
Contents

Part I Basic Mean-Field Theory for Bose–Einstein Condensates

1 Basic Mean-Field Theory for Bose–Einstein Condensates

<i>P.G. Kevrekidis, D.J. Frantzeskakis, and R. Carretero-González</i>	3
1.1 Introduction	3
1.2 The Gross–Pitaevskii (GP) Mean-Field Model	4
1.2.1 Origin and Basic Properties of the GP Equation	4
1.2.2 Repulsive and Attractive Interactions: Feshbach Resonance	5
1.2.3 The External Potential	6
1.3 Dimensionality Reduction	8
1.3.1 Length Scales	8
1.3.2 Derivation of Lower-Dimensional Models	9
1.3.3 The Discrete Nonlinear Schrödinger Equation	10
1.4 Ground State and Excitations	11
1.4.1 Ground State	11
1.4.2 Small-Amplitude Linear Excitations	12
1.4.3 Macroscopic Excitations: Solitons and Vortices	14
References	17

Part II Bright Solitons in Bose–Einstein Condensates

2 Bright Solitons in Bose–Einstein Condensates: Theory

<i>F.Kh. Abdullaev and J. Garnier</i>	25
2.1 Introduction	25
2.2 Bright Solitons in Quasi One-Dimensional BEC	27
2.2.1 The 1D Gross–Pitaevskii Equation	27
2.2.2 Adiabatic Soliton Compression	30
2.2.3 Transmission Through Nonlinear Barriers and Wells	32
2.2.4 Trapping by Dynamically Managed Linear Potentials	34

2.2.5	Controllable Soliton Emission by Spatial Variations of the Scattering Length	36
2.3	Bright Solitons in Nonlinear Optical Lattices	36
2.3.1	Propagation Through a Weak Nonlinear Periodic Potential	36
2.3.2	Propagation Through a Weak Random Nonlinear Potential	38
2.4	Multidimensional Bright Solitons in BECs	38
2.4.1	2D Bright Solitons in BECs with Time-Varying Scattering Length	38
2.4.2	2D Bright Solitons in BECs with Spatially-Varying Scattering Length	39
2.4.3	2D Bright Solitons in Dipolar BECs	40
2.4.4	3D Bright Solitons in Anisotropic Traps	40
2.5	Future Challenges	41
	References	41

3 Bright Solitons: Summary of Experimental Techniques

	<i>L. Khaykovich</i>	45
3.1	Introduction	45
3.2	Tunable Interatomic Interactions	46
3.2.1	Feshbach Resonance	46
3.2.2	Measure of the Interaction Strength	49
3.3	Optical Confinement of Bose–Einstein Condensates	50
3.3.1	BECs in Optical Traps	50
3.3.2	BECs in Low Dimensions	51
3.4	The Experiments	52
3.4.1	Formation of a Single Soliton	52
3.4.2	Soliton Trains	54
3.4.3	Formation of Solitons in Nearly 3D Traps	56
3.5	Origin of Higher Order Nonlinearity and its Impact on Soliton Dynamics	56
3.6	Conclusions	58
	References	59

Part III Dark Solitons in Bose–Einstein Condensates

4 Dark Solitons in Bose–Einstein Condensates: Theory

	<i>V. V. Konotop</i>	65
4.1	Condensate in an Elongated Trap	66
4.1.1	The Characteristic Scales	66
4.1.2	3D-to-1D Reduction of the Gross–Pitaevskii Equation	66
4.2	Dark Solitons in a Homogeneous Condensate	68
4.2.1	Dark Solitons of the Nonlinear Schrödinger Equation	68
4.2.2	On a Definition of a Dark Soliton	69

4.2.3	Dark Solitons and Sound Waves	71
4.2.4	Dark Solitons of the Quintic NLS Equation	72
4.3	Dark Solitons in a Trap	73
4.3.1	The Background Density Distribution	73
4.3.2	Landau Dynamics of a Soliton in an Inhomogeneous Condensate	74
4.3.3	Perturbation Theories and Long-Time Dynamics of a Dark Soliton	76
4.4	Nonconservative Dynamics of Dark Solitons	77
4.4.1	Effect of Dissipation on the Dark Soliton Evolution	77
4.4.2	Dark Solitons in Varying Traps	78
4.5	Theory of Generation of Dark Solitons	78
4.5.1	Phase Imprinting	79
4.5.2	Density Engineering	79
4.5.3	Generation of Trains of Dark Solitons	80
4.5.4	Feshbach-Resonance Management	80
	References	81

5 Dark Solitons in BECs: The First Experiments

<i>B.P. Anderson</i>	85
5.1	Introduction	85
5.2	Overview of Experimental Approaches	86
5.2.1	Bose–Einstein Condensates as a Nonlinear Medium	86
5.2.2	Soliton Creation Techniques	87
5.2.3	Imaging	88
5.3	Observations with Dark Solitons	89
5.4	Conclusions	95
	References	96

Part IV Nonlinear Localization of BECs in Optical Lattices

6 Nonlinear Localization of BECs in Optical Lattices

<i>E.A. Ostrovskaya, M.K. Oberthaler, and Y.S. Kivshar</i>	99
6.1	Introduction	99
6.2	Experimental Work Horse: Optical Potentials	100
6.3	BEC in a Periodic Potential: Theoretical Formalism	101
6.3.1	Mean-Field Model	101
6.3.2	Linear Bloch Waves	102
6.3.3	Nonlinear Bloch Waves	104
6.4	Dispersion/Diffraction Management: Experiment	106
6.5	Gap Solitons	109
6.5.1	Bright Solitons in Repulsive BEC	109
6.5.2	Soliton Trains and Anomalous Heating	113

6.6	Self-Trapped States	117
6.6.1	Observation of the Macroscopic Self-Trapping	117
6.6.2	Truncated Bloch States	119
6.7	Gap Vortices	121
6.8	Multi-Component Gap States	124
6.9	Conclusions and Acknowledgments	128
	References	128

Part V Multi-Dimensional Solitons in Bose–Einstein Condensates

7 Multidimensional Solitons: Theory

	<i>L.D. Carr and J. Brand</i>	133
7.1	Introduction	133
7.2	Dark Solitons and Solitary Waves in Higher Dimensions	134
7.2.1	Dark Band and Planar Solitons	134
7.2.2	Ring Dark Solitons and Spherical Shell Solitons	137
7.2.3	Solitary Waves in Restricted Geometries	139
7.2.4	Vortex Rings and Rarefaction Pulses	141
7.2.5	Multi-Component Bose–Einstein Condensates	142
7.3	Bright Solitons in Higher Dimensions	143
7.3.1	Instability, Metastability, Stability	143
7.3.2	Bright Soliton Engineering: Pulsed Atom Lasers and Other Applications	146
7.3.3	Solitons in a Thermal Bath	148
7.3.4	Soliton–Soliton Interactions	148
7.3.5	Bright Ring Solitons and Quantum Vortices	149
7.4	Summary and Acknowledgments	152
	References	152

8 Experiments on Multidimensional Solitons

	<i>J. Brand, L.D. Carr, and B.P. Anderson</i>	157
8.1	Dimensional Aspects of Soliton Experiments in BECs	157
8.2	Preparation of Non-equilibrium BECs	158
8.2.1	Dark Soliton Quantum State Engineering	158
8.2.2	Density Engineering by Slow Light	159
8.3	Decay and Formation of Multidimensional Solitons	162
8.3.1	Quantum Shock Wave Dynamics and Soliton Shedding	162
8.3.2	Snake Instability and Vortex Ring Generation	163
8.4	Interacting Dark Solitons and Hybrid Structures	166
8.5	Conclusions	168
	References	168

Part VI Vortices in Bose–Einstein Condensates

9 Vortices in Bose–Einstein Condensates: Theory

<i>N.G. Parker, B. Jackson, A.M. Martin, and C.S. Adams</i>	173
9.1 Quantized Vortices	173
9.1.1 Theoretical Framework	173
9.1.2 Vortex Structures	175
9.2 Nucleation of Vortices	176
9.2.1 Rotation	176
9.2.2 Nucleation by a Moving Object	177
9.2.3 Other Mechanisms and Structures	178
9.3 Dynamics of Vortices	178
9.4 Stability of Vortices	180
9.4.1 Thermal Instabilities	180
9.4.2 Hydrodynamic Instabilities	180
9.5 Dipolar BECs	183
9.5.1 The Modified Gross–Pitaevskii Equation	183
9.5.2 Vortex Energy	184
9.6 Analogs of Gravitational Physics in BECs	184
9.6.1 Superradiance	185
References	186

10 Vortices in Bose–Einstein Condensates: Experiments

<i>F. Chevy</i>	191
10.1 Introduction: Vortices and Superfluidity	191
10.2 Nucleation of Vortices	192
10.2.1 Vortices in Liquid Helium	192
10.2.2 Phase Imprinting	192
10.2.3 Stirring	194
10.2.4 Rotationless nucleation	197
10.3 Experimental Characterization	198
10.3.1 Vortex Profile, Vortex Bending and Decay	198
10.3.2 Vortex Charge	198
10.3.3 Kelvin Modes of a Single Vortex Line	203
10.4 Conclusions	205
References	206

Part VII Vortex Lattices

11 Vortex Lattices in Bose–Einstein Condensates: Theory

<i>M. Ueda and H. Saito</i>	211
11.1 Hydrodynamic Theory of Vortices	211
11.2 Vortices in a Bose–Einstein Condensate	214

11.3	Collective Modes of Vortices	218
11.3.1	Vortex Filament	218
11.3.2	Vortex Lattice	218
11.4	Dynamics of Vortex Nucleation	220
11.4.1	Scalar BEC	220
11.4.2	Spinor BEC	223
11.5	Fast Rotating BEC	225
	References	227

12 Vortex Lattices in Bose–Einstein Condensates:

Experiments

	<i>C. Raman</i>	229
12.1	Overview	229
12.2	Experimental Observation	230
12.3	Spinning Condensates	232
12.4	Lattice Basics	233
12.5	Lattice Dynamics	235
12.6	Seeing the Phase	237
12.7	The Rotating Speed Limit	240
12.8	Summary and Outlook	241
	References	242

Part VIII Optical Lattices

13 Optical Lattices: Theory

	<i>A. Smerzi and A. Trombettoni</i>	247
13.1	Introduction	247
13.2	Discrete Equations for the Dynamics	249
13.2.1	Effects of Transverse Confinement	251
13.3	Excitation Spectra	253
13.3.1	Bloch Spectrum	254
13.3.2	Bogoliubov Spectrum	255
13.4	Landau and Dynamical Instabilities	256
13.5	Wave Packet Dynamics	258
	References	262

14 Bose–Einstein Condensates in Optical Lattices:

Experiments

	<i>O. Morsch</i>	267
14.1	Introduction	267
14.1.1	Technical Considerations	268
14.1.2	Measurements, Observables and Calibration	269
14.2	Linear and Nonlinear Dynamics	271
14.2.1	Bloch Oscillations	271
14.2.2	Landau–Zener Tunneling	272

14.2.3	Josephson Effects	273
14.2.4	Instabilities	275
14.2.5	Dispersion Management and Solitons	276
14.3	Quantum Effects and the Mott Insulator Transition	277
14.4	Mixtures, Molecules and Fermions in Lattices	279
14.5	Perspectives	280
	References	280

Part IX Multi-Component Bose–Einstein Condensates

15 Multi-Component Bose–Einstein Condensates: Theory

<i>B. Malomed</i>		287
15.1	Introduction	287
15.2	Basic Models: Coupled Gross–Pitaevskii Equations	288
15.3	Immiscible Species in One Dimension: Domain-walls and a Transition to Miscibility in Boson Gases	290
15.4	Degenerate Binary Fermion Gases	295
15.5	Symbiotic Solitons in Binary BECs	297
15.6	Domain-wall Crosses and “Propellers” in Two Dimensions	299
15.7	More Complex Models	301
15.8	Conclusions	302
	References	303

16 Multi-Component Condensates: Experiment

<i>D.S. Hall</i>		307
16.1	Introduction	307
16.1.1	Imaging	308
16.1.2	Trapping	309
16.2	Pseudospinor Condensates	309
16.2.1	Component Separation and Domain Formation	311
16.2.2	Phase Coherence	314
16.2.3	Thermodynamics	315
16.2.4	Wavefunction Engineering	316
16.2.5	Spin Textures	317
16.3	Spinor Condensates	318
16.3.1	Basic Magnetic Properties	318
16.3.2	Spin Dynamics	320
16.3.3	Symmetry Breaking and Domain Formation	323
16.3.4	Thermodynamics	323
16.4	Future Prospects/Conclusions	324
	References	324

Part X Manipulations of Coherent Matter-Waves

17 Manipulations of Coherent Matter-Waves

<i>P.G. Kevrekidis, D.J. Frantzeskakis, and R. Carretero-González</i>	331
17.1 Introduction	331
17.2 General Aspects of Guidance and Driving of Matter-Waves	332
17.2.1 Matter-Wave Solitons in the Mean-Field Picture	332
17.2.2 Time-Dependent External Potentials	333
17.3 Matter-Waves and Localized Impurities	335
17.3.1 Bright Matter-Wave Solitons	335
17.3.2 Dark Matter-Wave Solitons	336
17.3.3 Matter-Waves in Optical Lattices and Subject to Localized Impurities	337
17.4 Driving Matter-Waves by Optical Lattices	338
17.4.1 Bright Matter-Wave Solitons	338
17.4.2 Dark Matter-Wave Solitons	339
17.4.3 Matter-Wave Solitons in Optical Superlattices	340
17.5 Manipulations of Vortices	341
17.5.1 Manipulating Vortices by Localized Impurities	341
17.5.2 Vortices in Optical Lattices and Vortex Lattices	342
17.5.3 Vortex and Dark Soliton Nucleation Induced by Moving Impurities	343
17.6 Manipulations of the s -Wave Scattering Length	344
Conclusions	345
References	345

Part XI Beyond Gross–Pitaevskii Mean Field Theory

18 Beyond Gross–Pitaevskii Mean-Field Theory

<i>N.P. Proukakis</i>	353
18.1 Introduction	353
18.2 Microscopic Derivation of the Gross–Pitaevskii Equation	354
18.3 Generalized Mean-Fields: Static Thermal Cloud	357
18.4 Generalized Mean-Fields: Dynamic Thermal Cloud	359
18.4.1 Time-Dependent Hartree–Fock–Bogoliubov	359
18.4.2 Theory of Zaremba–Nikuni–Griffin	361
18.5 Kinetic Theories based on Probability Distribution Functions	363
18.5.1 Stoof’s Non-Equilibrium Theory	363
18.5.2 The Gardiner–Zoller Quantum Kinetic Master Equation	364
18.6 Stochastic Approaches to Condensate Dynamics	365
18.6.1 Classical Field Methods	365
18.6.2 The Stochastic Gross–Pitaevskii Equation	367

18.7 The Role of System Dimensionality 368
 References 369

Part XII Asymptotic Reductions of the Gross–Pitaevskii Equation

19 Asymptotic Reductions of the Gross–Pitaevskii Equation

D.E. Pelinovsky 377
 19.1 Introduction 377
 19.2 Class of Periodic Potentials 378
 19.2.1 Small Strength: Coupled-Mode Equations 380
 19.2.2 Moderate Strength: Continuous NLS Equations 382
 19.2.3 Large Strength: Discrete NLS Equations 387
 19.3 Class of Decaying Potentials 391
 19.4 Class of Confining Potentials 394
 Conclusions 396
 References 397
Index 399