ELLSBERG REVISITED: AN EXPERIMENTAL STUDY

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Abstract

An extension to Ellsberg's experiment demonstrates that attitudes to ambiguity and compound objective lotteries are tightly associated. The sample is decomposed into three main groups: subjective expected utility subjects - who reduce compound objective lotteries and are ambiguity neutral, and two groups that exhibit different forms of association between preferences over compound lotteries and ambiguity - corresponding to alternative theoretical models that account for ambiguity averse or seeking behavior.

JEL Classification: D81, C91.

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1 Introduction

In 1961 Daniel Ellsberg [7] suggested several ingenious experiments which demonstrated that Savage's [30] normative approach, which allows to derive subjective probabilities from preferences, faces severe descriptive difficulties. Since then, several models that can accommodate Ellsberg type behavior have been proposed. The goal of this work is to compare the performance of these theories in a controlled experimental environment, which is an extension of the original Ellsberg experiment.

The main premise used in analyzing the sample is that the population may be heterogeneous: different subjects have different patterns of choice, that correspond to different theories. As a consequence, the analysis will not look for a unique theory that can explain the average decision maker, but will try to infer from the sample what are the common choice patterns in the population.

The theoretical focus of the experiment is to test the association between non-neutral attitude to ambiguity and violations of reduction of compound (objective) lotteries (ROCL). The former states that subjective uncertainty cannot be reduced to risk, or, in the language of Machina and Schmeidler [23], the agent is not "probabilistically sophisticated" (see Epstein [8]). Violation of ROCL implies that agents who face compound lotteries, do not calculate probabilities of final outcomes according to the laws of probability.

The results, as is evident from Table 1[1], reveal a tight association between ambiguity neutrality and reduction of compound lotteries - consistent with the subjective expected utility model. Further analysis clarifies that the structure of the association between non-neutral attitudes to ambiguity and violation of ROCL is not uniform in the population of subjects, as two different (about even in frequency) choice patterns emerge. One pattern corresponds to the theoretical predictions of the Recursive Non-Expected Utility (RNEU) model of Segal [32, 34], while the second may be generated by behavioral rules consistent with an environment of bundled risks (as in Halevy and Feltkamp [14]) and can be represented by the Recursive Expected Utility (REU) model (Klibanoff *et al* [21], Ergin and Gul [9], Ahn [1]) in which a decision maker does not reduce compound objective lotteries.

[1] Table 1 combines the two samples reported in this study (see Table 5.) Ambiguity neutrality is defined as $V1 = V2$, and ROCL as: $V1 = V3 = V4$

Table 1: The association between attitudes to ambiguity and compound objective lotteries

			ROCL		Total
			No	Yes	
Ambuguity Neutral	No	Count	113	1	114
		Expected	95.5	18.5	
	Yes	Count	6	22	28
		Expected	23.5	4.5	
Total			119	23	142

	Exact Sig. (2-sided)
Fisher's Exact Test	2.3E-19

2 Method

Two controlled experiments were conducted: an original experiment with moderate stakes, and a robustness experiment in which all payoffs were scaled by a factor of ten. In each experiment, the subjects were asked to state their reservation prices for four different lotteries through an incentive compatible elicitation mechanism.

2.1 Recruitment

104 subjects were recruited to the original experiment using ads posted on different locations on the campus of the University of British Columbia. They signed up for sequential time slots. 38 subjects participated in the robustness experiment. The recruitment of subjects to the second round was based on proportional sampling within each cohort of undergraduate students in the faculties of Arts and Science at UBC (about 12,500 students): 100 students were sent e-mail invitations, about half of them responded, out of which 38 students participated in the experiment (mainly due to scheduling conflicts).

2.2 Administration

During each experiment only one subject was present in the room. Following her arrival, each subject signed a consent form (Appendices A and B) which explained the experiment. In the first experiment, a research assistant was always in the room to answer any concerns, and to make sure the subject knew how to run the computer program that simulated the lotteries. The computer program was written by CASSEL's[2] staff. The second experiment was supervised by the author and all the lotteries were executed by physical randomization devices.

2.3 The Lotteries

Subjects were presented with four lotteries: the first two are the standard (two colors) Ellsberg urns, used to test ambiguity attitude. The latter two (together with the first one) test whether behavior satisfies the ROCL assumption for objective lotteries. A graphical presentation of the lotteries is presented in Appendix C. Follows is the description of the lotteries as presented to the subjects:
"There are 4 urns[3], each containing 10 balls, which can be either red or black. The composition of balls in the urns is as follows:

- **Urn 1:** Contains 5 red balls and 5 black balls.
- **Urn 2:** The number of red and black balls is unknown, it could be any number between 0 red balls (and 10 black balls) to 10 red balls (and 0 black balls).
- **Urn 3:** The number of red and black balls is determined as follows: one ticket is drawn from a bag containing 11 tickets with the numbers 0 to 10 written on them. The number written on the drawn ticket will determine the number of red balls in the third urn. For example, if the ticket drawn is 3, then there will be 3 red balls and 7 black balls.
- **Urn 4:** The color composition of balls in this urn is determined in a similar way to box 3. The difference is that instead of 11 tickets in the bag, there are 2, with the numbers 0 and 10 written on them. Therefore, the urn may contain either 0 red balls (and 10 black balls) or 10 red balls (and 0 black balls)."

Each participant was asked to place a bet on the color of the ball drawn from each urn (`Red` or `Black`) (eliminating problems arising from potentially asymmetric information, e.g. Morris [24]). If a bet on a specific urn is correct, the subject could win

[2] California Social Science Experimental Lab, which is a joint project of UCLA, California Institute of Technology, and the NSF.

[3] In the experiment, the word "box" was used, in order to minimize confusion among subjects who were not familiar with the word "urn."

2 Canadian Dollars, and if a bet is incorrect, the subject losses nothing. The total money which could have been earned is \$8 (plus \$2 paid for participation). Before balls were drawn from each urn (and before the tickets are drawn from the bags for urns 3 and 4), the subject had the options to "sell" each one of her bets. The Becker-Degroot-Marschak ([4], henceforth BDM) mechanism was used to elicit (an approximation to) the certainty equivalent of each bet: the subject was asked to state four minimal prices at which she was willing to sell each one of the bets (reservation prices). The subject set the selling prices by moving a lever on a scale between \$0 and \$2. For each urn, a random number between \$0 and \$2 was generated by the computer. The four random numbers were the "buying prices" for each one of the bets. If the buying price for an urn was higher than the reservation price the subject stated for that urn, she was paid the buying price (and her payoff did not depend on the outcome of her bet). However, if the buying price for the urn was lower than the minimal selling price reported for that urn, the actual payment depended on the outcome of her bet.

The BDM [4] mechanism has been used extensively in the "preference reversal" literature (e.g. Grether and Plott [11])[4]. Several researchers (Holt [16], Karni and Safra [19], Segal [33]) have pointed out that when preferences do not satisfy the axioms underlying expected utility theory (in particular, independence [16, 19] and reduction of compound lotteries [33]), the BDM mechanism may not elicit valuations accurately. Holt's [16] reservation - which applies to a situation in which several valuation are elicited, but the subject is paid the outcome of only one - has been fully accommodated in the current study since all outcomes are actually paid. Karni and Safra [19] showed that the "certainty equivalent" of a lottery elicited utilizing the BDM mechanism, respects the preference ordering if and only if preferences satisfy the independence axiom. Furthermore, they showed that there exists no incentive compatible mechanism that elicits the certainty equivalent and does not depend on the independence axiom. In the current experiment, the independence axiom does not play any role in the evaluation of urns 1, 3 and 4: if the subject calculates (reduces) probabilities, then she would view all of them as the same lottery. Segal [33]

[4] The main focus of this literature is the relative ranking of two lotteries: one with high probability of winning a moderate prize (P bet) and one with a low probability of winning a high prize (a p bet). Many agents choose the P bet over the p bet, but the valuation of the P bet is lower than the valuation of the p bet. The valuations were elicited using a BDM mechanism.

provided an example in which violation of ROCL results in a preference reversal[5]. The latter limitation of the BDM mechanism is important in the current experiment that focuses on the relation between ambiguity aversion and violations of ROCL. Furthermore, the BDM is complicated - and if subjects fail to understand it - the elicited values might reflect their confusion and not their evaluation. To minimize the confusion effect, the subjects received before the experiment an extensive explanation on the BDM mechanism and all experienced it in a trial round[6] before the actual experiment. Moreover, various aspects of the patterns of responses convince me that these concerns do not render the data useless. That is, even if the BDM mechanism elicits the valuation with some noise, the patterns in the data are extremely robust and consistent with some of the theoretical predictions[7].

The random numbers (which were generated by the computer program) and the outcomes of the draws were not revealed until all four reservation prices were set. The relevant data collected from each participant were the reservation prices she stated for each box (reported in Appendix D), as well as some personal information (available upon request).

2.4 Robustness Test

The original experiment as presented above may be subject to several imperfections: the price of \$2 may seem too small to give the subject sufficient incentives to think seriously of the problems at hand; although the research assistant tried to confirm the participants understood the BDM mechanism - there was no objective measure of his success; the subjects were asked to behave as "sellers" - a framing that might have influenced their reservation prices; the recruitment of the subjects was based on sign-up sheets - this convenience sampling technique might have introduced biases into the experiments' results. To counter these reservations it might be argued that non of the arguments is systematic, and if it introduces biases, there is no a priori reason to believe they have a differential effect on the reservation prices set for the four urns.

[5] Furthermore - Keller, Segal and Wang [20] show that under this latter interpretation, the certainty equivalent and the value elicited using BDM may lie on different sides of the expected value of the lottery. Therefore, risk attitude would be impossible to infer from the elicited value.

[6] The second (ambiguous) urn was a "new" urn, eliminating the possibility of learning from the trial round. This information was conveyed to the participants.

[7] This additional "measurement error" is minimized for theories like Maxmin Expected Utility, which satisfy both reduction of compound lotteries and the independence axiom when only objective probabilities are involved.

In particular, since the focus of the current study is on the relative reservation prices, the conjecture is that similar results would hold in an altered experiment in which these deficiencies would be corrected. To test this conjecture, a robustness experiment was conducted. The prizes were scaled from \$2 to \$20 per urn. The efficiency of the sampling has increased, thanks to the use of the proportional sampling within cohorts, as described above. In addition, while in the first round subjects were informed the range of possible payoffs ("earn up to \$10"), no definite amount was disclosed in the robustness round before the experiment itself (most subjects did not expect the payoffs to be as high).

The subjects received two paid opportunities to experience the operation of the BDM mechanism, before the trial round. They were given a two Canadian dollars coin (toonie) and were asked to set their minimal reservation price for it. This task was used to guide them how to "find" their minimal reservation price: they were prompted to consider if they would accept 5 cents less than their stated reservation price. If they accepted, the process was repeated until the minimal reservation price was achieved. Since in this task there is an objectively correct answer, the subjects learned what reservation prices are "too low" and which are "too high." Next, the subjects were given a pen (with a retail value of \$2.50 at UBC's bookstore, which was not reveled to the subjects) and were asked to set their minimal reservation price for it using the same mechanism. Only then, they were offered a trial round with the four urns. Throughout the experiment the subjects were reminded how to "find" their minimal reservation price. Furthermore, in the instructions to the experiment the terms "selling/buying price" and "true valuation" were not used.

In order to prevent any possibility the subjects suspect they are tricked, the implementation of the experiment was altered somewhat. The lotteries were physical and not computerized: there were four pouches containing beads that could be red or black. The composition of pouches 3 and 4 was determined by choosing at random a numbered token (one out of 11 or one out of 2, respectively), and the composition of pouch 1 could have been verified by the subject. The random numbers (between \$0 and \$20) were generated by a computer before the experiment. They were organized in a matrix, and the subjects chose at the beginning of the experiment 10 different coordinates[8], that were revealed sequentially after she set reservation prices for the

[8] The first two coordinates were used to teach the BDM elicitation mechanism (range of 0 to 4), while the latter 8 were used for the trial round and the paid experiment.

different urns. In addition, an order treatment was implemented: the subjects were randomly allocated to different order of urns: (1,2,3,4) - as in the original experiment, (2,3,4,1), (3,4,1,2) and (4,1,2,3). The goal of the random ordering was to test whether the reservation prices are influenced by alternative ordering schemes[9].

2.5 Related Experimental Literature

Two previous experimental studies added an objective two-stage lottery to the classic two color Ellsberg example. Yates and Zukowski [36] test the "range hypothesis"[10] by offering urns similar to the first three in the current study. Each subject was allowed to choose one urn out of the possible three pairs of urns. The value of the chosen lottery was elicited using the BDM mechanism. Yates and Zukowski found evidence that Urn 1 was weakly preferred to Urn 3, which was weakly preferred to Urn 2[11].

Chow and Sarin [6] test the distinction between known (risk), unknown and unknowable[12] uncertainties using urns 1, 2 and 3 respectively. They find that unknowable uncertainty is intermediate to the known and the unknown forms of uncertainties. They relate their findings to Fox and Tversky's [10] "comparative ignorance hypothesis," in which the availability of an informed agent (experimenter) decreases the attractiveness of a lottery.

3 Theoretical Predictions

This section will describe how different theories of choice under uncertainty predict individual choices in the experiment. The theories I focused on during the research include Subjective Expected Utility, Maxmin Expected Utility, Recursive Non-Expected Utility (RNEU), Recursive Expected Utility (REU) and the "bundling" rationale to

[9]Harrison *et al* [15] find a significant order effect (in addition to scale effect) in Holt and Laury's [17] study of the effect of higher scale of *real* incentives on risk aversion. In a follow-up study, Holt and Laury [18] show that the magnitude of the scale effect is robust to the elimination of the order effect.

[10]The range hypothesis claims that the range of the second order distribution is the critical element in accounting for the attractiveness of an "ambiguous" lottery. Hence, urn 3 has the largest range, and should be (weakly) inferior to the second (ambiguous) urn.

[11]Yates and Zukowski's evidence should be treated with care, since they average over different subjects who were offered different choice sets.

[12]The subject does not know the probability and believes that others, too, do not know the probability.

ambiguity aversion.

3.1 Subjective Expected Utility (SEU)

Partition the state space - S, into 11 events - representing different number of red (black) balls in urn 2 (0 to 10 red balls). Let s_i denote an event in which i red balls are in the second urn, and JL_i denote a bet on color $J \in \{R, B\}$ from urn $i \in \{1, \ldots 4\}$.

Preferences that are represented by a subjective expected utility (Savage [30]) function are of the form:

$$U_{SEU}(L_i) = \max_{J \in \{B,R\}} \sum_{s_i \in S} p(s_i) u(JL_i(s_i)) \tag{1}$$

where $p(s_i)$ is the subjective probability of event s_i, and $u(\cdot)$ is the decision maker's utility index. Although strictly speaking Savage's axioms are not stated in a dynamic framework, many works have shown that reduction of compound objective lotteries is a necessary part of (1) (e.g. Segal's [34] Theorem 3). Therefore, such a decision maker will state the same reservation price for urns 1, 3 and 4. It is possible that the decision maker might believe that urn 2 has more red or more black balls. That is, $p(\cdot)$ may not be symmetric around s_5 (the event in which there are 5 red and 5 black balls in the second urn). If this is the case, she will choose to bet on the more probable color (in which a higher subjective expected utility is attained), and will set a higher reservation price for it. Denote by Vi the reservation price for urn i, $i = 1, \ldots, 4$ respectively, then:

$$V1 = V2 = V4 \leq V2 \tag{2}$$

If the decision maker is an expected value maximizer then $\mathbb{E}(L_1) = V1 = V3 = V4$.

3.2 Maxmin Expected Utility (MEU)

A decision maker whose preferences are described by MEU (Gilboa and Schmeidler [12]) will have a set of prior beliefs (core of belief) and her utility of an act is the minimal expected utility on this set. The utility of betting on urn i is therefore:

$$U_{MEU}(L_i) = \max_{J \in \{B,R\}} \min_{p \in Core} \sum_{s_i \in S} p(s_i) u(JL_i(s_i)) \tag{3}$$

Since MEU is a generalization of expected utility, it allows for a pattern of reservation prices as in (2). If the core is not degenerate to a unique prior, it can accommodate the typical choice pattern suggested by Ellsberg: $RL_2 \sim BL_2 \prec BL_1 \sim RL_1$. For example, suppose that the core contains the two "pessimistic" non-symmetrical beliefs: that all balls are red and that all balls are black[13]. Then (if the prize is $\$x$) $U_{MEU}(L_2) = u(0) < 0.5u(0) + 0.5u(x) = U_{MEU}(L_1)$. However, within the realm of objective probabilities (urns 1, 3 and 4), MEU reduces to expected utility, and under the common interpretation, the decision maker reduces compound objective lotteries and is indifferent between bets on urns 1, 3, and 4. That is, if the core of belief includes a symmetric prior then:

$$V2 \leq V1 = V3 = V4 \tag{4}$$

where strict inequality follows if the core of belief is not degenerate.
Note that within the Choquet Expected Utility model (Schmeidler [31]) reduction of compound objective lotteries holds[14]. The case of convex capacity (which corresponds to ambiguity aversion) is a special case of the MEU model.

3.3 Recursive Non-Expected Utility (RNEU)

Segal [32, 34] relaxes the ROCL axiom and applies Rank Dependent Utility (RDU, or Anticipated Utility [27]), to evaluate the first and the second stage lotteries[15]. To better understand Segal's theory, let $x_1 \leqslant x_2 \leqslant \ldots \leqslant x_n$. The RDU of the lottery that gives x_i with probability p_i $i = 1, \ldots, n$ is:

$$U(x_1, p_1; x_2, p_2; \ldots; x_n, p_n) = u(x_1) + \sum_{i=2}^{n} [u(x_i) - u(x_{i-1})] f\left(\sum_{j=i}^{n} p_j\right) \tag{5}$$

[13] This is done for expositional purposes only. Nothing in the Gilboa-Schmeidler's [12] axioms underlying the MEU representation forces these extreme priors to be elements of the core.

[14] As with the SEU and MEU models, the axioms underlying the CEU representation are not, strictly speaking, dynamic. However, the common interpretation of the CEU framework is that when objective probabilities are involved - the representation reduces to expected utility.

[15] The RNEU model is not restricted to RDU: other models of decision making under risk, as weighted utility, could be applied and similar predictions would be attained. The critical assumptions are Segal's [34] "time neutrality" and "compound independence." It should be noted, however, that although the term "Non-Expected Utility" is commonly used to indicate a generalization of Expected Utility theory, the model suggested by Segal, imposes different (not weaker) restrictions on the data than the Recursive Expected Utility model (see below). Therefore a reader may wish to think of this model as "Recursive Rank Dependent Utility (RRDU)."

where $f : [0,1] \longrightarrow [0,1]$ and $f(0) = 0$ and $f(1) = 1$. The RD utility of the simple lottery that gives a prize of $\$x$ with probability p and $\$0$ with probability $1-p$ (after normalizing $u(0)$ to 0) is therefore:

$$U(x, p; 0, 1-p) = u(x) f(p) \tag{6}$$

and its certainty equivalent is:

$$CE(x, p; 0, 1-p) = u^{-1}(u(x) f(p)) \tag{7}$$

In order to demonstrate Segal's approach, assume that the decision maker's model of the ambiguous urn (L_2) is that with probability α it contains 10 red balls, with probability α it contains 0 red balls and with probability $1-2\alpha$ it contains 5 red balls. If the agent bets on red from urn 2, then she first evaluates the first stage lotteries: $(\$x, 1; \$0, 0)$, $(\$x, 0; \$0, 1)$ and $(\$x, 0.5; \$0, 0.5)$ using (6). Then, she evaluates the ambiguous lottery by substituting the certainty equivalents (calculated from (7)) as the prizes in (5):

$$\begin{aligned} U(RL_2) &= u\left[u^{-1}(v(0))\right] + \left[u\left(u^{-1}(u(x) f(0.5))\right) - u\left(u^{-1}(u(0))\right)\right] f(1-\alpha) \\ &+ \left[u\left(u^{-1}(u(x))\right) - u\left(u^{-1}(u(x) f(0.5))\right)\right] f(\alpha) \\ &= 0 + [u(x) f(0.5) - 0] f(1-\alpha) + [u(x) - u(x) f(0.5)] f(\alpha) \\ &= u(x)[f(0.5) f(1-\alpha) + (1 - f(0.5)) f(\alpha)] < u(x) f(0.5) = U(RL_1) \end{aligned}$$

where the last inequality follows from the convexity of f, which in this theory is a necessary condition for risk aversion, and reasonable properties of the transformation function $f(\cdot)$[16].

Segal's [32] novel interpretation of the Ellsberg paradox identifies ambiguity with a compound lottery, which she might fail to reduce. The critical feature of this model for the current experiment is that the certainty equivalent of a lottery is not monotone in the dispersion of the second order probability. In particular, the decision maker

[16] Segal [32] (Theorem 4.2) proved that if, in addition to convexity, f has non-decreasing elasticity and $\bar{f} = 1 - f(1-p)$ has non-increasing elasticity the decision maker will prefer a degenerate compound lottery (like L_1) to a subjective compound lottery like L_2 above.

is indifferent between a bet on Urn 1 and Urn 4. As before, the decision maker may believe that there are more red or more black balls in the second urn, and prefers to bet on the (subjectively) more probable color. Hence, the prediction of Segal's theory is that the decision maker will be indifferent between urns 1 and 4 and prefer them (under the conditions specified in the previous footnote) to a bet on urn 3. Indifference between the three objective urns results if f is the identity function (in which case RDU reduces to EU) and then reduction of compound lotteries holds. Hence, in terms of elicited valuations the theory's predictions are:

$$(V1 = V4) \text{ and } (V1 > V2 \ \Rightarrow V1 > V3) \text{ and } (V3 > V1 \Rightarrow V2 > V1) \tag{8}$$

That is, the recursive non-expected utility model predicts a negative correlation between $V21(= V2 - V1)$ and $V43(= V4 - V3)$, since ambiguity aversion $(V1 > V2)$ implies that $(V4 > V3)$, and $(V4 < V3)$ implies ambiguity seeking $(V1 < V2)$.

3.4 Recursive Expected Utility (REU)

Klibanoff, Marinacci and Mukerji [21] (henceforth KMM) study the preferences of a decision maker who is an expected utility on first and second stage lotteries, but her ambiguity attitude is determined by the relative concavities of the two utility functions. To understand KMM's model, consider the standard Ellsberg example (Urns 1 and 2 only) with a prize of x. The state space is $\Omega = \{RR, RB, BR, BB\}$ where, for example, state RB denotes a red ball drawn from the risky urn and a black ball from the ambiguous urn. Let π represent a probability distribution over Ω. Since the probability of drawing a red ball from the risky urn is 0.5, $\pi(\{RR, RB\}) = \pi(\{BR, BB\}) = 0.5$. For each π, the decision maker calculates its certainty equivalent according to a vN-M utility index u. The decision maker has a subjective prior μ over the possible π and evaluates an act using subjective expected utility according to the utility index v w.r.t. μ, substituting the certainty equivalents (calculated from u) for the objective lotteries for every π.

For example, suppose the support of μ, the set of possible objective probabilities, is composed of $\pi_1 = \left(\frac{1}{2}, 0, \frac{1}{2}, 0\right)$ - no blacks in urn 2, and $\pi_2 = \left(0, \frac{1}{2}, 0, \frac{1}{2}\right)$ - no reds in urn 2. The decision maker evaluates a bet on red from the ambiguous urn using the subjective prior $\mu = \left(\pi_1, \frac{1}{2}; \pi_2, \frac{1}{2}\right)$. That is, the *subjective* probability that urn 2 has only red balls (π_1), is equal to the subjective probability that it has all blacks (π_2) - which is equal to $\frac{1}{2}$ (similar to the *objective* urn 4). Given this belief, the decision maker's evaluation of a bet on either color from the ambiguous urn is:

$$U(JL_2) = \frac{1}{2} v\left(u^{-1}(u(x))\right) + \frac{1}{2} v\left(u^{-1}(u(0))\right) \tag{9}$$

Let $\phi = v \circ u^{-1}$ then:

$$U(JL_2) = \frac{1}{2}\phi(u(x)) + \frac{1}{2}\phi(u(0)) \tag{10}$$

KMM [21] generalize this representation, and show that the utility of an act f is given by the following functional $U(\cdot)$:

$$U(f) = \sum_{\pi\in\Delta} \phi\left(\sum_{s\in S} u(f(s)) \Pr(s|\pi)\right) \Pr(\pi) \tag{11}$$

where Δ is the set of all possible first stage objective lotteries. KMM define "Smooth Ambiguity Aversion" and show it is equivalent to ϕ being concave. Therefore, it is equivalent to aversion to mean preserving spreads of the expected utility values induced by the second order subjective probability (μ) and the act f. However, when μ is given objectively by $\widetilde{\mu}$, there is no behavioral reason to expect the decision maker to have differential risk attitudes in evaluating lotteries and second order acts, which induce *identical* objective probability distribution over outcomes. In this case, v would be an affine transformation of u, and reduction of compound lotteries (ROCL) will apply. As a result, a decision maker whose preferences are described by KMM will be indifferent between urns 1, 3 and 4:

$$V1 = V3 = V4 \tag{12}$$

However, being strictly formal, lotteries and second order acts (even when the second order distribution is objective) are different mathematical concepts. Hence, it is possible that $v(\cdot)$ would be more concave than $u(\cdot)$ even when μ is objective. If this is the case, the decision maker will evaluate urns 1, 3 and 4 in the following way:

$$\begin{aligned}
U\left(JL_1\right) &= \phi\left(0.5u\left(x\right)+0.5u\left(0\right)\right) \\
U\left(JL_3\right) &= \frac{1}{11}\sum_{r=0}^{10}\phi\left(\frac{r}{10}u\left(x\right)+\left(\frac{10-r}{10}\right)u\left(0\right)\right) \qquad (13)\\
U\left(JL_4\right) &= 0.5\phi\left(u\left(x\right)\right)+0.5\phi\left(u\left(0\right)\right)
\end{aligned}$$

and the reservation prices will satisfy[17]:

$$\left(V1\geq V3\geq V4\Rightarrow V2\geq V4\right)\text{ and }\left(V1\geq V2\Rightarrow V1\geq V3\geq V4\right) \qquad (14)$$
$$\text{and }\left(V1\leq V3\leq V4\Rightarrow V1\leq V2\right)$$

That is, if the subjective prior belief over the composition of the second urn is symmetric and non-degenerate around 0.5, we would expect a positive correlation between $V43(=V4-V3)$ and $V21(=V2-V1)$, since then $V4\leq V3$ if and only if $V1\geq V2$.

It is important to note that such an interpretation requires a behavioral argument why the decision maker should be more averse to second order acts than to lotteries. The bundling model (Halevy and Feltkamp [14]), presented in the following section, suggests one possible source for such divergence.

Ergin and Gul [9] suggest that ambiguity aversion is related to "issue preference"[18]. That is, an agent may prefer an act that depends on one issue (risk) over an act that depends on another issue (ambiguity). Ergin and Gul provide an axiomatic foundation for "second order probabilistically sophisticated" preferences - being able to assign subjective probabilities to the two issues, but allowing strict preference of a bet that depends on one issue over another. They show that if the agent's preferences satisfy the Sure Thing Principle or a comonotonic Sure Thing Principle then ambiguity aversion (in the sense of Schmeidler [31]) is equivalent to "second order risk aversion," which is aversion to mean preserving spreads in the subjective

[17] Note that if the subjective prior belief over the composition of the ambiguous (second) urn is not symmetric around 0.5 (5 red balls and 5 black balls), the decision maker may prefer to bet on the ambiguous urn over all the risky urns (1, 3 and 4).
If the subjective prior belief over the composition of the second (ambiguous) urn is symmetric around 5 red and 5 black balls, then the decision maker's ranking according to the REU model will be: $V1\geq V2\geq V4$. If it is degenerate on 5 red and 5 black balls then $V1=V2$, while if the subjective prior is extreme (as the objective fourth urn) then $V1=V4$.

[18] Similar preference have been named before "source preference" (e.g. Tversky and Wakker [35]).

belief. The representation derived is identical (in the case of expected utility) to KMM's. Formally, Ergin and Gul [9] could allow for issue preference even if the two issues are generated objectively, much in the same vein that Savage's approach could formally allow disparities between subjective and objective probabilities when the state is generated objectively. Therefore, although Ergin and Gul's [9] model is formally consistent with (14), justifying this pattern might be problematic: if both issues are objective, it is not clear why a decision maker would/should prefer one over the other. Again, the bundling model (Halevy and Feltkamp [14]) described in the next section offers one possible explanation for such pattern of preferences.

Kreps and Porteus' [22] model of decision making over temporal (objective) lotteries, does not concern ambiguity, but has exactly the same two-stage recursive structure, with expected utility at each stage, as in (11). Smooth ambiguity aversion (as in [21]) or second-order risk aversion (as in [9]) correspond to preference for late resolution in Kreps and Porteus' framework[19]. Segal [34] provides perspective on the relation between the RNEU and REU models: while Kreps-Porteus' REU model is derived by relaxing the time neutrality and ROCL axioms (maintaining mixture independence and compound independence), the RNEU is derived by relaxing the mixture independence and ROCL axioms (maintaining time neutrality and compound independence).

3.5 Bundling and "Rule Rationality"

A complementary "behavioral" perspective on ambiguity aversion is suggested by Halevy and Feltkamp [14]: if more than a single ball (bundle) may be drawn from each urn and the prize is determined as the sum (or average) of the correct bets, a decision maker who is averse to mean preserving spreads [28], will prefer a bet on the risky (first) urn to a bet on the ambiguous (second) urn[20]. Halevy and Feltkamp [14]

[19] Three other recent works generalize this recursive structure. Nau [25] allows for state dependent preferences; Chew and Sagi [5] study the possibility of maintaining probabilistic sophistication on separate domains while distinguishing between different sources of uncertainty, hence not being globally probabilistically sophisticated. Closely related to the current work is Ahn [1] - who does not impose an exogenous state space, and does not distinguish between subjective and objective uncertainty. As a result - he presents an axiomatic foundation for a representation similar to (11) where the interpretation of differentiating between sources of objective uncertainty emerges naturally.

[20] An alternative interpretation is of a decision maker who has to choose between two possible series of random outcomes: risky and ambiguous, and is constrained to decide ex-ante on a unique color to bet on in each series (that is, always has to bet on the same color).

claim that the behavior observed in the actual experiment (in which only one ball is drawn from each urn) may be a result of *rule rationality*: the criterion which prefers risk to ambiguity is appropriate in the environment of bundled risks, and since it is hard wired into the decision making process - it is applied to the standard experiment (in which the decision maker is actually indifferent between the urns).

To demonstrate the bundling rationale to ambiguity aversion, assume two draws with replacement from each urn. The payoff distributions from betting on either red or black in urns 1, 3 and 4 are given by:

$$\begin{array}{cccc} & L_{\mathbf{1(2)}} & L_{\mathbf{3(2)}} & L_{\mathbf{4(2)}} \\ \$\mathbf{2x} & 0.25 & 0.35 & 0.5 \\ \$\mathbf{x} & 0.5 & 0.3 & 0 \\ \$\mathbf{0} & 0.25 & 0.35 & 0.5 \end{array} \tag{15}$$

where $L_{i(2)}$ represents the random variable generated by 2 draws with replacement from urn i. These distributions are the result of averaging binomial distributions, using the second order probabilities. To be more specific, let k denote the number of red balls in urn i. Then the probability of drawing two red balls when betting on red is $\left(\frac{k}{10}\right)^2$. Averaging over $k = 0, 1, \ldots, 10$ using the respective second order probabilities for the different urns results in:

$$\begin{aligned} \Pr\left\{L_{1(2)} = 2x\right\} &= 1 \cdot \left(\frac{5}{10}\right)^2 = 0.25 \\ \Pr\left\{L_{3(2)} = 2x\right\} &= \sum_{k=0}^{10} \frac{1}{11} \left(\frac{k}{10}\right)^2 = 0.35 \\ \Pr\left\{L_{4(2)} = 2x\right\} &= \frac{1}{2} \cdot 1^2 + \frac{1}{2} \cdot 0^2 = 0.5 \end{aligned} \tag{16}$$

The decision maker may have any second order belief over the composition of the second (ambiguous) urn. As long as it is symmetric and not a degenerate distribution around five red balls, she will exhibit ambiguity aversion, that is - prefer to bet on the first urn rather the second urn. Furthermore, if the decision maker is averse to mean preserving spreads then, for any second order belief over the composition of the second urn, she will weakly prefer a bet on the second urn to a bet on the fourth urn. As a result, the predictions of the bundling theory coincide with an interpretation of the REU model (14) in which reduction of compound objective probabilities is violated. It is important to emphasize that the bundling rationale to ambiguity aversion does not depend on a distinction between objective and subjective second order probabilities (similarly to Segal [32]), and could explain why a decision maker exhibits smooth ambiguity aversion [21] or issue (source) preference/second order risk aversion [9] with second order objective probabilities.

4 Results

The 104 subjects who participated in the first round of the experiment were paid a total of 613 Canadian Dollars, which is about 5.9 Dollars on average. The 38 subjects in the robustness experiment were paid a total of $1,948 (about $51 on average). The analysis will concentrate on the reservation prices ($V1, V2, V3, V4$) set by the subjects[21]. Descriptive statistics are reported in Table 2[22]. The table reveals the

Table 2: Descriptive Statistics

	V1	V2	V3	V4	AVE(V1,...,V4)	STD(V1,...,V4)	Max(V1,...,V4)	Min(V1,...,V4)	V21	V43	V41	V31
First Round												
Mean	1.061	0.878	0.929	0.948	0.954	0.194	1.170	0.749	-0.183	0.018	-0.113	-0.132
SE	0.033	0.032	0.030	0.037	0.025	0.017	0.031	0.032	0.032	0.041	0.033	0.030
Median	1	0.99	0.99	1	0.989	0.158	1.045	0.8	-0.1	0	0	-0.025
Mode	1	1	1	1	1	0	1	1	0	0	0	0
SD	0.336	0.324	0.304	0.377	0.257	0.176	0.316	0.325	0.330	0.417	0.337	0.309
Kurtosis	1.397	0.933	1.646	1.215	0.946	0.888	0.968	-0.250	3.662	2.501	7.261	1.931
Skewness	0.036	-0.081	0.314	0.287	0.200	1.129	0.826	-0.393	-0.282	0.019	0.921	-0.122
Minimum	0.1	0	0.13	0.06	0.2	0	0.4	0	-1.26	-1.39	-1	-1
Maximum	2	1.83	2	2	1.725	0.739	2	1.5	1.2	1.4	1.6	1.02
Count	104	104	104	104	104	104	104	104	104	104	104	104
Robustness Round												
Mean	8.37	6.66	7.25	7.74	7.51	2.00	9.89	5.62	-1.70	0.50	-0.63	-1.12
SE	0.49	0.49	0.51	0.57	0.41	0.20	0.47	0.44	0.50	0.58	0.56	0.56
Median	10.00	6.00	6.00	9.00	7.50	1.95	10.00	5.00	-2	0	0	-1
Mode	10.00	5.00	5.00	10.00	7.50	0.00	10.00	5.00	-5*a	0	0	0
SD	3.02	3.00	3.13	3.54	2.51	1.24	2.89	2.69	3.08	3.56	3.42	3.47
Kurtosis	-0.18	-0.80	0.49	-0.12	-0.86	-0.61	0.65	-1.03	1.36	0.68	1.81	0.21
Skewness	-0.32	0.20	0.88	-0.09	-0.06	0.12	0.16	0.12	0.81	-0.26	-0.15	0.08
Minimum	2	1	2	2	2.75	0	4	1	-7	-8	-10	-10
Maximum	15	13	15	17	12	4.72	17	10	8	9	8	7
Count	38	38	38	38	38	38	38	38	38	38	38	38

a Multiple modes exist. The smallest value is shown

anticipated pattern in the aggregate on both rounds: the average reservation price set by the subjects for Urn 1 is higher than those set for the other urns, and the average

[21] Although data on age, gender, exposure to mathematics and economics courses and years of study were collected, none (except one which will be discussed below) of these variables seem to be related to the reservation prices in general and measures of ambiguity aversion, in particular.

[22] Vi is the reservation price set for the i^{th} urn. AVE, STD, MAX and MIN are the average, standard deviation, maximum and minimum respectively, in the four urns. $Vij = Vi - Vj$.

price set for the ambiguous urn (Urn 2) is the lowest. Moreover, in both samples the distribution of reservation prices for the first risky urn ($V1$) first order stochastically dominates the distribution of reservation prices for the ambiguous urn ($V2$)[23,24]. To statistically test whether the valuations of the urns are different, a Friedman test[25] is performed to test the null hypothesis that the four valuations came from the same distribution. In the first experiment, the Friedman test is $\chi^2(3, N = 104) = 29.55$, with $p < 0.001$, hence the hypothesis that the assignment of the valuations is random is rejected. A similar test that is performed on the valuations of urns 2, 3 and 4 (the ambiguous and the non-degenerate compound lotteries) could not reject the null hypothesis that the valuations of the three urns came from the same distribution ($\chi^2(2, N = 104) = 0.33$, $p = 0.847$[26]). In the robustness test, the non-parametric Friedman test rejects the null hypothesis that the four reservation prices came from the same distribution ($\chi^2(3, N = 38) = 13.7$, $p < 0.0033$). When comparing urns 2, 3 and 4 in the second round, the Friedman test is inconclusive ($\chi^2(2, N = 38) = 7.43$, $p < 0.0245$)[27]. To sum up both rounds: the reservation prices set for Urn 1 are significantly higher than the rest of the urns, and even FOSD the reservation prices for Urn 2.

The only important difference between the first round and the robustness round is the fact that the average reservation price for Urn 1 is lower than the expected value of the lottery - implying risk aversion. I believe it reflects not only increase in risk aversion as the stakes have increased [17], but the more careful design of the experiment: the sampling was more efficient (attracting less individuals who just wanted to earn the ten dollars or enjoyed the gambling aspect of the experiment);

[23] In both samples - $V1$ does not FOSD $V3$ or $V4$, and the latter two do not FOSD $V2$.

[24] If one studies the average reservation prices, a possible interpretation could be that the ranking is based on perceived simplicity of the lotteries (according to the order: 1, 4, 3, 2). However, as one studies response patterns (for examples, in Table 6 below) it is clear that this perception is not universal, and the patterns correspond closely to some of the theories tested. Moreover, it could be that "simplicity" is a complementary measure to the concepts of compound lotteries and ambiguity.

[25] A nonparametric test that compares several paired groups. The Friedman test first ranks the valuation for each subject from low to high (separately). It then sums the ranks for each urn. If the sums are very different, the test will tend to reject the null hypothesis that the valuations of different urns came from the same distribution.

[26] A parametric test, such as the repeated measures ANOVA, that allows for heterogeneity between subjects, could not reject the null hypothesis that the mean valuation of urns 2, 3 and 4 are equal, at 10% significance level.

[27] However, the parametric repeated measures ANOVA cannot reject this latter hypothesis at a significance level of 10%.

there was no framing in terms of “selling price” (that might cause a subject to state a higher reservation price[28]); the subjects were repeatedly prompted to find their *minimal* reservation price; and the operation of the BDM elicitation mechanism was better demonstrated. It seems that the effect was uniform across the urns (the average reservation price for all of the urns increased in about eight fold), hence the effect on the relative attractiveness of the urns was minimal. To sum up, I believe that even if $V1 > \mathbb{E}(L_1)$ (a frequent observation in the first round), it does not necessarily imply the subject is risk seeking [29]. Moreover, the factors that may have inflated $V1$ in the first round had similar effect on the other urns, hence the results that follow - which focus on the differences in the elicited values - continue to hold.

The random order treatment in the robustness round tests whether the higher reservation price for Urn 1 in the original sample is a consequence of it being a simple one stage objective lottery, or a consequence of Urn 1 being the first task the subject confronted in the original experiment, and in the following tasks the subject exhibited higher risk aversion, and hence lower reservation prices. The subjects were randomly treated with alternative orders of urns: (1,2,3,4), (2,3,4,1), (3,4,1,2) and (4,1,2,3)[30]. The only significant order effect found in the sample is that the urn which was the first task received a significantly *lower* reservation price than under alternative orders in which this urn was not the first (Friedman test's value of $\chi^2(3, N = 38) = 8.8$, $p = 0.032$.) This order effect seems to operate in an opposite direction to Harrison *et al* [15] who found that in late tasks people are more risk averse (lower reservation price.) Table 3 reports the average reservation price for each urn, as a function of its order. For example, ((V1,3rd);(V2,4th);(V3,1st);(V4,2nd)) correspond to the average reservation prices of the 10 subjects who were treated with the order (3,4,1,2). The conclusion from the order treatment is that the significantly higher reservation price for Urn 1 in the original experiment (in which setting the reservation price for Urn 1 was always the first task) could not be attributed to an order effect (indeed it persisted in the robustness test).

[28] The endowment effect may be responsible in part for the high reservation prices in the first round, although a recent study by Plott and Zeiler [26] found it to be insignificant, when sufficient controls were introduced (which may be the case in the second round).

[29] Furthermore, Keller *et al* [20] showed that even theoretically, when subject's preferences do not satisfy ROCL, the true certainty equivalent and the elicited value may lie on opposite sides of the expected value.

[30] Because of sample size limitations only four treatments were considered. These allowed each urn to be in every ordered place.

Table 3: Variation in the reservation prices as a function of order of urns.

	1st	2nd	3rd	4th
V1	7.28	8.90	7.35	10.00
V2	6.44	6.67	7.65	5.88
V3	5.55	6.77	8.56	8.20
V4	6.70	8.10	8.25	8.00
Average	**6.49**	**7.61**	**7.95**	**8.02**

The focus of the analysis in this work is to identify (possibly heterogeneous) patterns of choice in the subjects' population. Table 4 shows the relatively high positive correlation between the reservation prices set for the four urns in both rounds. This

Table 4: Rereservation prices' correlation matrices

First round

	V1	*V2*	*V3*	*V4*
V1	1			
V2	0.5011	1		
V3	0.54026	0.449509	1	
V4	0.557401	0.369145	0.266104	1

Robustness round

	V1	*V2*	*V3*	*V4*
V1	1			
V2	0.4787	1		
V3	0.3637	0.711	1	
V4	0.4645	0.5548	0.4353	1

positive correlation could be related to Ariely *et al* [3] "coherent arbitrariness": a subject may find it difficult to evaluate each urn separately, but easier to compare two or more lotteries. Fox and Tversky [10] compared valuations of risky (Urn 1) and ambiguous (Urn 2) lotteries, and found that when the subjects were not comparing the lotteries, the valuations where not significantly different. However, as argued above - this may be exactly the reflection of the "arbitrariness". Ellsberg type behavior exists especially when the decision maker compares an ambiguous lottery to a risky one. The environment in the current study is comparative, and is enriched by the existence of objective compound lotteries.

The difference between the prices set by the subjects for Urn 2 (ambiguous) and Urn 1 (one stage risky) - the (negative of) ambiguity premium - is used as a measure of ambiguity aversion. Therefore, if this variable is negative (positive) it implies ambiguity aversion (seeking). Similarly, the difference in the reservation prices set for urns 1, 3 and 4 (separately) measure the subject's attitude to (objective) second order risk. Since all of: $(V2 - V1)$, $(V3 - V1)$ and $(V4 - V1)$ are defined relative to $V1$, they will always be positively correlated. This observation does not apply to

$(V4 - V3)$, and therefore this will be the main variable on which the test of theories that do not abide by ROCL will be based[31].

4.1 Ambiguity Neutrality and Reduction

One of the main characteristics of the population of subjects is the strong association between ambiguity neutrality ($V1 = V2$) and reduction of compound objective lotteries ($V1 = V3 = V4$). This behavior is evident in both rounds, as described in Table 5[32].

Table 5: The association between ambiguity neutrality and ROCL

First round

			V1=V3=V4 No	V1=V3=V4 Yes	Total
V1=V2	No	Count	80	1	81
		Expected	66.98	14.02	
	Yes	Count	6	17	23
		Expected	19.02	3.98	
Total			86	18	104

	Exact Sig. (2-sided)
Fisher's Exact Test	1.24E-13

Robustness round

			V1=V3=V4 No	V1=V3=V4 Yes	Total
V1=V2	No	Count	33	0	33
		Expected	28.66	4.34	
	Yes	Count	0	5	5
		Expected	4.34	0.66	
Total			33	5	38

	Exact Sig. (2-sided)
Fisher's Exact Test	1.99E-06

In the original sample: 18 subjects set $V1 = V3 = V4$, and more than 94% of them (17 subjects) asked for no ambiguity premium (set $V2 = V1$). This is more than four times the expected frequency under a null hypothesis of independence. Out of the 86 subjects that did not abide by ROCL, only 6 were ambiguity neutral (less than one third of the expected frequency under the null hypothesis of independence). In the scaled sample: 5 subjects set $V1 = V3 = V4$ and *all* 5 of them set $V2 = V1$ (more than seven times the expected frequency under independence). Out of 33 subjects who did not abide by ROCL, *none* was ambiguity neutral (compared to expected frequency of 4.34 under independence). In the first round, there was a group of 13 subjects who set prices of \$1 for all four urns, and in the second round this group consisted of four subjects. These subjects are responsible for a substantial part of the association. We can only speculate what are these subjects' preferences: it could be that \$1 (\$10 in the second round) is a focal point. Alternatively, it could be that these subjects are expected value maximizers. Some indication is given by the fact that

[31] Note that the use of these measures imply some cardinality, but since I would like to quantify ambiguity aversion, this cardinality is necessary.

[32] Table 1 in the Introduction aggregates the information in the two tables reported here.

taking at least one advanced (second year or higher) mathematics course increases the probability of choosing (1,1,1,1) from 10% to 21% in the first round. Similarly, the conditional probability in the second round of being an expected value maximizer increases from 6% to 40% when controlling for taking advanced mathematics course.

The conclusion derived from Table 5 is that there is a very tight association between ambiguity neutrality $\{V2 = V1\}$ and reduction of compound lotteries, that is: $\{V1 = V3 = V4\}$[33]. Therefore, a descriptive theory that accounts for ambiguity aversion, should account - at the same time - for violation of reduction of compound *objective* lotteries.

4.2 Attitude towards mean preserving spreads in probabilities and ambiguity

As discussed in Section 3, alternative theories that can account for non-neutral attitude towards ambiguity by relaxing ROCL in objective probabilities (recursive non-expected utility and bundling/possible interpretation of recursive expected utility), have different predictions on the relative attractiveness of urns 1, 3 and 4. Furthermore, their predictions on the sign of the correlation between ambiguity premium (as measured by $V21 = V2 - V1$) and the premium to dispersion in the second order distribution (as measured by $V43 = V4 - V3$) differ.

If one looks at the "average" subject who does not satisfy reduction of compound lotteries[34] it seems that this latter correlation is very weak[35]. However, the data clearly exhibit different patterns of reservation prices for urns 1, 3 and 4 - that conform to the two alternative models. Therefore, the absence of significant correlation between ambiguity premium ($V21$) and the premium to dispersion in the second order objective probability ($V43$) may be a result of averaging the two sets of subjects

[33] In the first sample: in addition to the 13 subjects who are expected value maximizers, two subjects' reservation prices are equal and higher than 1, and two subjects' reservation prices are equal and smaller than 1.
In the scaled sample: in addition to the 4 expected value subjects, one subject set all reservation prices to \$5.

[34] That is, the subset of 83 subjects in the original experiment and 33 subjecs in the robustness experiment who do not conform to SEU or MEU (3 subjects who set reservation prices within 2 cents away from the expected value predictions are included in the set of 21 subjects who satisfy the ROCL in the original sample.)

[35] In the first sample: the Pearson correlation is -0.1 ($p = 0.35$), and Spearman's ρ is -0.07 and insignificantly different from zero. Similar "average" behavior is exhibited in the scaled sample: the Pearson correlation is -0.2 ($p = 0.24$) and the Spearman ρ is -0.23 ($p = 0.18$).

(that conform to bundling/REU and RNEU), which exhibit approximately opposite correlations between the two variables. To test whether the data is consistent with this explanation, it is necessary first to classify the subjects. Second, to test whether the classification is internally consistent[36]. Third, to test whether ambiguity aversion, as measured by the ambiguity premium, could be accounted for using these theories. The objectivity of this methodology relies crucially on the fact that the first two stages do not use any information containing $V2$, but rely solely on subjects' preferences over mean preserving spreads of the objective second order distribution (that is: urns 1, 3 and 4 only). The third step is a test of whether the classification produces the correlations between $V21$ and $V43$ predicted by the theories.

Ranking of urns 1, 3 and 4 may exhibit 13 possible ordinal ranking schemes. The classification is based on the following criteria:

- If $V1 = V3 = V4$ the subject reduces compound objective lotteries, and therefore consistent with Subjective Expected Utility [30] or Maxmin Expected Utility [12] / Choque Expected Utility [31].
- If $V1 \geq V3 \geq V4$ or $V1 \leq V3 \leq V4$ (where at least one of the inequalities is strict) the preferences are consistent with the the bundling rationale [14] or the Recursive Expected Utility model [1, 9, 21].
- If $V1 = V4 \neq V3$ then the subject is consistent with the Recursive Non-Expected Utility model of Segal [32].

This classification leaves four ordinal ranking[37] (which include 36 and 11 subjects in the original and scaled samples, respectively) that are not consistent with the above theories. Acknowledging possible noise/error/randomness in assigning reservation prices allows to classify these subjects. It captures human error in assigning reservation prices using the graphical interface of the experiment, may result from the use of the BDM mechanism, from difficulties understanding the mechanism, and other sources. A possible avenue to model this randomness may be by providing a random utility model as in Gul and Pesendorfer [13][38]. They provide an axiomatic foundation

[36] Especially in the case of Segal [32]: does $\mathbb{E}\left(V4 | V1\right) = V1$ hold?

[37] That is: $V3 < V4 < V1$, $V1 < V4 < V3$, $V3 < V1 < V4$ and $V4 < V1 < V3$

[38] The decision maker's behavior of choosing from finite menus is described by a random choice rule, which assigns to each possible menu a probability distribution over feasible choices. A random utility function is a probability measure on some set of utility functions. The random choice rule

that allows a representation of choices by a random expected utility function. Here, however, the space of one stage lotteries is replaced by compound lotteries, and the axioms (especially linearity, which is comparable to the standard independence axiom) have to be modified. This is an important and challenging task, which is beyond the scope of the current paper. An intermediate solution can be adopted from the logistic choice literature (see Anderson *et al* [2].) For an urn L_i with possible prize of $\$x$, let the element of choice be the reservation price - a number in the interval $[0, x]$. For every $t \in [0, x]$ let $U_k(L_i, t)$ be the utility of the decision maker whose preference correspond to theory k (one of those discussed in Section 3) of choosing reservation price t for urn $i = 1, \ldots, 4$. Unlike [13], this approach assumes a possible error in comparing utility of different reservation prices, that influences the probability of choosing the number (reservation price) with the highest utility. The density of choosing a reservation price t for urn i if the decision maker's preferences are described by theory k is:

$$f_k(L_i, t) = \frac{\exp\left(U_k(L_i, t)/\xi\right)}{\int_0^x \exp\left(U_k(L_i, s)/\xi\right) ds} \tag{17}$$

where ξ is an error parameter that determines the importance of an error term (which in this case is logistic). A small ξ implies that the choices of reservation prices are close to the one predicted by the respective theory, k. For example, low ξ for a decision maker described by the RNEU model of Segal [32] implies that $V1$ would be relative close to $V4$, while if the decision maker's preferences originate in the bundling rationale (or described by the REU model) the difference between the two would be relatively large. The goal in classifying the remaining subjects is to choose, for every subject, the theory that is consistent with the lowest ξ. To better understand how allowing for noise allows to classify the subjects, consider an observation of $V3 < V1 < V4$: it may belong to a decision maker described by Segal [32] who, without error, had a ranking of $V3 < V1 = V4$, or to an agent described by the bundling rationale (whose preferences may be represented by the REU) whose "before noise" ranking is $V1 < V3 < V4$. To separate between the alternative explanations, attention is focused on the relative reservation prices of urns 1 and 4 for low values of ξ: under Segal's theory a decision maker is indifferent between urns 1 and 4, hence we would expect a small cardinal difference between the valuations of the two

maximizes the random utility function if for every possible menu, the random choice rule coincides with the probability that the random utility function attains its maximum on the corresponding alternatives.

urns. Under the bundling rationale and REU, the cardinal difference between the evaluations of urns 1 and 4 should be relatively larger. How "small" or "large" is the cardinal differentiation will be measured by the absolute difference between $V1$ and $V4$, normalized by the standard deviation of the reservation prices determined in urns 1, 3 and 4.

Following the above principle subjects with the patterns of $V3 < V4 < V1$, $V1 < V4 < V3$, and $V3 < V1 < V4$ - that exhibited uniformly (in the original sample) low values of $|V41| / STD_{V1,V3,V4}$ - were classified as recursive non-expected utility (Segal [32]). The group of subjects with the pattern of $V4 < V1 < V3$ exhibited higher variability with respect to this measure and therefore three of them were classified as RNEU while the rest were classified as bundling [14]/REU [21, 9, 1] subjects[39]. It is important to note that the classification method used, did not employ any information on the reservation prices for the ambiguous (second) urn.

The partition is reported in Table 6. It results in 21 (5 in the scaled sample)

Table 6: Classification summary

	$2 Sample						**$20 Sample**					
Theory	***# of obs***	***V1***	***V2***	***V3***	***V4***	***corr(V21,V43)***	***# of obs***	***V1***	***V2***	***V3***	***V4***	***corr(V21,V43)***
Rule/REU	42	1.04	0.78	0.96	0.79	0.447	16	7.66	5.34	6.36	6.47	0.592
Consistent[1]	**31**	**1.01**	**0.84**	**0.90**	**0.73**	**0.644**	**12**	**7.88**	**5.33**	**5.83**	**5.50**	**0.571**
Optimist[2]	1	1.10	1.20	1.00	1.00		0					
MREU[3]	7	1.19	0.62	1.23	0.89		1	13	9.5	14	10	
Inconsistent[4]	3	0.89	0.39	0.92	1.14		3	5	4	5.92	9.17	
RNEU	41	1.11	0.91	0.85	1.07	-0.620	17	8.85	7.22	7.57	8.57	-0.771
Consistent[1]	**32**	**1.17**	**0.86**	**0.85**	**1.12**	**-0.735**	**15**	**9.37**	**7.12**	**7.58**	**8.98**	**-0.886**
Optimist	7 (for 6[2])	0.89	1.17	0.75	0.87		1	2	10	5	3	
Inconsistent[5]	2	0.92	0.77	1.27	0.92		1	8	6	10	8	
SEU	**20**	**1.03**	**1.03**	**1.03**	**1.03**		**5**	9	9	9	9	
Expected Value	13	1	1	1	1		4	10	10	10	10	
EV with noise[6]	3	0.99	1.00	0.99	0.99		0					
Risk Averse[7]	2	0.90	0.90	0.90	0.90		1	5	5	5	5	
Risk Seeking[8]	2	1.40	1.40	1.40	1.40		0					
MEU[9]	**1**	**0.90**	**0.80**	**0.90**	**0.90**		**0**					
Total	104	1.06	0.88	0.93	0.95	-0.104	38	8.37	6.66	7.25	7.74	-0.211

[1] includes subjects averse or seeking MPS in the second order objective distribution
[2] averse to MPS in the second order distribution but *V2>V1*
[3] Maxmin REU: V*2<V4<V1* [4] REU-inconsistent: V*2<V1<V3<V4* [5] RNEU-inconsistent: V*2<V1=V4<V3*
[6] within 1-2 cents of the EV predictions [7] V*i<EV* for *i=1,...,4* [8] V*i>EV* for *i=1,...,4* [9] *V2<V1=V3=V4*

subjects who reduce compound lotteries, 20 (all 5) of them are consistent with the

[39] Similar partition was performed in the scaled sample: 8 subjected were classified as RNEU ($U3 < U4 < U1$, $U1 < U4 < U3$ and 2 observations with $U4 < U1 < U3$) and 3 subjects were classified as bundling/REU (a single observation with $U3 < U1 < U4$ and two observations with $U4 < U1 < U3$).

subjective expected utility model[40], 1 (no) subject that corresponds to the maxmin expected utility predictions, 41 (17) subjects that correspond the recursive non-expected utility model of Segal [32], and 42 (16) subjects that correspond to the bundling rationale of Halevy and Feltkamp [14] and to the interpretation of the recursive expected utility model (Klibanoff *et al* [21], Ergin and Gul [9], Ahn [1]) in which the decision maker does not reduce compound objective lotteries.

4.2.1 Cardinal Analysis

The current section tests whether the classification method presented in Table 6 is internally consistent, and does it account for ambiguity attitudes in a way that is consistent with the theories.

Recursive Non-Expected Utility (Segal [32]) The average reservation prices for urns 1 and 4 in this sub-sample of subjects are very close ($1.11 and $1.07 respectively in the original sample, and $8.85 and $8.57 in the scaled sample). Non-parametric (Friedman) and parametric (repeated measures ANOVA) tests cannot reject the null hypothesis that the two series came from the same underlying distribution [41]. However, the theory imposes a stricter restriction on the series: differences between $V1$ and $V4$ should be due only to non-systematic noise/error. In other words, the expected value of $V4$ conditional on $V1$ should be $V1$. The model estimated is:

$$V4 = \alpha + \beta V1 + \varepsilon \tag{18}$$

and the null composite hypothesis tested is therefore: $\alpha = 0$ and $\beta = 1$. The value of the $F(2, 39)$ statistic is 1.02 ($p = 0.37$), hence the hypothesis cannot be rejected, and this group is internally consistent with the theoretical predictions of the RNEU model of Segal [32]. The group of the RNEU subjects in the robustness sample exhibits similar consistency ($F(2, 15) = 0.83$, $p = 0.45$).

Once internal consistency of the RNEU group is established, the focus shifts to

[40] As noted above, this includes: 13 subjects who behaved as expected value maximizers, 3 subjects within 2 cents of the theoretical prediction of the expected value model, two subjects who set the four reservation prices higher than $1 ($1.3 and $1.5) and two subjects who set the four reservation prices lower than $1 ($0.8 and $0.99). In the sclaed sample 4 subjects were expected value maximizers and one subject set all reservation prices to $5.

[41] P-values of the Friedman test is 0.34 in the first sample (0.16 in the second sample) and of the repeated measures ANOVA is 0.27 in the first sample (0.31 in the robustness sample.)

the predictive behavior of the theory on ambiguity aversion. Out of 41 subjects in the first sample who were classified to this group based on their reservation prices for the three objective urns (see Table 6), two subjects are inconsistent with the theoretical predictions of the model concerning ambiguity aversion, since their reservation prices satisfy: $V2 < V1 = V4 < V3$[42]. Seven more subjects in the original sample hold "optimistic" beliefs over the composition of the second urn, when six of them exhibited $V3 < \min\{V1, V4\} \leq \max\{V1, V4\} \leq V2$[43]. Although those subjects are consistent with the RNEU model - since the theory does not impose restrictions on the decision maker's beliefs over the ambiguous urn (the prior may not be symmetric around 5 red and 5 black balls) - they were removed from the quantitative analysis. Some support to a view that this pattern of reservation price is due to "mistake" or "carelessness" may be found in the fact that when the payoffs were scaled up by a factor of 10, this pattern of optimistic choice almost disappeared. The prediction of the RNEU model - that the ambiguity premium ($-V21$) and the premium for mean preserving spread in the second order distribution ($-V43$) are negatively correlated - is tested on the remaining 32 subjects. The correlation between these two sequences is -0.735 and is significantly different from zero ($p = 1.62 * 10^{-6}$), conforming the theory's explanation of ambiguity aversion within this subset of subjects. In the robustness test this correlation is -0.886 and significantly different from zero ($p < 0.00025$). Appendix E.1 further quantifies the relation between ambiguity premium and the premium to mean preserving spread in the second order probability within this population, via a simple regression of $V21$ on $V31$ or $V43$. The results are consistent with the theoretical predictions of the RNEU model (in both the original and the scaled sample): there is a strong association between subjects' attitude to urn 3 (relative to urns 1 or 4) and their ambiguity premium (urn 2 relative to urns 1 or 4).

Bundling (Rule Rationality)/Recursive Expected Utility The bundling rationale [14] and the interpretation of the recursive expected utility model [1, 9, 21] (which is consistent with violation of reduction in objective compound lotteries) impose several restrictions (14) on the ordinal ranking of the ambiguous urn. Out of the 42 subjects in the $2 sample that belong to this group (based on their prefer-

[42] That is, they like mean preserving spreads in the objective second order distribution, but are ambiguity averse. In the robustness sample, only one subject exhibited this inconsistency.

[43] One more subject (who set $V3 > \max\{V1, V4\}$) set a reservation price for the ambiguous urn that was seven times higher than Urn 1.

ences over two-stage objective lotteries), 10 subjects do not satisfy these restrictions (see Table 6). Three subjects exhibit the following pattern of reservation prices - $V2 < V1 \leq V3 \leq V4$ (where one of the two right inequalities is strict): that is, although they seem to set monotonically higher reservation prices for urns with higher dispersion in the second order objective distribution, they dislike ambiguity. Seven other subjects set: $V2 < V4 < V1$, violating the restriction that under the bundling rationale/REU, if a subject is averse to mean preserving spread in the second order distribution, the valuation of the ambiguous urn is bounded from below by the valuation of Urn 4. These subjects' valuation of the ambiguous urn, has the "pessimistic" flavour of the maxmin expected utility model, or, alternatively their preferences are based on other criteria as "simplicity". One additional subject in the $2 sample had an "optimistic" valuation of Urn 2: $V2 > \max\{V1, V4\}$. As noted for the RNEU model, this pattern does not contradict (14), since the decision maker may hold belief over the composition of the ambiguous urn that are not symmetric around 5 red and 5 black balls. This leaves 31 subjects (out of the 42 subjects), with valuation of the ambiguous that are consistent with the theoretical predictions of the bundling/REU model. In the robustness sample 4 out of 16 subjects are not consistent with the theoretical predictions of the bundling/REU model (see Table 6.) Among the remaining 31 and 12 subjects (in the $2 and $20 samples, respectively), the correlation between $V43$ (the premium to mean preserving spread in the second order distribution) and $V21$ (the ambiguity premium) is positive and significantly different from zero. The Pearson correlations are 0.644 and 0.571 respectively, and are statistically different from zero ($p < 0.0001$ and $p = 0.053$, respectively)[44]. Appendix E.2 further quantifies the relation between reduction of compound objective probabilities and ambiguity aversion, using linear regression of $V21$ on $V31$ and $V43$ (or their sum - $V41$). The ambiguity premium is a positive function of the premium to mean preserving spread in the second order probability: from Urn 1 to Urn 3, and from Urn 3 to Urn 4. It is important to note that even when controlling for the former ($V31$), the latter's ($V43$) effect on the ambiguity premium ($V21$) is positive and significant ($p < 0.0001$ in both samples). The regression results for this group indicate that, similarly to the subjects who belong to Segal's group, those subjects identify the ambiguous urn with an urn that has non-degenerate second order distribution. They assign *subjective*

[44] The rank-based Spearman's ρ are 0.62 for the original sample and 0.514 for the scaled sample ($p = 0.0002$ and $p = 0.08$, respectively.)

second order belief over the composition of the second urn, that could be anything between the first urn (degenerate second order belief) and the fourth urn (extreme second order belief). Their ambiguity preferences is therefore associated with their preferences over compound lotteries like urns 3 and 4.

5 Conclusions

The experimental design used in this study allows to relate ambiguity attitudes of individuals to their attitudes towards compound objective lotteries. This design allows a clear empirical test of theories that model ambiguity, while assuming reduction of compound objective lotteries (Schmeidler [31]; Gilboa and Schmeidler [12]). Furthermore, theories that model ambiguity aversion as a phenomenon that is associated with a violation of reduction of compound lotteries (Segal [32, 34], Halevy and Feltkamp [14], Ergin and Gul [9], Klibanoff *et al* [21], Ahn [1]) are evaluated empirically based on their predicted pattern of preference among objective lotteries with varying amount of dispersion in the second order probability.

The results reveal a tight association between ambiguity neutrality and reduction of compound *objective* lotteries, consistent with the subjective expected utility model: subjects who reduced compound lotteries were almost always ambiguity neutral, and most subjects who were ambiguity neutral reduced compound lotteries appropriately (15-20% of the subjects). The reminder of the subjects exhibit violation of ROCL and ambiguity aversion, but there is no unique theory that can accommodate the different choice patterns in the population. The population is heterogeneous, and two choice patterns, which account for approximately 70% of all subjects, emerge. In particular, about half (35%) exhibit ambiguity aversion (seeking) together with aversion (love) to mean preserving spreads in the second order distribution. These preferences can be traced back to a "rational rule", which originates in an environment of choice among bundles of lotteries [14], and are consistent with an interpretation of the recursive expected utility models [1, 9, 21] that allows a decision maker not to reduce (or differentiate) different sources of objective risk. The other half (35%) of the subjects exhibit pattern of preferences consistent with Segal's [32, 34] theory of recursive non-expected utility, where the decision maker evaluates two-stage lotteries (including ambiguous lotteries) using, recursively, rank dependent utility.

The findings point to the fact that currently there is no unique theoretical model that universally captures ambiguity preferences. In this sense the current work confirms Epstein's [8] approach of defining ambiguity aversion as a behavior which is not probabilistically sophisticated, without committing to a specific functional model. However, the results suggest that not reducing compound (objective) lotteries is the underlying factor of the Ellsberg paradox.

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A Consent Form - First Round

Principal Investigator:

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Purpose:

Ambiguity is characterized by a situation in which decision is made when consequences of actions are uncertain, and it is hard to describe them in a simple probabilistic form. The purpose of this study is to compare different explanations of individual's behaviour in these situations.

Study Procedures:

You will be offered to bet in four different situations, and allowed to set a minimal selling price for each bet. The selling price you set should reflect your true valuation for each bet, at which you will agree to exchange the bet for a certain payment. The selling mechanism used guarantees you cannot profit by misreporting your valuations. All randomizations are performed using a computerized random number generator. The experiment takes up to 15 minutes. Your compensation will be random (see below).

Confidentiality:

Any information resulting from this research study will be kept strictly confidential. Participants will not be identified by name in any reports of the completed study. Access to data records that are kept on a computer hard disk, and will require a password.

Remuneration/Compensation:

In order to defray the costs of your participation, you will receive the right to participate in four lotteries. For each lottery you win, you will be paid \$2. You can potentially win 4 lotteries. Please note that the final payment you receive, will depend on the selling prices you state, the buying prices offered and the outcomes of the lotteries.

Contact:

If you have any questions or desire further information with respect to this study, you may contact Yoram Halevy at 604-822-2202.
If you have any concerns about your treatment or rights as a research subject you may contact the Director of Research Services at the University of British Columbia at 604-822-8598.

Consent:

I understand that my participation in this study is entirely voluntary and that I may refuse to participate or withdraw from the study at any time without jeopardy to my class standing etc.

I have received a copy of this consent form for my own records.

I consent to participate in this study.

____

Subject Signature Date

Please tell us about yourself

This information is required for us to identify you for payment purposes, and will be kept confidential.

Name: ____________________________

E-mail Address: ____________________________

Student ID: ______________ Age: _______ Gender: M F

Number of years of university completed: 0 1 2 3 4+

Major ______________

Number of 200+ courses in : Economics _______ Mathematics _______

Consider the following scenario. There are 4 boxes, each containing 10 balls, which can be either red or black. The composition of balls in the boxes is as follows:

Box 1: Contains 5 red balls and 5 black balls.

Box 2: The number of red and black balls is unknown, it could be any number between 0 red balls (and 10 black balls) to 10 red balls (and 0 black balls).

Box 3: The number of red and black balls is determined as follows. One ticket is drawn from a bag containing 11 tickets with the numbers 0 to 10 written on them. The number written on the drawn ticket will determine the number of red balls in the third box. For example, if the ticket drawn is 3, then there will be 3 red balls and 7 black balls.

Box 4: The composition of balls in this box is determined in a similar way to box 3. The difference is that instead of 11 tickets in the bag, there are 2, with the numbers 0 and 10 written on them. Therefore, the box may contain either 0 red balls (and 10 black balls) or 10 red balls (and 0 black balls).

You are asked to place a bet on the colour of the ball drawn from each box (note that for boxes 3 and 4, you do not know what ticket is drawn from the bag when you place your bet). If your bet on a specific box is correct, you could win \$2. If your bet is incorrect, nothing will happen. For example, if your bets on boxes 1 and 3 are correct, but your bets on boxes 2 and 4 are incorrect, you will win a total of \$4.

Before balls are drawn from each box (and before the tickets are drawn from the bags for boxes 3 and 4), you may sell each one of your bets. You are asked to state 4 **minimal** prices at which you are willing to sell each one of the bets. For each box, a random number between \$0 and \$2 will be generated. The 4 random numbers will be the buying prices for each one of the bets.

If the buying price for a box is higher than the minimal selling price you stated for that box, you will be paid the buying price (and will not have to wait for the outcome of your bet). However, if the buying price for the box is lower than the minimal selling price you stated for that box, your payment will depend on the outcome of your bet.

Note that it is in your best interest not to overstate your selling prices since this lowers the chances you will be able to sell your bet, and does not increase the buying price.

Likewise, it is in your best interest not to understate your selling price, since this may force you to sell a bet at a price that is lower than your valuation of the bet.

For example, suppose you want to sell a \$1 coin you have. Clearly - its value is exactly \$1. If you state a selling price higher than \$1 (say \$1.50) you might not be able to sell it even if the buying price is as high as \$1.49 - a profitable transaction. Likewise, if you state a selling price lower than \$1 (say \$0.75), you might be forced to sell your coin at a loss (if the buying price is between 75 and 99 cents). The only way you are sure not to lose, is if you state a selling price of exactly your valuation (\$1 in this case).

It is important that your stated selling prices will reflect how attractive each bet is: the more attractive it is for you to participate in a bet, the higher the selling price you should state.

B Consent Form - Robustness Round

Comment: This is the consent form for the order treatment: (1,2,3,4).

Principal Investigator:

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Phone: 604-822-2202
E-mail: yhalevy@interchange.ubc.ca

Purpose:

Ambiguity is characterized by a situation in which decision is made when consequences of actions are uncertain, and it is hard to describe them in a simple probabilistic form. The purpose of this study is to compare different explanations of individual's behaviour in these situations.

Study Procedures:

You will be offered to bet in four different situations, and allowed to set a minimal reservation price for each bet. The reservation price you set should reflect your true valuation for each bet: this is the **minimal** amount of sure payment you will agree to exchange for the bet. The elicitation mechanism used guarantees you cannot gain by setting a reservation price which is higher or lower than the minimal reservation price above. All randomizations are performed using a computerized random number generator. The experiment takes up to 15 minutes. Your compensation will be random (see below).

Confidentiality:

Any information resulting from this research study will be kept strictly confidential. Participants will not be identified by name in any reports of the completed study. Access to data records that are kept on a computer hard disk will require a password.

Remuneration/Compensation:

In order to defray the costs of your participation, you will receive the right to participate in four lotteries with a potential prize of $20 in each, and offered to substitute a sure payment for each lottery. Please note that the final payment you receive, will depend on the minimal reservation prices you set, random numbers and the outcomes of the lotteries.

Contact:

If you have any questions or desire further information with respect to this study, you may contact Yoram Halevy at 604-822-2202.
If you have any concerns about your treatment or rights as a research subject you may contact the Research Subject Information Line in the UBC Office of Research Services at 604-822-8598.

Consent:

I understand that my participation in this study is entirely voluntary and that I may refuse to participate or withdraw from the study at any time without jeopardy to my class standing etc.

I have received a copy of this consent form for my own records.

I consent to participate in this study.

Subject Name Subject Signature Date

Explanation:

Consider the following scenario. There are 4 boxes, each containing 10 chips, which can be either red or black. The composition of chips in the boxes is as follows:

Box 1: Contains 5 red chips and 5 black chips.

Box 2: The number of red and black chips is unknown, it could be any number between 0 red chips (and 10 black chips) to 10 red chips (and 0 black chips).

Box 3: The number of red and black chips is determined as follows: one ticket is chosen from a bag containing 11 tickets with the numbers 0 to 10 written on them. The number written on the drawn ticket will determine the number of red chips in the third box. For example, if the ticket drawn is 3, then there will be 3 red chips and 7 black chips.

Box 4: The composition of chips in this box is determined in a similar way to box 3, but instead of 11 tickets in the bag, there are 2, with the numbers 0 and 10 written on them. Therefore, the box may contain either 0 red chips (and 10 black chips) or 10 red chips (and 0 black chips).

You are asked to place a bet on the colour of the ball drawn from each box. Note that for boxes 3 and 4, you do not know the colour composition of the box (what ticket is drawn from the bag) when you place your bet. If your bet on a specific box is correct, you could win $20. If your bet is incorrect, nothing will happen. For example, if your bets on boxes 1 and 3 are correct, but your bets on boxes 2 and 4 are incorrect, you will win a total of $40.

Before chips are drawn from each box (and before the tickets are drawn from the bags for boxes 3 and 4), you are asked to set **4 minimal** amounts of money you are willing to substitute for each bet. These will be called your "reservation prices."

For each box, a random number between $0 and $20 will be generated.

If the random number for a box is higher than the reservation price you set for that box, you will be paid the random number (and your payment will not depend on the outcome of your bet). However, if the random number for a box is lower than the reservation price you stated for that box, your payment will depend on the outcome of your bet.

It is in your best interest not to overstate your reservation price since this lowers the chances you will substitute a high random number for the lottery.
Likewise, it is in your best interest not to understate your reservation price, since the random number might be lower than your actual reservation price, forcing you to substitute the low random number for the bet.
For example, suppose you want to set your reservation price for a \$5 bill you have. Clearly - its value is exactly \$5. If you state a reservation price higher than \$5 (say \$8) you will not be able to substitute a high random number (between \$5.01 and \$7.99) for the bill, resulting in losing a profitable transaction. Likewise, if you state a reservation price lower than \$5 (say \$4), and the random number is between \$4.01 and \$4.99, you might be forced to substitute the \$5 for a lesser amount. The only way you are sure not to lose (potential gains or portion of the \$5), is if you set a reservation price that exactly equals your valuation (\$5 in this case).

Before you will be asked to set your minimum reservation prices for the 4 lotteries, you will be given a 2 dollars coin and a pen. You are asked to set your minimum reservation prices for these items. The random number for each of them will be between \$0 and \$4.
Then, you will be given an (unpaid) trial round with the 4 boxes, and then a paid round with the 4 boxes.

Please tell us about yourself

This information is required for us to identify you for payment purposes, and will be kept confidential.

Name: ______________________________

E-mail Address: ______________________________

Student ID: _______________ Age: ________ Gender: M F

Number of years of university completed: 0 1 2 3 4+

Major _______________

Number of 100 level courses in : Economics _____ Mathematics _____

Number of 200+ level courses in : Economics _____ Mathematics _____

Please answer the following questions:

- What is your minimum reservation price for the \$2 coin? _______________ (between 0 and 4)

  Random number _______________

- What is your minimum reservation price for the pen? _______________ (between 0 and 4)

  Random number _______________

Trial Round:

1. What is your bet for box 1? (circle one) red black

   What is your minimal reservation price for the bet on box ? _______________ (between 0 and 20)

   Random number _______________

   Outcome if Random Number<Reservation Price: _______________

2. What is your bet for box 2? (circle one) red black

   What is your minimal reservation price for the bet on box ? _______________ (between 0 and 20)

   Random number _______________

   Outcome if Random Number<Reservation Price: _______________

3. What is your bet for box 3? (circle one) red black

   What is your minimal reservation price for the bet on box ? _______________ (between 0 and 20)

   Random number _______________

   Outcome if Random Number<Reservation Price: _______________

4. What is your bet for box 4? (circle one) red black

   What is your minimal reservation price for the bet on box ? ______________ (between 0 and 20)

   Random number ______________

   Outcome if Random Number<Reservation Price: ______________

Paid Round:

1. What is your bet for box 1? (circle one) red black

   What is your minimal reservation price for the bet on box ? ______________ (between 0 and 20)

   Random number ______________

   Outcome if Random Number<Reservation Price: ______________

2. What is your bet for box 2? (circle one) red black

   What is your minimal reservation price for the bet on box ? ______________ (between 0 and 20)

   Random number ______________

   Outcome if Random Number<Reservation Price: ______________

3. What is your bet for box 3? (circle one) red black

   What is your minimal reservation price for the bet on box ? ______________ (between 0 and 20)

   Random number ______________

   Outcome if Random Number<Reservation Price: ______________

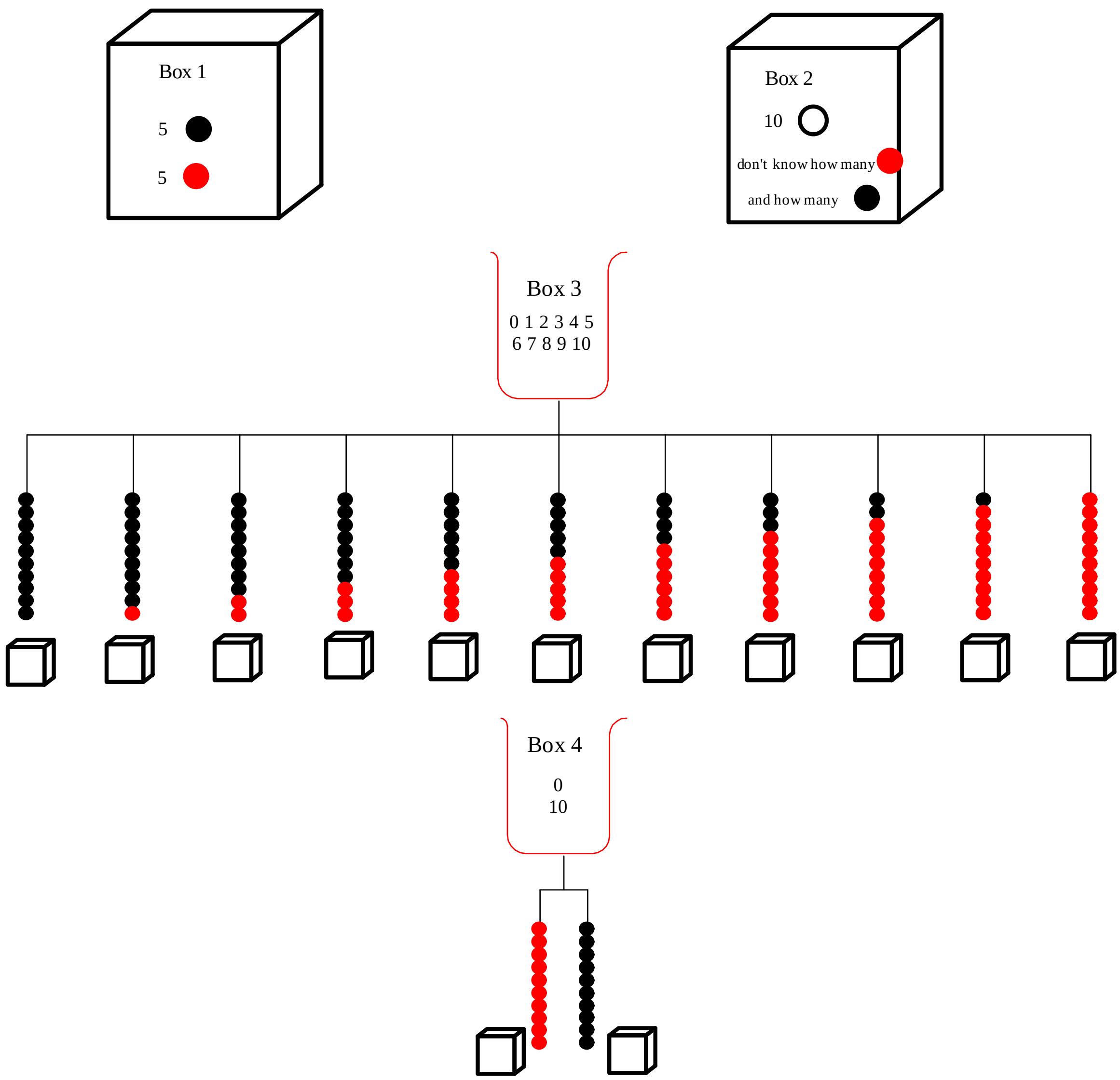
4. What is your bet for box 4? (circle one) red black

   What is your minimal reservation price for the bet on box ? ______________ (between 0 and 20)

   Random number ______________

   Outcome if Random Number<Reservation Price: ______________

C The Lotteries



D Data

$2 Round

Index	V1	V2	V3	V4	Index	V1	V2	V3	V4
1	0.90	0.80	0.90	0.90	53	1.00	0.73	0.80	0.67
2	1.20	0.90	0.80	0.80	54	1.00	0.50	0.80	2.00
3	1.00	1.00	0.98	1.00	55	1.01	0.42	0.69	0.90
4	1.26	0.58	1.00	1.00	56	2.00	1.50	1.20	1.00
5	1.00	1.00	1.00	1.00	57	0.66	0.84	0.37	0.66
6	1.00	0.60	0.80	0.40	58	1.20	1.20	1.20	0.90
7	1.00	1.00	1.00	0.79	59	1.37	0.80	1.43	1.19
8	1.00	0.80	0.80	1.00	60	1.00	0.81	0.79	0.20
9	0.75	0.75	0.75	0.90	61	1.22	1.20	1.10	1.08
10	1.80	1.80	1.50	1.80	62	0.99	0.90	1.41	0.76
11	1.00	1.00	1.00	1.00	64*	0.70	0.86	0.51	0.70
12	1.23	0.86	0.71	1.39	65	1.00	1.00	1.00	1.00
13	1.25	1.00	0.80	1.00	66	1.00	1.00	1.00	1.00
14	1.20	0.99	0.88	0.79	67	0.99	0.88	1.24	0.52
15	1.25	1.00	0.85	1.00	68	1.60	0.94	0.96	1.60
16	0.99	1.00	0.90	1.00	69	1.50	1.00	0.99	1.30
17	1.00	0.74	0.88	0.71	70	0.90	0.73	0.77	0.77
18	1.29	1.16	1.00	1.00	71	1.00	0.51	0.13	1.00
19	1.00	0.90	0.89	0.78	72	1.00	1.00	1.00	1.00
20	1.00	0.40	1.20	0.06	73	1.00	1.00	1.00	1.00
21	0.99	0.80	0.90	1.00	74	1.30	0.80	0.70	0.30
22	1.00	1.00	1.00	1.00	75	0.99	0.99	0.99	0.99
23	1.10	1.20	1.00	1.00	76	0.99	1.00	1.00	1.00
24	1.00	0.54	0.39	0.80	77	1.50	1.50	1.50	1.50
25	1.34	1.00	0.90	1.00	78	1.20	0.99	1.02	1.40
26	1.24	1.20	1.22	1.30	79	1.40	0.14	0.40	1.30
27	1.10	1.01	0.90	1.19	80	1.00	1.00	1.00	1.00
28	1.00	1.00	1.00	1.00	81	0.82	0.76	0.66	0.66
29	1.12	0.73	0.68	0.92	82	1.30	1.10	1.00	1.06
30	0.80	0.80	0.80	0.80	83	1.02	0.99	1.06	0.97
31	0.20	1.40	0.40	0.11	84	1.50	1.20	1.00	1.20
32	0.92	0.61	1.16	0.68	85	2.00	1.00	1.00	2.00
33	1.00	0.34	1.00	0.20	86	1.00	0.41	1.00	0.26
34	0.37	0.52	0.67	0.81	87	0.79	0.22	0.98	0.39
35	0.90	1.00	1.10	0.90	88	1.00	0.88	0.76	1.06
36	0.26	0.56	0.36	0.61	89	1.61	0.39	1.78	1.00
37	1.00	1.00	1.00	1.00	90	1.00	0.78	0.51	0.68
38	0.69	0.48	0.24	0.76	91	1.00	0.00	1.02	1.48
39	1.00	0.44	0.60	1.00	92	1.30	1.30	1.30	1.30
40	1.44	1.39	1.39	1.00	93	1.00	1.00	1.00	1.00
41	1.22	0.82	0.82	0.82	94	0.60	0.80	0.60	0.80
42	0.80	0.68	0.78	0.68	95	0.50	0.50	0.40	0.40
43	0.81	0.94	1.00	0.80	96	1.50	1.00	1.50	1.22
44	0.98	1.00	0.99	0.98	97	0.98	0.58	0.98	1.03
45	1.50	1.40	1.60	1.50	98	1.00	1.00	1.00	1.00
46	1.36	1.20	1.00	0.90	99	0.70	0.60	0.75	0.90
47	1.42	1.00	0.97	0.81	100	1.50	1.00	1.00	1.50
48	0.98	1.18	2.00	0.61	101	1.49	0.99	1.02	1.48
49	0.87	1.19	0.61	0.84	102	0.40	0.60	0.60	2.00
50	1.00	1.00	1.00	1.00	103	1.00	1.12	0.97	1.01
51	0.33	0.13	0.93	0.33	104	1.87	1.83	1.20	1.66
52	0.10	0.20	0.40	0.10	105	0.98	1.00	1.11	1.00

*#63's reservation prices were deleted due to computer crash

$20 Round

Index	V1	V2	V3	V4
1	8	6	10	8
2	10	10	10	10
3	10	5	6	9
4	2	10	5	3
5	5.5	6.5	8	10
6	13	9.5	14	10
7	2	1	4	10
8	10	10	10	10
9	5	2	10	2
10	10	5	5	5
11	10	10	10	10
12	10	4	6	5
13	13	10	9.15	12.75
14	10	4	5	10
15	10	6	5	2
16	10	8	8	10
17	7	6	7.75	9.5
18	10	5	5	10
19	6	5	6	8
20	6	7	8	9
21	5	5	5	5
22	10	5	5	10
23	10	5	6	10
24	10	13	8	17
25	3	3.75	4	2
26	6.5	5	3.5	6
27	5	2	2	2
28	12	8	8	12
29	5	3	4	4
30	15	8	5	5
31	10	7	6	8
32	8	12	15	10
33	10	11	15	7
34	8	10	10	8
35	5	3	4	2
36	10	10	10	10
37	10	8	8	10
38	8	4.5	5	3

E Regression results

E.1 Recursive Non Expected Utility (Segal [32])

Two alternative variables can be used: $V31$ and $V43$[45]: both options are presented in Table 7[46]. The estimated alternative (given that $\mathbb{E}(V4|V1) = V1$) models are:

$$V21 = \alpha_{31} + \beta_{31}V31 + \varepsilon \tag{19}$$

$$V21 = \alpha_{43} + \beta_{43}V43 + \varepsilon' \tag{20}$$

Table 7: The ambiguity premium as a function of alternative estimates of aversion to MPS for the RNEU (Segal) group

	V31	**V43**	**V31**	**V43**
Statistics	**$2 sample**		**$20 sample**	
Multiple R	0.788	0.735	0.948	0.886
R Square	0.621	0.541	0.899	0.785
Adjusted R Square	0.608	0.526	0.882	0.768
Standard Error	0.197	0.216	1.032	1.444
Observations	32	32	15	15

		Coef	***SE***	***t Stat***	***P-value***	***Lower 95%***	***Upper 95%***
$2 sample	Intercept	-0.110	0.045	-2.451	0.020	-0.202	-0.018
	V31	0.620	0.089	7.004	8.78E-08	0.439	0.801
$20 sample	Intercept	-0.925	0.311	-2.970	0.011	-1.597	-0.252
	V31	0.740	0.076	9.705	2.54E-07	0.576	0.905
$2 sample	Intercept	-0.170	0.045	-3.806	0.001	-0.262	-0.079
	V43	-0.502	0.085	-5.946	1.62E-06	-0.675	-0.330
$20 sample	Intercept	-1.295	0.398	-3.258	0.006	-2.155	-0.436
	V43	-0.679	0.099	-6.885	1.11E-05	-0.892	-0.466

E.2 Bundling (Rule Rationality)/Recursive Expected Utility

Two alternative and equivalent formulations are possible. One possibility is that the variables on the right hand side are $V31$ and $V43$, which measure the subjects'

[45] Using both will result in multicollinearity.

[46] A test whether $V1$ has a significant effect beyond $V31$ or $V43$ reveals that it is insignificant at 5%. That is, the ambiguity premium could be explained by agents' attitudes to mean preserving spread in the second order distribution.

aversion to mean preserving spreads in the second order distribution moving from urn 1 to 3 and from the latter to urn 4, respectively. An alternative way is to place the sum of the two - $V41$, on the right hand side of the regression equation[47,48]. The two alternative models estimated are therefore:

$$V21 = \theta + \gamma_{41} V41 + \varepsilon \tag{21}$$

$$V21 = \theta' + \gamma_{31} V31 + \gamma_{43} V43 + \varepsilon' \tag{22}$$

Table 8 summarizes the results of estimating these models.

Table 8: The ambiguity premium as a function of alternative estimates of aversion to MPS for the bundling/REU group

	V31,V43	V41	V31,V43	V41
Statistics	**$2 sample**		**$20 sample**	
Multiple R	0.806	0.803	0.951	0.951
R Square	0.650	0.644	0.905	0.904
Adjusted R Square	0.625	0.632	0.883	0.894
Standard Error	0.151	0.150	1.018	0.970
Observations	31	31	12	12

		Coef	***SE***	***t Stat***	***P-value***	***Lower 95%***	***Upper 95%***
$2 sample	Intercept	-0.057	0.032	-1.821	0.079	-0.122	0.007
	V31	0.454	0.105	4.341	0.0002	0.240	0.668
	V43	0.372	0.063	5.888	2.48E-06	0.243	0.501
$20 sample	Intercept	-1.178	0.338	-3.484	0.007	-1.942	-0.413
	V31	0.579	0.078	7.388	0.00004	0.401	0.756
	V43	0.549	0.080	6.833	7.61E-05	0.367	0.730
$2 sample	Intercept	-0.060	0.031	-1.950	0.061	-0.124	0.003
	V41	0.393	0.054	7.249	5.55E-08	0.282	0.504
$20 sample	Intercept	-1.202	0.312	-3.847	0.003	-1.898	-0.506
	V41	0.564	0.058	9.683	2.13E-06	0.434	0.694

[47] Since they are linearly dependent, one cannot use the three of them.

[48] The effect of $V1$ on ambiguity premium (beyond its effect on $V31$ and $V41$) is insignificant at 10%.