 repelling q_3 .

Darlene initiates the conversation by asserting that the charge on q_1 must be “negative Q .” The negative sign in this case standing for her realization that q_1 and q_2 will have opposite effects on q_3 . Alisa elaborates on this point by articulating that q_2 exerts an influence on q_3 , which she identifies as a force, that is either repelling or attracting, and that q_1 exerts the opposite influence on q_3 . The semantic content contained in Alisa’s explanation can be summarized in the following facet: ‘the attractive effect of q_2 on q_3 cancels the repulsive effect of q_1 on q_3 .’ The abstract reasoning primitive underlying this facet is *canceling*. In this case, *canceling* is an appropriately mapped primitive, because in fact the two forces acting on q_3 do cancel, which results in there being no net electrostatic force on q_3 .

From Alisa’s initial cursory comment (“we thought [the charge on q_1] would be twice as much [as the charge on q_2]”) it appears that she has the reasoning primitives *more is more* and *balancing* activated. That is, since the two influences acting on q_3 balance, q_1 must have more charge because there is more distance between q_1 and q_3 than there is between q_2 and q_3 .

It cannot be confirmed whether Alisa has *more is more* and *balancing* activated, because the direction of the conversation changes. Darlene contends with the other students, because it appears she has activated a different reasoning primitive: *blocking*.

Darlene: How is it repelling when it's got this charge in the middle?

Alisa: Because it's still acting. Like if it's bigger, than q_2 it can still, because they're fixed. This isn't going to move to its equilibrium point. So, it could be being pushed this way.

Darlene: Oh, I see what you're saying.

Alisa: Or, pulled. You know, it could be being pulled more, but it's not moving.

Darlene: Um-huh.

The orientation of the charges cues the reasoning primitive of *blocking*, because q_2 is between q_1 and q_3 . From the superposition principle we know the effect of q_1 on q_3 does not get blocked by the presence of q_2 , so the activation of *blocking* is an unnecessary distraction. In contrast to the reasoning primitive of *canceling* that was activated earlier in this strip, *blocking* does not get mapped into a productive facet for solving this problem. (This is not to say that *blocking* is ‘wrong’; rather, in this particular instance the activation of *blocking* does not lead to a productive facet.)

Bonnie continues Alisa’s line of reasoning by explaining why the value of q_1 has to be twice as big as that of q_2 .

Alisa: So, we—we were thinking it was like negative two Q or something like that.

Bonnie: Yeah. Cause it has to be like big enough to push away.

Darlene: Push away q_3 .

Bonnie: Yeah, which we—which I figured out negative two.

Darlene: Cause it's twice the distance away than q_2 is?

Bonnie: Yeah.

Darlene: I agree with that.

It appears that Alisa draws on *overcoming* when she explains that ‘Like if it's bigger, than q_2 it can still [have an effect]’ and Bonnie restates this as ‘[q_1] has to be like big enough to push away [q_3].’ That is, q_1 has to have enough charge to overcome the influence of q_2 . The tacit conclusion from this assertion is that the charge of q_1 must have a larger magnitude than that of q_2 . This is particularly interesting since Alisa later shows (see below) that she understands Coulomb’s law and superposition. But in the context of *Physical Mechanism* she generates an (incorrect) argument in support of her sense that the force from both source charges must be included using reasoning primitives and facets. This is a clear example of the e-game she is playing limiting the responses that she considers appropriate within the context of this particular game.

Bonnie and Darlene quantify this conclusion by using the reasoning primitive *more is more* and the symbolic form *dependence* (which has the symbol template $\square = [\dots x \dots]$) to assert that the charge on q_1 has to be twice the magnitude of q_2 . *More is more* and *dependence* get mapped into the facet *twice the distance is twice the charge*. Bonnie’s argument stays within the rules of the local e-game. Because *physical mechanism* does not include moves that access formal knowledge, they do not invoke the formal knowledge that says *blocking* is irrelevant. We will see later that they (and Alisa in particular) indeed do have the relevant formal knowledge.

The students’ problem-solving activities during this entire strip have the ontology of *Physical Mechanism*. While playing this game the students draw on their intuitive knowledge base rather than their formal knowledge to support their claims. During this strip the students use various reasoning

primitives and do not mention any formal mathematics or physics principles. The epistemic form in the *Physical Mechanism Game* involves a coherent, physical description that is either verbal or imagistic. These students are actively seeking physical causes for the effects that are described in the problem.

Playing this game helps the students become oriented to this problem, but the solution to this problem necessarily involves physics equations (in particular Coulomb's Law). Since *Physical Mechanism* does not include mathematical expressions or equations, it cannot ultimately lead them to the correct answer. In the next strip, a comment from the TA helps them reframe the problem, which activates other resources they have, and cues them to play another epistemic game.

Playing the Pictorial Analysis Game

In the last strip we saw the students making sense of the problem by using their intuitive reasoning primitives in the context of *Physical Mechanism* game. At the end of the strip, the students appear to have difficulty focusing their collective attention. Although they achieve the correct conclusion (which forces are relevant to consider), a question asked by one of them causes the others to begin the discussion anew, losing the ground they had gained through the discussion.⁸⁰ This happens twice. To assist them, the TA (Tuminaro) offers a suggestion.

Darlene: I think they all have the same charge.
Bonnie: You think they all have the same charge? Then they don't repel each other.
Darlene: Huh?
Bonnie: Then they would all repel each other.
Darlene: That's what I think is happening.
Bonnie: Yeah, but q_3 is fixed. If it was being repelled—
Alisa: No, it's not. q_3 is free to move.
Bonnie: I mean, q_3 is not fixed. That's what I meant.
Darlene: Right.
Bonnie: So, like...
Darlene: So, the force of q_2 is pushing away with is only equal to d .
Bonnie: Yeah, but then...
Darlene: These two aren't moving.
Bonnie: Wouldn't this push it somewhat?
Alisa: Just because they're not moving doesn't mean they're not exerting forces.
Darlene: I know.
Alisa: What do you think?
TA: Can I make a suggestion?
Darlene: Uh-huh.
TA: You guys are talking about like a lot of forces and stuff. And, one thing I've suggested in previous semesters, if you write it down and say, what forces do you think are acting here, you can all talk about it.
Darlene: Where did the marker go?
TA: That's a suggestion—a general suggestion—that I might make.

In the first few lines above, it seems as though the students take a step back. Earlier, they appeared to have established the major aspect of the problem: two influences act on q_3 , which exactly cancel each other. In this strip, the students restate the set up of the problem (“these two are moving”) and recite remembered facts (“just because they’re not moving doesn’t mean they’re not exerting forces”). While these things are important to keep straight, this discussion does not appear to push the problem-solving process forward.

The suggestion to write on the whiteboards has two effects on the students. First, it nudges them into playing a different epistemic game, *pictorial analysis*.⁸¹ Second, the introduction of this new epistemic game and a new e-form reframes the students’ interactions and helps them focus their collective attention and clarifies their communication.

Alisa attempts to make an external representation of this problem on the white board while Bonnie and Darlene offer their assistance:

Darlene: You're trying to figure out what q_1 is, right?
Bonnie: Oh, yeah.
Alisa: Because this [q_3] is in equilibrium, there's some force...
Darlene: Pulling it that way and some force pulling equally back on it.
Bonnie: Yeah.
Alisa: And, they're equal?
Bonnie: Yes.
Darlene: Same with up and down. Not that that matters, really.
Bonnie: We'll just stick with...
Darlene: Horizontal.
Bonnie: Yeah, one dimension.

In this strip the students are deciding which features mentioned in the problem should be included in their diagram — a move within *Pictorial Analysis*. The structure of this game is similar to *Physical Mechanism*; however, the ontological components of *Physical Mechanism* and *Pictorial Analysis* are different. The epistemic form in *Pictorial Analysis* involves a coherent, physical description and an external representation; the epistemic form for *Physical Mechanism* only involves a coherent, physical description.

The external representation generated in the *Pictorial Analysis* epistemic game activates additional resources in the students, which help them better understand this problem. In particular, the students draw on the interpretive device of *physical change* to conclude that q_1 and q_2 have to have opposite charges.

Alisa: So, maybe this is pushing...
Darlene: That's [q_2] repelling and q_1 's attracting?
Bonnie: Yeah, it's just that whatever q_2 is, q_1 has to be the opposite. Right?

Alisa: Not necessarily.
 Darlene: Yeah.
 Bonnie: OK, like what if they were both positive?
 Alisa: Well, I guess you're right, they do have to be different, because if they were both positive...
 Bonnie: Then, they'd both push the same way.
 Alisa: And, if this were positive it would go zooming that way.
 Darlene: They would both push.
 Alisa: And, if this were negative it would go there.
 Bonnie: It would go zooming that way.
 Alisa: And, if they were negative...
 Darlene: It would still—they'd all go that way.
 Alisa: It would be the same thing.

Bonnie claims that the charge on q_1 has to be the opposite of q_2 , but the other students do not initially agree, despite the fact that this claim was agreed upon in the context of a different game in the previous strip. Bonnie's suggestion to verify, or falsify, her claim involves the interpretive strategy of *physical change*. That is, she considers the affect of an actual physical alteration to the system ("OK, like what if they were both positive?"). From this move the students almost immediately conclude that the charges on q_1 and q_2 must be different, or else q_3 would go 'zooming' away, since both forces would push in the same direction.

Switching to *Pictorial Analysis* turns out to be a very effective strategy for this group of students. By decomposing the forces in space and creating an external representation, they are able to physically justify why q_1 and q_2 have to have opposite charge. This strip also illustrates that the students' problem does not stem from *lack* of knowledge or skills; rather, the epistemic game the students play in their initial approach (*Physical Mechanism*) does not help them adequately articulate the physical relationship between the charges. The external representation they collectively generate in *Pictorial Analysis* cues resources they already possess (*physical change*), which helps them make progress on this problem (i.e., conclude with confidence that q_1 and q_2 have opposite charges).

Although the students' external representation and conclusion marks progress, they have yet to solve the problem. In fact, they have not even identified the necessary physics principle (*Coulomb's Law*), which is what happens in the next strip.

Playing the Mapping Mathematics to Meaning Game

So far the students have drawn a diagram representing which forces act and in what direction, and they have concluded that q_1 and q_2 have opposite charges. However, they have not yet solved the problem. In this strip we see Alisa spontaneously reframe the problem-solving process by drawing on a new set of resources: *formal mathematics knowledge*.

Alisa: Are we going to go with that? [That q_1 is twice as big as q_2 and opposite.]

Bonnie: I think it makes sense.
 Darlene: That makes...
 Alisa: Well, I don't know, because when you're covering a distance you're using it in the denominator as the square.
 Bonnie: Oh! Is that how it works?
 Alisa: And (?) makes a difference.
 Bonnie: Yeah, you're right.
 TA: So, how do you know that?
 All: From the Coulomb's Law.
 Bonnie: So, it should actually be negative four q ? Or what? Since it has...
 Alisa: Cause we were getting into problems in the beginning of the problem with [the 2D-three-charge problem, Appendix, problem #2], because I thought that like if you move this a little bit to the right the decrease for this would make up for the increase for this. But, then we decided it didn't. So, that's how I know that I don't think it would just increase it by a factor of two.

Alisa is not only attempting to introduce an equation, she is negotiating a shift in how this problem is being viewed — asking the group to play a new epistemic game. All the previous reasoning relied on intuitive reasoning primitives, without any explicit reference to physics principles or equations. Alisa's introduction of Coulomb's Law is the first mention of a physics principle during this entire problem-solving process. In addition, it's the first time anyone explicitly makes reference to an equation ("when you cover a distance you use it in the denominator as the square"). Alisa's use of formal physics principles and explicit reference to equations is (tacitly) asking the other students to play *Mapping Mathematics to Meaning*.

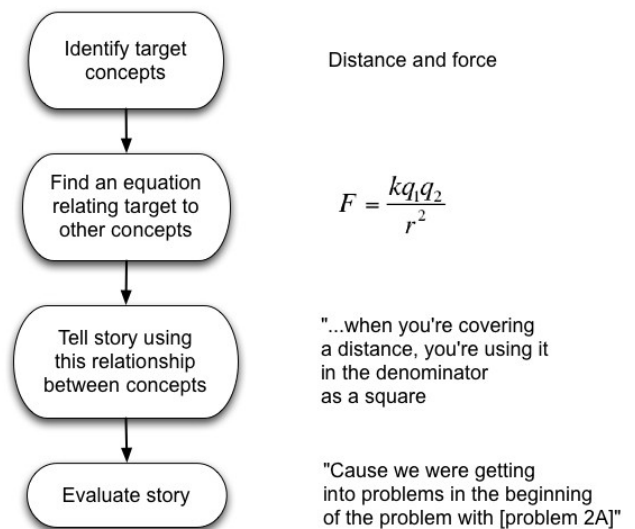


Fig. (8): Schematic map of Alisa's moves within Mapping Mathematics to Meaning

Alisa's discussion follows all the moves within *Mapping Mathematics to Meaning* (see figure (8)). First, the distance

and force are identified as the relevant concepts in this problem. Second, she identifies Coulomb's Law, $F = kq_1q_2/r^2$, as an equation that relates the target concept to other concepts. Third, she develops a story using this relationship between concepts: "When you're covering a distance you're using it in the denominator as the square." Fourth, she evaluates the validity of her story by referencing a previous problem. She acknowledges that her intuitive reasoning had failed her on the previous problem, which justifies for her the need for Coulomb's Law on this problem: "I thought that like if you move this a little bit to the right the decrease for this would make up for the increase for this. But, then we decided it didn't."

Alisa's use of Coulomb's Law is significant progress on this problem, but all the other students don't know how to apply this new piece of information. In fact, the introduction of Coulomb's Law cues Darlene to play a new (and counter-productive) epistemic game.

Playing the Transliteration to Mathematics Game

Although it appears the students are making progress on this problem, they take a detour and attempt to use another problem as a prototype for solving this problem. Alisa has suggested that Coulomb's Law is an important concept. It appears that Darlene does not initially know how to apply this new information. She attempts to find a different problem that uses Coulomb's Law in its solution, and then map the solution pattern from the other problem to the Three-Charge Problem. The problem that Darlene identifies as using Coulomb's Law in the solution is the two-charge problem (Appendix, problem #3).

Darlene: Where is that other problem? Three times as far apart as they were now what is the magnitude of the force?

Bonnie: I think it should be four times.

Darlene: If it's three times as far apart it's...you divide...uh! I think it's q over two.

Bonnie: Q over two? So, if you think of it as half the force of q_2 .

Darlene: Look at this one.

Bonnie: Is this one you're talking about?

Darlene: Uh-huh. If you increase the distance that they are from each other it's decreasing by the same amount. I thought it was four (?), but they said it was (?). I don't know why. Just three times...does it matter? I'm looking at this one. Number three, isn't that like the same thing?

Alisa: Three was an estimation problem.

Darlene: No, no with the q and four q and all that, you know how there was this question that asked when you move the charges three times further apart than they originally were, what the resulting force is.

Alisa: OK.

Darlene: And, you said it was—we said it was four—the charge would be like q , or nine, but it would get three

times as far apart. Why it's not three I don't understand, but that's all right. So—

Alisa: Well, 'cause in the equation you square this—the distance between them. Like if you're multiplying by three...

Darlene: Oh! So, I would think this one would be q over four—negative q over four. Cause it's twice as far away, opposite charge. Does that make sense?

Alisa: But, then it's a smaller charge than this.

Bonnie: Yeah.

Alisa: So, I don't understand how it would be pushing three or pulling three whatever it's doing.

In the Force-Distance Two-Charge Problem, the students had found that if the force between two charges for a given distance is F , tripling the distance results in a force between the two charges that is decreased by a factor of nine (see Appendix, problem #3), in compliance with Coulomb's Law. Darlene is attempting to match the quantities in the Three-Charge Problem with quantities from the Force-Distance Charge Problem, so the solution pattern can be transferred; *i.e.* she is playing the *Transliteration to Mathematics* epistemic game.

One piece of evidence that Darlene is playing *Transliteration to Mathematics* comes when she says, "Why it's not three I don't understand, but that's all right." Darlene is explicitly indicating that she does not understand the previous problem, but conceptual understanding is not a move in the *Transliteration to Mathematics* epistemic game. All that is important is that the problems have enough similar features that the solution from one problem can be transferred to the other.

Darlene's metacognitive statement ("Why it's not three I don't understand, but that's all right.") stands in stark contrast to Alisa's meta-cognitive statement ("I thought that like if you move this a little bit to the right the decrease for this would make up for the increase for this."). Darlene simply admits she does not understand and slavishly transfers the solution pattern from the previous problem anyway. In contrast, Alisa's metacognitive statement leads to her justification for using Coulomb's Law.

Darlene's *Transliteration to Mathematics* approach doesn't help her with the Three-Charge Problem. She says, "If you increase the distance that they are from each other it's decreasing by the same amount." The problem with Darlene's approach is that she is unaware of the two meanings that she attributes to the pronoun 'it.' In the previous problem the pronoun stands for 'force,' so that the statement would read, "If you increase the distance that they are from each other, then *the force* is decreasing by the same amount." However, Darlene tacitly maps this into the statement, "If you increase the distance that they are from each other, then *the charge* is decreasing by the same amount." The *Transliteration to Mathematics* game is not helpful in this case because force and charge are not related to distance in the same way in Coulomb's Law. The charge has to be found from the

balance of two forces. This is not to say that the *Transliteration to Mathematics* game is wrong. Rather, it does not work in this situation because of Darlene’s inappropriate mapping of force and charge. The components (ontology and structure) of *Transliteration to Mathematics* can also be found as a part of the richer *Mapping Mathematics to Meaning* game – just as is true for *Recursive Plug-and-Chug*, but the goals (perceived endstates) of the games differ.

Playing the Mapping Meaning to Mathematics Game

In this strip the students finally come to the solution of the problem. Alisa summarizes her final solution as the other students listen. Alisa’s problem-solving activities follow the Mapping Meaning to Mathematics epistemic game (see figure (9)). First, she develops a conceptual story describing the physical situation. This conceptual story relies heavily on the reasoning primitives of balancing.

TA: *What did you do there?*

Alisa: *What did I do there?*

TA: *Yeah, can I ask?*

Alisa: *All right, so because this isn't moving, the two forces that are acting on it are equal: the push and the pull.*

Alisa correctly maps ‘force’ as the two influences that balance in this physical situation. *Second*, Alisa uses the idea of *identity* (a symbolic form,⁷ with the symbol template $\square = \dots$) to translate her conceptual story into mathematical expressions:

Alisa: *So, the F—I don't know if this is the right F symbol—but, the F q_2 on q_3 is equal to this (see Equation 2). And, then the F q_1 on q_3 is equal to this (see Equation 3), because the distance is twice as much, so it would be four d squared instead of d squared.*

$$F_{q_2 \rightarrow q_3} = \frac{kQq_3}{d^2} \tag{2}$$

$$F_{q_1 \rightarrow q_3} = \frac{kxQq_3}{4d^2} \tag{3}$$

Alisa explains why she wrote the charge on q_1 as ‘ xQ ,’ by drawing on the reasoning primitive of *scaling* (another symbolic form,⁷ this one with the syntax $x\square$).

Alisa: *And, then I used xQ like or you can even do—yeah— xQ for the charge on q_1 , because we know in some way it's going to be related to Q like the big Q we just got to find the factor that relates to that.*

In the third step in the *Mapping Meaning to Mathematics*, Alisa relates the mathematical entities that she derived in step 2 with her conceptual story that she developed in step 1:

Alisa: *Then, I set them equal to each other...*

Fourth, she manipulates the mathematical expression to arrive at the desired solution:

Alisa: *... and I crossed out like the q_2 and the k and the d squared and that gave me Q equals xQ over four. And, then Q equals four Q , so x would have to be equal to four. That's how you know it's four Q .*

Fifth, the other students evaluate Alisa’s problem-solving approach and conclusion.

Bonnie: *Well, shouldn't it be—well equal and opposite, but...*

Alisa: *Yeah, you could stick the negative.*

Bonnie: *Yeah.*

Darlene: *I didn't use Coulomb's equation, I just—but it was similar to that.*

Bonnie: *That's a good way of proving it.*

Darlene: *Uh-huh.*

Bonnie: *Good explanation.*

Alisa: *Can I have my A now?*

Darlene and Bonnie accept Alisa’s approach is “a good way of proving it.” In fact, Alisa must realize that this is a good way to prove this, since she self-evaluates her solution and asks for an “A now.”

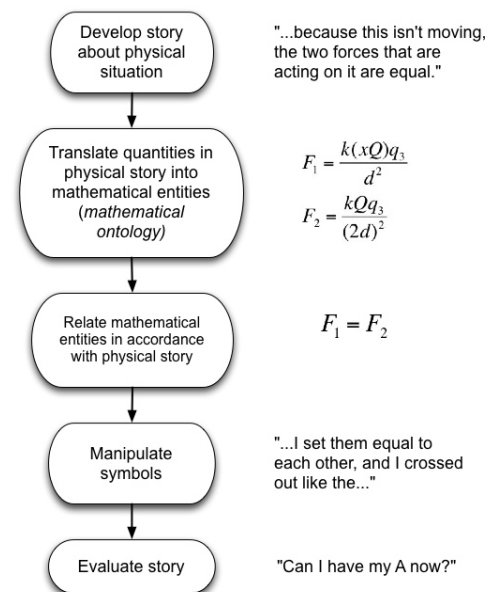


Fig. (9): Schematic map of Alisa’s moves within Mapping Meaning to Mathematics.

Case 2: Playing Recursive Plug-and-Chug

In our second case study, a group of three students is attempting to solve the Air Pressure problem (Appendix, Problem #4). The episode occurs near the end of the first semester of the class. The problem is a fairly straightforward estimation problem in which the students have to come up with numerical data on their own from their personal experience. There has been such a problem on nearly every homework assignment and on every exam throughout the semester. Nonetheless, some students bring into the class what appear

to be quite robust “misconceptions of expectations” and have considerable difficulty in getting into the spirit of estimation problems. For most of the recorded episode, one student (code named Susannah) dominates the discussion. Part way into the episode, a teaching assistant attempts to help get her on the right track.

An instructor’s solution to this problem is straightforward. The problem asks the student to estimate the pressure difference between the floor and ceiling of their dorm room. The equation that governs the pressure difference is

$$P_{\text{floor}} - P_{\text{ceiling}} = \rho gh \quad (4)$$

where ρ is the density of the air, g is the gravitational field constant (~ 10 N/kg) and h is the height of the ceiling above the floor. The value of h can be easily estimated by knowing your own height, visualizing yourself in your dorm room, and estimating how many times taller than yourself the ceiling is. A reasonable height for many rooms is 3 m. Putting this together with the density of the air (given as ~ 1 kg/m³) gives the result

$$P_{\text{floor}} - P_{\text{ceiling}} \approx (1 \text{ kg/m}^3)(10 \text{ N/kg})(3 \text{ m}) = 30 \text{ N/m}^2 = 30 \text{ P}$$

The solution appears quite straightforward – if you choose the correct equation and if you use your own experience as a source of quantitative data. The student in this episode does neither.

At the beginning of the episode, we transcribe the following interesting strip.

Martha: We're saying that the pressure...

Susannah: Right.

Martha: Well pressure's supposed to be higher at the bottom, isn't it?

Susannah: Hmm?

Martha: Pressure is supposed to be higher at the bottom.

Susannah: I think there's more at the bottom, because the thing, because the gravitation.

Martha: And, there's pressure pushing down on it.

Susannah: Um-huh.

Martha: OK.

Susannah: Pressure's equal to the radius times the moles of the gas times the temperature divided by the volume.

So, what we need to do, we know the pressure find the volume from this. Density is equal to...

Martha: Are you using pV equals nRT ?

Susannah: Huh?

Martha: Are you using pV equals nRT ?

Susannah: Yeah, or yeah.

Martha: Or.

Susannah: Or p equals R times nT ...

Martha: Over V .

Susannah: Over V .

What’s going on in this exchange can be nicely described in terms of e-games. Martha begins with a statement that proposes they play an e-game involving mechanism and a qualitative story, perhaps *physical mechanism*. We can’t say for sure, since Susannah quickly makes a move to establish a

different e-game, one that, it soon becomes clear, is *recursive plug-and-chug*. Her choice of the form of the law ($p = nRT/V$) suggests that she began with the identification of a variable – the pressure, rather than the pressure difference. And she knows an equation containing that variable – the ideal gas law.

The next strip is also remarkable.

Susannah: OK. So, if let's say it's [the density] equal to mass over volume, then [to another student] yeah. No I just found the formula to do it. So, this is equal to mass over volume, then the mass is equal to... So, basically we just found the formula that p is equal to the radius times the moles times the temperature over the volume. So, if we have the density we can find the volume.

Martha: Is R the radius?

Daphne: I don't think R is the radius.

Susannah: It's not? The radius of the...

Martha: R isn't radius. R is...

Susannah: Or, whatever R is.

Daphne: Some number.

Martha: It's not radius.

Susannah: Is it a constant?

Alice: Yeah, it's a constant. It's a constant.

Martha: It's a constant. It's...

Susannah: Awesome. One less thing for us to find.

Susannah has focused on the fact that the density is given and is trying to use that number to generate the value of other unknowns in her formula. She admits to being deeply mistaken about the meaning of one of the symbols in her equation (“*It's not? The radius of the...?*”). This doesn’t bother her; on the contrary, she is very pleased (“*Awesome. One less thing for us to find.*”). This is strong evidence that she is playing *recursive plug-and-chug*, an e-game with a strict and very limited set of allowed moves. It is not an admissible move in this game to evaluate whether the equation makes sense in terms of mechanism, and (as we see below), it is not an admissible move to get quantitative information from anywhere but an authoritative source tied to the class.

After some discussion, Susannah and Martha have decided that they need to find the volume of the dorm room. The TA is passing by and in the next strip they ask for help. He asks how far they have gotten and they explain their position. Following the guiding principles we set down for TAs, he does not tell them they have the wrong equation. Rather, he encourages them to reason it through fully. Susannah decides she knows both the mass and the volume of the dorm room.

Susannah: Well, it's one kilogram per for meters cubed, so it's kind of easy.

TA: That's the density?

Susannah: Yeah, but that's kind of giving it to us easy, right, 'cause it's...

Martha: It's saying that mass is one, one kilogram. Is that what you're saying?

Susannah: One kilogram per one meter cubed.

TA: Right, so if you lived in a room that was this big [He gestures in space describing a one meter cube.], one meter cubed there would be one kilogram of air there.

Susannah: Yeah.

TA: I don't think you live in a room that big.

Susannah: Yeah, I feel silly. OK. So, it's one kilogram...

TA: So, what um...

Martha: So, the mass is one kilogram, is what you're saying?

TA: Would you agree with me this is an estimation problem?

Susannah: Um.

Martha: Yes.

TA: OK.

Susannah: To a certain extent, yeah.

TA: What this problem is about a dorm room. How big is a dorm room?

Susannah: Oh!

Martha: Not big at all.

Susannah: He gave it in another problem. Like another homework.

In this strip, Susannah shows serious resistance to the idea that the data she is to use in a physics problem can come from her own experience.⁸² In watching the video, her facial expressions, cadence, intonation, and body language give clear clues to what she is thinking. At the beginning of the strip, she feels that she has found the answer – there is a mass and a volume given in the problem that she has overlooked: the density (of air) is specified as a mass over a volume. Since she appears to be looking for a number given in the problem and since interpretation of the number is not an allowed move in her e-game, she is satisfied with finding a mass (1 kg) and a volume (1 m³). Indeed, she comments, “it’s kind of easy.” Her later statement “I feel silly” does not appear to be in response to the TA’s statement, “I don’t think you live in a room that big.” On the contrary, she appears to be ignoring the TA and not attending to his statement at all. This is confirmed by her immediate continuation (without a pause), “so it’s 1 kilogram...” The TA, apparently realizing that Susannah is not thinking about a dorm room at all but just looking for numbers tries to get her to play the right game by saying, “Would you agree with me that this is an estimation problem?” Susannah, apparently not fully attending to the question, looks up and makes a non-committal sound. Martha quickly and firmly agrees. In response to further pressure, Susannah suggests they look back in their notes for previous information.

Susannah’s response to, “How big is a dorm room?” is “He gave it in another problem” and she starts to look in her notebook. This response makes no sense unless we accept the idea that Susannah is operating under a particular pair of local rules: all information to be used in a problem must

come from a course-related authority and it is not necessary (or even allowed) to think about what is going on in terms of mechanism or conceptual story. Susannah is stuck playing recursive plug-and-chug and even specific hints from the TA are ignored in this context.

What We Learn from These Case Studies

These two studies are reasonably typical of the 11 hours of video that we have studied of students in algebra-based physics class authentically solving physics problems. They demonstrate two points.

First, this population of students often works on problems within an invisible and often unstated web of constraining expectations we refer to as epistemic games. These expectations limit the resources these students are willing to use at a given time in the context of a given task. This limitation is in itself neither good nor bad. One cannot access all one’s knowledge at any given time. One must restrict to a set of tools appropriate to the given task.

Second, a problem does arise when a student’s perception of the tools appropriate to the task are not what we intend as instructors. If we only consider the epistemic form – a derivation, say, of an equation or result – and not the intuitive sense-making the instructor expects to go on behind it, we can be misled as to what the students are doing and the students can misinterpret what we are trying to teach. They wind up learning to play the wrong game.

The study reported in this paper focuses on students at the university level and most of them were upper division students who had already taken many science courses at the university. As a result, we make no claims concerning the origin of the games the students were playing. However, we often saw students playing games that were clearly learned somewhere. Students playing recursive plug-and-chug, for example, had a very strong sense of what they thought they were supposed to be doing — both the goals of their local activities and what they ought to be doing to get there. We can well imagine the students being taught to “identify the variable to be found,” to “find an equation containing that variable,” and so forth. This strongly suggests that we need to be aware that when we are teaching our students processes that can produce effective results in situations with a particular limited class of problems to be solved, we may also unintentionally be teaching them to play particular epistemic games without helping them to develop a good sense of when those games might (or might not) be appropriate. Such instruction might help students get through the vicissitudes of a particular course but might have unintended negative consequences at later stages in the students’ education.

6. Implications for Instruction

Far too often in physics instruction, physics teachers focus on the content and the answers to physics problems rather than on what it is they really want their students to learn — how to think about physics and how to solve problems effectively. When we choose our content as learning Newton’s or Kirchoff’s Laws, when we give students a syllabus consist-

ing of particular chapters in an encyclopedic text, and when we permit them to take a card consisting of all the “necessary equations” into an exam, we are sending an unintended message that what matters in physics are the equations and answers rather than the processes of generating and evaluating them.

Physics teachers generally know that this is not the case and may stress understanding and reasoning in their lectures. But students may not understand what this means. They may ignore the derivations and reasoning the teacher presents, box the final results in their notes for memorization, and ignore the process that generated them.

Research in physics education documents that students bring understandings of the physical world into their physics class that may contradict and confound their attempt to make sense of what is being taught. This can lead them to emphasize memorization — which in some environments works even when you have little understanding of what you are memorizing.

Physics teachers often have the sense that “problem solving is where you learn to actually do physics” — where students should learn the process and sense-making that exemplifies good physical thinking. We assign many problems and in our lectures we model the processes we want our students to follow. But until we have a good understanding of what kinds of thinking the students need to activate for solving problems, it is difficult to understand why some students seem to get so little out of solving large numbers of problems.⁸³

As experts, our knowledge exists in compiled and well-structured form, whereas the students’ knowledge does not. As instructors, we may not be aware of all the knowledge and reasoning that goes into solving a problem, if we can see the solution easily and quickly. Decomposing students’ problem-solving sessions in terms of epistemic games and resources allows us to “see” and examine the knowledge and reasoning that is involved in solving a problem in more detail. With increased understanding of the knowledge and reasoning involved in a seemingly simple problem, we can begin to develop teaching environments and interventions that more effectively and efficiently cue the appropriate resources and epistemic games. This in turn could help students become better and more efficient problem solvers.

The study reported on here gives two examples of how cognitive modeling helps increase our understanding of what our students need to learn. The specific resources and games we describe are not the only ones students use or play. Our intention is to introduce a new kind of structure for analyzing students’ thoughts on problem solving. In addition, we have studied a particular population in a particular course. Further research is needed to determine whether the games that we have observed to be common among our students occur commonly among other student populations. Although it is true that, in the end, an epistemic game lives in the head of an individual student, and, therefore, each student may possess a unique set of games, we expect that, since many stu-

dents experience similar learning environments, a limited set of games will prove useful in describing the behavior of large numbers of students.

Our focus has been on structures in the cognitive model of the individual student, but it is clear that two additional factors play essential roles and also require further research. First, the student’s decisions (tacit or conscious) about which games to play have a critical role. Second, the interaction of the students in their group games is extremely important and the structures proposed here could be of considerable help in understanding a group’s negotiation of how to approach and solve a problem. We expect that the description of student problem solving behavior in terms of epistemic games will be useful both in understanding how to teach strategies and metacognition in problem solving and in analyzing group behavior in the context of problem solving.

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Appendix

1. Three-Charge Problem

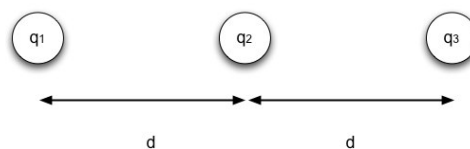


Figure (10): The figure for the three-charge problem.

In the figure above three charged particles lie on a straight line and are separated by distances d . Charges q_1 and q_2 are held fixed. Charge q_3 is free to move but happens to be in equilibrium (no net electrostatic force acts on it). If charge q_2 has the value Q , what value must the charge q_1 have?

2. 2-D Three-Charge Problem

Suppose you have a particle with a negative charge $-q$ exactly between two identical particles with equal, positive charge Q , as shown in the figure below.

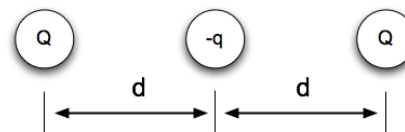


Fig. (11): The figure for the 2D three charge problem.

A. If you moved the particle in the middle a tiny bit to the right, what direction would the total force be on it by the other two charges?

B. Start with that particle back dead center again, and now move it a tiny bit up. In what direction would the total force be?

3. Two-Charge Problem⁸⁴

Two small objects each with a net charge of Q (where Q is a positive number) exert a force of magnitude F on each other. We replace one of the objects with another whose net charge is $4Q$. If we move the Q and $4Q$ charges to be 3 times as far apart as they were, what is the magnitude of the force on the $4Q$?

(a) $F/9$ (b) $F/3$ (c) $4F/9$ (d) $4F/3$ (e) other

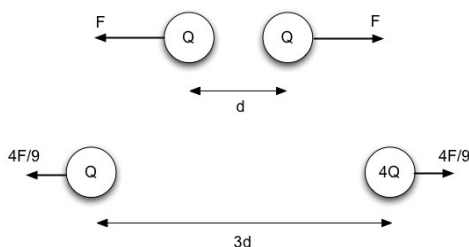


Figure (12). Figure for the force-distance two-charge problem (with the answer shown).

4. Air-Pressure Difference Problem

Estimate the difference in pressure between the floor and the ceiling in your dorm room. (Note: You may take the density of air to be about 1 kg/m^3 .)

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⁴⁵ The term "activate" here plays the role of the term more commonly used in physics education research, "elicit."

⁴⁶ E. Kandel, J. Schwartz, and T. Jessell, *Principles of Neural Science*, 4th Ed (McGraw Hill, 2000).

⁴⁷ D. O. Hebb, *The Organization of Behavior* (John Wiley & Sons, 1949).

⁴⁸ C. Kopec and R. Malinow, "Matters of Size," *Science* **314**, 1554-55 (8 December, 2006).

⁴⁹ A. Baddeley, *Human Memory: Theory and Practice, Revised Edition* (Allyn & Bacon, 1998).

⁵⁰ J. Fuster, *The Prefrontal Cortex: Anatomy, Physiology, and Neuropsychology of the Frontal Lobe*, 3rd Ed. (Lippincott, Williams, & Wilkins, 1997).

⁵¹ E. Goldberg, *The Executive Brain: Frontal Lobes and the Civilized Mind* (Oxford U. Press, 2002).

⁵² A. C. Roberts, T. W. Robbins, and L. Weiskrantz, eds., *The Prefrontal Cortex: Executive and Cognitive Functions* (Oxford U. Press, 1998).

⁵³ The metaphor here is of computer code. Code that is written in a high level language, such as FORTRAN or C, must be interpreted into a form that the computer's processor can use. The translation of code into machine language is typically referred to as compilation and results in a much faster processing than line-by-line translation (viz., interpreted vs. compiled code).

⁵⁴ A. Baddeley, *Human Memory: Theory and Practice* (Allyn & Bacon, 1998).

⁵⁵ E. F. Redish, R. E. Scherr, and J. Tuminaro, "Reverse engineering the solution of a "simple" physics problem: Why learning physics is harder than it looks," submitted to *The Physics Teacher*, 2005.

⁵⁶ A. Elby, "Helping physics students learn how to learn," *Am. J. Phys. PER Suppl.* **69:S1** (2001) S54-S64.

⁵⁷ J. Fuster, ref. 42, p. 14.

⁵⁸ J. Schwarz and B. Sherin, "Describing science content: Bridging the gap between content and process," in *Keeping Learning Complex: Proceedings of the Fifth International Conference of the Learning Sciences (ICLS)* (EuroPIA, 2002) 421-428.

⁵⁹ S. Loper and B. White, "Talking Inquiry: 'Hypothetical Narratives' in Middle-School Students' Science Classroom Discourse," interactive poster presented at NARST Conference, Dallas, TX, Session 3B, Strand 2, April 5, 2005.

⁶⁰ Indeed, all reasoning in the end may depend on such phenomenologically developed intuitions. See for example, G. Lakoff and M. Johnson, *Metaphors We Live By* (U. of Chicago Press, 1980) and L. Carroll, "What the Tortoise Said to Achilles," *Mind* **4(14)**, 278-280 (April 1895).

⁶¹ In cognitive neuroscience such phenomenological bound resources are referred to as "reflexive reasoning" (Fuster, ref. 42).

⁶² S. Dehaene, *The Number Sense: How the Mind Creates Mathematics* (Oxford U. Press, 1997).

⁶³ G. Lakoff and R. E. Núñez, *Where Mathematics Comes From* (Basic Books, 2000).

⁶⁴ E. Berne, *Games People Play: The Basic Handbook of Transactional Analysis* (Ballentine Books, 1996);.

⁶⁵ O. Morganstern and J. von Neuman, *Theory of Games and Economic Behavior* (Princeton U. Press, 1980).

⁶⁶ D. Hinsley and J. Hayes, "From words to equations: Meaning and representation in algebra word problems," in P. Carpenter and M. Just (Eds.), *Cognitive Processes in Comprehension*, (Lawrence Erlbaum, 1977) 89-106.

⁶⁷ We talk about these expectations and categorizations in terms of control structures we call *epistemological framing*. This process is discussed in a paper in preparation (Redish, Scherr, and Tuminaro) and in ref. 14.

⁶⁸ We use the terms "conceptual" and "concepts" here loosely to mean any collection of interpretive ideas about a physical quantity or mechanism.

⁶⁹ In some sense, *Pictorial Analysis* is a collection of games rather than a single game. Each distinct kind of diagram — free-

body diagram, circuit diagram, phase diagram, etc. — represents a distinct epistemic form and has its own distinct moves, conceptual resources, and endstate. Refinement of this general game into more specific games will require further research.

⁷⁰ T. Ben-Zeev, “When Erroneous Mathematical Thinking is Just as ‘Correct’: The Oxymoron of Rational Errors,” in R. Sternberg and T. Ben-Zeev (Eds.) *The Nature of Mathematical Thinking* (Lawrence Erlbaum, 1996) 55-79.

⁷¹ T. Ben-Zeev, “Rational Errors and the Mathematical Mind,” *General Review of Psychology* **2** (1998) 366-383.

⁷² *American Heritage Dictionary* (Houghton Mifflin, 2000).

⁷³ This turns out to be the case with epistemological state measured by pre-post MPEX and conceptual state measured by fractional gains on the FCI. Strong gains were obtained in both measures. These results will be documented elsewhere.

⁷⁴ D. Hestenes, M. Wells and G. Swackhamer, “Force Concept Inventory,” *Phys. Teach.* **30**, 141-158 (1992).

⁷⁵ R.K. Thornton and D.R. Sokoloff, “Assessing student learning of Newton’s laws: The Force and Motion Conceptual Evaluation,” *Am. J. Phys.* **66**(4), 228-351 (1998)

⁷⁶ E. F. Redish, J. M. Saul, and R. N. Steinberg, “Student Expectations In Introductory Physics,” *Am. J. Phys.* **66** 212-224 (1998)

⁷⁷ E. Redish, *Teaching Physics with the Physics Suite* (John Wiley & Sons, Inc., 2003).

⁷⁸ F. Marton & S. Booth, *Learning and Awareness* (Lawrence Erlbaum Associates, 1997), chapter 6.

⁷⁹ A fourth student (a male) is present during this session but he contributes little and does not speak during the selected excerpts.

⁸⁰ This illustrates the fact that these students, having only recently learned Newton’s laws and the use of forces, are not only solving the problem at hand in their discussion; they are taking steps in the process of compiling their Newtonian knowledge. See ref. 55 for more discussion of this point.

⁸¹ At the time of the instructional intervention, Tuminaro was not consciously attempting to nudge “the students into playing a different epistemic game.” It is only in the analysis, not in the actual event, that he used the concept of epistemic games to describe this episode.

⁸² At an earlier point in the discussion, she accepts the idea that the temperature needed in the gas formula should be room temperature, even though it is not given. She says, “I’m assuming it’s room temperature since it’s not specified.” This may not contradict our hypothesis since many classes in chemistry and physics take “standard temperature and pressure” as the assumed value when it is unspecified.

⁸³ E. Kim and S.-J. Pak, “Students do not overcome conceptual difficulties after solving 1000 traditional problems,” *Am. J. Phys.* **70**, 759-765 (2002).

⁸⁴ This problem is adapted from D. P. Maloney, T. L. O’Kuma, C. J. Hieggelke, and A. Van Heuvelen, “Surveying Students’ Conceptual Knowledge of Electricity and Magnetism,” *Am. J. Phys. PER Suppl.* **69**(7S), S12-S23 (2001).