

# Elements of the Theory of Functions and Functional Analysis

VOLUMES 1 AND 2  
TWO VOLUMES BOUND AS ONE

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