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ELUTRIATION OF FINES FROM FLUIDIZED BED*

— Study of Transport Disengaging Height —

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Transport Disengaging Height (T.D.H.) was studied theoretically and experimentally for both continuous and batch fluidized beds.
The theoretical elutriation rate of fines above the T.D.H., V_t, was calculated by use of the Maxwell-Bolzmann energy distribution, and it showed good agreement with the experimental value for the continuous fluidized bed.
The theoretical values of the T.D.H., which were defined as Z at $V=1.01V_t$, coincided fairly well with the present and reported experimental results.
The elutriation height coefficients a were found empirically to increase with increase in particle diameter and with decrease in superficial velocity.

Introduction

It is important to study elutriation of fine particles from fluidized beds used for reaction, drying and classifying.

In our previous work[5], the elutriation rate above the T.D.H., V_t, that is, the elutriation rate at a column height larger than the T.D.H., was studied experimentally and the modified elutriation rate coefficients K_s* and K* obtained in gas-solid and liquid-solid systems and in batch and continuous fluidized beds were correlated within ±25% deviation by a dimensionless equation.

In this work, the T.D.H., an important factor in the design of fluidized beds, was studied experimentally and analytically for continuous and batch fluidized beds. For the maintenance of steady state, the latter was operated with total reflux of elutriated fines.

There are several previous investigations. Zenz and Weil[8] have shown how to determine the T.D.H. from the figure obtained from an experimental study.

Andrews[1] has derived an analytical relationship between the elutriation rate V and the height from the dense bed surface Z by use of the Maxwell-Bolzmann energy distribution as follows:

$$-\partial V/\sqrt{Z}\,\partial Z = \lambda A\beta\rho\sqrt{2g}\,mg\exp(-\beta mgZ) \quad (1)$$

where $\lambda A\beta\rho\sqrt{2g}\,mg$ and βmg must be determined experimentally. The relationship between these factors and the particle characteristics or the operating conditions was not given.

Blyakher and Pavlov[2] studied the dependence of the T.D.H. on grids located above a conical fluidized bed.

Lewis *et al.*[4] have proposed an empirical equation as follows:

$$V=K\exp(-aZ),\quad K=cuA\exp[-(b/u)^2] \quad (2)$$

where a, b and c are the experimental coefficients, but the reported values of c are scattered. The above coefficient a is called the elutriation height coefficient especially by the authors.

Kunii and Levenspiel[3] have derived an analytical equation as follows

$$V/V_0=1-(1-V_s/V_0)[1-\exp(-aZ)] \quad (3)$$

The elutriation rate above the T.D.H. which they used was the saturation carrying flow rate V_s that was the maximum elutriation rate observed under pneumatic transport conditions of the single particle-size system by Zenz *et al.*[8]. However, it is found that the elutriation rate above the T.D.H., V_t, in this study is much smaller than V_s.
They gave no experimental values of a and V_0.

In this work, the elutriation rate above the T.D.H., V_t, has been analysed by use of the Maxwell-Bolzmann energy distribution. This V_t should be used in place of V_s in Eq.(3). The elutriation height coefficient a proposed by Lewis *et al.*[4] and Kunii *et al.*[3] was measured experimentally and an equation giving the value of the T.D.H. has been derived analytically.

* Received on April 7, 1971
Presented at the 35th Annual Meeting of the Society of Chem. Engrs., Japan, at Tokyo, April 4, 1970 and the Kyushu Meeting of the Society of Chem. Engrs., Japan, at Fukuoka, Sept. 16, 1970

Table 1 Fluidized particles

No.	Particles	Density ρ_s[g/cm^3]	Terminal velocity u_t[cm/sec]	Diameter D_p[μ]
1	Neobeads	1.54	64	141
	Soma sand	2.67	588	718
2	Toyoura sand	2.67	127	178
	Soma sand	2.67	588	718
3	Glass beads	2.50	87	135
	Soma sand	2.67	588	718
4	Glass beads	2.50	62	106
	Soma sand	2.67	588	718

* D_p=median diameter. Refer to Table 1 in the authors' report[5].

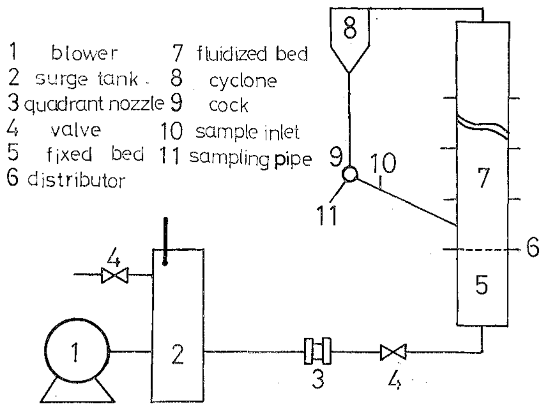


Fig. 1 Schematic diagram of the batch fluidized bed with total reflux

1. Experimental Apparatus and Method

1-1 Experimental apparatus and method for a continuous fluidized bed

The experimental apparatus and method are similar to those in our previous work[5].

The several column heights from the overflow pipe were used to measure the effects of column height on elutriation rate, and column diameter was kept constant at 67 mm*.

1-2 Experimental apparatus and method for a batch fluidized bed with total reflux

The experimental apparatus is shown in **Fig. 1** and the experimental method is reported in the previous work[5].

For the maintenance of steady state, all elutriated fines collected by the cyclone were returned to the bed (total reflux operation) and the holdup of solids in the bed was kept constant.

The fluidized particles used are listed in **Talbe 1**.

2. Analysis of Elutriation Rates

For analysing elutriation of fines from a fluidized bed, the Maxwell-Bolzmann energy distribution is applied to the dilute phase of fluidized bed. For the application of this energy distribution, the following four conditions are necessary.

1. Steady-state conditions must be maintained.
2. The total energy of the particles must remain constant.
3. The particles must be identical and indistinguishable from each other.
4. The total number of particles in the system must remain unchanged.

The above four conditions seem to be almost satisfied in the dilute phases of a continuous fluidized bed and a batch bed with total reflux.

Maxwell-Bolzmann energy distribution[1] is shown by

$$f_i=\beta \exp(-\beta\epsilon_i) \tag{4}$$

from which the elutriation rate of i-component $U_i d\epsilon_i$ is followed by

$$U_i d\epsilon_i=\gamma\rho AC_s U_i f_i d\epsilon_i \tag{5}$$

In the case of the height above the T.D.H., the minimum energy for elutriation of fines is $\epsilon_t=mu_t^2/2$ and the elutriation rate becomes

$$V_t=\int_{\epsilon_t}^{\infty} U_i d\epsilon_i=\int_{\epsilon_t}^{\infty}\gamma\rho AC_s u_i f_i d\epsilon_i \tag{6}$$

assuming that fines with an energy above ϵ_t are elutriated. The substitution of $\epsilon_i=mu_i^2/2$ into Eq.(6) gives

$$V_t=\gamma\rho AC_s\{u_t\exp(-\beta m u_t^2/2)+\sqrt{\pi}\ \mathrm{erfc}(\sqrt{\beta m/2}\,u_t)/2\sqrt{\beta m/2}\} \tag{7}$$

$\beta m/2$ in Eq.(7) is shown as follows:

$$\beta m/2=\frac{m}{2}\frac{N}{\frac{1}{2}Nmu_z^2}=1/u_z^2 \tag{8}$$

Substituting Eq.(8) into Eq.(7), one gets

$$V_t=\gamma\rho AC_s u_z\left[\left(\frac{u_t}{u_z}\right)\exp\left\{-\left(\frac{u_t}{u_z}\right)^2\right\}+\frac{\sqrt{\pi}}{2}\ \mathrm{erfc}\left(\frac{u_t}{u_z}\right)\right] \tag{9}$$

Eq.(9) is a theoretical equation of the elutriation rate above the T.D.H..

According to Eq.(2) in the case of the height below the T.D.H.,

$$V=C_1\exp(-aZ) \quad \text{where} \quad C_1\neq K \tag{10}$$

To apply Eq.(10) to the arbitrary column height, it is rewitten as

$$V=C_1\exp(-aZ)+C_2 \tag{11}$$

with boundary conditions

$$\text{at} \quad Z=0, \quad V=V_0 \tag{12}$$

$$\text{at} \quad Z=\infty, \quad V=V_t \tag{13}$$

where Z is the height from the elutriation surface (which seems to be equivalent to the dense bed surface) and V_0 is the elutriation rate assumed at the elutriation surface (Z=0 i.e. H=H_0) as follows

$$V_0=\gamma\rho AC_s u_z \tag{14}$$

In the case of the continuous fluidized bed, Eq.(14) becomes

$$V_0=C_f F_{op}=\gamma\rho AC_s u_z \tag{15}$$

F_{op}, called the optimum feed rate by the authors, is mentioned later in detail.

* In the previous work[5,6], the column diameters of 31, 67 and 150 mm were used, but the effect on the elutriation rate coefficients was observed to be insignificant.

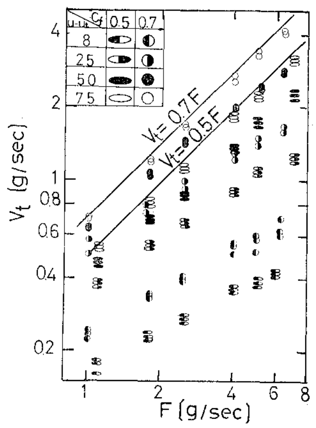


Fig. 2 Effects of feed rate on the elutriation rate at column height above T.D.H. (Particles No. 2)

From Eqs.(11)~(13), $C_1=V_0-V_t$ and $C_2=V_t$ are given and the following equation is obtained.

$$(V-V_t)/(V_0-V_t)=\exp(-aZ) \qquad (16)$$

Substituting Eqs.(9) and (14) into Eq.(16) and rearranging, one gets

$$V/V_0=V/\gamma\rho AC_s u_z=\exp(-aZ)+\left[\left(\frac{u_t}{u_z}\right)\exp\left\{-\left(\frac{u_t}{u_z}\right)^2\right\}+\frac{\sqrt{\pi}}{2}\mathrm{erfc}\left(\frac{u_t}{u_z}\right)\right]\left[1-\exp(-aZ)\right] \qquad (17)$$

The elutriation height coefficient a has been shown by Lewis *et al.*[4] and Kunii *et al.*[3] as follows

$$\alpha=au \qquad (18)$$

Eq.(17) is a theoretical equation of the elutriation rate at an arbitrary column height.

3. Experimental Results and Discussion

3-1 Experimental results and discussion for a continuous fluidized bed

The relationship between the elutriation rate V_t and the feed rate F at $Z=180$ cm (which was higher than the T.D.H., as mentioned later) is shown in **Fig. 2**. When all fines fed in the bed are elutriated, the elutriation rate is given by the solid line in Fig. 2. It is found that the slope of the line showing the relationship between V_t and F in Fig. 2 is not always unity. Therefore, V_t/C_fF is affected by F as shown in **Fig. 3**.

On the other hand, the theoretical values of V_t/C_fF_{op} derived from Eqs.(9) and (15) are not affected by F and are dependent upon u_t/u_z only.

This difference seems to be caused mainly by the dependence of the elutriation surface height on $u-u_t$ and F. It may be considered that Eq.(15) is valid when the elutriation surface height is regarded as the height of the overflow pipe, that is, the elutriation surface is kept constant.

Thus, to find the condition at which the height of the elutriation surface is equal to that of the overflow pipe was investigated. **Fig. 4** shows qualitative tendencies explaining the effects of F and u on the

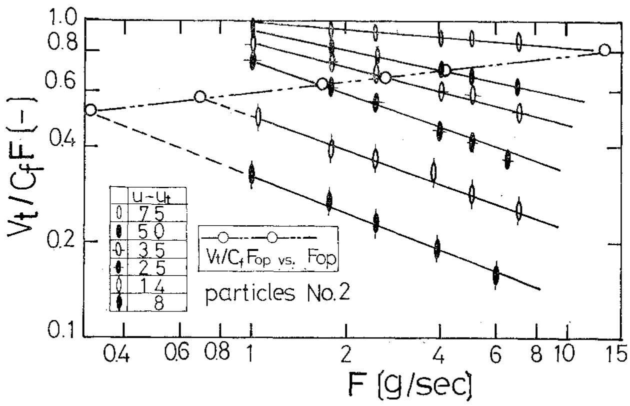


Fig. 3 Effects of feed rate on V_t/C_fF

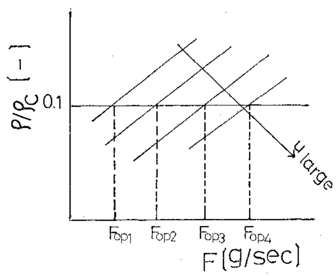


Fig. 4 Qualitative diagram of elutriation surface where ρ/ρ_c is assumed as 0.1

elutriation surface height. It is assumed that the elutriation surface appears when ρ/ρ_c becomes a certain constant value. According to Urabe *et al.*[7], it appears when ρ/ρ_c reduced to a value below 0.1. It seems that ρ/ρ_c at a certain height in the bed increases with F and inversely with $u-u_t$.

To keep ρ/ρ_c at a certain constant value, therefore, $u-u_t$ must be increased when F is increased.

Above all, to make the height of the elutriation surface equal to that of the overflow pipe, that is, to keep ρ/ρ_c constant at that height, F is naturally determined when $u-u_t$ is chosen. F_{op} refers to F for this situation.

Actually, when $u-u_t$ is given, V_t/C_fF_{op} is calculated from Eqs.(9) and (15) by the use of $u=u_z$ and the calculated values are shown by the mark ○ in Fig. 3. It is found from Eqs.(9) and (15) that V_t/C_fF_{op} increases with u_z/u_t, that is, V_t/C_fF_{op} increases as $u-u_t$ increases. This tendency is shown by the two-dotted chain line in Fig. 3.

Fig. 5 shows the experimentally obtained relationship between F_{op}/ρ_s and $u-u_t$. Theoretical values of V_t calculated from Eqs.(9) and (15) by the use of the relationship in Fig. 5 coincide with experimental ones (refer to Fig. 10 in **3-2**).

The values of F_{op} in Fig. 5 were measured at the height above the T.D.H.. Since F_{op} is the feed rate for keeping the elutriation surface height (equal to the height of the overflow pipe) constant, F_{op} seems to be independent of the column height, and therefore F_{op} may also exist for the case of a height below the T.D.H. as well as above the T.D.H..

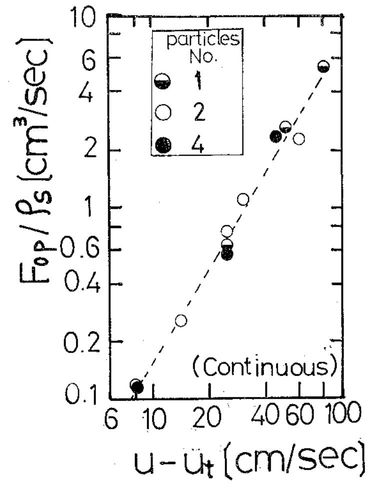


Fig. 5 Relationship between optimum feed rate and gas velocity

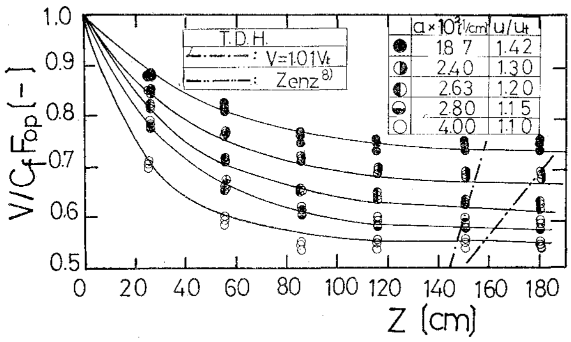


Fig. 6 Relationship between V/C_fF_{op} and $Z=H-H_0$, height from elutriation surface (Particles No. 2)

The relationship between the elutriation rate V and the height from the elutriation surface Z was measured by changing the column height at F_{op}, which was obtained from Fig. 5 by the use of ρ_s and $u-u_t$ under the experimental conditions, and the result is shown in **Fig. 6**.

The solid lines in Fig. 6 were obtained by applying the values of elutriation height coefficient a, determined by the trial and error method until Eq.(17) coincided with the experimental data. **Fig. 7** shows $\alpha(=au)$ calculated from the above determined a and Eq.(18). It is found from Fig. 7 that α increases linearly with D_p.

When the T.D.H. is defined as Z at $V=1.01V_t$ and V/V_t is calculated from Eqs.(17) and (9), the following equation is derived.

$$\exp(-aZ)=\frac{0.01\left[\left(\frac{u_t}{u_z}\right)\exp\left\{-\left(\frac{u_t}{u_z}\right)^2\right\}+\frac{\sqrt{\pi}}{2}\operatorname{erfc}\left(\frac{u_t}{u_z}\right)\right]}{1-\left[\left(\frac{u_t}{u_z}\right)\exp\left\{-\left(\frac{u_t}{u_z}\right)^2\right\}+\frac{\sqrt{\pi}}{2}\operatorname{erfc}\left(\frac{u_t}{u_z}\right)\right]} \tag{19}$$

The T.D.H. can be calculated from Eq.(19) by the use of u_t/u_z $(u_z=u)$ and a. The calculated values of the T.D.H. are shown by the chain line in Fig. 6.

The experimental result of the T.D.H. reported by Zenz *et al.*[8)] is also shown by the two-dotted chain line in Fig. 6. It is found from Fig. 6 that the chain line is close to the two-dotted chain line. This supports the definition that the T.D.H. is Z at $V=1.01V_t$.

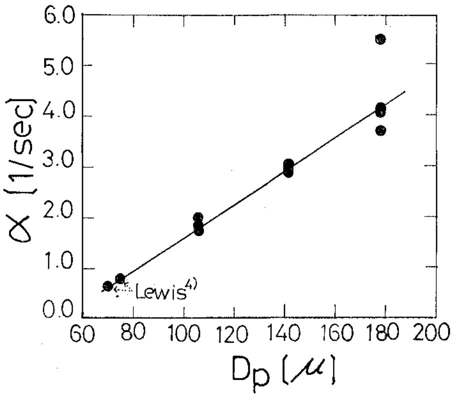


Fig. 7 Effects of particle diameter on α in Eq.(18)

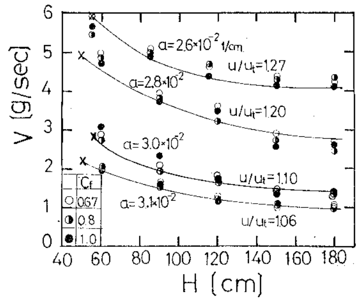
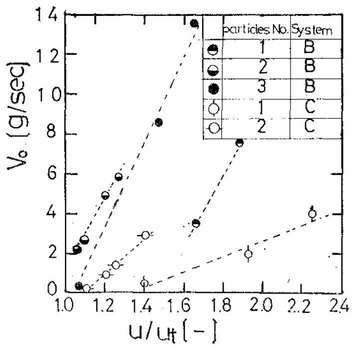


Fig. 8 Relationship between elutriation rate and height from the distributor (Particles No. 2)

3-2 Experimental result and discussion for a batch fluidized bed with total reflux

Fig. 8 shows the relationship between the column height from the distributor H and the elutriation rate V. No effect of C_f appears, as shown in Fig. 8, and this seems to be caused mainly by the fact that the weight of fines remained at 200g in the bed and the weight of coarse particles was added to the bed for the desired C_f. A detailed discussion of this result will be reported in future. The experimental data plotted in Fig. 8 are compared with Eq.(17) as follows. First, the elutriation height coefficient a is determined from Eq.(18) by the use of u and α in Fig. 7. Next, the theoretical line (solid line) in Fig. 8 is obtained by the trial and error method as follows. Assuming that V at $H=180$ cm is V_t, V_0 is calculated from Eqs.(9) and (14) by the use of the assumed V_t and $u_t/u_z(u_z=u)$. A theoretical relationship between Z and V is given from Eq.(17) by applying the calculated value of V_0, u_t/u_z $(u_z=u)$ and the above determined a, and this theoretical relationship is compared with the experimental result in Fig. 8. This procedure is continued until the former coincides with the latter.



H_0=height of elutriation surface
B: Batch, C: Continuous

Fig. 9 Effects of gas velocity on V_0 at H_0

Extrapolating the above calculated theoretical line to $V=V_0$, the marks × as shown in Fig. 8 are obtained. The abscissas of these points give the elutriation surface height, that is, $Z=0$ $(H=H_0)$.

Fig. 9 shows an empirical relationship between V_0 and u/u_t. It is found from Fig. 9 that V_0 increases linearly with u and this tendency agrees with that of Eq.(14). It is also shown in Fig. 9 that V_0 becomes a different value owing to the variations of γ and ρ with the fluidized particles at the same value of u/u_t. V_0 in the continuous fluidized bed is equal to $C_f F_{op}$ (which can be calculated from Fig. 5) and its values are also shown in Fig. 9.

Applying V_0 in Fig. 9 to Eqs.(9), (14) and (15), the theoretical values of V_t can be calculated and are compared with experimental data in **Fig. 10**. Fig. 10 shows that the former coincides with the latter in the case of a continuous fluidized bed. The experimental values of V_t were not measured in the batch bed with total reflux, but V at $H=180$ cm in Fig. 8 are shown in Fig. 10 instead of V_t in the batch bed, which also almost agree with the theoretical values of V_t.

Fig. 11 shows the experimental relationship between the elutriation surface height H_0 and u/u_t. It is found from Fig. 11 that H_0 increases with u/u_t in a batch fluidized bed with total reflux while it remains constant in a continuous bed under the condition of $V_0=C_f F_{op}$.

Conclusions

(1) Applying the Maxwell-Bolzmann energy distribution to the elutriation rate from fluidized beds, the theoretical equations of elutriation rates Eqs.(9) and (17) were introduced. Eq.(9) and Eq.(17) are applicable to the height above the T.D.H. and to an arbitrary height, respectively. These theoretical equations showed good agreement with the experimental data.

(2) It seems that F_{op} exists to keep the elutriation surface height constant in a continuous fluidized bed.

(3) The theoretical values of the T.D.H. were defined as the height at $V=1.01\ V_t$ and calculated from Eq.(19). It was found that they coincide with the experimental values of the T.D.H. obtained in this and other works.

(4) The elutriation height coefficient a can be calculated by Eq.(18) and Fig. 7. The effects of column diameter and the properties of fluidizing fluid on the coefficient a must be examined in future.

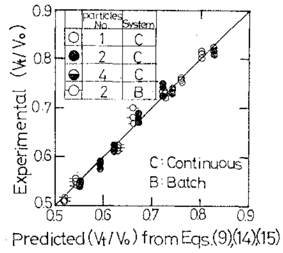
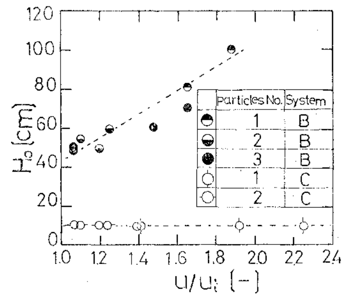


Fig. 10 Comparison of experiments with theoretical predictions.



B: Batch, C: Continuous

Fig. 11 Effects of gas velocity on the height of elutriation surface

Nomenclature

A	= cross-sectional area of fluidized bed	[cm²]
a	= elutriation height coefficient	[1/cm]
b, c	= experimental coefficients in Eq.(2)	[cm/sec], [g/cm³]
C_f	= fines concentration in feed	[—]
C_s	= constant fines concentration in bed	[—]
D_p	= particle diameter	[μ]
F	= feed rate	[g/sec]
F_{op}	= optimum feed rate	[g/sec]
f_i	= mass fraction of particle with energy ϵ_i	[sec²/g·cm²]
H	= height from the bottom of fluidized bed	[cm]
H_0	= height of elutriation surface from distributor	[cm]
K^*	= modified elutriation rate coefficient for batch fluidized bed	[g/cm²·sec]
K_s^*	= modified elutriation rate coefficient for continuous fluidized bed	[g/cm²·sec]
m	= mass of particle	[g]
N	= number of particles	[—]
U_i	= elutriation rate per unit energy	[sec/cm²]
u_i	= velocity of fluidized particle with a diameter of i-component	[cm/sec]
u	= superficial gas velocity	[cm/sec]
u_t	= terminal velocity	[cm/sec]
u_z	= average velocity of fluidized particles	[cm/sec]
V	= elutriation rate	[g/sec]
V_0	= elutriation rate at H_0	[g/sec]
V_s	= saturation carrying flow rate of fines	[g/sec]
V_t	= elutriation rate above T.D.H.	[g/sec]
Z	= height from the elutriation surface	[cm]
α	= au, see Eq.(18)	[1/sec]
β	= inverse of average energy of particles	[sec²/g·cm²]
γ	= fraction of elutriation	[—]
ϵ_i	= energy of fluidized particle with a diameter of	

	i-component	[g·cm²/sec²]
ϵ_t	$= mu_t^2/2$	[g·cm²/sec²]
ρ	= bulk density of fluidized bed	[g/cm³]
ρ_c	= bulk density of fixed bed	[g/cm³]
ρ_s	= particle density	[g/cm³]
λ	= experimental coefficient in Eq.(1)	[—]

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ANALYSIS OF THE DISPERSING PROCESS OF LIQUID-LIQUID DISPERSED SYSTEM BY USE OF A NEW SMALL TYPE OF HIGH-FREQUENCY INDUCTANCE TORQUE METER*

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Using a new type of torque meter with an extremely high sensitivity, some attempts have been made to get more detailed information than ever about coalescence and dispersion in a stirred liquid-liquid system. A dimensionless empirical equation could be derived relating the perfect-dispersing time, defined from the transient characteristics of torque received by a stirrer's shaft, to the several representative dimensionless groups. Furthermore it was made clear how much portions of the entire energy given to the liquid-liquid system are directly responsible for the dispersing process itself.

1. Introduction

In common with the role of the temperature of the environment in the case of usual chemical reaction, even in the case of droplet interactions in a stirred tank the turbulent-field strength may be regarded as a very important factor controlling the rates of coalescence or dispersion[2,4,12,18]. Already, in order to make clear the role of turbulent energy in a stirred vessel both experimentally and theoretically, various attempts have been made from different standpoints[5,9,10].

However, little work has yet been undertaken to elucidate in a quantitative sense in what manner the mixing energy furnished by a rotating impeller contributes to the dispersing process, and also to what extent the same would be consumed due to the viscous friction within liquids. The one purpose of the present study was numerical estimation of the contribution of mixing energy to the dispersing process and the establishment of a correlation between operating conditions and perfect-dispersing time. For this purpose, authors utilized a small new type of high-sensitive torque meter attachable to a low-power motor.

The slip-ring part and the pick-up part of this transducer are designed to be relatively small as compared with those of the usual torque meter commercially available. By measuring exactly the mixing torque acting on the impeller shaft, it was made clear that on the one hand the entire energy necessary for agitation was consumed to give rise to the motion of fluids and eventually to overcome the viscous friction within the liquid, and on the other hand the same was changed to the mechanical friction energy generated in the apparatus (principally in the ball-bearing support). The energy consumption due to the viscous friction within the fluid could be estimated as compared with that obtained by measuring the actual-temperature rise of the agitated liquid under adiabatic conditions. As for the liquid-liquid dispersed system, the same procedure was used to determine what portion of the mixing energy given to the fluid would be consumed to produce the dispersed phase.

Moreover, attempt were made to relate the perfect-dispersing time, which was newly defined

* Received on April 5, 1971
Presented at Tohoku Meeting, Yamagata, September, 1970 and the 36th Annual Meeting of the Society of Chemical Engineers, Japan, April 1971