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Elimination of nonlinear polarization rotation in twisted fibers

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It has been previously reported that it is possible to eliminate nonlinear polarization rotation in elliptically birefringent optical fibers. We show that the same can hold true in twisted optical fibers, even though the local eigenmodes are linearly polarized, if the twist length becomes equal to $\sqrt{2}$ times the beat length. The physical requirements are that the beat length must be short compared with the walk-off length, the dispersive scale length, and the nonlinear scale length. When the twist length becomes comparable with the beat length, the twisting leads to a linear coupling between the two polarizations through their time derivatives.

In Ref. 1 Menyuk showed that the equations that govern light-pulse evolution in a birefringent Kerr medium may be written as

$$i\,\frac{\partial u}{\partial \xi} + i\delta\,\frac{\partial u}{\partial s} + \frac{1}{2}\,\frac{\partial^2 u}{\partial s^2} + (|u|^2 + B|v|^2)u = 0\,,$$

$$i\,\frac{\partial u}{\partial \xi} - i\delta\,\frac{\partial v}{\partial s} + \frac{1}{2}\,\frac{\partial^2 v}{\partial s^2} + (B|u|^2 + |v|^2)v = 0\,, \qquad (1)$$

where u and v are the normalized envelopes of the two polarizations, ξ and s are appropriately normalized distance and time, δ is proportional to the birefringence strength, and B depends on the ellipticity of the birefringent medium. In the derivation of Eqs. (1) it is assumed that the birefringence is sufficiently large that terms that are rapidly varying because of a mismatch between the wave numbers of the two polarizations can be neglected. This assumption is always valid in present-day optical fibers when solitons are considered.[2] It is not always valid when quasi-cw pulses are considered,[3] but it will be valid when the birefringence is large, as is the case in many important applications.[4] The coefficient B is given by the formula

$$B = \frac{2a + 2b\sin^2\theta}{2a + b\cos^2\theta}\,, \qquad (2)$$

where θ is the angle of ellipticity of the birefringent medium and a and b are the Kerr coefficients.[1,5] We must be able to neglect contributions of the time derivatives of the electric field to the nonlinear portion of the polarization field in order for Eqs. (1) to be valid. The most important effect of this sort in optical fibers is the Raman effect,[6] which limits the validity of Eqs. (1) to pulse durations that are at least several hundred femtoseconds long. For pulses of this duration and longer we may treat the Kerr effect as instantaneous, so that $a = b$.

Menyuk[1] pointed out previously that when $\theta = \tan^{-1}(1/\sqrt{2}) \simeq 35°$, $B = 1$ and Eqs. (1) become the Manakov equation. Under these circumstances solitons will no longer generate shadows when they collide, which may be important for certain proposed switches.[7] Of even greater importance is that nonlinear polarization rotation can be eliminated. This effect is a nuisance in sensor applications, and its elimination would be helpful. In principle one could make an elliptically birefringent fiber by elliptically deforming the core and subjecting it to helical stresses. It would be very difficult to make such a fiber, and to date it has not been done.[4,8] A more promising route is to twist a fiber that is linearly birefringent because of an elliptically deformed core. Under these circumstances the local eigenmodes are still linearly polarized, but one can find global "eigenmodes" that are elliptically polarized in a frame that rotates along with the fiber's axes of birefringence. (We have written the word "eigenmodes" in quotes because, strictly speaking, there are no eigenmodes since the fiber is inhomogeneous; i.e., its properties change along the axis of propagation. Nonetheless, when measured in the rotating frame, these "eigenmodes" are unchanging, and the distinction between these "eigenmodes" and true eigenmodes is of no concern to us.) It is not altogether obvious that Eqs. (1) remain valid in a twisted fiber. Even if Eqs. (1) remain valid, it is not obvious what the coefficient B should be. Is it governed by the local or by the global eigenmodes?

We show in this Communication that under the right physical conditions, which require a sufficiently large birefringence, Eqs. (1) remain valid and B is governed by the global eigenmodes, so that it is possible to obtain $B = 1$ fibers and eliminate nonlinear polarization rotation. A similar issue has been briefly addressed in the different context of counterpropagating cw beams, in which the effects of dispersion can be ignored.[9] The assumption that the birefringence is big enough to allow us to neglect

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terms whose wave numbers are mismatched plays an even larger role in the derivation of Eqs. (1) in twisted fiber than it does in homogeneous fiber, considered in Ref. 1. In homogeneous fiber this assumption leads to the neglect of several nonlinear terms. In twisted fiber we find that there is an additional linear coupling between the time derivatives of the two polarizations, which can be neglected only when this assumption is valid.

Our starting point, as in Ref. 1, is Maxwell's equation, which in the plane-wave approximation may be written as

$$\frac{\partial^2 \mathbf{E}}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 \mathbf{D}}{\partial t^2} = 0, \tag{3}$$

where $\mathbf{E}$ is the electric field, $\mathbf{D}$ is the electric displacement, c is the speed of light, z is the distance along the fiber axis, and t is time. In the plane-wave approximation $\mathbf{E}$ and $\mathbf{D}$ are two-dimensional vectors, oriented in a direction transverse to the propagation axis. Use of the plane-wave approximation drastically reduces the amount of algebra required without changing the result in any significant way.[1]

We begin by determining the relationship between $\mathbf{D}$ and $\mathbf{E}$, neglecting the nonlinear contribution, which we will take into account later. We recall that

$$\mathbf{D} = \mathbf{E} + 4\pi \mathbf{P}, \tag{4}$$

where

$$\mathbf{P}(z,t) = \int_{-\infty}^{t} \chi(t-t') \cdot \mathbf{E}(z,t')\mathrm{d}t' \tag{5}$$

is the polarization field and χ is the polarization tensor. Making the slowly varying approximation, we first separate $\mathbf{E}$, $\mathbf{P}$, and $\mathbf{D}$ into positive and negative frequency portions: $\mathbf{E} = \mathbf{E}^+ + \mathbf{E}^-$, $\mathbf{P} = \mathbf{P}^+ + \mathbf{P}^-$, and $\mathbf{D} = \mathbf{D}^+ + \mathbf{D}^-$, where the plus superscripts refer to the positive frequency portions and the minus superscripts refer to the negative frequency portions. The positive and the negative frequency portions are complex conjugates of each other, which guarantees the reality of the total field.[1] We next write

$$\begin{aligned}
\mathbf{E}^+(z,t) &= \mathbf{U}(z,t)\exp(ik_0 z - i\omega_0 t),\\
\mathbf{P}^+(z,t) &= \mathbf{\Pi}(z,t)\exp(ik_0 z - i\omega_0 t),\\
\mathbf{D}^+(z,t) &= \mathbf{\Delta}(z,t)\exp(ik_0 z - i\omega_0 t),
\end{aligned} \tag{6}$$

where $\mathbf{U}$, $\mathbf{\Pi}$, and $\mathbf{\Delta}$ are all assumed to vary slowly. If one defines the Fourier transform

$$\tilde{X}(z,\omega) = \int_{-\infty}^{\infty} X(z,t)\exp(i\omega t)\mathrm{d}t, \tag{7}$$

where $X(z,t)$ is any quantity, then one finds

$$\mathbf{\Pi}(z,t) = \tilde{\chi}(\omega_0)\cdot \mathbf{U}(z,t) + i\tilde{\chi}'(\omega_0)\cdot \frac{\partial \mathbf{U}(z,t)}{\partial t} - \frac{1}{2}\tilde{\chi}''(\omega_0)\cdot \frac{\partial^2 \mathbf{U}(z,t)}{\partial t^2}, \tag{8}$$

where $\tilde{\chi}'(\omega^0) \equiv \partial\tilde{\chi}/\partial\omega|_{\omega_0}$ and $\tilde{\chi}''(\omega^0) \equiv \partial^2\tilde{\chi}/\partial\omega^2|_{\omega_0}$. To obtain Eq. (8), we expand $\chi(\omega)$ in a Taylor series and truncate the series after the third term.[1] For this truncation to make sense physically, we must assume that the spectral width of the field $\mathbf{E}^+$ is small compared with ω_0 or, equivalently, that $\mathbf{U}$ is slowly varying. This approximation is just the usual slowly varying envelope approximation. Letting $\tilde{\epsilon}(\omega) = \mathsf{I} + 4\pi\tilde{\chi}(\omega)$, we obtain

$$\mathbf{\Delta}(z,t) = \tilde{\epsilon}(\omega_0)\cdot \mathbf{U}(z,t) + i\tilde{\epsilon}'(\omega_0)\cdot \frac{\partial \mathbf{U}(z,t)}{\partial t} - \frac{1}{2}\tilde{\epsilon}''(\omega_0)\cdot \frac{\partial^2 \mathbf{U}(z,t)}{\partial t^2}, \tag{9}$$

where $\tilde{\epsilon}'(\omega) = 4\pi\tilde{\chi}'(\omega)$ and $\tilde{\epsilon}''(\omega) = 4\pi\tilde{\chi}''(\omega)$. From Eqs. (6) and (9), we obtain

$$\frac{1}{c^2}\frac{\partial^2 \mathbf{D}^+}{\partial t^2} = \left(-\frac{\omega_0{}^2}{c^2}\tilde{\epsilon}\cdot\mathbf{U} - 2ik_0\boldsymbol{\beta}\cdot\frac{\partial \mathbf{U}}{\partial t} + k_0\boldsymbol{\gamma}\cdot\frac{\partial^2\mathbf{U}}{\partial t^2}\right) \times \exp(ik_0 z - i\omega_0 t), \tag{10}$$

where

$$\begin{aligned}
\boldsymbol{\beta} &= \frac{1}{2k_0}\left(\frac{\omega_0{}^2}{c^2}\tilde{\epsilon}' + 2\frac{\omega_0}{c^2}\tilde{\epsilon}\right),\\
\boldsymbol{\gamma} &= \frac{1}{k_0}\left(\frac{\omega_0{}^2}{2c^2}\tilde{\epsilon}'' + 2\frac{\omega_0}{c^2}\tilde{\epsilon}' + \frac{1}{c^2}\tilde{\epsilon}\right).
\end{aligned} \tag{11}$$

We assume that the local eigenmodes of the optical fiber are linearly polarized and rotate as a function of z, so that $\tilde{\epsilon}(\omega)$ has two orthogonal eigenvectors, $\hat{\mathbf{e}}_1(z) = \hat{\mathbf{e}}_x\cos(\Omega z) + \hat{\mathbf{e}}_y\sin(\Omega z)$ and $\hat{\mathbf{e}}_2(z) = -\hat{\mathbf{e}}_x\sin(\Omega z) + \hat{\mathbf{e}}_y\cos(\Omega z)$, where $\hat{\mathbf{e}}_x$ and $\hat{\mathbf{e}}_y$ are fixed. We also assume that $\hat{\mathbf{e}}_1$ and $\hat{\mathbf{e}}_2$ are independent of ω over the bandwidth of interest—a reasonable assumption in many practical problems.[10] In the frame of reference that twists with the fiber, $\tilde{\epsilon}$, $\boldsymbol{\beta}$, and $\boldsymbol{\gamma}$ are all diagonal operators. We may usefully write them in the form

$$\begin{aligned}
\tilde{\epsilon} &= \bar{\epsilon}\mathsf{I} + \Delta\epsilon\boldsymbol{\sigma}_3,\\
\boldsymbol{\beta} &= \bar{\beta}\mathsf{I} + \Delta\beta\boldsymbol{\sigma}_3,\\
\boldsymbol{\gamma} &= \bar{\gamma}\mathsf{I},
\end{aligned} \tag{12}$$

where $\mathsf{I} = \hat{\mathbf{e}}_1\hat{\mathbf{e}}_1{}^* + \hat{\mathbf{e}}_2\hat{\mathbf{e}}_2{}^*$ is the identity tensor and $\boldsymbol{\sigma}_3$ is one of the three Pauli tensors

$$\boldsymbol{\sigma}_1 = \hat{\mathbf{e}}_2\hat{\mathbf{e}}_1{}^* + \hat{\mathbf{e}}_1\hat{\mathbf{e}}_2{}^*, \qquad \boldsymbol{\sigma}_2 = i(\hat{\mathbf{e}}_2\hat{\mathbf{e}}_1{}^* - \hat{\mathbf{e}}_1\hat{\mathbf{e}}_2{}^*), \qquad \boldsymbol{\sigma}_3 = \hat{\mathbf{e}}_1\hat{\mathbf{e}}_1{}^* - \hat{\mathbf{e}}_2\hat{\mathbf{e}}_2{}^*. \tag{13}$$

If we represent the vector $\mathbf{U} = U_1\hat{\mathbf{e}}_1 + U_2\hat{\mathbf{e}}_2$ with the column vector $(U_1, U_2)^t$, then the three Pauli tensors given above are represented by the usual Pauli matrices

$$\mathsf{I} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}, \qquad \boldsymbol{\sigma}_1 = \begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}, \qquad \boldsymbol{\sigma}_2 = \begin{bmatrix}0 & -i\\ i & 0\end{bmatrix}, \qquad \boldsymbol{\sigma}_3 = \begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}. \tag{14}$$

Since $\hat{\mathbf{e}}_1$ and $\hat{\mathbf{e}}_2$ are real, it is not necessary to include the complex conjugation in Eqs. (13). We do so because with the complex conjugation the definitions that we have given generalize to situations in which the fiber is elliptically polarized. The elements $\overline{\epsilon}$ and $\overline{\beta}$ that appear in Eqs. (12) are larger than $\Delta\epsilon$ and $\Delta\beta$ by a factor of the order of $\Delta n/n$, where Δn is the index difference that is due to the birefringence. It is also the case that $|(\omega_0{}^2/k_0c)\overline{\epsilon}\cdot U| \gg |\boldsymbol{\beta}\cdot\partial\mathbf{U}/\partial t| \gg |\boldsymbol{\gamma}\cdot\partial^2\mathbf{U}/\partial t^2|$ on average, consistent with the slowly varying envelope approximation. The term $(\omega_0{}^2/c^2)\Delta\epsilon\boldsymbol{\sigma}_3\cdot\mathbf{U}$ is of the same order of magnitude as $2ik_0\overline{\beta}\mathbf{I}\cdot\partial\mathbf{U}/\partial t$, and the term $2ik_0\Delta\beta\boldsymbol{\sigma}_3\cdot\partial\mathbf{U}/\partial t$ is of the same order of magnitude as $k_0\overline{\gamma}\mathbf{I}\cdot\partial^2\mathbf{U}/\partial t^2$; which term dominates in both these comparisons depends on the details of the physical situation.[1] We have not included the $\Delta\gamma$ contribution in Eqs. (12) because it is of the same order of magnitude as the third-order dispersion, which was neglected in the truncation leading to Eq. (8). Using the relations

$$\frac{d\hat{\mathbf{e}}_1}{dz} = \Omega\hat{\mathbf{e}}_2, \qquad \frac{d\hat{\mathbf{e}}_2}{dz} = -\Omega\hat{\mathbf{e}}_1, \tag{15}$$

we obtain

$$\frac{\partial^2\mathbf{E}^+}{\partial z^2} = \left[-(k_0{}^2+\Omega^2)\mathbf{U} + 2ik_0\frac{\partial\mathbf{U}}{\partial z} + \frac{\partial^2\mathbf{U}}{\partial z^2} + 2k_0\Omega\boldsymbol{\sigma}_2\cdot\mathbf{U} - 2i\Omega\boldsymbol{\sigma}_2\cdot\frac{\partial\mathbf{U}}{\partial z}\right]\exp(ik_0z - i\omega_0t). \tag{16}$$

Substituting Eqs. (10), (12), and (16) into the positive-frequency version of Eq. (3),

$$\frac{\partial^2\mathbf{E}^+}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2\mathbf{D}^+}{\partial t^2} = 0, \tag{17}$$

and using the dispersion relation

$$k_0{}^2 - \frac{\omega_0{}^2}{c^2}\overline{\epsilon} - \Omega^2, \tag{18}$$

we obtain

$$2ik_0\frac{\partial\mathbf{U}}{\partial z} + \frac{\omega_0{}^2}{c^2}\Delta\epsilon\boldsymbol{\sigma}_3\cdot\mathbf{U} + 2k_0\Omega\boldsymbol{\sigma}_2\cdot\mathbf{U} + 2ik_0\overline{\beta}\frac{\partial\mathbf{U}}{\partial t} - 2i\Omega\boldsymbol{\sigma}_2\cdot\frac{\partial\mathbf{U}}{\partial z} + 2ik_0\Delta\beta\boldsymbol{\sigma}_3\cdot\frac{\partial\mathbf{U}}{\partial t} + \frac{\partial^2\mathbf{U}}{\partial z^2} - k_0\overline{\gamma}\frac{\partial^2\mathbf{U}}{\partial t^2} = 0. \tag{19}$$

When the beat length is the shortest scale length in the optical fiber and the twist length is comparable with the beat length, then the second and the third terms in Eq. (19) will dominate the evolution of $\mathbf{U}$. The third term, which is due to the fiber's twisting, will lead to coupling between U_1 and U_2. It is useful to find a set of modes that are not coupled by the second and the third terms of Eq. (19) and are thus eigenmodes of the restricted evolution equation, which consists of the first three terms of Eq. (19). We first define $\alpha = \omega_0{}^2\Delta\epsilon/2k_0c^2$, $\kappa = (\alpha^2+\Omega^2)^{1/2}$, $\cos\theta = \alpha/\kappa$, and $\sin\theta = \Omega/\kappa$. Using the column-vector representation that we mentioned in the above paragraph, we find that if we set

$$\begin{pmatrix}V_1\\V_2\end{pmatrix} = \mathbf{R}\begin{pmatrix}U_1\\U_2\end{pmatrix}, \tag{20}$$

where $\mathbf{R} = \cos(\theta/2)\mathbf{I} - i\sin(\theta/2)\boldsymbol{\sigma}_1$, then the evolution of the components V_1 and V_2 is not coupled by the second and the third terms in Eq. (19). Still using the column vector representation and the relations

$$\mathbf{R}\boldsymbol{\sigma}_2\mathbf{R}^{-1} = (\sin\theta)\boldsymbol{\sigma}_3 + (\cos\theta)\boldsymbol{\sigma}_2 = \frac{\Omega}{\kappa}\boldsymbol{\sigma}_3 + \frac{\alpha}{\kappa}\boldsymbol{\sigma}_2,$$

$$\mathbf{R}\boldsymbol{\sigma}_3\mathbf{R}^{-1} = (\cos\theta)\boldsymbol{\sigma}_3 - (\sin\theta)\boldsymbol{\sigma}_2 = \frac{\alpha}{\kappa}\boldsymbol{\sigma}_3 - \frac{\Omega}{\kappa}\boldsymbol{\sigma}_2, \tag{21}$$

we find that Eq. (19) becomes

$$i\frac{\partial\mathbf{V}}{\partial z} + \kappa\boldsymbol{\sigma}_3\cdot\mathbf{V} + i\overline{\beta}\frac{\partial\mathbf{V}}{\partial t} - i\frac{\Omega}{k_0}\left(\frac{\Omega}{\kappa}\boldsymbol{\sigma}_3 + \frac{\alpha}{\kappa}\boldsymbol{\sigma}_2\right)\cdot\frac{\partial\mathbf{V}}{\partial z} + i\Delta\beta\left(\frac{\alpha}{\kappa}\boldsymbol{\sigma}_3 - \frac{\Omega}{\kappa}\boldsymbol{\sigma}_2\right)\cdot\frac{\partial\mathbf{V}}{\partial t} + \frac{1}{2k_0}\frac{\partial^2\mathbf{V}}{\partial z^2} - \frac{\overline{\gamma}}{2}\frac{\partial^2\mathbf{V}}{\partial t^2} = 0. \tag{22}$$

It is most convenient to view the transformation from $(U_1, U_2)^t \rightarrow (V_1, V_2)^t$ given by Eq. (20) as a coordinate transformation so that $\mathbf{U} = \mathbf{V}$. In this case, $\mathbf{V} = V_1\hat{\mathbf{a}}_1 + V_2\hat{\mathbf{a}}_2$, where

$$\hat{\mathbf{a}}_1 = \hat{\mathbf{e}}_1\cos(\theta/2) + i\hat{\mathbf{e}}_2\sin(\theta/2),$$

$$\hat{\mathbf{a}}_2 = i\hat{\mathbf{e}}_1\sin(\theta/2) + \hat{\mathbf{e}}_2\cos(\theta/2). \tag{23}$$

In this vector representation the $\boldsymbol{\sigma}_j$ in Eq. (22) are the Pauli tensors

$$\boldsymbol{\sigma}_1 = \hat{\mathbf{a}}_2\hat{\mathbf{a}}_1{}^* + \hat{\mathbf{a}}_1\hat{\mathbf{a}}_2{}^*, \qquad \boldsymbol{\sigma}_2 = i(\hat{\mathbf{a}}_2\hat{\mathbf{a}}_1{}^* - \hat{\mathbf{a}}_1\hat{\mathbf{a}}_2{}^*),$$

$$\boldsymbol{\sigma}_3 = \hat{\mathbf{a}}_1\hat{\mathbf{a}}_1{}^* - \hat{\mathbf{a}}_2\hat{\mathbf{a}}_2{}^*. \tag{24}$$

The complex conjugation is required in Eqs. (24). Because $\mathbf{V} = \mathbf{U}$, we may write

$$\mathbf{E}^+ = \mathbf{V}\exp(ik_0z - i\omega_0t) = (V_1\hat{\mathbf{a}}_1 + V_2\hat{\mathbf{a}}_2)\exp(ik_0z - i\omega_0t). \tag{25}$$

This expression will be of use when we consider the nonlinear contribution to the polarization field.

To obtain a first-order evolution equation for $\mathbf{V}$, we now eliminate the z derivatives in the fourth and the sixth terms of Eq. (22) in favor of t derivatives. Noting that $\Omega/k_0 \ll 1$, $\Delta\beta/\overline{\beta} \ll 1$, and that to be consistent with the slowly varying envelope approximation we must assume that the terms with second derivatives in t and z are small, we find that the lowest-order contributions to $\partial\mathbf{V}/\partial z$ and $\partial^2\mathbf{V}/\partial z^2$ are given by

$$\frac{\partial\mathbf{V}}{\partial z} = i\kappa\boldsymbol{\sigma}_3\cdot\mathbf{V} - \overline{\beta}\frac{\partial\mathbf{V}}{\partial t},$$

$$\frac{\partial^2\mathbf{V}}{\partial z^2} = -\kappa^2\mathbf{V} - 2i\kappa\overline{\beta}\boldsymbol{\sigma}_3\cdot\frac{\partial\mathbf{V}}{\partial t} + \overline{\beta}^2\frac{\partial^2\mathbf{V}}{\partial t^2}. \tag{26}$$

Using Eqs. (26) in Eq. (22), we finally obtain

$$i\frac{\partial\mathbf{V}}{\partial z} + \frac{1}{2k_0}(2\Omega^2 - \kappa^2)\mathbf{V} + \kappa\boldsymbol{\sigma}_3\cdot\mathbf{V} + i\frac{\alpha\Omega}{k_0}\boldsymbol{\sigma}_1\cdot\mathbf{V} + i\overline{\beta}\frac{\partial\mathbf{V}}{\partial t} + i\frac{\alpha}{\kappa}(\Delta\beta - \overline{\beta}\alpha/k_0)\boldsymbol{\sigma}_3\cdot\frac{\partial\mathbf{V}}{\partial t} - i\frac{\Omega}{\kappa}(\Delta\beta - \overline{\beta}\alpha/k_0)\boldsymbol{\sigma}_2\cdot\frac{\partial\mathbf{V}}{\partial t} + \frac{1}{2}(\overline{\beta}^2/k_0 - \overline{\gamma})\frac{\partial^2\mathbf{V}}{\partial t^2} = 0. \tag{27}$$

The second term in Eq. (27) leads to a slight change in the dispersion relation, Eq. (18), while the fourth term leads to a slight change in the orientation of the electric field's polarization ellipse with respect to the twisted fiber's polarization axes. Neither term adds anything qualitatively new to the equation, and both are too small to be experimentally detectable. We therefore drop them. By contrast, the seventh term in Eq. (27), the term proportional to $\sigma_2 \cdot \partial \mathbf{V}/\partial t$, will lead to coupling between V_1 and V_2 through their time derivatives, which is a qualitatively new effect. From Eq. (27) we find that the birefringent beat length is given by π/κ, while the walk-off length, the length over which pulses in the two polarizations shift significantly in time with respect to each other, is given by $\pi/|\Delta\beta - \overline{\beta}\alpha/k_0|$. When the beat length is short compared with the walk-off length, the third term in Eq. (27), $\kappa\sigma_3 \cdot \mathbf{V}$, will lead to a phase mismatch between V_1 and V_2. In this case the term proportional to $\sigma_2 \cdot \partial\mathbf{V}/\partial t$ is rapidly oscillating and can be neglected. In feasible present-day experiments involving solitons the beat length is always short compared with the walk-off length.[2] This will not necessarily be true when broad-bandwidth, quasi-cw light is launched into a fiber, and this coupling could then play a significant role.

Because it is our purpose to determine the circumstances under which Eqs. (1) hold, we will henceforward assume that the beat length is short compared with the walk-off length and drop the term proportional to $\sigma_2 \cdot \partial\mathbf{V}/\partial t$.

We now turn to consideration of the nonlinear contribution. We first recall[1,5] that the contribution of the Kerr effect to the polarization field is given by

$$\mathbf{P}_{\text{nl}}^{+} = 2a(\mathbf{E}^{+} \cdot \mathbf{E}^{-})\mathbf{E}^{+} + b(\mathbf{E}^{+} \cdot \mathbf{E}^{+})\mathbf{E}^{-}, \tag{28}$$

where $\mathbf{P}_{\text{nl}}^{+}$ is the nonlinear contribution to $\mathbf{P}^{+}$, a and b are the usual Kerr coefficients, and we assume that contributions of the time derivatives of the $\mathbf{E}$ field to the nonlinear portion of the polarization field are negligible. In optical fibers we may also assume that the Kerr response of the medium is instantaneous, so that $a = b$. From Eqs. (23), (25), and (28) we obtain

$$\begin{aligned}\boldsymbol{\Pi}_{\text{nl}}^{+} = {} & (2a + b\cos^2\theta)(|V_1|^2V_1\hat{\mathbf{a}}_1 + |V_2|^2V_2\hat{\mathbf{a}}_2) \\ & + (2a + 2b\sin^2\theta)(|V_2|^2V_1\hat{\mathbf{a}}_1 + |V_1|^2V_2\hat{\mathbf{a}}_2) \\ & + b\cos^2\theta(V_2{}^2V_1{}^*\hat{\mathbf{a}}_1 + V_1{}^2V_2{}^*\hat{\mathbf{a}}_2) \\ & + ib(\cos\theta\sin\theta) \\ & \times [(2|V_1|^2V_2 - |V_2|^2V_2 - V_1{}^2V_2{}^*)\hat{\mathbf{a}}_1 \\ & + (2|V_2|^2V_1 - |V_1|^2V_1 - V_2{}^2V_1{}^*)\hat{\mathbf{a}}_2],\end{aligned} \tag{29}$$

where $\mathbf{P}_{\text{nl}}^{+} = \boldsymbol{\Pi}_{\text{nl}}^{+}\exp(ik_0z - i\omega_0t)$. Under the assumption that the beat length is short compared with the nonlinear scale length, we find that only the first two terms in Eq. (29) give phase-matched contributions to $\partial\mathbf{V}/\partial z$.[1] The other terms are rapidly oscillating and can be neglected.

Keeping only phase-matched terms and including the nonlinear contribution, we find that Eq. (27) becomes

$$\begin{aligned} i\frac{\partial V_1}{\partial z} &+ \kappa V_1 + i\overline{\beta}\frac{\partial V_1}{\partial t} + i\frac{\alpha}{\kappa}\left(\Delta\beta - \frac{\overline{\beta}\alpha}{k_0}\right)\frac{\partial V_1}{\partial t} \\ &+ \frac{1}{2}\left(\frac{\overline{\beta}^2}{k_0} - \overline{\gamma}\right)\frac{\partial^2 V_1}{\partial t^2} + \frac{2\pi\omega_0{}^2}{k_0c^2}\left(2a + \frac{b\alpha^2}{\kappa^2}\right)|V_1|^2V_1 \\ &+ \frac{2\pi\omega_0{}^2}{k_0c^2}\left(2a + \frac{2b\Omega^2}{\kappa^2}\right)|V_2|^2V_1 = 0,\end{aligned} \tag{30a}$$

$$\begin{aligned} i\frac{\partial V_2}{\partial z} &- \kappa V_2 + i\overline{\beta}\frac{\partial V_2}{\partial t} - i\frac{\alpha}{\kappa}\left(\Delta\beta - \frac{\overline{\beta}\alpha}{k_0}\right)\frac{\partial V_2}{\partial t} \\ &+ \frac{1}{2}\left(\frac{\overline{\beta}^2}{k_0} - \overline{\gamma}\right)\frac{\partial^2 V_2}{\partial t^2} + \frac{2\pi\omega_0{}^2}{k_0c^2}\left(2a + \frac{2b\Omega^2}{\kappa^2}\right)|V_1|^2V_2 \\ &+ \frac{2\pi\omega_0{}^2}{k_0c^2}\left(2a + \frac{b\alpha^2}{\kappa^2}\right)|V_2|^2V_2 = 0,\end{aligned} \tag{30b}$$

where we have written the two components of $\mathbf{V}$ separately. As our first step toward reducing Eqs. (30) to the form of Eqs. (1), we transform V_1 and V_2 by shifting their wave numbers. We let

$$W_1 = V_1\exp(-i\kappa z), \qquad W_2 = V_2\exp(i\kappa z). \tag{31}$$

We also define a retarded time $\tau = t - \overline{\beta}z$. These changes have the effect of eliminating the second and the third terms in both Eqs. (30a) and (30b). It is of great physical significance that the wave-number shifts in Eqs. (31) are opposite in sign. It is this fact, plus the assumption that κ is large, that leads to phase mismatches in many terms that would otherwise contribute to Eqs. (30). To reduce Eqs. (30) to Eqs. (1), we define the following normalized quantities:

$$\begin{aligned} s &= \tau/t_0, \\ \xi &= (\overline{\beta}^2/k_0 - \overline{\gamma})z/t_0{}^2, \\ u &= \left(\frac{2\pi\omega_0{}^2t_0{}^2}{k_0c^2}\,\frac{2a + b\alpha^2/\kappa^2}{\overline{\beta}^2/k_0 - \overline{\gamma}}\right)^{1/2} W_1, \\ v &= \left(\frac{2\pi\omega_0{}^2t_0{}^2}{k_0c^2}\,\frac{2a + b\alpha^2/\kappa^2}{\overline{\beta}^2/k_0 - \overline{\gamma}}\right)^{1/2} W_2, \\ \delta &= \frac{\alpha}{\kappa}\,\frac{\Delta\beta - \overline{\beta}\alpha/k_0}{\overline{\beta}^2/k_0 - \overline{\gamma}}\,t_0, \\ B &= \frac{2a + 2b\Omega^2/\kappa^2}{2a + b\alpha^2/\kappa^2}.\end{aligned} \tag{32}$$

The parameter t_0 is arbitrary and may be chosen as convenient in any application. We find that $B = 1$, and that nonlinear polarization rotation is eliminated when $\Omega = \alpha/\sqrt{2}$; i.e., the twist length is $\sqrt{2}$ times the beat length.

In this Communication we have shown that Eqs. (1) remain valid in twisted optical fibers under certain physical conditions and that B is still given by Eq. (2), where $\tan\theta = \Omega/\alpha$, the ratio of the beat length to the twist length. The physical requirements for Eqs. (1) to hold are that the beat length must be small in comparison with the walk-off length, the dispersive scale length, and the nonlinear scale length. We have found that when the walk-off length becomes comparable with the beat length, twisting leads to a coupling of the two polarizations through their time derivatives. Understanding the effect of this coupling would be an interesting topic for future investigation. When Eqs. (1) and (2) hold, we have found that it is possible to eliminate nonlinear polarization rotation by twisting the fiber at a rate $\Omega = \alpha/\sqrt{2}$.

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