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An Empirical Fit to the Nucleon Electromagnetic Form Factors*

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Abstract

We have made an empirical fit to the world data for the proton elastic electromagnetic form factors G_{Ep}, G_{Mp} for $0 < Q^2 < 30$ (GeV/c)^2, and to the neutron electromagnetic form factors G_{En}, and G_{Mn} in the range $0 < Q^2 < 10$ (GeV/c)^2.

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The elastic electromagnetic form-factors of the nucleon are fundamental quantities that embody the probability for a nucleon to absorb a virtual photon of four-momentum squared Q^2. In the non-relativistic limit, $G_{Ep}(Q^2)$ and $G_{En}(Q^2)$ describe the distribution of electric charge for the proton and neutron, respectively, while $G_{Mp}(Q^2)$ and $G_{Mn}(Q^2)$ reflect the distribution of magnetization current. A rich body of experiments to determine these form factors dates back to the 1960s, and continues to the present day with ever-improving experimental techniques. Most of the data is for the spacelike region ($Q^2 = -q^2 > 0$), but recently the timelike region ($Q^2 < 0$) has been investigated as well. Many models and theories have been developed to fit and/or predict the nucleon form factors, but none provides a good description of all the data. Since the nucleon form factors enter into calculations of most reactions involving electron scattering from nucleons or nuclei (including deep-inelastic scattering through the radiative tails), there is a need for a simple but reliable model to describe the present body of data. We present such a model for the spacelike region $Q^2 > 0$ (GeV/c)^2.

Early experiments quickly revealed a simple parametrization, known as the dipole fit, which described available data to the 20% level or so. Defining

$$G_D(Q^2) = (1 + Q^2/0.71)^{-2} \tag{1}$$

it was observed that

$$G_{Ep}(Q^2) \approx \frac{G_{Mp}(Q^2)}{\mu_p} \approx \frac{G_{Mn}(Q^2)}{\mu_n} \approx G_D(Q^2) \tag{2}$$

$$G_{En}(Q^2) \approx 0, \tag{3}$$

where the magnetic moments $\mu_p \approx 2.793$ nm and $\mu_n \approx -1.913$ nm, and we use Q^2 in units of (GeV/c)^2 throughout. The strong similarity in the Q^2 dependence of G_{Ep}, G_{Mp}, and G_{Mn} is known as form-factor scaling.

A compilation of world form-factor data is shown in Fig. 1. The data for $G_{Mp}/\mu_p G_D$ and G_{Ep}/G_D for $Q^2 \leq 7$ (GeV/c)^2 are from the global analysis of Walker *et al.* [1] (solid circles). The global analysis results for $Q^2 \geq 3$ (GeV/c)^2 are essentially determined by the recent SLAC experiment NE11 [2]. The final results from NE11 [3] are slightly different from the original ones, but not enough to change the global analysis significantly. This global analysis [1] used improved radiative corrections to the older data, and normalized the earlier data to the more precise recent experiments. We observe that in the measured Q^2 range, form-factor scaling $G_{Ep}(Q^2) \approx G_{Mp}(Q^2)/\mu_p$ seems to work reasonably well, so for $Q^2 > 9$ (GeV/c)^2 we have used this assumption to extract values of G_{Mp} from the forward-angle, elastic cross section measurements of E136 [4] (open squares).

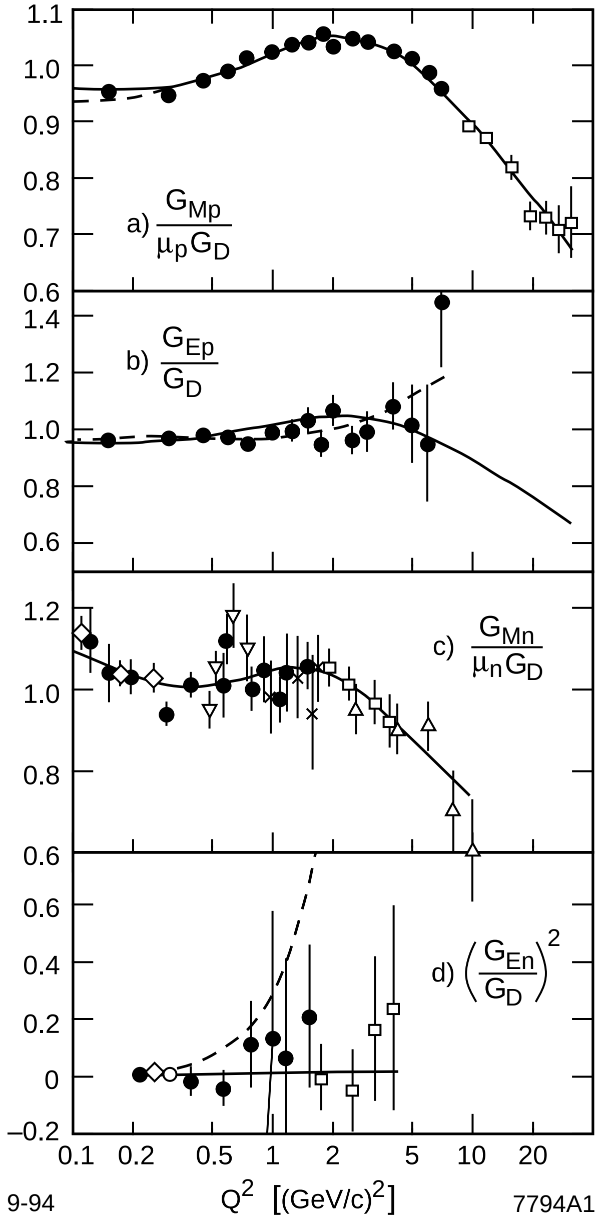

Fig. 1. Compilation of data for a) $G_{Mp}/\mu_p G_D$, b) G_{Ep}/G_D, c) $G_{Mn}/\mu_n G_D$, and d) $(G_{En}/G_D)^2$. The symbols and curves are described in the text.

Most of the information on neutron form factors comes from analyses of quasi-elastic electron scattering from deuterium. No recent global analysis is available, and in most cases the raw spectra (before radiative corrections) are no longer available, so such an analysis would be very difficult. The best compilation of early data appears to be that of Bartel *et al.* [5] (solid circles) from 1973. We have not used the compilation of Hanson *et al.* [6] from the same year. Note that in these experiments, G_{En}^2 is the experimentally measured quantity, and in some cases is found to be negative. The recent (1992) experiment NE11 [7] (open squares) provides reliable separations of G_{En}^2 and G_{Mn}^2 in the range $1.75 \leq Q^2 \leq 4$ (GeV/c)^2, and showed that $G_{En}^2 \ll G_{Mn}^2$ in this Q^2 range. Using this observation, we have extracted G_{Mn} from the forward-angle E133 experiment [8] (triangles) assuming $G_{En} = 0$. This is the highest Q^2 data available, extending to $Q^2 = 10$ (GeV/c)^2, but it should be remembered that at high Q^2 the quasi-elastic peak is only visible as a slight shoulder in the data, and the form-factor extraction becomes increasingly dependent on models of Fermi-smearing the resonance region and deep-inelastic contributions. We have included the results of two backward-angle quasi-elastic experiments in the moderate Q^2 range, NE4 [9] (crosses) and Esaulov *et al.* [10] (inverted triangles), which measured G_{Mn} directly, since there are negligible contributions from G_{En} at backward angles. Finally, recent experiments at Bates have determined both G_{Mn} [11] and G_{En} [12] (diamonds) at low Q^2 using quasielastic $d(e, e'n)$, where for G_{En} a polarized electron beam was used and the polarization transferred to the neutron was measured. An even more recent measurement of G_{En} from MAMI [13] (open circle) was made using quasi-elastic scattering of polarized electrons from polarized ^{3}He. All of the results for G_{Mn} and G_{En} depend to varying degrees on the treatments of final-state interactions (FSI), meson-exchange currents (MEC), and relativistic effects, and the error bars include a rough attempt to include these theoretical uncertainties. Results for G_{En} extracted from elastic electron-deuteron scattering are even more model dependent, and while not shown in Fig. 1, are discussed further below.

We first examine the proton form factors, plotted relative to the dipole fit in Fig. 1a, b. Several empirical forms were tried. Functional forms similar to the dipole fit $(1 + aQ^2 + bQ^4 + cQ^6 + ...)^{-1}$ have the advantage of a well-defined derivative at $Q^2 = 0$, but are not able to describe the oscillations of the data about the dipole fit clearly seen in Fig. 1a. Instead, we found that a good description can be obtained using a polynomial expansion in terms of $Q = \sqrt{Q^2}$. The best fits to G_{Ep} and G_{Mp} individually are shown as the dashed curves in Figs. 1a, b and are given by

$$G_{Ep}(Q^2) = \frac{1}{1 + 0.62Q + 0.68Q^2 + 2.80Q^3 + 0.83Q^4} \tag{4}$$

$$\frac{G_{Mp}(Q^2)}{\mu_p} = \frac{1}{1 + 0.35Q + 2.44Q^2 + 0.50Q^3 + 1.04Q^4 + 0.34Q^5} \ . \tag{5}$$

We have included the constraints that $G_{Ep}(0) = 1$ and $G_{Mp}(0)/\mu_p = 1$. The number of free parameters was increased until good fits were obtained (χ^2/d.f.=0.86 for G_{Ep}, χ^2/d.f.=0.65 for G_{Mp}). The fit for G_{Ep} is only valid up to $Q^2 = 7$ (GeV/c)^2. We also tried a fit assuming form-factor scaling for the proton to obtain

$$G_{Ep}(Q^2) = \frac{G_{Mp}(Q^2)}{\mu_p} = \frac{1}{1 + 0.14Q + 3.01Q^2 + 0.02Q^3 + 1.20Q^4 + 0.32Q^5}, \tag{6}$$

shown as the solid curve in Figs. 1a, b. The fit is not quite as good (χ^2/d.f.=1.18), but given the correlated errors between the various experimental points, this is not sufficient to make the fits to the individual form factors preferable. Given the assumption of proton form-factor scaling, Eq. (6) is valid up to $Q^2 = 30$ (GeV/c)^2. The error in the fit is approximately 3% for determining cross sections (proportional to $G_{Ep}^2 + \tau G_{Mp}^2$, where $\tau = Q^2/4M^2$ and M is the proton mass) up to $Q^2 = 10$ (GeV/c)^2, dominated by overall normalization error in the data, increasing to 20% at $Q^2 = 30$ (GeV/c)^2, where the statistical accuracy dominates.

We tried fitting all three form factors G_{Ep} ,G_{Mp}, and G_{Mn} with a single function assuming form-factor scaling, and were not able to obtain a good fit. This is because $G_{Mn}/\mu_n G_D > 1$ at low Q^2, while $G_{Mp}/\mu_p G_D < 1$. We therefore fit G_{Mn} alone to obtain

$$\frac{G_{Mn}(Q^2)}{\mu_n} = \frac{1}{1 - 1.74Q + 9.29Q^2 - 7.63Q^3 + 4.63Q^4} \tag{7}$$

with a χ^2/d.f.=0.93 (solid curve on Fig. 1c). Based on the scatter in the data set, the fit is estimated to be good to about 5% for $Q^2 < 4$ (GeV/c)^2, increasing to about 20% at $Q^2 = 10$ (GeV/c)^2.

The neutron electric form factor is the least well-known of the four nucleon form factors: its small size makes it very difficult to measure. However, the slope at $Q^2 = 0$ is known quite well from thermal neutron scattering from electrons [14]:

$$\frac{dG_{En}}{dQ^2}|_{Q^2=0} = 0.511 \pm 0.008 \tag{8}$$

In addition, G_{En} has been extracted [15,16] from measurements of $A(Q^2)$ up to 1 (GeV/c)^2, where $A(Q^2)$ is the forward-angle, elastic form factor of the deuteron. The most recent measurements [16] have small statistical errors, but large theoretical uncertainties due to the lack of knowledge of the relativistic, deuteron wave function and FSI and MEC corrections. For the Paris potential, Platchkov *et al.* [16] found a best fit given by

$$G_{En}(Q^2) = \frac{-a\mu_n \tau G_D(Q^2)}{1 + b\tau} \tag{9}$$

with $a = 1.25 \pm 0.13$ and $b = 18.3 \pm 3.4$. Comparing this fit (solid line) to the quasi-elastic data shown in Fig. 1d, we find a reasonable fit (χ^2/d.f.=1.14). If the slope of G_{En} is constrained to match the thermal neutron data ($a = 0.94$, $b = 10.4 \pm 0.6$), the fit to the $A(Q^2)$ data remains quite good [16], and for the quasi-elastic data the χ^2/d.f. remains almost unchanged at 1.17. While the quasi-elastic data do not exclude $G_{En} = 0$ (χ^2/d.f.=1.32), the elastic data preclude this possibility for the range of wave functions studied in Ref. [16], and this would disagree with the measured slope at $Q^2 = 0$. We also examined the commonly used prescription $G_{En}(Q^2) = -\mu_n \tau G_D(Q^2)$ (equivalent to $F_{1n} = 0$), shown as the dashed curve in Fig. 1d. The recent data from NE11 [7] clearly rule out this possibility, with a χ^2/d.f.=44. We conclude that until more precise data become available, the fit Eq. (9) with either choice for (a, b) is a reasonable description of G_{En}, with a relative error of about 40% for $Q^2 < 1$ (GeV/c)^2, increasing to over 100% at $Q^2 = 4$ (GeV/c)^2.

In summary, Eq. (6) provides a good description of the proton form factors for $0 < Q^2 < 30$ (GeV/c)^2, under the assumption of form-factor scaling, which seems to be valid within experimental systematic errors up to $Q^2 = 7$ (GeV/c)^2. Equation (7) provides a good fit to G_{Mn} in the range $0 < Q^2 < 10$ (GeV/c)^2, and Eq. (9) with $a = 1.25$ and $b = 18.3$ is consistent with existing data for G_{En} up to $Q^2 = 4$ (GeV/c)^2. It is hoped that these fits will provide a useful empirical description of present-day knowledge of the nucleon form factors.

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