International Conference on Dynamical Systems
in honour of Michał Misiurewicz on his 60th birthday

Będlewo, Poland, June 30 – July 5, 2008

Embedding odometers in cellular automata

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(joint work with Reem Yassawi)

We consider the problem of embedding odometers ("+1" maps on countable products of finite cyclic groups, addition with "carrying") in one-dimensional, two-sided cellular automata.

Theorem 1. *Every odometer such that*

$$\{p \text{ prime} : p \text{ divides the size of one of the groups}\}$$

is finite can be embedded in a cellular automaton with local rule $x_i \mapsto x_i + x_{i+1} \bmod n$ *(*$i \in \mathbb{Z}$*), where* n *depends on the odometer. Conversely, these are the only odometers that can be embedded in cellular automata with these local rules.*

(Cellular automata with these local rules are the simplest non-trivial cellular automata.)

Theorem 2. *Every odometer can be be embedded in a "gliders bouncing off walls" cellular automaton, which one depending on the odometer.*