# Eliminating Harmonics in a Multilevel Converter using Resultant Theory 

John Chiasson, Leon Tolbert, Keith McKenzie and Zhong Du<br>ECE Department<br>The University of Tennessee<br>Knoxville, TN 37996-2100<br>chiasson@utk.edu, tolbert@utk.edu,kmc18@utk.edu, zdu1@utk.edu


#### Abstract

A method is given to determine conditions for which the switching angles in a multilevel converter can be chosen to produce the required fundamental voltage while at the same time cancel out higher order harmonics. A complete analysis is given for a 7- level converter where it is shown that for a range of the modulation index $m_{I}$, the switching angles can be chosen to produce the desired fundamental $V_{1}=m_{I}\left(s 4 V_{d c} / \pi\right)$ while making the $5^{t h}$ and $7^{\text {th }}$ harmonics identically zero.


Keywords-multilevel inverter, multilevel converter, resultants, hybrid electric vehicle, motor drive, cascade inverter

## I. Introduction

Designs for heavy duty hybrid-electric vehicles (HEVs) that have large electric drives such as tractor trailers, transfer trucks, or military vehicles will require advanced power electronic inverters to meet the high power demands ( $>100$ $\mathrm{kW})$ required of them. Development of large electric drive trains for these vehicles will result in increased fuel efficiency, lower emissions, and likely better vehicle performance (acceleration and braking).

Transformerless multilevel inverters are uniquely suited for this application because of the high VA ratings possible with these inverters [9]. The multilevel voltage source inverter's unique structure allows it to reach high voltages with low harmonics without the use of transformers or series-connected, synchronized-switching devices. The general function of the multilevel inverter is to synthesize a desired voltage from several levels of dc voltages. For this reason, multilevel inverters can easily provide the high power required of a large electric traction drive. For parallelconfigured HEVs, a cascaded H-bridges inverter can be used to drive the traction motor from a set of batteries, ultracapacitors, or fuel cells. The use of a cascade inverter also allows the HEV drive to continue to operate even with the failure of one level of the inverter structure [14][15][16].

Multilevel inverters also have several advantages with respect to hard-switched two-level pulse width modulation (PWM) adjustable-speed drives (ASDs). Motor damage and failure have been reported by industry as a result of some ASD inverters' high voltage change rates $(d V / d t)$, which produced a common-mode voltage across the motor windings. High-frequency switching can exacerbate the problem because of the numerous times this common mode voltage is impressed upon the motor each cycle. The main problems reported have been "motor bearing failure" and
"motor winding insulation breakdown" because of circulating currents, dielectric stresses, voltage surge, and corona discharge [1][4][13].

## II. Cascaded H-bridges

Cascade multilevel inverter consists of a series of H bridge (single-phase full-bridge) inverter units. The general function of this multilevel inverter is to synthesize a desired voltage from several separate dc sources (SDCSs), which may be obtained from batteries, fuel cells, or ultracapacitors in a HEV. Figure 1 shows a single-phase structure of a cascade inverter with SDCSs [9]. Each SDCS is con-


Fig. 1.
nected to a single-phase full-bridge inverter. Each inverter level can generate three different voltage outputs, $+V_{d c}, 0$ and $-V_{d c}$ by connecting the dc source to the ac output side by different combinations of the four switches, $S_{1}, S_{2}, S_{3}$ and $S_{4}$. The ac output of each level's full-bridge inverter is connected in series such that the synthesized voltage waveform is the sum of all of the individual inverter outputs. The number of output phase voltage levels in a cascade multilevel inverter is then $2 s+1$, where $s$ is the number of dc sources. An example phase voltage waveform for an 11level cascaded multilevel inverter with five $\operatorname{SDCSs}(s=5)$ and five full bridges is shown in Figure 2. The output phase voltage is given by $v_{a n}=v_{a 1}+v_{a 2}+v_{a 3}+v_{a 4}+v_{a 5}$.


Fig. 2.

With enough levels and an appropriate switching algorithm, the multilevel inverter results in an output voltage that is almost sinusoidal. For the 11 - level example shown in Figure 2, the waveform has less than 5\% THD with each of the active devices of the H -bridges active devices switching only at the fundamental frequency. Each H-bridge unit generates a quasi-square waveform by phaseshifting its positive and negative phase legs' switching timings. Each switching device always conducts for $180^{\circ}$ (or $\frac{1}{2}$ cycle) regardless of the pulse width of the quasi-square wave so that this switching method results in equalizing the current stress in each active device.

## III. Switching Algorithm for the Multilevel Converter

The Fourier series expansion of the (stepped) output voltage waveform of the multilevel inverter as shown in Figure 2 is $[14][15][16]$

$$
\begin{align*}
& V(\omega t)=\sum_{n=1,3,5, \ldots}^{\infty} \\
& \frac{4 V_{d c}}{n \pi}\left(\cos \left(n \theta_{1}\right)+\cos \left(n \theta_{2}\right)+\cdots+\cos \left(n \theta_{s}\right)\right) \sin (n \omega t) \tag{1}
\end{align*}
$$

where $s$ is the number of dc sources. Ideally, given a desired fundamental voltage $V_{1}$, one wants to determine the switching angles $\theta_{1}, \cdots, \theta_{n}$ so that (1) becomes $V(\omega t)=$ $V_{1} \sin (\omega t)$. In practice, one is left with trying to do this approximately. Two predominate methods in choosing the switching angles $\theta_{1}, \cdots \theta_{n}$ are (1) eliminate the lower frequency dominant harmonics, or (2) minimize the total harmonic distortion. The more popular and straightforward of the two techniques is the first, that is, eliminate the lower dominant harmonics and filter the output to remove the higher residual frequencies. Here, the choice is also to eliminate the lower frequency harmonics.

The goal here is to choose the switching angles $0 \leq \theta_{1}<$ $\theta_{2}<\cdots<\theta_{s} \leq \pi / 2$ so as to make the first harmonic equal
to the desired fundamental voltage $V_{1}$ and specific higher harmonics of $V(\omega t)$ equal to zero. As the application of interest here is a three-phase motor drive, the triplen harmonics in each phase need not be canceled as they automatically cancel in the line-to-line voltages. Consequently, the desire here is to cancel the $5^{t h}, 7^{t h}, 11^{t h}, 13^{\text {th }}$ order harmonics as they dominate the total harmonic distortion.

The mathematical statement of these conditions is then

$$
\begin{align*}
\frac{4 V_{d c}}{\pi}\left(\cos \left(\theta_{1}\right)+\cos \left(\theta_{2}\right)+\cdots+\cos \left(\theta_{s}\right)\right) & =V_{1} \\
\cos \left(5 \theta_{1}\right)+\cos \left(5 \theta_{2}\right)+\cdots+\cos \left(5 \theta_{s}\right) & =0 \\
\cos \left(7 \theta_{1}\right)+\cos \left(7 \theta_{2}\right)+\cdots+\cos \left(7 \theta_{s}\right) & =0  \tag{2}\\
\cos \left(11 \theta_{1}\right)+\cos \left(11 \theta_{2}\right)+\cdots+\cos \left(11 \theta_{s}\right) & =0 \\
\cos \left(13 \theta_{1}\right)+\cos \left(13 \theta_{2}\right)+\cdots+\cos \left(13 \theta_{s}\right) & =0
\end{align*}
$$

This is a system of 5 transcendental equations in the unknowns $\theta_{1}, \theta_{2}, \cdots, \theta_{s}$ so that at least 5 steps are needed $(s=5)$ if there is to be any chance of a solution. One approach to solving this set of nonlinear transcendental equations (2) is to use an iterative method such as the NewtonRaphson method [3][14][15][16]. The correct solution to the conditions (2) would mean that the output voltage of the 11 -level inverter would not contain the $5^{t h}, 7^{t h}, 11^{\text {th }}$ and $13^{\text {th }}$ order harmonic components.

The fundamental question is "When does the set of equations (2) have a solution?". As will be shown below, it turns out that a solution exists for only specific ranges of the modulation index ${ }^{1} m_{I} \triangleq V_{1} /\left(s 4 V_{d c} / \pi\right)$. This range does not include the low end or the high end of the modulation index. A method is now presented to find the solutions when they exist. This method is based on the theory of resultants of polynomials [5]. To proceed, let $s=5$, and define

$$
\begin{aligned}
x_{1} & =\cos \left(\theta_{1}\right) \\
x_{2} & =\cos \left(\theta_{2}\right) \\
x_{3} & =\cos \left(\theta_{3}\right) \\
x_{4} & =\cos \left(\theta_{4}\right) \\
x_{5} & =\cos \left(\theta_{5}\right)
\end{aligned}
$$

Using the trigonometric identities

$$
\begin{aligned}
\cos (5 \theta)= & 5 \cos (\theta)-20 \cos ^{3}(\theta)+16 \cos ^{5}(\theta) \\
\cos (7 \theta)= & -7 \cos (\theta)+56 \cos ^{3}(\theta)-112 \cos ^{5}(\theta) \\
& +64 \cos ^{7}(\theta) \\
\cos (11 \theta)= & -11 \cos (\theta)+220 \cos ^{3}(\theta)-1232 \cos ^{5}(\theta)+ \\
& 2816 \cos ^{7}(\theta)-2816 \cos ^{9}(\theta)+1024 \cos ^{11}(\theta) \\
\cos (13 \theta)= & 13 \cos (\theta)-364 \cos ^{3}(\theta)+2912 \cos ^{5}(\theta)- \\
& 9984 \cos ^{7}(\theta)+16640 \cos ^{9}(\theta)- \\
& 13312 \cos ^{11}(\theta)+4096 \cos ^{13}(\theta)
\end{aligned}
$$

[^0]the conditions (2) become
\[

$$
\begin{align*}
p_{1}(x) \triangleq & x_{1}+x_{2}+x_{3}+x_{4}+x_{5}-m=0 \\
p_{5}(x) \triangleq & \sum_{i=1}^{5}\left(5 x_{i}-20 x_{i}^{3}+16 x_{i}^{5}\right)=0 \\
p_{7}(x) \triangleq & \sum_{i=1}^{5}\left(-7 x_{i}+56 x_{i}^{3}-112 x_{i}^{5}+64 x_{i}^{7}\right)=0 \\
p_{11}(x) \triangleq & \sum_{i=1}^{5}\left(-11 x_{i}+220 x_{i}^{3}-1232 x_{i}^{5}+\right. \\
& \left.2816 x_{i}^{7}-2816 x_{i}^{9}+1024 x_{i}^{11}\right)=0 \\
p_{13}(x) \triangleq & \sum_{i=1}^{5}\left(13 x_{i}-364 x_{i}^{3}+2912 x_{i}^{5}-9984 x_{i}^{7}\right. \\
& \left.+16640 x_{i}^{9}-13312 x_{i}^{11}+4096 x_{i}^{13}\right)=0 \tag{3}
\end{align*}
$$
\]

where $x=\left(x_{1}, x_{2}, x_{3}, x_{4}, x_{5}\right)$ and $m \triangleq V_{1} /\left(4 V_{d c} / \pi\right)$. This is a set of five equations in the five unknowns $x_{1}, x_{2}, x_{3}, x_{4}, x_{5}$. The interest here is to find solutions $x$ for $m \in[0, s]$ which satisfy $0 \leq x_{5}<\cdots<x_{2}<x_{1} \leq 1$. This development has resulted in a set of polynomial equations rather than trigonometric equations. Though the degree is high, the theory of resultants of polynomials [5] provides a systematic way to determine all the zeros of the set of polynomials (3).

## A. Seven Level Case

To illustrate the procedure of using the theory of resultants to solve the system (3), the seven level case is considered. The conditions are

$$
\begin{align*}
& p_{1}(x) \triangleq x_{1}+x_{2}+x_{3}-m=0, \quad m \triangleq \frac{V_{1}}{4 V_{d c} / \pi}=s m_{I} \\
& p_{5}(x) \triangleq \sum_{i=1}^{3}\left(5 x_{i}-20 x_{i}^{3}+16 x_{i}^{5}\right)=0  \tag{4}\\
& p_{7}(x) \triangleq \sum_{i=1}^{3}\left(-7 x_{i}+56 x_{i}^{3}-112 x_{i}^{5}+64 x_{i}^{7}\right)=0
\end{align*}
$$

Substitute $x_{3}=m-\left(x_{1}+x_{2}\right)$ into $p_{5}, p_{7}$ to get

$$
\begin{aligned}
p_{5}\left(x_{1}, x_{2}\right)= & 5 x_{1}-20 x_{1}^{3}+16 x_{1}^{5}+5 x_{2}-20 x_{2}^{2}+16 x_{2}^{5} \\
& +5\left(m-x_{1}-x_{2}\right)-20\left(m-x_{1}-x_{2}\right)^{3} \\
& +16\left(m-x_{1}-x_{2}\right)^{5} \\
p_{7}\left(x_{1}, x_{2}\right)= & -7 x_{1}+56 x_{1}^{3}-112 x_{1}^{5}+64 x_{1}^{7}-7 x_{2} \\
& +56 x_{2}^{3}-112 x_{2}^{5}+64 x_{2}^{7}-7\left(m-x_{1}-x_{2}\right) \\
& +56\left(m-x_{1}-x_{2}\right)^{3}-112\left(m-x_{1}-x_{2}\right)^{5} \\
& +64\left(m-x_{1}-x_{2}\right)^{7}
\end{aligned}
$$

The goal here is to find solutions of

$$
\begin{aligned}
& p_{5}\left(x_{1}, x_{2}\right)=0 \\
& p_{7}\left(x_{1}, x_{2}\right)=0
\end{aligned}
$$

For each fixed $x_{1}, p_{5}\left(x_{1}, x_{2}\right)$ can be viewed as a polynomial of (at most) degree 5 in $x_{2}$ whose coefficients are polynomials of (at most) degree 5 in $x_{1}$. For example ${ }^{2}$,

$$
\begin{aligned}
p_{5}\left(x_{1}, x_{2}\right)= & 5 m-20 m^{3}+16 m^{5}+60 m^{2} x_{1}-80 m^{4} x_{1} \\
& -60 m x_{1}^{2}+160 m^{3} x_{1}^{2}-160 m^{2} x_{1}^{3}+80 m x_{1}^{4} \\
& +\left[60 m^{2}-80 m^{4}-120 m x_{1}+320 m^{3} x_{1}\right. \\
& \left.+60 x_{1}^{2}-480 m^{2} x_{1}^{2}+320 m x_{1}^{3}-80 x_{1}^{4}\right] x_{2} \\
& +\left[-60 m+160 m^{3}+60 x_{1}\right. \\
& \left.-480 m^{2} x_{1}+480 m x_{1}^{2}-160 x_{1}^{3}\right] x_{2}^{2} \\
& +\left[-160 m^{2}+320 m x_{1}-160 x_{1}^{2}\right] x_{2}^{3} \\
& +\left[80 m-80 x_{1}\right] x_{2}^{4}
\end{aligned}
$$

This is often written as $p_{5}\left(x_{1}, x_{2}\right) \in \Re\left[x_{1}\right]\left(x_{2}\right)$ to emphasize that $p_{5}$ is being viewed as a polynomial in $x_{2}$ whose coefficients are in the ring of polynomials $\Re\left[x_{1}\right]$. Similarly, $p_{7}\left(x_{1}, x_{2}\right) \in \Re\left[x_{1}\right]\left(x_{2}\right)$ is a polynomial of degree 7 in $x_{2}$ whose coefficients are polynomials of (at most) degree 7 in $x_{1}$.

A pair $\left(x_{10}, x_{20}\right)$ is a simultaneous solution of $p_{5}\left(x_{10}, x_{20}\right)=0, p_{7}\left(x_{10}, x_{20}\right)=0$, if and only if the corresponding resultant polynomial $r_{5,7}\left(x_{10}\right)=0$. (The reader is referred to [5] for an explanation of resultants and their computation.) Consequently, finding the roots of the resultant polynomial $r_{5,7}\left(x_{1}\right)=0$ gives candidate solutions for $x_{1}$ to check for common zeros of $p_{5}=p_{7}=$ 0 . Here, the resultant polynomial $r_{5,7}\left(x_{1}\right)$ of the pair $\left\{p_{5}\left(x_{1}, x_{2}\right), p_{7}\left(x_{1}, x_{2}\right)\right\}$ was found with Mathematica ${ }^{\circledR}$ using the Resultant command. The polynomial $r_{5,7}\left(x_{2}\right)$ turned out to be a $22^{n d}$ order polynomial. The algorithm is as follows:

## Algorithm for the 7 Level Case

1. Given $m$, find the roots of $r_{5,7}\left(x_{1}\right)=0$.
2. Discard any roots that are less than zero, greater than 1 or that are complex. Denote the remaining roots as $\left\{x_{1 i}\right\}$. 3. For each fixed zero $x_{1 i}$ in the set $\left\{x_{1 i}\right\}$, substitute it into $p_{5}$ and solve for the roots of $p_{5}\left(x_{1 i}, x_{2}\right)=0$.
3. Discard any roots (in $x_{2}$ ) that are complex, less than zero or greater than one. Denote the pairs of remaining roots as $\left\{\left(x_{1 j}, x_{2 j}\right)\right\}$.
4. Compute $m-x_{1 j}-x_{2 j}$ and discard any pair $\left(x_{1 j}, x_{2 j}\right)$ that makes this quantity negative or greater than one. Denote the triples of remaining roots as $\left\{\left(x_{1 k}, x_{2 k}, x_{3 k}\right)\right\}$.
5. Discard any triple for which $x_{3 k}<x_{2 k}<x_{1 k}$ does not hold. Denote the remaining triples as $\left\{\left(x_{1 l}, x_{2 l}, x_{3 l}\right)\right\}$. The switching angles that are a solution to the three level system (4) are

$$
\left\{\left(\theta_{1 l}, \theta_{2 l}, \theta_{3 l}\right)\right\}=\left\{\left(\cos ^{-1}\left(x_{1 l}\right), \cos ^{-1}\left(x_{2 l}\right), \cos ^{-1}\left(x_{3 l}\right)\right)\right\}
$$

A. 1 Minimization of the $5^{t h}$ and $7^{t h}$ Harmonic Components

For those values of $m$ for which $p_{5}\left(x_{1}, x_{2}\right), p_{7}\left(x_{1}, x_{2}\right)$ do not have common zeros satisfying $0 \leq x_{1} \leq 1,0 \leq x_{2} \leq 1$,

[^1]the next best thing is to minimize the error
$$
c\left(x_{1}, x_{2}\right)=p_{5}^{2}\left(x_{1}, x_{2}\right) / 25+p_{7}^{2}\left(x_{1}, x_{2}\right) / 49
$$

This was accomplished by simply computing the values of $c(j \Delta x, k \Delta y)$ for $j, k=0,1,2, \ldots, 1000$ with $\Delta x=$ $.001, \Delta y=.001$ and then choosing the minimum value.

## A. 2 Results for the 7 Level Inverter

The results are summarized in Figures 3, 4 and 5. These three figures show the switching angles $\theta_{1}, \theta_{2}, \theta_{3}$ vs. $m$ for those values of $m$ in which the system (4) has a solution. Note that for $m$ in the range from approximately 1.49 to 1.85, there are two different sets of solutions that solve (4). On the other hand, for $m \in[0,0.8], m \in[0.83,1.15]$ and $m \in[2.52,2.77]$ there are no solutions to (4). Interestingly, for $m \approx 0.8, m \approx 0.82$ and $m \approx 2.76$ there are (isolated) solutions.


Fig. 3. $\theta_{1}$ vs $m$


Fig. 4. $\theta_{2}$ vs $m$


Fig. 5. $\theta_{3}$ vs $m$

As pointed out above, for $m \in[0,0.8], m \in[0.83,1.15]$, $m \in[2.52,2.77]$ and $m \in[2.78,3]$ there are no solutions satisfying the conditions (4). Consequently, for these ranges of $m$, the switching angles were determined by minimizing $\sqrt{\left(p_{5} / 5\right)^{2}+\left(p_{7} / 7\right)^{2}}$. Figure 6 shows a plot of the resulting minimum error $\sqrt{\left(p_{5} / 5\right)^{2}+\left(p_{7} / 7\right)^{2}}$ vs. $m$ for these values of $m$. As Figure 6 shows, when $m \approx 0.81$ and $m \approx 2.76$, the error is zero corresponding to the isolated solutions to (4) for those values of $m$. For $m=1.15$ and $m=2.52$, the error goes to zero because these values correspond to the boundary of the exact solutions of (4). However, note, e.g., when $m=0.25$, the error is about 0.25 , that is, the error is the same size as $m$. Other than close to the endpoints of the two intervals $[0,0.8],[2.78,3]$ the minimum error $\sqrt{\left(p_{5} / 5\right)^{2}+\left(p_{7} / 7\right)^{2}}$ is too large to make the corresponding switching angles for this interval of any use. Consequently, for $m$ in this interval, one must use some other approach (e.g., PWM) in order to get reduced harmonics. For the other two intervals [0.83, 1.15], [2.52, 2.77], the minimum error $\sqrt{\left(p_{5} / 5\right)^{2}+\left(p_{7} / 7\right)^{2}}$ is around $5 \%$ or less so that it might be satisfactory to use the corresponding switching angles for these intervals.

## IV. Experimental Work

A prototype three-phase 11-level wye-connected cascaded inverter has been built using $100 \mathrm{~V}, 70$ A MOSFETs as the switching devices [19]. A battery bank of 15 SDCSs of 48 Volts DC each feed the inverter (5 SDCSs per phase). In the experimental study here, this prototype system was configured to be a 7 -level ( 3 SDCSs per phase) converter with each level being 12 Volts. A 50 pin ribbon cable provides the communication link between the gate driver board and the real-time processor. In this work, the OpalRT ${ }^{\circledR}$ real-time computing platform [8] was used to interface the computer (which generates the logic signals) to this cable. The OpalRT ${ }^{\circledR}$ system allows one to write


Fig. 6. Error $=\sqrt{\left(p_{5} / 5\right)^{2}+\left(p_{7} / 7\right)^{2}}$ vs. $m$
the switching algorithm in Simulink ${ }^{\circledR}$ which is then converted to $C$ code using RTW ${ }^{\circledR}$. The OpalRT ${ }^{\circledR}$ software provides icons to interface the Simulink ${ }^{\circledR}$ model to the digital I/O board and converts the $C$ code into executables. Using the XHP ${ }^{\circledR}$ (extra high performance) option in OpalRT ${ }^{\circledR}$ as well as the multiprocessor option to spread the computation between two processors, an execution time of 16 microseconds was achieved.

Experiments were performed to validate the theoretical results of section III-A.2. Due to space limitations, only data for $m=0.5$ and 2 are presented. The first value $m=$ 0.5 corresponds to the case where the $5^{t h}$ and $7^{t h}$ harmonics cannot be eliminated while the second value $m=2$ is a case in which these harmonics can be eliminated. In this set of data, the angles were chosen by taking $\theta_{1}, \theta_{2}$ according to the upper curves in Figures 3 and 4, respectively and the corresponding $\theta_{3}$ from the lower curve in Figure 5. The frequency was set to 60 Hz in each case and the program was run in real time with a 16 microseconds sample period, i.e., the logic signals were updated to the gate driver board every 16 microseconds.

The voltage was measured using a high speed data acquisition oscilloscope every $T=5$ microseconds resulting in the data $\{v(n T), n=1, \ldots, N\}$ where $N=$ $3(1 / 60) /\left(5 \times 10^{-6}\right)=10000$ samples corresponding to three periods of the 60 Hz waveform. A fast Fourier transform was performed on this voltage data to get $\left\{\hat{v}\left(k \omega_{0}\right), k=1, \ldots, N\right\}$ where the frequency increment is $\omega_{0}=(2 \pi / T) / N=2 \pi(20) \mathrm{rad} / \mathrm{sec}$ or 20 Hz . The number $\hat{v}\left(k \omega_{0}\right)$ is simply the Fourier coefficient of the $k^{t h}$ harmonic (whose frequency is $k \omega_{0}$ with $\omega_{0}=\frac{2 \pi}{N} \frac{1}{T}$ ) in the Fourier series expansion of the phase voltage signal $v(t)$. With $a_{k}=\left|\hat{v}\left(k \omega_{0}\right)\right|$ and $a_{\text {max }}=\max _{k}\left\{\left|\hat{v}\left(k \omega_{0}\right)\right|\right\}$, the data that is plotted is the normalized magnitude $a_{k} / a_{\text {max }}$.

Figure 7 is the plot of the phase voltage for $m=0.5$ and the corresponding FFT of this signal is given in Figure 8 . Figure 8 show a 0.225 normalized magnitude
of the $5^{t h}$ harmonic and a 0.15 normalized magnitude of the $7^{\text {th }}$ harmonic for a total normalized distortion of $\sqrt{(0.225)^{2}+(0.15)^{2}}=0.27$ due to these two harmonics. Figure 6 shows an error of about 0.125 at $m=0.5$ for a normalized magnitude of $0.125 / 0.5=0.25$ because of these two harmonics, which is in close agreement.


Fig. 7. Phase voltage when $m=0.5$


Fig. 8. Normalized FFT $a_{k} / a_{\max }$ vs frequency for $m=0.5$
Figure 9 is the plot of the phase voltage for $m=2$. The corresponding FFT of this signal is given in Figure 10. Figure 10 shows $5^{\text {th }}$ and $7^{\text {th }}$ harmonics are zero as predicted in Figure 6.

## V. Conclusions and Further Work

A full solution to the problem eliminating the $5^{t h}$ and $7^{\text {th }}$ harmonics in a seven level multilevel inverter has been given. Specifically, resultant theory was used to completely


Fig. 9. Phase voltage when $m=2$


Fig. 10. Normalized FFT $a_{k} / a_{\max }$ vs frequency for $m=2$
characterize for each $m$ when a solution existed and when it did not (in contrast to numerical techniques such as Newton-Raphson). Futher, it was shown that for a range of values of $m$, there were two sets of solutions and these values were also completely characterized. The solution set that happened to minimize the $11^{\text {th }}$ and $13^{\text {th }}$ harmonics was chosen. Experimental results were also presented and corresponded well to the theoretically predicted results. Further work is now underway to consider the case studied by Cunnyngham [3] where the separate dc sources do not all provide equal voltages $V_{d c}$.

## VI. Acknowledgements

Dr. Tolbert would like to thank the National Science Foundation for partially supporting this work through contract NSF ECS-0093884 and both Drs. Chiasson and Tol-
bert would like to thank Oak Ridge National Laboratory for partially supporting this work through the UT/Battelle contract no. 4000007596.

## References

[1] Bonnett, A. H. "A comparison between insulation systems available for PWM-Inverter-Fed Motors," IEEE Trans. Industry Applications, vol. 33, no. 5, pp. 1331-1341, Sep./Oct. 1997.
[2] Chen, C.T., Linear Systems Theory and Design, Third Edition, Oxford Press, 1999.
[3] Cunnyngham, Tim, Cascade multilevel inverters for large hybrid-electric vehicle applications with variant dc sources, M.S. Thesis, University of Tennessee, 2001.
[4] Erdman, J., R. Kerkman, D. Schlegel, G. Skibinski, "Effect of PWM inverters on AC motor bearing currents and shaft voltages," IEEE Trans. Industry Applications, vol. 32, no. 2, pp. 250-259, Mar./Apr. 1996.
[5] Kailath, T., Linear Systems, Prentice-Hall, 1980.
[6] Klabunde, M., Y. Zhao, T. A. Lipo, "Current control of a 3 level rectifier/inverter drive system," Conf. Rec. 1994 IEEE IAS Annual Meeting, pp. 2348-2356, Oct. 1994.
[7] Sinha,G. and T. A. Lipo, "A four level rectifier-inverter system for drive applications," Conf. Rec. 1996 IEEE IAS Annual Meeting, pp. 980-987, Oct. 1996.
[8] Opal-RT Technologies, Inc., See http://www.opal-rt.com/
[9] Lai, J.S., F. Z. Peng, "Multilevel converters - a new breed of power converters," IEEE Trans. Industry Applications, vol. 32, no. 3, pp. 509-517, May/June 1996.
[10] Peng, F.Z., J. S. Lai, J. W. McKeever, J. VanCoevering, "A multilevel voltage-source inverter with separate dc sources for static var generation," IEEE Trans. Industry Applications, vol. 32, no. 5, pp. 1130-1138, Sept. 1996.
[11] Peng, F.Z. and J. S. Lai, "Dynamic performance and control of a static var generator using cascade multilevel inverters," IEEE Trans. Industry Applications, vol. 33, no. 3, pp. 748-755, May 1997.
[12] Steinke, K. "Control strategy for a three phase ac traction drive with three level GTO PWM inverter," 1988 IEEE PESC, pp. 431-438.
[13] Bell, S. and J. Sung, "Will your motor insulation survive a new adjustable frequency drive?", IEEE Trans. Industry Applications, vol. 33, no. 5, pp. 1307-1311, Sept. / Oct. 1997.
[14] Tolbert, L.M., F. Z. Peng, T. G. Habetler, "Multilevel Converters for Large Electric Drives," IEEE Trans. Industry Applications, vol. 35, no. 1, pp. 36-44, Jan./Feb. 1999.
[15] Tolbert, L.M., T. G. Habetler, "Novel Multilevel Inverter Carrier-Based PWM Methods," IEEE Transactions on Industry Applications, vol. 35, no. 5, Sept./Oct. 1999, pp. 1098-1107.
[16] Tolbert, L.M., F. Z. Peng, T. G. Habetler, "Multilevel PWM Methods at Low Modulation Indexes," IEEE Transactions on Power Electronics, vol. 15, no. 4, July 2000, pp. 719-725.
[17] Tolbert, L.M. and Fang Z. Peng, "Multilevel Converters as a Utility Interface for Renewable Energy Systems," IEEE Power Engineering Society Summer Meeting, July 15-20, 2000, Seattle, Washington, pp. 1271-1274.
[18] Tolbert, L. M., F. Z. Peng, T. G. Habetler, "A Multilevel Converter-Based Universal Power Conditioner," IEEE Transactions on Industry Applications, vol. 36, no. 2, pp. 596-603, March/April 2000.
[19] Tolbert, L.M., F. Z. Peng, T. Cunnyngham, and J. Chiasson, "Charge Balance Control Schemes for Cascade Multilevel Converter in Hybrid Electric Vehicles" to appear in IEEE Transactions on Industrial Electronics, Oct 2002.
[20] Zhang, J., "High performance control of a three level IGBT inverter fed ac drive," Conf. Rec. 1995 IEEE IAS Annual Meeting, pp. 22-28.


[^0]:    ${ }^{1}$ Each inverter has a dc source of $V_{d c}$ so that the maximum output voltage of the multilevel inverter is $s V_{d c}$. A square wave of amplitude $s V_{d c}$ results in the maximum fundamental output possible of $V_{1 \max }=4 s V_{d c} / \pi$. The modulation index is therefore $m_{I} \triangleq V_{1} / V_{1 \max }=V_{1} /\left(s 4 V_{d c} / \pi\right)$.

[^1]:    ${ }^{2}$ In this case, it turns out that the coefficient of the $x_{2}^{5}$ is zero so that $p_{5}\left(x_{1}, x_{2}\right)$ has degree 4 in $x_{2}$.

