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ABSTRACT

This paper examines the novel problem solving actions of a pair of college students. The analysis highlights the role of the solvers' inferential processes including abductions, deductions, and inductions as structuring resources that contribute to both their understanding of the problems they face and the emerging novelty that constitutes their viable solution activity. The purpose of this research is to clarify the processes by which learners construct new knowledge in mathematical problem solving situations, with particular focus on instances where the learner's emerging abductions help to facilitate development of novel problem solution activity. Findings indicate that: (1) abduction is characterized as an ongoing, sense-making process that constitutes the problem solver's source of ideas as to how to proceed when unexpected problems occur; (2) problem solvers' abductions aided their novel explorations, serving to organize and structure their subsequent solution activity; and (3) novelty demonstrated by the problem solvers through their abductive inferences suggests the need to rethink views toward teaching problem solving. Contains 23 references. (DDR)

Emergence of Abductive Reasoning in Mathematical Problem Solving

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Introduction

This paper examines the novel problem solving actions of a pair of college students. The analysis highlights the role of the solvers' inferential processes (abductions, deductions, inductions) as structuring resources that contribute to both their understanding of the problems they face and the emerging novelty that constitutes their viable solution activity.

Theoretical Rationale

The philosopher and logician Charles Saunders Peirce (1839-1914) asserted that there occurs in science and everyday life a pattern of reasoning wherein explanatory hypotheses are constructed to account for unexplained data or facts. Peirce called this kind of reasoning ***abduction***, distinguishing the process from the two traditionally recognized inferential types of reasoning, induction and deduction. Specifically, abduction furnishes the reasoner with a novel hypothesis to account for surprising facts; it is the initial proposal of a plausible hypothesis on probation to account for the facts, whereas deduction explicates hypotheses, deducing from them the necessary consequences, which may be tested inductively. According to Peirce, abduction is the only logical operation which introduces any new ideas, "for induction does nothing but determine a value, and deduction merely evolves the necessary consequences of a pure hypothesis" (as quoted in Fann, 1970, p. 10).

Since Peirce argued that abduction covers "all the operations by which theories and conceptions are engendered" (as quoted in Fann, 1970, p. 8), it appears that abduction may play a prominent role in the mathematical knowledge that learners construct while in the process of solving a problem. Of particular interest here is the role of abduction in the process of sense-making, which aids solvers in "getting a handle on" or developing understanding about the problems they face. In this context, the solver's hypotheses may include novel ideas about the problem, that pave a way for them to make conjectures about both potential courses of action to carry out as well as the result(s) of those actions.

The work of Polya (1945) is based on ideas consistent with the view that problem solvers reason hypothetically in the course of solving a problem. Specifically, Polya identified heuristic reasoning as "reasoning not regarded as final and strict but as provisional and plausible only, whose purpose is to discover the solution of the present problem" (Polya, 1945, p. 113). Further, Polya cited the usefulness of varying the problem when solvers fail to achieve progress towards their goals because the solvers' consideration of new questions serves to "unfold untried possibilities of contact with our previous knowledge" (Polya, 1945, p. 210). Hence, solvers who reason through hypotheses are (1) cautious in their reflections about appropriate courses of action to carry out; (2) always looking to monitor the usefulness of the activity they plan to carry out; and, (3) willing to adopt a new perspective of the problem situation when their progress is impeded.

The importance of such reflective activity has been emphasized by Burton (1984), identifying the process of making conjectures as a component of mathematical thinking through which "a sense of any underlying pattern is explored" (Burton, 1984, p. 38). More recently, Mason (1995) has remarked on the importance of examining where learners' conjectures come from, suggesting that a fresh examination of the abduction process is warranted. Nevertheless, the researcher agrees with Anderson's (1995) contention that the process of abduction is transitory and slippery, difficult to foster, impossible to teach, and probably easy to discourage.

Objectives

The purpose of the study was to clarify the processes by which learners construct new knowledge in mathematical problem solving situations, with particular focus on instances where the learner's emerging abductions or hypotheses help to facilitate development of novel solution activity. The perspective taken here is that problem solving situations involve much more than merely carrying out a set of procedures to complete a task. Specifically, <u>problematic situations</u> (Pask, 1985) are self-generated by solvers, arising from their interpretations of the tasks given to them. The solvers' interpretations may suggest to them questions and uncertainties, the consideration of which helps them construct goals for purposeful action. For example, solvers might face a problematic situation when they try to "make sense" of, or understand statements that describe a specific algebra word problem. Alternatively, the solver's <u>problem</u> might be to understand why a particular solution method led to unanticipated success, or why two different solution methods led to the same result. These situations arise unexpectedly for solvers in the course of their goal directed activity, yet can serve as learning opportunities for them (Pask, 1985; Cobb, Yackel, & Wood, 1989). Further, the problem solving activity initiated to resolve these problems is of a conceptual nature and often precedes the introduction of formal algorithms by the solver. Sáenz-Ludlow and Walgamuth (1996) emphasized the importance of teachers considering their students' "idiosyncratic ways of operating", arguing that the conceptual growth of children is fostered when they have opportunities to connect their real-life experiences with classroom mathematical experiences. The importance of informal reasoning in mathematical activity is echoed by Cai, Moyer, and Laughlin (in press), asserting that while "the algorithms that students invent, in many cases, are different from the algorithms usually taught in school", these "invented algorithms" form the conceptual foundation from which solvers can introduce more formal algorithmic procedures (Cai, et al., in press, p. 3).

Successful completion of problem solving tasks may involve many cognitive constructions of the type described above, all generated in the course of on-going activity and each monitored for its usefulness by the solver. In this way, problem solving can be viewed as a form of hypothetical reasoning, where solvers continually explore and try out viable strategies to relieve cognitive tension, involving no less than their ability to form conceptions of, transform, and elaborate the problematic situations they face.

Few studies of mathematical problem solving have specified precisely the role of hypothetical actions in the novel solution activity of solvers. However, the research on "problem posing" (Silver, 1994; Silver and Cai, 1996; Brown and Walter, 1990) suggests ways that hypotheses play a prominent role in solvers' novel solution activity. According to Brown and Walter (1990), problem posing and problem solving are naturally related in the sense that new questions emerge as one is problem solving, that "we need not wait until *after* we have solved a problem to generate new questions; rather, we logically obligated to generate a new question or pose a new problem in order to solve a problem *in the first place* (Brown and Walter, 1990, p. 114). Furthermore, Silver (1994) asserted that this kind of problem posing, "problem formulation or re-formulation, occurs within the process of problem solving" (Silver, 1994, p. 19). Finally, the cognitive activity of "within-solution posing, in which one reformulates a problem as it is being solved" (Silver and Cai, 1996, p. 523) may aid the solver to consider novel questions and situations that are "hypothesis-based" (Silver and Cai, 1996, p. 529). This illustrates the dynamic, yet tentative nature of solvers' solution activity as well as the propensity of solvers to entertain new ideas about the problem when surprising results are generated.

In an earlier study (Cifarelli and Sáenz-Ludlow, 1996), examples of hypothetical reasoning activity were discussed, highlighting its mediating role and its transformational influence in the mathematical activity of learners. The current study sought to extend these results by specifying more precisely the ways that learners' self-generated hypotheses serve to organize and transform (or re-organize) their mathematical actions while resolving problematic situations.

Methodology

Data Source. A total of 21 subjects participated in the study and came from two sources. Nine first-year calculus students at the University of California at San Diego participated in the study. In addition, twelve graduate students in Mathematics Education from a Linear Algebra class taught by the researcher at the University of North Carolina at Charlotte participated in the study. The problem solving activity of college students has been examined in many prominent studies over the last thirty years (Marshall, 1995; Schoenfeld, 1994; Schoenfeld, 1985; Silver, 1985). However, no studies have examined the self-generating process of abduction and its role in the novel actions of solvers.

Data Collection. Subjects were interviewed as they solved sets of mathematical word problems. These interviews took the form of problem solving sessions, where subjects solved a variety of algebraic and non-algebraic word problems while "thinking aloud". All interviews were videotaped for subsequent analysis. In addition to the video protocols, written transcripts of the subjects' verbal responses as well as their paper-and-pencil activity were used in the analysis.

Subjects from the first institution participated in a single interview, solving a set of similar algebra word problems (see Table 1). (These subjects participated in an earlier study conducted by the researcher and were included for analysis as part of the current study.) In contrast, the subjects from the second institution were interviewed on 3 occasions while enrolled in the Linear Algebra class. In addition to solving the set of similar algebra word problems, these subjects solved several non-algebraic word problems (see Table 2). The use of non-algebraic tasks in the study enabled the researcher to observe solvers grappling with problems which were unfamiliar to them, requiring novel solution activity.

Insert Table 1 about here

Insert Table 2 about here

Data Analysis. Based on the examination of the video, verbal, and written protocols, a case study was prepared for each solver. First, the solvers' protocols were examined to identify episodes where they faced genuinely problematic situations. Previous studies conducted by the researcher characterized the conceptual knowledge of solvers in terms of their ability to build mental structures from their solution activity (Cifarelli, 1991; Cifarelli, in press). For example, solvers were inferred as having constructed re-presentations when their solution activity suggested they had combined mathematical relationships in thought and mentally acted on them (e.g., they could reflect on their solution activity as a unified whole, mentally "run through" proposed solution activity, and anticipate the results without resorting to pencil-and-paper actions). Of particular interest in the current study were the solvers' novel actions that precede and form the foundation for these constructions, with particular focus on the role that hypothetical reasoning played as a structuring resource for solvers. For example, in solving a particular problem, a solver may initiate activity, reflect on the results, and then use the results re-formulate the problem they are trying to solve; their activity might be called hypothetical in the sense that the reformulation is seen by the solver only as a plausible route to a possible solution, and still needs to be carried out, explored, and verified.

Analysis

This section describes and discusses episodes from the problem solving interviews conducted with John and Marie. These cases were chosen because they illustrate two prominent roles hypothetical reasoning can play in solution activity. Specifically, John's solution activity demonstrates hypothetical reasoning as problem formulation and re-formulation (within a single task) to build understanding (i.e., his "sense-making" that precedes his construction of formal solution procedures). In contrast, Marie's solution activity demonstrates hypothetical reasoning through her determination of "problem sameness" (across several tasks) to build efficacy knowledge (i.e., her evolving awareness of why and how well her methods work across similar tasks).

John's solution activity will be described and discussed in the following sections, followed by description and discussion of Marie's solution activity. Following the discussion of these cases, a list of conclusions will be presented.

<u>About John</u>. John was an aspiring secondary mathematics teacher and proved to be a strong mathematics student in the Linear Algebra class, achieving high scores on all exams and assignments throughout the course. He demonstrated strong problem solving activity throughout the interviews, as indicated by the novelty of his actions in completing the tasks.

<u>About the task</u>. During the interviews, John was given both algebraic and non-algebraic tasks. One of the non-algebraic tasks involved exploring an array of letters that could be used to spell out the palindrome **WAS IT A CAT I SAW** (see below).

WAS IT A CAT I SAW

An early edition of <u>Alice in Wonderland</u> included the spatial diagram shown here.

See how many mathematical problems you can make up and solve using the array.

```
            W
          W A W
        W A S A W
      W A S I S A W
    W A S I T I S A W
  W A S I T A T I S A W
W A S I T A C A T I S A W
  W A S I T A T I S A W
    W A S I T I S A W
      W A S I S A W
        W A S A W
          W A W
            W
```

The organization of the array of letters suggests many conceptually rich problems. For example, if the problem is to count the number of ways the palindrome can be spelled in the array, the problem could be solved noting that (1) all appropriate paths must go through the 'center C', and (2) the symmetry of the array reduces the problem to examining paths in a single quadrant. Accordingly, it suffices to count the number of paths from all **W**s on the boundary in a single quadrant to the 'center C'(i.e., the number of ways to get in), and multiply the results appropriately to capture all exit paths (including those that may exit in other quadrants). While counting the number of paths from all **W**s on the boundary in a single quadrant to the 'center **C**' can be done using a combinatorics "block walking" counting scheme devised by Polya[1] (Tucker, 1995), the counting can also be done manually, if one has the patience to trace out all of the possibilities and is careful not to produce duplicate paths in the count.

The purpose in using this task is to give students opportunity to generate their own problems and questions. Having used this task in classroom situations on several occasions, I have observed that students are apt to lock on particular kinds of problems (e.g., how many letters are there in the array) while at the same time failing to see others (e.g., how many ways can the palindrome be spelled). In my role as teacher, I try to limit my interventions to questions that I believe (1) have the potential to facilitate continued development of the students' conceptions, and (2) have the potential to induce additional problematic situations for them to explore. In this way, I continually look to support their current explorations and at the same time, help them develop conceptually rich new problems that may not have occurred to them. For the current study, I encouraged subjects to explore the array as long as they wished, constructing problems as they went. If when they were completed their lists of problems, I intervened with questions, hoping to call their attention to aspects of the array they may have overlooked.

The case of John includes one such intervention, the result of which John is able to exploit to his advantage. Of particular interest is the way John is able to extend and elaborate "his problems" The following section describes the problem solving activity of John as he solves the palindrome task.

<u>John's Inferential Processes</u>. Upon reading the instructions for the palindrome task, John interpreted that "his problem" was to <u>make up</u> and <u>solve</u> mathematical problems. As he began to formulate problems to solve, John remarked on how the task differed from other problem solving tasks he had encountered:

John: *Well, this is unusual. I think of problem solving as usually here is one problem, there is a problem you solve it. This is like you make a problem, it's like... orders of abstraction because I get, the problem is <u>to make up</u> problems. It's a little difficult.*

[1] I am grateful to the mathematician Harold Reiter for pointing out to me this interesting result by Polya.

Despite his comment about the difficulty of the task, John routinely generated five problems, all of which had to do with counting letters and words:

Table 3: John's Problem Solving - Part 1

1. How many of each type of letter ?

2. How many times does a particular word show up ?

3. How many words total ?

4. How many letters in the array ?

5. How many different patterns of counting the letters ?

As he was working, John remarked on the superficial quality of the problems he had constructed. For example, in formulating problems #2 and #5, John commented:

John: *How many mathematical problems can you make up and solve using the array? ... Hm,you could ask how many times a particular word shows up* ***(writes it on problem sheet as problem #2[2])****. The answer will depend on the word. Like, 'WAS' is going to show up a lot of times. 'IT' shows up fewer times. ... They're problems, but they're so, I don't know, so unsatisfying. I'd like to find something interesting.*

John: *You might want to come up with how many different ways to count the letters. You could move around ... how many different... sort of basically different patterns of counting can you establish* ***(writes it on problem sheet as problem #5)****? You could also do rows and 1, 3, 5, 7 and you would be adding odds duplicate 2 ones, 2 threes, 2 five's, 2 sevens, 2 nines, 2 eleven's, 2 thirteen's, do it that way. It reminds me in a way of Pascal's Triangle, but I don't think you can do anything with that because they're not offset the right way. Let's see ... that's about all.*

The episodes above are noteworthy for the following reasons. First, John's comments indicate a genuine lack of interest on his part regarding his newly constructed problems: he sees his made-up problems as "unsatisfying", and that he "would like to find something interesting". Second, John's comment about Pascal's Triangle appears to be a reference to an idea that he sees as both interesting and potentially useful. (His idea about Pascal's Triangle will re-appear later in the interview when he formulated a new problem.) Third, after generating problem #5, he has given up his quest to pose additional problems and looks to the interviewer for direction. The interviewer then prompts John:

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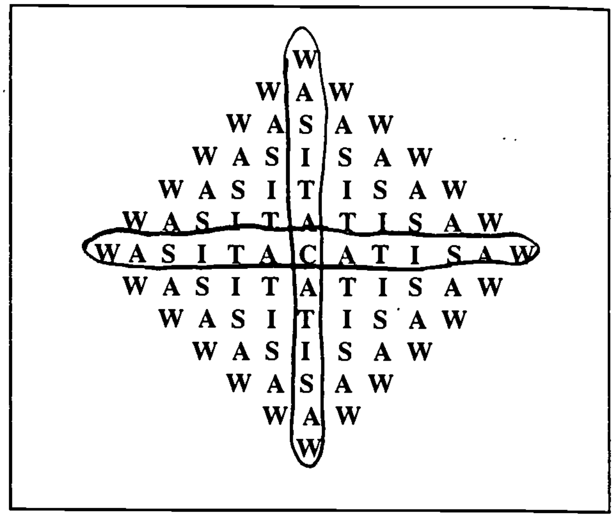
[2] **Comments in boldface describe the non-verbal actions of the solver.**

Interviewer: *Can I give you one?*

John: *Yeah.*

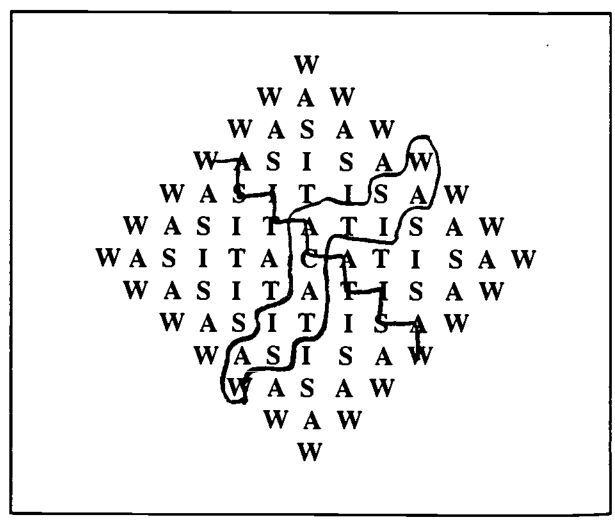
Interviewer: *Okay, the palindrome Was It A Cat I Saw, can you give me some ways that you could spell it in the array?*

John: *Oh, to spell out the <u>whole</u> thing? Oh, okay, well, I mean you obviously got the 2... you could... all diagonals.* ***(solver traces the palindrome along vertical and horizontal paths, followed by many seconds of reflection)*** *..... Interesting!*



With the comments above, John initiates a shift in his reasoning activity. He has a new problem to solve, his curiosity has been aroused, and he begins to become more engaged in the problem situation. He then adopts a pattern of hypothetical reasoning activity, generating provisional explanations of what his new problem <u>might</u> be about. He proceeds to explore the array of letters in a more focused manner, with a view towards learning about the different ways he could spell out the palindrome. For example, after tracing several paths through the array, each spelling out the palindrome, John generates a hypothesis:

John: *If you come in anywhere, you take a turn and finish it* ***(traces several zigzag paths on the array)****. From below, you turn up. At every place you got ... options of <u>switching</u>. It's like you've got <u>a network type problem</u> or something like that. You get little nodes and <u>you're going through nodes</u> or something.*



John's hypothesis about the array of letters constituting a network of switching options helped organize and direct his further explorations. Further, his hypothesis, while providing him with only an initial idea about the different paths, served as a source from which he was able to elaborate and derive more sophisticated properties about the different ways of spelling out the palindrome. For example, after traversing several more paths through the array, John hypothesized a property that all appropriate paths must possess:

John: *Hm, ... Was It A Cat I Saw. I could finish here, that will get down to the C but I could come up or finish that way, or I could come up and finish that way, or finish that way, or finish that way, or that one. So,* ***(several seconds of reflection)*** *it's all of them have to go through here because that's your only* ***C****. And if you're going to get the palindrome, you've got to go through the* ***C****, so they all have to go through the center.*

W
W A W
W A S A W
W A S I S A W
W A S I T I S A W
W A S I T A T I S A W
W A S I T A C A T I S A W
W A S I T A T I S A W
W A S I T I S A W
W A S I S A W
W A S A W
W A W
W

John then summarizes his ideas, using the following analogy:

John: *Kind of reminds me ... of Chinese Checkers or something... it's like regardless of where you start, you've got to diagonally move your way in ... and ...in some way or another... work your way out.*

John elaborates on his analogy as he further explores the array:

John: *You can ... you can make a move down from this position, you know, down or right, down or right, down or right ...* ***(solver traces a zigzag path through the array)*** *... Here, you can't go right anymore. You've got to go down either way as you work you way through here. It looks like kind of like a bus. You know, the kind of problems where they talk city blocks and how many they could get from here to here, you know, I get what 2 choices here, then you get, then here you got 2 choices, so to get here, you got 4 choices to get here ... 8 choices to get here, but then these 2, those are straight in, these you got 2 choices ... get 1, 2, ... It is just basically, I mean, it's like 8 paths to get in here from here* ***(solver points to 4 "I"s in triangular block)*** *... Because you've 2, 2, 2 ... I don't know, I'm thinking out loud.*

W
W A W
W A S A W
W A S I S A W
W A S I T I S A W
W A S I T A T I S A W
W A S I T A C A T I S A W
W A S I T A T I S A W
W A S I T I S A W
W A S I S A W
W A S A W
W A W
W

From this elaboration of his original analogy, John proceeds to look for a pattern to help him to count the paths:

John: *I don't know...if I play with a little more it turns a pattern 1, 2... may be its more than 8... 1,2,... Yes, it has to be more than 8. But that's just from* <u>*this*</u> *position !!*

John's sudden surprise was a realization that there might be many more paths to spell out the palindrome than he initially anticipated. He reflects on this unexpected result, and continues to press forward towards a solution:

John: *And you can come in from* <u>*all*</u> *these other positions. But I mean this has the most choices* ***(points to middle W on the boundary)****. If you start here* ***(points to horizontal and vertical paths)****, you've got to come straight in, and you could either go straight out ... or straight out if you come here you get one choice. At some point you've got to branch over to main channels, sort of speak, sooner or later you get a branch over the main channel... two possibilities... it seems you could work something out with that. I mean in terms of like literally* <u>*how many paths could you come up with the form of the palindrome*</u> *...* <u>*a lot*</u> *!!*

```
            W
          W A W
        W A S A W
      W A S I S A W
    W A S I T I S A W
  W A S I T A T I S A W
W A S I T A C A T I S A W
  W A S I T A T I S A W
    W A S I T I S A W
      W A S I S A W
        W A S A W
          W A W
            W
```

John then generates a hypothesis about a possible solution to counting the paths:

John: *I feel* <u>*it has something to do with powers of two*</u> *because you've got choices of 2 in each of these nodes. Even if you can work your way up from here, still you've got 2 choices of each node, but you're basically working your way in, and then you've got to work your way out. Let's see, ...* ***(many seconds of reflection)*** *... actually working your way out you get like 3 choices from here but then once you've got to one of those ...* ***(more reflection)*** *... you could, ... its kind of mindboggling!!*

John continues to explore, now looking to use "powers of two" as part of his solution.

John: *But it does remind me like, you know, city streets or patterns or even like probability in it like from here to here there is one path, from here to here, there is, you know, 1, 2, 3, 4, 5, you could count may be, I don't know, or it might have to do with is how many letters there are in 8 paths. I'm not sure.* ***(many seconds of reflection here)*** *1, 2, 3, 4, 5, 6 ... something like that and count from here, same thing, you probably find a pattern.* ***(more reflection here before exclaiming)*** *Oh,* <u>*this*</u> *would be would be Pascal's Triangle, 1, 6, 10, 15, 10, 6,1*[3] *...* ***(writes sequence on boundary of array);***

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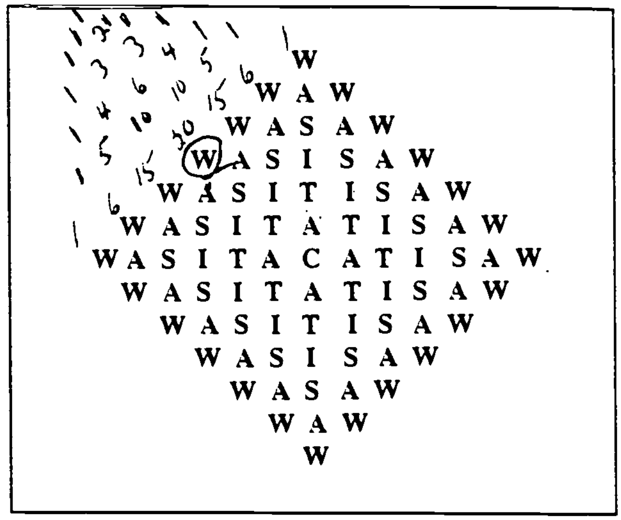
[3] **while the solver's conjecture was viable, these numbers do not correspond to a row of numbers in Pascal's Triangle; he appeared to make an error in recalling the rows of Pascal's Triangle, an error he later identifies and corrects.**

John's sudden and emphatic statement relating Pascal's Triangle is noteworthy because: (1) it emerged as a result of his persistence to find a pattern for counting the paths (i.e., came out of a genuine problem solving opportunity that he had initiated and sustained); and, (2) signaled a newfound confidence for him, placing a high level of certainty on both his prior as well as future actions. John continued his activity of counting the paths, using his knowledge of Pascal's Triangle to organize his actions.



John: *I'm trying to figure out Pascal's Triangle in my head. It reminds me of a problem I had in class one time. It's like you leave your house, go to Pizza Hut or Taco Bell, you have to make a turn at every corner. But yeah, it's like this, there's more paths from here than from anywhere else* ***(points to middle W on the boundary)****, and I can just do Pascal's Triangle, you know, I'll double check my arithmetic's, OOPS, I'm off. So, 1, 6, 15, 20, 15, 6, 1, so I'd say, ..., you know, there's that many ways to get into the center once you pick where you're going to start, and then* ***(traces a path and appears surprised when he gets to the center C)*** *to finish is like, do you let yourself repeat? I mean can you come back* <u>*this way*</u> ***(points to the same quadrant in which he started)*** *to finish palindrome? or you have to go out to each sector, or you have to kind of spell it out or it's like... What if you come back out in same sector? Just with in here this way. So you'd have this many ways to get in, but then from here out* ***(several seconds of reflection here)*** *... it'd probably be the sum of this* ***(points to sequence of numbers on boundary)****, 2 to the 6^{th} ... 64, so I'm guessing basically there would be 64 ways to come back out. No, it is not just a guess, it's an* <u>*intuition*</u> *because there is basically 64 ways to come in, spread out in these different positions, so they reach 64 ways to go out because once you get to the center you have more choice in finishing the letter.*

In asserting his "intuition", John has placed some certainty on his reasoning, confident that he is close to a solution to the problem. John continues to explore and elaborate his hypothesis:

John: *To start it you're kind of locked into a particular place to start. So, I guess what you could come in to the C as I'm figuring 64 different ways, and you could come out 64 different ways. So just within the sector, there are 64^2 different ways you come out. And if you start, I mean multiply here by 4... another 16, something to get the whole thing. If you get to go in here then you get 4 choices coming out here. 4 times 4 I think 16 times, that'll give you like total ways.*

John: *That's a good question!* ***(solver points to the 5 prior problems he made up and had written down)*** *These really didn't satisfy me so much.*

Discussion

The results characterize John as an assertive, aggressive sense-maker, continuously looking to make sense of the situations he finds himself in, and at the same time, aggressively projecting results of his problem solving actions towards the solution of other questions and problems. It was inferred that John's hypotheses, while providing ideas that contributed to his solution activity, also served to create new questions or problems for him to address, which were then actualized in the form of particular explorations. In this way, his evolving hypotheses concerning what the problem was about, went hand-in-hand with his continuously changing goals of what he was trying to achieve through his actions. In other words, as John solves his problems, new problems arise for him that need to be addressed.

Table 4 summarizes the researcher's inferences about the relationship between John's goals and purposes (his problems) and hypotheses that contributed to his knowledge about his problems (his solved problems).

Table 4: John's Problem Solving - Part 2

Goals and Purposes: His Problems	John's Hypotheses: His Solved Problems
to explore some ways to get the palindrome (what constitutes a path?)	H_1 :need to get onto a diagonal to spell out the palindrome
to explore properties of the paths (what properties are common to all paths ?)	H_2 :all paths need to go through the center C and back out again to spell the palindrome
to look for some efficient way to count paths (what is the pattern ?)	H_3:number of ways to spell out the palindrome appears related to Pascal's Triangle
to look for to make a generalization (is going out the same as going in ?)	H_4:the number of ways into the center C is the same as the number of ways out to the boundary

From Table 4 it is clear that John's hypotheses evolved continuously in the course of his actions as he determined how many ways there were to spell the palindrome. With each hypothesis, John solved a problem, the result of which fueled his understanding of the overall situation. The linear appearance of Table 2 is not meant to suggest a linear progression of problem formulation followed by hypothesis generation; rather, the researcher posits a relationship where the solver's problems and problem solving activity continually feed and nourish each other, each providing sources of action.

In more theoretical terms, John's problem solving performance constituted a confluence or flowing together of his evolving hypotheses, deductions and inductions. Specifically, John hypotheses, while serving to answer questions that arose for him in the course of his on-going solution activity, also served as conceptual springboards to (1) provide structure for his potential actions (i.e., by structure I mean he could organize his potential activity in ways that were compatible with his goals), and (2) actualize hypothetical relationships in solution activity (i.e., self-generate particular trials that could feedback to his conjectures). For example, he started with the relatively primitive question of what constitutes a path, generating a hypothesis ($\mathbf{H_1}$) that enabled him to inductively generate and examine several actual paths. Results of these inductive trials provided him with feedback that enabled him to abduce more sophisticated properties about appropriate paths through the array ($\mathbf{H_2}$-$\mathbf{H_4}$), with each successive hypothesis suggesting new questions to explore.

Do abductions depend on the structure and/or sequence of tasks given or presented to the solver ? Since John had not seen the task before, it could be argued that this realization on his part put him in a mindset ripe for exploring aspects of the array, and that his abductions were just a natural consequence of an aggressive problem solver generating novel explorations of the array of letters. However, it should be noted that John's explorations were never random trial and error; rather, his trials were systematically generated and ongoing, suggesting that his knowledge evolved from his reflections on the results of trials (and not the trials themselves). Nevertheless, the researcher was interested in studying abductions in a variety of tasks and formats.

The following sections describe the problem solving activity of Marie. In contrast to the single task presented to John, Marie is presented with a series of tasks, thus enabling the researcher to examine her abductions across several situations.

About Marie. Marie was a first-year student enrolled in the third quarter of the university calculus sequence. Though undeclared in her academic major at the time of the interview, she eventually pursued and completed a degree in Physics.

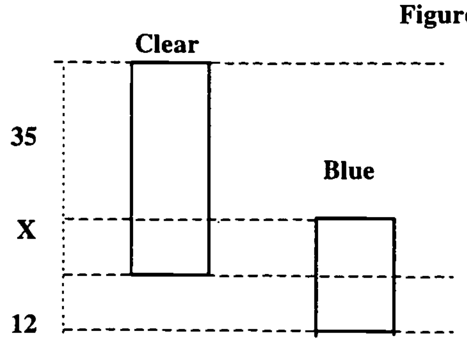
About the tasks. The tasks were designed by Yackel (1984) to illicit data about solvers' problem representations (see Table 1). The researcher's purpose in using these tasks was to induce problem solving situations for solvers, with a view towards examining the ways they construct and then generalize their solution activity across similar situations. Specifically, Marie was given the following algebra word problem to solve:

The surface of Clear Lake is 35 feet above the surface of Blue Lake. Clear Lake is twice as deep as Blue Lake. The bottom of Clear Lake is 12 feet above the bottom of Blue Lake. How deep are the two lakes?

Marie was then asked to solve eight follow-up tasks, each a variation of the lakes problem (e.g., similar but containing superfluous information, similar but containing insufficient information, similar but more general, etc.). These tasks provided opportunities for Marie to compare and contrast her actions across a range of similar problem solving situations. Her ongoing examination and re-examination of previously constructed strategies in the face of new problem situations made possible a context within which she could explore the effectiveness of her previously constructed strategies in new, yet similar situations. In this way, Marie's evolving hypotheses constituted a means by which she framed and developed novel explorations about the efficacy of her prior solution strategies in new situations, with particular emphasis on ways her strategies needed to be revised and adapted to fit new situations.

Marie's Inferential Processes. Marie constructed a solution to Task 1, using the variable **x** to represent the portion common to the two lakes (see Figure 1), an approach she successfully used while solving later tasks.

Figure 1: Marie's Solution to Task 1



$$2(X+12) = X + 35$$

$$X = 11$$

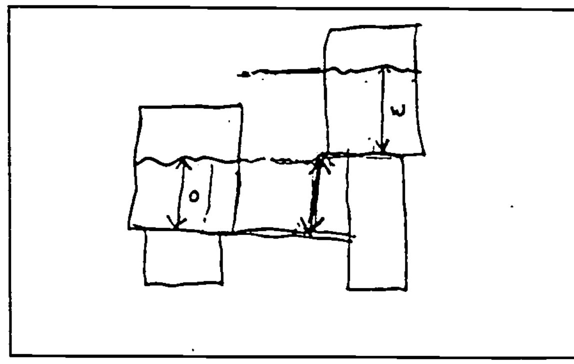
Clear Lake = X + 35 = 46 ft.
Blue Lake = X + 12 = 23 ft.

Despite constructing a solution, Marie's solution activity was by no means routine. For example, she initially interpreted the task as an "algebra word problem in two variables" and generated a system of several equations, no two of which were consistent. When she realized that this approach did not lead to a solution, she pursued an alternate solution method incorporating a graphical approach (i.e., diagrams of the lakes were constructed side-by-side and relevant lengths from the diagrams were translated to a vertical axis which served as a reference aid in constructing relationships). This solution activity eventually led to a correct solution.

After solving Task 1, Marie interpreted each successive task as similar to Task 1, anticipating that she would solve them much the same way as she solved Task 1. She experienced problematic situations whenever her anticipations of what to do proved inappropriate when actual activity was carried out. It was during these instances that Marie appeared to make abductions which had the impact of both changing the impetus of her solution activity and at the same time presenting novel opportunities for her to develop increased awareness about the efficacy of her previously constructed methods. For example, after solving Tasks 1 and 2 by using similar diagrams and solution methods, Marie attempted to solve Task 3, a problem with insufficient information, in the same way. However, she soon found herself faced with a problematic situation she had not anticipated:

Task 3: An oil storage drum is mounted on a stand. A water storage drum is mounted on a stand that is 8 feet taller than the oil drum stand. The water level is 15 feet above the oil level. What is the depth of the oil in the drum? Of the water?

Marie: *I am going to draw a picture. Here is my oil stand ... water stand. And we have a water storage 8 feet taller. And here's level water. And here's the oil level. (Long reflection) So, solve it ... the same way.* ***(She smiles, then displays a facial expression suggesting sudden puzzlement)*** *Impossible!! It strikes me suddenly that there might not be enough information to solve this problem. (She re-reads and reflects on her work) I suspect I'm going to need to know the height of one of these things (Solver points to both containers in her diagram). I don't know though, so I am going to go over here all the way through.*



Marie's anticipation that "the same way" would not work was followed by her hypothesis that the problem did not contain enough information, later refined to the hypothesis that she needed more information about the relative heights of the unknowns. By generating an hypothesis as a plausible reason for the unanticipated problem she faced, Marie adopted a new perspective in her activity that enabled her to consider aspects of the problem situation she had never before contemplated. While her hypothesis contained an element of uncertainty, it nevertheless helped her to organize and structure her subsequent solution activity, whereupon she explored and tested its plausibility as an explanatory device. More importantly, these novel

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actions served as a learning source from which she eventually developed more abstract criteria to evaluate her potential solution activity when solving later tasks.

In what ways did Marie's abductions help to structure her solution activity while solving Tasks 4-9 ? A partial answer to this question is that she became more cautious in her activity, spending increased time reflecting on her potential activity. As a result, her contemplations about potential solution activity took on a hypothetical quality in the sense that she could imagine states of the problem from which she could generate novel conjectures. For example, while solving Task 4, Marie quickly noticed the omission of a question from the problem statement yet was able to hypothesize potential problems for her to solve from the information:

Marie: *There's no question!*

Interviewer: *Is there a problem to solve?*

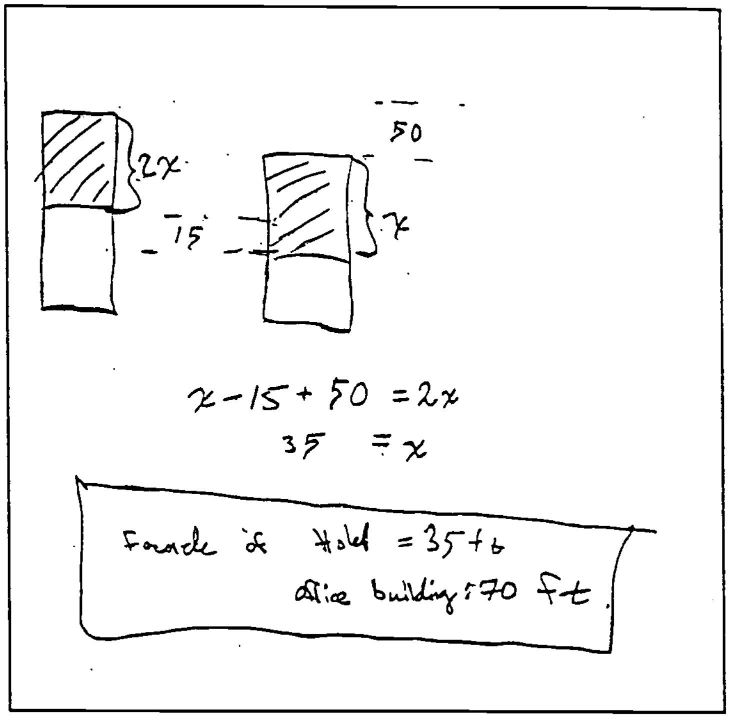
Marie: **(Long reflection here)** ... *The things they could ask for are things like ...* **(ANTICIPATION)** ... *the height of one of the buildings but ...* **(ANTICIPATION)** ... *there's not enough information to get that. ...* **(ANTICIPATION)** ... *The only thing we have information about is ...* **(ANTICIPATION)** ... *Ah, the relative heights of the two facades. So, if I were ... if somebody wanted me to solve any problem, that's probably what they're asking for.*

This episode illustrates the solver's developing flexibility and control of her potential solution activity. When prompted by the Interviewer, she engaged in a long period of reflection, the result of which she formulated a series of conjectures evaluating the potential problems that could be solved using the information contained within the problem statements. In particular, she determined that of two potential problems to solve, it made more sense to her to solve the problem of finding the heights of the two facades rather than finding the heights of the two buildings. Furthermore, her reflections included anticipatory activity. Specifically, she could mentally "call up" and explore potential solution activity and evaluate the usefulness of its results to achieve her goals. In particular, she could re-present or "run through" her potential solution activity in thought and use the results in an evaluative way, as a means with which to evaluate the viability of her potential solution activity to achieve her goals. Moreover, the solver demonstrated this highly abstract activity prior to constructing her diagrams of the two buildings.

The solver proceeded to construct a solution to the problem she had generated, utilizing diagrams to construct relationships, in much the same way as she had done to solve earlier tasks.

Marie: *Okay. Let's see if there is anything here that will at least give me information.* **(Long reflection here)** *Okay, the hotel is 50 feet shorter than the office building. So we have distance here which is 50. The facade of the hotel extends 15 feet below the facade of the office building. That distance would be 15. The height of the facade on the office building is twice that on the hotel.* **(Long reflection here)** *So I call this distance X, this distance here is 2X. All right!*

Marie: *And then I can say that X minus ... I'm trying to find a relationship between these two. And I know that ... X minus 15 plus 50 is going to equal 2X. So, 35 equals one X. So that would indicate that the facade on the hotel is 35 feet. On the office building is 70 feet.*



The solver continued to develop her solution activity while solving Tasks 5-9.

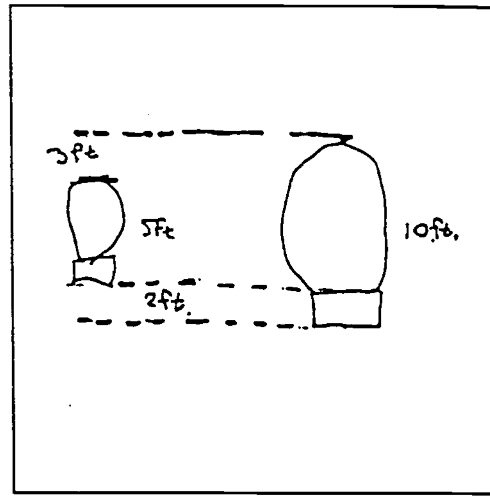
The solver's solution activity in Task 9 indicated that she had re-organized her solution activity at a higher level of abstraction, to the extent that she could reflect on her potential solution activity, anticipate its results, and evaluate the usefulness of the results for the current situation without the need to carry out the activity with paper-and-pencil. In other words, she could reflect on her potential solution activity, determine appropriate relationships, and evaluate the efficacy of those relationships for the situation she faced. The task required the solver to make up a problem which had a solution method similar to the prior tasks.

Marie: *Okay, ...* **(ANTICIPATION)** *... I'm thinking of something with different heights. Oh, ...* **(ANTICIPATION)** *... bookshelves in a bookcase. **(Long Reflection)** No, ...* **(ANTICIPATION)** *... that's no good. ... How about hot air balloons!*

The solver ran through potential solution activity for the particular situation she proposed (i.e., bookshelves) and anticipated its results (i.e., that it would not work for "bookshelves" but that she could solve it for "hot air balloons"). In this way, she ran through potential solution activity in thought, produced its results, and drew inferences from the results. Her subsequent actions in completing the task, leading to a formal statement of an appropriate similar problem, were routine and indicated she was very confident that she had constructed a correct solution.

Marie: *Okay if I were going to draw a picture of the problem I'd have one hot balloon that looks like ...* **(DRAWS BALLOON)**. *And a bigger hot air balloon that looks like that. And I'll make this distance ... 3 feet. I'll make this distance 2 feet. And I'll make this height 10 feet high 'cause that makes this 12 feet and that makes this one twice this one which is useful. So, I'll just say the top of one hot air balloon, HAB being the abbreviation for that, is 3 feet. I'll make it a yellow hot air balloon which will make it easier, above a green hot air balloon.*

Marie: *The bottom of the yellow hot air balloon is 2 feet below the bottom of the green hot air balloon. The yellow balloon is twice the height of the green balloon. Let's make that a lake. What are the heights of the balloons?*



Discussion

Marie's solution activity can be summarized as follows. She constructed a viable solution method while solving Task 1. Tasks 2-9 gave her opportunities to elaborate and refine her solution methods across situations she interpreted as similar to the initial task.

Table 5 contains a task-by-task summary of Marie's hypotheses, together with the researcher's inferences and interpretations of the role they played as structuring resources in Marie's solution activity.

Insert Table 5 about here

From Table 5 it is seen that Marie generates hypotheses when she finds herself faced with problems she had not anticipated. These problems can be viewed as challenges to her interpretations of "problem sameness". While Marie's hypotheses constituted plausible explanations of the problems she faced (i.e., why they not be the same), they suggested structure for her potential solution activity (i.e., specific actions to carry out to explore the situation). For example, hypothesis $\mathbf{H_1}$ is a plausible explanation of the unanticipated problem she experienced when she tried to "do the same thing" to solve Task 3 as she did while solving Task 1. In addition, hypothesis $\mathbf{H_1}$ signaled a change of perspective in her solution activity, which both called into question the viability of her prior solution methods for the current situation ("there might not be enough information") and, at the same time suggested possibilities for future action ("I think I am going to need to know one of these things"). This testing activity led her to refine and revise her initial hypothesis as $\mathbf{H_2}$; the result of this activity was that Marie achieved a heightened awareness over her solution methods (i.e., why and how well they work). Such mathematical knowledge has been referred to as self-efficacy (Pajares and Kranzler, 1994) or control of solution activity (Schoenfeld, 1985). The term self-efficacy is preferred here. According to Pajares and Kranzler (1994), self-efficacy "may serve students well when solving math problems, not because it causes them to be better problem solvers, but because it engenders greater interest in and attention to working the problems, increased effort, and greater perseverance in the face of adversity" (Pajares and Kranzler, 1994, p. 9).

Conclusions

The analysis indicated some prominent ways that the solvers' abductive reasoning influenced their solution activity. First, abduction was characterized as an ongoing, sense-making process that constituted the solvers' source of novel ideas about how to proceed when they found themselves faced with unexpected problems. Second, the solvers' abductions aided their novel exporations, serving to organize and structure their subsequent solution activity. Specifically, the solvers' self-generated hypotheses went hand-in-hand with their conception and carrying out of purposive activity designed to test the viability of their hypotheses. This result is consistent with the view that abductive reasoning connects hypothetical ideas with inductively generated courses of action that test the plausibility of those ideas and confers a degree of certainty on them (Batts, Cook, and Lincourt, 1979). Third, the novelty demonstrated by the solvers' through their abductive inferences suggests the need to re-think some of our views about "teaching" problem solving. Specifically, the ideas generated by the solvers were their own, self-generated to help them "make sense" of the situations they faced, and seen by them as plausible explanations of the problems. However, from these intial ideas evolved conceptually rich ideas that included new problem formulations and re-formulations, and conjectures about how potential solution activity would work out.

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Table 1: SET OF SIMILAR ALGEBRA WORD PROBLEMS

TASK 1: Solve the Two Lakes Problem

The surface of Clear Lake is 35 feet above the surface of Blue Lake. Clear Lake is twice as deep as Blue Lake. The bottom of Clear Lake is 12 feet above the bottom of Blue Lake. How deep are the two lakes?

TASK 2: Solve a Similar Problem Which Contains Superfluous Information

The northern edge of the city of Brownsburg is 200 miles north of the northern edge of Greenville. The distance between the southern edges is 218 miles. Greenville is three times as long, north to south as Brownsburg. A line drawn due north through the city center of Greenville falls 10 miles east of the city center of Brownsburg. How many miles in length is each city, north to south?

TASK 3: Solve a Similar Problem Which Contains Insufficient Information

An oil storage drum is mounted on a stand. A water storage drum is mounted on a stand that is 8 feet taller than the oil drum stand. The water level is 15 feet above the oil level. What is the depth of the oil in the drum? Of the water?

TASK 4: Solve a Similar Problem In Which the Question is Omitted

An office building and an adjacent hotel each have a mirrored glass facade on the upper portions. The hotel is 50 feet shorter than the office building. The bottom of the glass facade on the hotel extends 15 feet below the bottom of the facade on the office building. The height of the facade on the office building is twice that on the hotel.

TASK 5: Solve a Similar Problem Which Contains Inconsistent Information

A mountain climber wishes to know the heights of Mt. Washburn and Mt. McCoy. The information he has is that the top of Mt. Washburn is 2000 feet above the top of Mt. McCoy, and that the base of Mt. Washburn is 180 feet below the base of Mt. McCoy. Mt. McCoy is twice as high as Mt. Washburn. What is the height of each mountain?

TASK 6: Solve a Similar Problem Which Contains the Same Implicit Information

A freight train and a passenger train are stopped on adjacent tracks. The engine of the freight is 100 yards ahead of the engine of the passenger train. The end of the caboose of the freight train is 30 yards ahead of the end of the caboose of the passenger train. The freight train is twice as long as the passenger train. How long are the trains?

TASK 7: Solve a Similar Problem that is a Generalization

In constructing a tower of fixed height a contractor determines that he can use a 35 foot high base, 7 steel tower segments and no aerial platform. Alternatively, he can construct the tower by using no base, 9 steel tower segments and a 15 foot high aerial platform. What is the height of the tower he will construct?

TASK 8: Solve a Similar Simpler Problem

Green Lake and Fish Lake have surfaces at the same level. Green Lake is 3 times as deep as Fish Lake. The bottom of Green Lake is 40 feet below the bottom of Fish Lake. How deep are the two lakes?

TASK 9: Make Up a Problem Which has a Similar Solution Method

Table 2: EXAMPLES OF NON-ALGEBRA WORD PROBLEMS USED IN THE STUDY

- The Big 8 Conference was getting ready for its basketball season. There are 8 teams in the Conference and each team plays every one of the 7 other teams in the conference twice during the season. How many games will be played during the season ?

Sally, an avid canoeist, decided one day to paddle upstream 6 miles. In 1 hour, she could travel 2 miles upstream, using her strongest stroke. After such strenuous activity, she needed to rest for 1 hour, during which time the canoe floated downstream 1 mile. In this manner of paddling for 1 hour and resting for 1 hour, she traveled 6 miles upstream. How long did it take her to make this trip ? Suppose after 4 hours on the river, Sally took a lunch break for 1 hour, during which time she floated downstream. How long did it take her to go the 6 miles up the river ?

Joe and Rhoda bought some items in the local pharmacy. All the items they bought cost the same amount, and they bought as many items as the number of cents in the cost of one single item. If Joe and Rhoda spent exactly $6.25, how many items did they buy ?

The new school was almost completed, and Richard's job was to put numbers on the lockers. The lockers were to be numbered consecutively beginning with 1. The numbers were to be put up 1 digit at a time. Richard used 492 digits in all. How many lockers were put in the school?

Steve is responsible for keeping the fish tanks in the Seaside Aquarium Shop filled with water. One of their 50-gallon tanks has a small leak, and along with evaporation, loses 2 gallons of water each day. Every three days, Steve adds 5 gallons of water to the tank, and on the 30th day, he fills the tank. How much water will he have to add on the 30th day to fill the tank ?

Ryland Homes has just completed a development of 369 family homes along a ten mile stretch of Griswell Road. The homes are to be numbered consecutively starting with the number **1** and ending with the number **369**. Each house will have its address adorning the post outside the front door. One of Ryland's contractors is in charge of purchasing the single-digit brass numerals that will make up each address. If the cost of each brass numeral is $2.50, how much will the contractor have to spend to purchase the needed digits to number all 369 houses ?

At a Chinese dinner every 4 guests shared a dish of rice, every 3 guests shared a dish of vegetables, and every 2 guests shared a dish of meat. There were 65 dishes in all. How many guests were there ?

Suppose you are given a collection of 12 coins that look identical but you are told that one of the coins weighs less than the others, and is counterfeit. If you have a balance scale, how can you determine in at most 3 weighings which is the fake coin ? What is the most number of coins you can have, given one "light" counterfeit coin, and be able to determine the fake coin in at most 3 weighings ?

WAS IT A CAT I SAW

While perusing a rare early edition of Alice in Wonderland, Professor Cifarelli came across an illustration which included the spatial diagram below. See how many mathematical problems you can make up and solve using the array.

(Once solvers had generated some of their own problems, they were asked to solve the following problem if they had not thought of it themselves): Starting at any **W** on the boundary, how many ways can you spell out **Was It a Cat I Saw** ?

```
            W
          W A W
        W A S A W
      W A S I S A W
    W A S I T I S A W
  W A S I T A T I S A W
W A S I T A C A T I S A W
  W A S I T A T I S A W
    W A S I T I S A W
      W A S I S A W
        W A S A W
          W A W
            W
```

Table 5: Marie's Abductions, Deductions, and Inductions

Task	Nature of Problem	Hypothesis	Result of Hypothesis
Task 3	same method will not work	**H$_1$:** Not enough information	Generates trial to test **H$_1$** Generates feedback and refines **H$_1$** (solved problem)
		H$_2$: May need to know one of the individual heights	No new trials (she is not sure how she can test it) (problem unresolved)
Task 4	needs to formulate problem that can be solved using the given information	**H$_3$:** They <u>could</u> ask for height of 2 buildings	generates trial to test **H$_3$** (runs through thought and sees results problematic (solved problem)
		H$_4$: They <u>could</u> ask for height of facade and top of building	generates trial to test **H$_4$** (runs through thought and sees results compatible with her goals (solved problem)
		H$_5$: It's now the same !	carries out solution activity with paper and pencil
Task 9	Make up a similar problem	**H$_6$:** How about bookshelves?	generates trail to test **H** (runs through thought and sees results problematic
		H$_7$: How about air balloons?	carries out solution activity with paper and pencil

Induction

Deduction

Abductive inference indicated by **H$_i$**

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[1]If you are an AERA chair or discussant, please save this form for future use.