Emergent universe in a Jordan-Brans-Dicke theory

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- S. del Campo, R. Herrera and P. Labrana, "Emergent universe in a Jordan-Brans-Dicke theory," JCAP **0711** (2007) 030.
- S. del Campo, R. Herrera and P. Labrana, "On the Stability of Jordan-Brans-Dicke Static Universe"

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Emergent Universe Models

Independent of the successes of the Standard Cosmological Model and the inflationary paradigm in order to explain the cosmological data (as was mentioned previously by Gabriela Baremboim), there are still important question to be address in modern cosmology.

One of this fundamental questions is whether the universe had a definite origin or whether it is past eternal.

- Recent work seems to indicate that the answer to this intriguing question is no.
- Singularity theorems have been devised that apply in the inflationary context, showing that the universe necessarily had a beginning. In other words, according to these theorems:

> The initial singularity cannot be avoided in the past even if inflation takes place.

[1] Borde A. and Vilenkin A., Eternal inflation and the initial singularity, 1994 Phys. Rev. Lett. **72** 3305.
[2] Borde A., Guth A. H. and Vilenkin A., Inflationary space-times are incompletein past directions, 2003 Phys. Rev. Lett. **90** 151301.

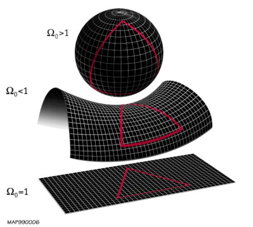
However, recently, the search for singularity free inflationary models has led to the development of Emergent Universe models.

[3] Ellis G. F. R. and Maartens R., The emergent universe: Inflationary cosmology with no singularity, 2004 Class. Quant. Grav. **21** 223.
[4] Ellis G. F. R., Murugan J. and Tsagas C. G., The emergent universe: An explicit construction, 2004 Class. Quant. Grav. **21** 233.

These models do not satisfy the geometrical assumptions of [1,2], were it is assumed that either

- The universe has open space sections, implying that the universe is open or flat.
- The Hubble expansion rate $H = \dot{a}/a$ is bounded away from zero in the past, $H > 0$, where a is the scale factor.

Closed universe, Opend universe,Flat universe



Emergent Universe Models

The original idea of an emergent universe is a simple closed inflationary model in which the universe emerges from an Einstein Static (ES) state with radius $a_0 >> l_p$ (where l_p is the Planck length), inflates and is then subsumed into a hot Big Bang era.

- Such models are appealing since they provide specific examples of non-singular (geodesically complete) inflationary universes.
- Also, these models could avoid an initial quantum-gravity stage if the static radius is larger than the Planck length.

Emergent Universe Models

Summarize, in these models, the universe is positively curved and initially in a past eternal classical static state that eventually evolves into a subsequent inflationary phase.
We can identify three important regimens in these models:

- First stage: Past eternal static universe.
- Second stage: Leaving the static regimen.
- Third stage: Inflation.

Jordan-Brans-Dicke Theory

The Jordan-Brans-Dicke (JBD) theory is a class of models in which the effective gravitational coupling evolves with time. The strength of this coupling is determined by a scalar field, the so-called Brans-Dicke field, which tends to the value G^{-1}, the inverse of the Newton's constant.
In modern context, Brans-Dicke theory appears naturally in supergravity models, Kaluza-Klein theories and in all the known effective string actions.

We consider the following JBD action for a self-interacting potential and matter, given by

$$S = \int d^4x \sqrt{-g}\left[\frac{1}{2}\,\Phi\,R - \frac{1}{2}\,\frac{\omega}{\Phi}\,\nabla_\mu\Phi\,\nabla^\mu\Phi + V(\Phi) + \mathcal{L}_m\right] \quad (1)$$

where $\mathcal{L}_m$ denote the Lagrangian density of the matter, R is the Ricci scalar curvature, Φ is the JBD scalar field, ω is the JBD parameter and $V(\Phi) = V$ is the potential associated to the field Φ. In this theory $1/\Phi$ plays the role of the gravitational constant, which changes with time.

Scalar-Tensor Theories of Gravity

The original Brans-Dicke model corresponds to $V(\Phi) = 0$. However, non-zero $V(\Phi)$ is better motivated and appears in many particle physics models. In particular, $V(\Phi)$ can be chosen in such a way that Φ is forced to settle down to a non-zero expectation value, $\Phi \to m_p^2$, where $m_p = 10^{19} GeV$ is the value of the Planck mass today. On the other hand, if $V(\Phi)$ fixes the field Φ to a non-zero value, then time-delay experiments place no constraints on the Brans-Dicke parameter ω.

In particular, if we choose the JBD potential in such a way that Φ will be forced to stabilize at a constant value Φ_f at the end of the inflationary period. Then, we can recover General Relativity by setting $\Phi_f = m_p^2$, therefore whatever we choose for w in our model, it does not contradict the solar system bound on ω.

Emergent Universe in a Jordan-Brans-Dicke Theory

Jordan- Brans-Dicke Theory

$$S = \int d^4x \sqrt{-g}\left[\frac{1}{2}\,\Phi\,R - \frac{1}{2}\,\frac{\omega}{\Phi}\,\nabla_\mu\Phi\,\nabla^\mu\Phi + V(\Phi) + \mathcal{L}_m\right]$$

The universe is dominated by a scalar field Ψ (Inflaton).

$$\mathcal{L}_m = \frac{1}{2}\partial_\mu\Psi\partial^\mu\Psi - U(\Psi)$$

The energy density and the pressure are given by:

$$\rho = \frac{\dot{\Psi}^2}{2} + U(\Psi)$$

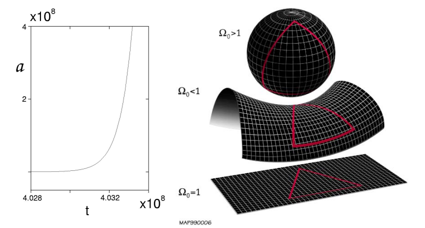
$$P = \frac{\dot{\Psi}^2}{2} - U(\Psi)$$

The universe is consider as a closed universe. Then the Friedmann-Robertson-Walker metric becomes:

$$ds^2 = dt^2 - a(t)^2 \left[\frac{dr^2}{1 - r^2} + r^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2 \right) \right] , \qquad (2)$$

where $a(t)$ is the scale factor, t represents the cosmic time.

Emergent Universe Model: Evolution of the scale factor



Field Equation

$$H^2 + \frac{1}{a^2} + H\frac{\dot{\Phi}}{\Phi} = \frac{\rho}{3\,\Phi} + \frac{w}{6}\left(\frac{\dot{\Phi}}{\Phi}\right)^2 + \frac{V}{3\,\Phi}, \qquad (3)$$

$$2\frac{\ddot{a}}{a} + H^2 + \frac{1}{a^2} + \frac{\ddot{\Phi}}{\Phi} + 2\,H\frac{\dot{\Phi}}{\Phi} + \frac{w}{2}\left(\frac{\dot{\Phi}}{\Phi}\right)^2 - \frac{V}{\Phi} = -\frac{P}{\Phi} \qquad (4)$$

$$\ddot{\Phi} + 3H\dot{\Phi} = \frac{(\rho - 3P)}{(2w+3)} + \frac{2}{2w+3}\left[2\,V - \Phi\,V'\right] \qquad (5)$$

and the conservation of energy-momentum implies that

$$\ddot{\Psi} + 3H\dot{\Psi} = -\frac{\partial U(\psi)}{\partial \Psi} \qquad (6)$$

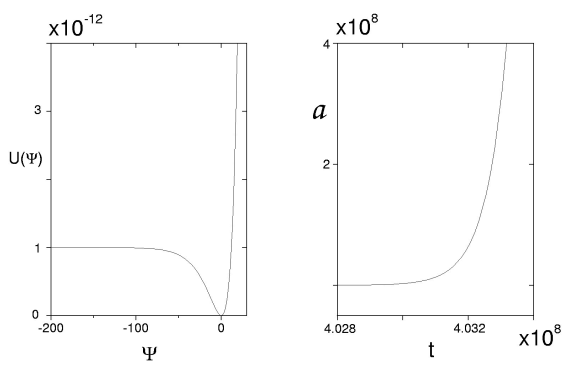
Emergent Universe Models

- First stage: Static universe.
- Second stage: Leaving the static regimen.
- Third stage: Inflation.

In order to obtain this behavior for the scale factor we consider the following potential $U(\Psi)$ for the inflaton.

The potential for the inflaton field and *Graceful Entrance to Inflation*

Inflaton Potential



- We consider a general class of potentials that approach a constant U_0 as $\Psi \to -\infty$ and rise monotonically once the value of the scalar field exceeds a certain value.
- Then during the static regimen, the matter potential $U(\Psi)$ is consider as a flat potential and the scalar field rolls along this potential with a constant velocity $\dot{\Psi}_0$.
- After that the overall effect of increasing the potential is to distort the equilibrium behavior of the static regimen breaking the static solution. In particular, the field Ψ decelerates as it moves further up the potential, subsequently reaching a point of maximal displacement and then rolling back down. If the potential has a suitable form in this region, slow-roll inflation will occur. *Graceful Entrance to Inflation*.

Static Regimen

Static universe in the context of JBD theory is characterized by the conditions $a = a_0 = Const.$, $\dot{a}_0 = 0 = \ddot{a}_0$ and $\Phi = \Phi_0 = Cte.$, $\dot{\Phi}_0 = 0 = \ddot{\Phi}_0$.
We can find a static solution given by:

$$a_0^2 = \frac{3}{V_0'}, \tag{7}$$

$$\rho_0 = V_0' \Phi_0 - V_0, \tag{8}$$

$$\frac{\Phi_0}{a_0^2 \rho_0} = 2\left(1 - \frac{U_0}{\rho_0}\right). \tag{9}$$

$$\dot{\Psi}_0^2 = 2\,\frac{\Phi_0}{a_0^2}. \tag{10}$$

Stability of the static universe

We have studied the stability of this static solution against isotropic and anisotropic perturbation. We fund that this static solution is stable (center) if the JBD potential and the JBD parameter ω satisfy:

$$0 < a_0^2 \Phi_0 \, V_0'' < \frac{3}{2} \, , \tag{11}$$

and

$$-\frac{3}{2} < \omega < -\frac{1}{4}\left[\sqrt{9 - 6a_0^2 \Phi_0 V_0''} + (3 + a_0^2 \Phi_0 V_0'')\right] . \tag{12}$$

This situation is different respect to GR where the static solution is unstable.

Other static solution

It is interesting to note that we can obtain a static and stable solution for universes dominated by other perfect fluids, for example for dust.
If we consider a standard perfect fluid with equation of state $P = (1 - \gamma)\rho$ where γ is constant.

Ej. $\gamma = 1$ dust, $\gamma = 4/3$ radiation.

The stability conditions are:

$$\frac{2}{3} < \gamma < \frac{4}{3}\,,\, or \;\; \frac{4}{3} < \gamma$$

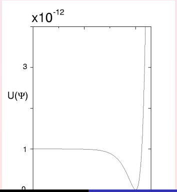
$$-\frac{3}{2} < w < -18\,\frac{(\gamma-1)}{(3-3\gamma)^2}\,,$$

$$2(6+w)-3(3+w)\gamma+\sqrt{3}\,|4-3\gamma|\sqrt{3+2w} \; < a_0^2\,\Phi_0\,V_0'' \; < \frac{6}{3\gamma-2}\,.$$

From these inequalities we can conclude that for a Universe dominated by an standard perfect fluid it is possible to have a static and stable universe, in the context of a JBD theory, for a prefect fluid which satisfies $\gamma > 2/3$, whit the only exception of radiation ($\gamma = 4/3$) which is explicitly excluded by the inequalities.

Leaving the static regimen and Inflation

The overall effect of increasing the potential is to distort the equilibrium behavior. The inclusion of the derivative term in the equation of the scalar field Ψ produce changes in its equilibrium velocity breaking the static solution. In particular, the field Ψ decelerates as it moves further up the potential, subsequently reaching a point of maximal displacement and then rolling back down. If the potential has a suitable form in this region, slow-roll inflation will occur.



On the other hand, in the slow-roll regimen the inflaton potential evolves slowly. In that case we can consider $U(\Psi) \sim const. = U_{inf}$. Then, Eqs. (3) and (5) have an exact static solution for a particular value of Φ, driving a de Sitter expansion. This occurs when the right hand side of Eq. (5) becomes zero. Denominating this quasi-static value of the JBD field as $\tilde{\Phi}$, it satisfies the following condition

$$4U_{inf} + 4V(\tilde{\Phi}) - 2\tilde{\Phi}\, V'(\tilde{\Phi}) = 0\,. \tag{13}$$

Then, once the scalar field starts to move in the slow-roll regime, the JBD field goes to the value $\tilde{\Phi}$ and the universe begins a de Sitter expansion with

$$H^2 = \frac{1}{3\tilde{\Phi}}\left[U_{inf} + V(\tilde{\Phi})\right]. \tag{14}$$

For example, the JBD potential $V(\Phi) = \lambda(\Phi^2 - \nu)^2$ satisfies this condition. Finally, during the evolution of Ψ over $U(\Psi)$ to zero, the JBD field evolves slowly to its final value Φ_f, at which the expression $2V - \phi\, V'$ vanishes. We consider the value Φ_f as the current value of the JBD field.
Also, we can determinate the existence of the inflationary period by introduce the dimensionless slow-roll parameter

$$\epsilon = -\frac{\dot{H}}{H^2} \simeq \frac{2}{(3+2w)} + \frac{\Phi}{2}\left(\frac{U_{,\psi}}{V+U}\right)^2 + \frac{V'\Phi}{(3+2w)(V+U)}\left[\frac{V'\Phi}{V+U} - 3\right]$$

Then, the inflationary regime takes place if the parameter ϵ satisfies the inequality $\epsilon < 1$, a condition analogous to the requirement $\ddot{a} > 0$. We note that if Eq. (13) is satisfied (i.e. $\Phi = \tilde{\Phi}$) and the scalar potential $U(\Psi)$ satisfies the requirement of an inflationary potential, we get $\epsilon < 1$.

A specific model of an emergent universe

Motivated by the former discussion, we consider the following inflaton potential as an example:

$$U(\Psi) = U_0 \left[\exp\left(\beta\Psi/\sqrt{3}\right) - 1\right]^2,$$

which exhibits the generic properties described previously. As an example of a Brans-Dicke potential that satisfies the condition of static solution, we consider the following polynomial potential

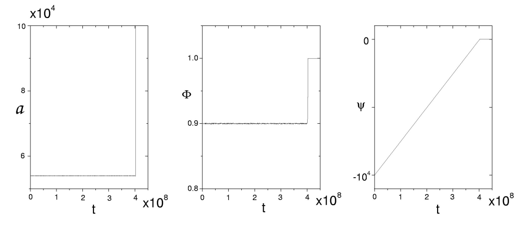
$$V(\Phi) = V_0 + A\,(\Phi - \Phi_0) + \frac{1}{2}\,B\,(\Phi - \Phi_0)^2 + \frac{1}{4!}\;C\,(\Phi - \Phi_0)^4\ ,$$

where Φ_0 correspond to the value of the JBD field at the static solution.

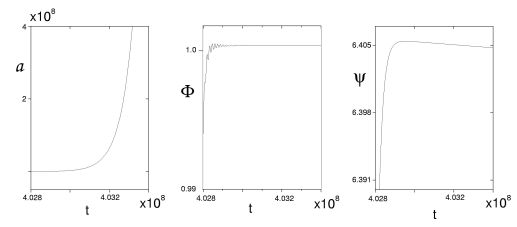
The parameters V_0, A, B are fixed in order to obtain a static solution at $\Phi = \Phi_0$, $a = a_0$ and $\rho = \rho_0$.
In order to obtain a numerical solution we take the following values for the parameters in the JBD potential $\Phi_0 = 0.9$, $\Phi_f = 1$, $a_0 = 5.4 \times 10^4$, $\chi = 1$ and $\omega = -1.45$, where we have used units in which $8\pi G = 1$. These particular parameters satisfy all the constraints discussed previously. On the other hand, in order to consider the model just at the classical level we have to be out of the Planck era. This imposes the following conditions: $\rho < \Phi(t)^2$ and $V(\Phi) < \Phi(t)^2$, which are satisfied with the values of the parameters mentioned above.

Numerical solution

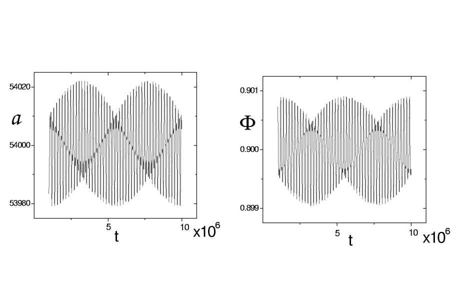
We consider a numerical solution corresponding to a universe starting from an initial state close to the static solution. We can notice that during the part of the process where the scalar field Ψ moves in the asymptotically flat region of the potential $U(\Psi)$, the universe remains static, due to the form of the JBD potential.



The static regimen finishes when the scalar field moves past the minimum of its potential (near the value $\Psi \sim 0$) and begins to decelerate as it moves up its potential. During this period the static equilibrium is broken, and the scale factor and with the JBD field start to evolve. Finally, the inflaton field starts to go down the potential $U(\Psi)$ in the slow-roll regimen.



On the other hand, a numerical solution corresponding to a universe starting from an initial state not in the static solution but close to it, presents small oscillations around the equilibrium values.



Summarize

- The search for singularity free inflationary models has led to the development of Emergent Universe models. However, this model suffers from the problem of instability of the Einstein static state.
- We have provided an explicit construction of an emergent universe scenario, which presents a stable past eternal static solution.
- Contrary to classical general relativity we have found that JBD static universe dominated by a perfect fluid could be stable against isotropic and anisotropic perturbations for some sort of perfect fluids, for example for dust.
- Now we are working on the study of different mechanism to leave the static regimen.

THANKS