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Berlin, December 2010

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Empirical Strategies to Eliminate Life-Cycle Bias in the Intergenerational Elasticity of Earnings Literature

Jan Leonard Stuhler[*]

December 23, 2010

Abstract

I argue that the empirical strategies for estimation of the intergenerational elasticity of lifetime earnings that are currently employed in the literature might not eliminate bias arising from life-cycle effects. Specifically, I demonstrate that procedures based on the generalized errors-in-variables model suggested by Haider and Solon (2006) or the consideration of differential earnings growth rates across subpopulations may not yield unbiased or consistent estimates. I further argue that instrumental variable estimators will not identify an upper bound for the true population parameter.

Introduction

The intergenerational transmission of economic status within families is often measured by the intergenerational elasticity between parents' and children's lifetime earnings. A large and growing literature has estimated this parameter in order to analyze the extent of intergenerational mobility across countries. Unfortunately, the empirical identification strategies employed in the early literature have been shown to yield biased estimates[1] and successive improvements of the methodology led to large scale corrections in estimates. For example, the intergenerational elasticity of earnings for fathers and sons in the U.S. was estimated to be less than 0.2 among early studies (surveyed in Becker and Tomes (1986)), ranged between about 0.3 and 0.5 in the studies surveyed in Solon (1999), and is estimated to be around 0.6 or above in more recent studies like Mazumder (2005) and Gouskova, Chiteji, and Stafford (2009).

While estimates in early empirical studies suffered primarily from a severe attenuation bias arising from measurement error in short spans of earning data, more recently Haider and Solon (2006); Grawe (2006) and Vogel (2006) extended on the analysis of Jenkins (1987) and identified systematic changes in earnings over the life-cycle as additional source of inconsistencies. They each propose refined estimation methods in order to eliminate such life-cycle bias and to reach consistent estimates of the intergenerational elasticity parameter. The generalized errors-in-variable model expressed in Haider and Solon (2006) has since been widely adopted as motivation for empirical strategies in the intergenerational mobility literature.

My objective is to demonstrate that these refined methods might still suffer from life-cycle bias and might therefore not yield unbiased or consistent estimates. Moreover, I show that instrumental variable estimators will not yield an upper bound for the true population parameter. While I focus on the identification problem in the context of the literature on the intergenerational elasticity of lifetime earnings, the discussion should be generally relevant for any application in which interest lies on parameters that are based on within-group (e.g. within-family) correlation of partially observed characteristics that possibly change over the life-cycle.[2]

The first section describes the general methodology and reviews the identifying assumptions employed in the early literature. The following sections review estimation procedures which are based on more recent methodological improvements: the generalized errors-in-variables model suggested by Haider and Solon (2006) in section 2, the structural estimation of life-cycle patterns

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[1] See Solon (1999) for a comprehensive evaluation of the early empirical literature.

[2] For example, identification of the correlation in lifetime earnings of siblings or of the correlation in consumption preferences within families will be complicated by similar methodological problems. See Black and Devereux (2011) for a review of research on various family characteristics.

as proposed by Vogel (2006) in section 3, instrumental variable methods in section 4.

1 Literature Review

In intergenerational mobility studies, interest often lies on the linear regression of (real) log lifetime earnings of the father[3] in family i, $y_{f,i}^*$, on log lifetime earnings of his son, $y_{s,i}^*$,

$$y_{s,i}^* = \alpha + \beta y_{f,i}^* + \epsilon_i \tag{1}$$

where ϵ_i is uncorrelated with the regressor $y_{f,i}^*$. As of the log-specification, the coefficient β can be interpreted as the intergenerational elasticity of son's lifetime labor earnings with respect to the father's lifetime labor earnings, and a one percent increase in father's earnings raises expectations on son's earnings by β percents. The coefficient reflects correlation and does not allow for any specific interpretation of causality. The intercept allows for shifts of mean labor earnings of sons compared to mean labor earnings of fathers[4] and captures the share of mean lifetime earnings of sons which is not explained by variation in the lifetime earnings of fathers.

As currently employed data sets do not contain complete lifetime earnings profiles, the crucial methodological challenge stems from the requirement to obtain approximations of lifetime earnings y_i^* in a first step. Note that the availability of better data would not generally solve the identification problem, since data sets will never contain complete lifetime earnings profiles for contemporary populations.

Approximation of Lifetime Earnings

Let y_i be some observed proxy for unobserved log lifetime earnings of an individual in family i, e.g. the logarithm of a single yearly earning observation, an average measure of multiple earning observations, or a more complex estimate based on such yearly earning observations. Let

$$y_{s,i} = y_{s,i}^* + u_{s,i}$$

with $y_{s,i}^*$ as unobserved true log lifetime earnings of the son in family i and $u_{s,i}$ as measurement error. Similarly, denote

$$y_{f,i} = y_{f,i}^* + u_{f,i}$$

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[3]Most of the literature focuses on the intergenerational elasticity of earnings for fathers and sons since the stronger labor market participation of men facilitates the analysis,

[4]Lifetime earnings have to be discounted to a common base time if the year of birth varies among sampled fathers. Otherwise β will contain earning correlations between fathers and sons arising from shared experience of economic growth shocks through the correlation between father's and son's year of birth.

with $u_{f,i}$ as the measurement error in log lifetime earnings of fathers. Generally, the probability limit of the ordinary least square estimator of a linear regression of approximated log lifetime earnings of sons y_s on approximated log earnings of fathers y_f can be expressed as

$$plim \; \hat{\beta}_{OLS} = \frac{Cov(y_f, y_s)}{Var(y_f)}$$

$$= \frac{Cov(y_f^* + u_f, y_s^* + u_s)}{Var(y_f^* + u_f)}$$

$$= \frac{\beta \, Var(y_f^*) + Cov(y_f^*, u_s) + Cov(y_s^*, u_f) + Cov(u_s, u_f)}{Var(y_f^*) + Var(u_f) + 2 \, Cov(y_f^*, u_f)} \quad (2)$$

where for the last step I used equation (1) to substitute for $y_{s,i}^*$ and applied the covariance restriction $Cov(y_{f,i}^*, \epsilon_i) = 0$. It follows that the OLS estimator of the intergenerational elasticity in lifetime earnings could be down- or upward biased, and that the covariance between measurement errors u_s and u_f as well as between measurement errors and true earnings y_s^* and y_f^* has a crucial impact on consistency. The evolution of identification strategies employed in the literature in the last decades can be broadly categorized in terms of changes in the assumptions on this covariance structure.

First Generation Literature

Many studies in the early literature on intergenerational earnings mobility, surveyed in Section V of Becker and Tomes (1986), neglected the problem of measurement errors in lifetime earnings status. Often just single-year earning observations were used as proxy for lifetime earnings, thereby implicitly assuming that

$$Cov(y_f^*, u_s) = Cov(y_s^*, u_f) = Cov(u_s, u_f) = Cov(y_f^*, u_f) = 0$$

and

$$Var(u_f) = 0$$

Existence of measurement errors in lifetime earning status violates the latter assumption, so that estimates of the elasticity of lifetime earnings suffered from a severe attenuation bias. Resulting estimates of the intergenerational elasticity of earnings were therefore too low, estimates of the intergenerational earnings mobility too large.

Second Generation Literature

The problem that measurement error in lifetime earnings of fathers will lead to an attenuation bias was recognized in the second wave of studies in the literature, which has been surveyed in Solon (1999). As Solon notes in respect to the early literature, "*Most of the studies [...] used single-year earnings or income measures, and, in many, the resulting attenuation inconsistency was aggravated by reliance on peculiarly homogeneous samples.*"

But the conventional assumption remained that the measurement errors u_s and u_f are random noise, independent of each other and of true earnings y_s^*, y_f^*. The fact that earning growth over age had to be accounted for was recognized, but it was assumed that the inclusion of age controls in the regression equation would be sufficient to account for both generations' life-cycle variation in annual earnings. Assumptions on the covariances in equation (2) were therefore that

$$Cov(y_f^*, u_s) = Cov(y_s^*, u_f) = Cov(u_s, u_f) = Cov(y_f^*, u_f) = 0$$

and

$$Var(u_f) \neq 0$$

If these assumptions hold, the probability limit of the estimator reduces to

$$plim \ \hat{\beta}_{OLS} = \beta \ \frac{Var(y_f^*)}{Var(y_f^*) + Var(u_f)}$$

This is the classical errors-in-variables model for scalar regressors; inconsistencies are limited to the classical attenuation bias in $\hat{\beta}$ towards zero, introduced by measurement error in the *earnings of fathers* only. An increase in the number of earning observations of sampled fathers increases the signal-to-noise ratio in y_f and thereby decreases the attenuation bias. In contrast, measurement error in the earnings of sons is not a source of inconsistency in such model. The parameter β is identified if the number of earning observations for fathers is sufficiently large, if a consistent estimator for the attenuation factor can be derived, or if moment restrictions on the measurement errors can be justified, e.g. by inferring the distribution of the measurement errors from a different data set. Researchers typically used averages of multiple earning observations for fathers to increase the signal-to-noise ratio while giving less attention to measurement errors in lifetime earnings of sons.

Recent Literature

More recently the focus in the literature has been on the existence of non-classical measurement error that arises from approximation of lifetime earnings by current earnings at specific stages of individuals' life-cycles. An early

theoretical discussion can be found in Jenkins (1987). Analyzing a simple model of life-cycle earnings, Jenkins finds that approximation of lifetime earnings by yearly earnings in equation (1) will bias the estimate β as of life-cycle effects. He concludes that theoretically the direction of this life-cycle bias is ambiguous, that the bias can be large, and that it will not necessarily be smaller when one uses samples of fathers and sons in the same age.

Further evidence that a textbook errors-in-variables model is not appropriate in the intergenerational mobility context has been given by Björklund (1993) and Reville (1995). Björklund shows that the correlation between current and transitory earnings varies strongly across the life-cycle in Sweden, which implies that estimates of intergenerational mobility will be sensitive to the age distribution of sampled fathers and sons. Reville shows that estimates for the intergenerational elasticity of earnings increase with the age of sampled sons. Solon's (1999) survey of the intergenerational mobility literature confirms this pattern - the studies that estimate the smallest elasticities tend to be those that observe sons' earnings early in their careers. Solon (2002) concludes that "*this pattern arises because the measurement error in son's early earnings as a proxy for his long-run earnings is not of the classical textbook variety*".

Employing a simple model of earnings formation, Haider and Solon (2006) show that the common practice of controlling for the central tendency of earnings growth in the population will therefore not suffice, as heterogeneous variation around the average earnings growth rate will bias intergenerational elasticity estimates.[5] Specifically, the inclusion of age or experience controls in a mean regression like equation (1) will control for mean age or experience effects only, and will not capture heterogeneous variation of earning growth around the mean growth rate.

This argument has also been illustrated by Vogel (2006). Vogel argues that the earning growth of highly educated workers is often steeper than the mean earnings growth in the population. Since available data tends to cover annual earning observations for sons at early and for fathers at later age the approximation of log lifetime earnings y_i^* with annual earnings data will result in lifetime earnings of highly educated sons to be underestimated and lifetime earnings of highly educated fathers to be overestimated, even after accounting for the central tendency of earnings growth in the population. The expectation of measurement error conditional on high education E will be non-zero; $E(u_f|E_f = high) > 0$, $E(u_s|E_s = high) < 0$, and vice versa for lowly educated individuals. If educational achievement is correlated within families and if higher education tends to lead to higher lifetime earnings we have

$$Cov(y_f^*, u_s) < 0\,,\ Cov(y_s^*, u_f) < 0\,,\ Cov(u_s, u_f) < 0\ \text{and}\ Cov(y_f^*, u_f) > 0$$

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[5]These arguments are also discussed in Grawe (2006).

and it follows from equation (2) that the non-classical measurement errors would bias $\hat{\beta}_{OLS}$ further towards zero than implied by the simple errors-in-variables model with independent measurement errors.

Indeed, in extreme cases the probability limit of the OLS estimator might be negative. Various refined estimation procedures have been proposed and applied in the literature to address this life-cycle bias. I proceed to discuss the most popular refinement in detail.

2 Age Restrictions in Sample Selection

Haider and Solon (2006) formulate a generalized errors-in-variables model for the relationship between current and lifetime earnings that incorporates some life-cycle related departures from a textbook errors-in-variables model. While the model is more general in scope, the authors cite the intergenerational mobility literature as one of the potential application fields, demonstrating the need for the literature to move beyond the classical errors-in-variables model. Analyzing the generalized errors-in-variables model Haider and Solon conclude that left-side measurement error is innocuous for consistency in OLS estimation of the intergenerational elasticity if unobserved lifetime earnings of sons are proxied by observed annual earnings at a certain age.

This generalized errors-in-variable model has been widely adopted as motivation for empirical strategies in the intergenerational mobility literature and the procedure of measuring earnings around mid-life has since been frequently applied.[6]

Following Haider and Solon's analysis step by step I intend to show that the model might not be readily applicable to the intergenerational mobility literature since one of the underlying assumptions is potentially violated in this application field. The procedure of approximating lifetime earnings by annual earnings at a certain age does then not necessarily remove the life-cycle bias, and might thus not lead to consistent or unbiased estimates of the intergenerational elasticity parameter.

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[6]Among others, in Dahl and DeLeire (2008) and Gouskova, Chiteji, and Stafford (2009) for the United States; in Eisenhauer and Pfeiffer (2008) and Schnitzlein (2008) for Germany; in Bohlmark and Lindquist (2006), Björklund, Lindahl, and Plug (2006), Björklund, Jäntti, and Solon (2007) and Björklund, Jäntti, and Lindquist (2009) for Sweden; in Nilsen, Vaage, Aakvik, and Jacobsen (2008) for Norway; in Raaum, Bratsberg, Røed, Österbacka, Eriksson, Jäntti, and Naylor (2007) for Denmark, Finland, Norway, the UK and the US; in Nicoletti and Ermisch (2007) for the UK; in Cervini Pla (2009) for Spain; in Piraino (2007) and Mocetti (2007) for Italy; in Lefranc, Ojima, and Yoshida (2008) for Japan and France.

The Generalized Errors-in-Variables Model

Haider and Solon employ a simple model of earnings formation in which y_{it}, the log real earnings of an individual in family i in year t of his career, follows

$$y_{it} = \eta_i + \gamma_i t \qquad (3)$$

Both initial log earnings η_i and the earnings growth rate γ_i vary across the population with variance σ_η^2 and α_γ^2, respectively. For simplicity zero covariance between η_i and γ_i, infinite lifetimes, and a constant real interest rate $r > \gamma_i$ are assumed. The log of the present value of lifetime earnings y_i^* is thereby

$$
\begin{aligned}
y_i^* &= log\left(\sum_{s=0}^{\infty} exp(\eta_i + \gamma_i s)^s (1+r)^{-s}\right) \\
&= \eta_i + log(1+r) - log(1+r - exp(\gamma_i)) \qquad (4) \\
&\cong \eta_i + log(1+r) - log(r - \gamma_i)
\end{aligned}
$$

It follows that the slope coefficient in the regression of current log annual earnings y_{it} on the log lifetime earnings y_i^* (simplifying further by $y_i^* \cong \eta_i + r - log\, r + \frac{\gamma_i}{r}$) equals

$$\lambda_t = \frac{Cov(y_i^*, y_{it})}{Var(y_i^*)} \cong \frac{\sigma_\eta^2 + t\sigma_\gamma^2/r}{\sigma_\eta^2 + \sigma_\gamma^2/r^2}$$

Haider and Solon note that the association between current and lifetime earnings varies systematically across the life-cycle, contrary to a textbook errors-in-variables model in which measurement error is assumed to be independent of true values. Measurement error in current earnings as a proxy for lifetime earnings can therefore lead to inconsistency in intergenerational elasticity estimates even when the errors are on the left hand side of a regression model as given in equation (1).

Haider and Solon's proposed solution is to approximate lifetime earnings by annual earnings at a certain age. Intuition is given in figure 1 in Haider and Solon (2006), in which the authors depict that for two individuals with different earning trajectories there will nevertheless exist an age t^* for which the difference between current log earnings and the log of annuitized present discounted value of lifetime earnings (hereinafter referred to as log annuitized lifetime earnings) is the same for both individuals. While at age t^* current earnings might be a misrepresentation of lifetime earnings, the distance between the current log earning trajectories of both workers equals the distance between their log annuitized lifetime earnings. The authors conclude that the textbook errors-in-variables model holds at this age t^*.

This insight can then be applied to the regression model given in equation (1). Haider and Solon first focus on left hand side measurement error

and assume that $y_{s,i}^*$ is not observed and hence proxied by $y_{s,it}$, log annual earnings of sons at age t. Their generalized errors-in-variables model is given by

$$y_{s,it} = \lambda_t y_{s,i}^* + u_{s,it}$$

where λ_t is the slope coefficient in the linear projection of $y_{s,it}$ on $y_{s,i}^*$, which varies over the life-cycle. By construction $u_{s,it}$ is uncorrelated with $y_{s,i}^*$. Using the regression equation (1) to substitute for $y_{s,i}^*$ gives

$$y_{s,it} = \lambda_t \alpha + \lambda_t \beta y_{f,i}^* + \lambda_t \epsilon_i + u_{s,it} \tag{5}$$

Then, if ordinary least square is applied to the regression of $y_{s,it}$ on $y_{f,i}^*$ the probability limit of the estimated coefficient reduces to

$$plim \; \hat{\beta}_{OLS} = \frac{\beta \, \lambda_t \, Var(y_f^*) + Cov(y_f^*, u_{s,t})}{Var(y_f^*)}$$

Haider and Solon make the assumption that the error $u_{s,it}$ is uncorrelated with the regressor $y_{f,i}^*$ and conclude that left-side measurement error in lifetime earnings is innocuous for consistency if the sample is restricted to annual earning observations of sons around an age t for which λ_t is sufficiently close to one.

Applicability in the Intergenerational Mobility Literature

I argue that this generalised error-in-variables model might not be readily applicable in the context of the intergenerational mobility literature since the correlation between the error $u_{s,it}$ and fathers' log lifetime earnings $y_{f,i}^*$ should not be expected to be zero.

First, note that for more than two individuals we will generally *not* find some age t^* at which annual earnings are an undistorted approximation for lifetime earnings. Figure 1 illustrates this argument by plotting life-cycle trajectories of log earnings for workers 1 and 2 (as in figure 1 in Haider and Solon (2006)) and an additional worker 3. The horizontal lines depict log annuitized lifetime earnings, the difference between workers' log lifetime earnings is therefore given by the vertical distance between the horizontal lines. At age t_1^* the distance between current earnings trajectories equals the distance between the horizontal lines for worker 1 and worker 2, at age t_2^* for worker 1 and worker 3. There exists no age for which these distances are equal for all three workers at the same time. Note that this result does not depend on any peculiarities in the earnings growth process. Even for a simple linear formation of log annual earnings, as given by equation (3), the difference between log earnings y_{it} and the log annuitized lifetime earnings depends on the earnings growth rate γ_i; will have the same value for at most two

Figure 1: Illustrative Example of Log Annual Earning Trajectories
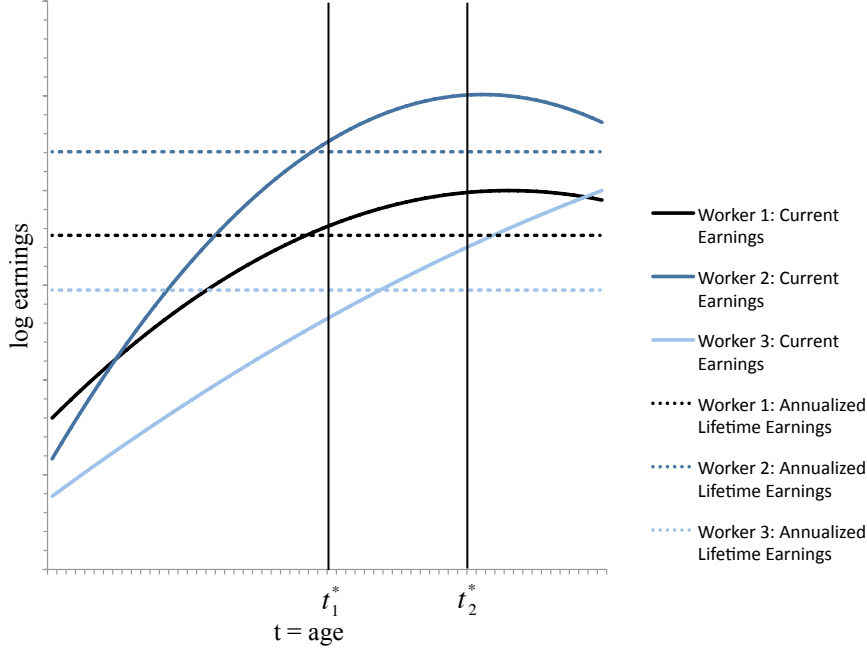
Notes: Illustrative Example. For each worker, the upward-sloping line depicts log annual earnings by age, the horizontal line depicts log annuitized lifetime earnings.

distinctive realizations of γ_i; and will therefore systematically differ across individuals at any age t (see proof in Appendix A1).

This insight clarifies that the slope coefficient λ_t is merely a population parameter that reflects how differences in current earnings and differences in lifetime earnings relate *on average in the population* at age t. Individuals will nevertheless deviate from this population average relationship as their annual earnings still over- or understate their lifetime earnings. Decisive for intergenerational mobility studies is that the parameter λ_t contains no information on if such idiosyncratic deviations correlate within families.

The assumption that $u_{s,it}$ is uncorrelated with $y^*_{f,i}$ corresponds to the conjecture that these idiosyncratic deviations do not correlate within families, so that individuals have the same expected relation between current and lifetime earnings regardless of family background. It excludes, for example, the possibility that a father's lifetime earnings $y^*_{f,i}$ correlate with career outcomes and therefore the shape of earning trajectories of his children (besides uniform shifts or transformations of trajectories that do not affect the relation of current to lifetime earnings at age t).

Technically, the conjecture that the error $u_{s,it}$ is uncorrelated with the regressor $y^*_{f,i}$ can be examined by deriving the elements of $u_{s,it}$ for a given

earnings formation model and analyzing its relation to $y_{f,i}^*$. Denote the individual-specific ratio of log annual earnings to log lifetime earnings by λ_{it} and express log annual earnings as $y_{s,it} = \lambda_{it} y_{s,i}^*$. Rewrite the error $u_{s,it}$ as $u_{s,it} = (\lambda_{it} - \lambda_t) y_{s,i}^*$ and substitute for $u_{s,it}$ in equation (5),

$$y_{s,it} = \lambda_t \alpha + \lambda_t \beta y_{f,i}^* + \lambda_t \epsilon_i + (\lambda_{it} - \lambda_t) y_{s,i}^*$$

I demonstrate in Appendix A.2 that λ_{it} varies with respect to individuals' earnings growth rates across all t, even for simple log-linear earning trajectories as generated by equation (3). Furthermore, fathers' lifetime earnings $y_{f,i}^*$ depend on fathers' earning growth rates. If we suspect that lifetime earnings correlate within families we might also expect that earning growth rates correlate within families. Combination of these arguments implies that $Cov(u_{s,it}, y_{f,i}^*) \neq 0$, as formally shown in Appendix A.3.

I conclude that the probability limit of the estimated coefficient from an OLS regression of sons' current log earnings $y_{s,it}$ on the regressor $y_{f,i}^*$ does not necessarily equal $\lambda_t \beta$ since correlation between $u_{s,it}$ and $y_{f,i}^*$ might cause an omitted variable bias; this bias is the familiar life-cycle bias, as described in Jenkins (1987). Estimates of β might also suffer from life-cycle bias in the case of right hand side measurement error in which unobserved log lifetime earnings of fathers are approximated by log annual earnings at age t for which $\lambda_t = 1$ while lifetime earnings of sons are perfectly observed (see Appendix A.4). The size of the life-cycle bias can be derived for specific functional forms of the earnings growth process.

Size of the Life-Cycle Bias

The size of the life-cycle bias depends on the nature of the variation in individual life-cycle earnings trajectories. Even in the presence of heterogeneous variation in the shape of life-cycle earning trajectories we might expect that log earning trajectories tend to reach their log annuitized lifetime earnings equivalent close to some point in mid-life.[7] If individuals merely differ with respect to their average earnings growth rates then log earnings around age t for which $\lambda_t = 1$ can be a good approximation of log annuitized lifetime earnings for all individuals; the life-cycle bias in intergenerational elasticity estimates will be relatively small. But if individuals differ with respect to the average growth of their earnings growth rates then log earnings around that age will severely misrepresent log annuitized lifetime earnings of some individuals; the life-cycle bias can then be relatively large.

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[7]By the intermediate value theorem, if a function $y = f(x)$ is continuous on the interval $[a, b]$, and u is a number between $f(a)$ and $f(b)$, then there exists a $c \in [a, b]$ such that $f(c) = u$. Let y denote yearly earnings, x age, and u annuitized lifetime earnings. If we assume that $f''(x) = 0$ then $f(x)$ is a straight line and a point c where yearly earnings equate annuitized lifetime earnings will lie exactly in the middle of the age interval. More generally, the smaller $f''(x)$ is in absolute terms (e.g. earnings growth rates change gradually over age), the closer point c will lie to the middle of the age interval.

Simulation Results

These theoretical arguments can be quantitatively assessed by applying the proposed procedure of measuring annual earnings around mid-life as proxy for unobserved lifetime earnings on a simulated data set, in which life-cycle earning trajectories are generated according to some earnings formation model. I consider two simple specifications.

First, using equation (3) as data generating process I simulate linear log earning life-cycle trajectories for a large number of families across T years. I assume that intergenerational elasticity of earnings arises through correlation of individuals' earning growth rates γ_i within families. Specifically I assume that earning growth rates are joint normally distributed within families according to

$$\begin{pmatrix} \gamma_i^{son} \\ \gamma_i^{father} \end{pmatrix} \overset{d}{\sim} N\left(\begin{pmatrix} \mu_\gamma \\ \mu_\gamma \end{pmatrix}, \begin{pmatrix} \sigma_\gamma^2 & \sigma_\gamma^2 \rho \\ \sigma_\gamma^2 \rho & \sigma_\gamma^2 \end{pmatrix} \right)$$

For simplicity I hold η_i constant and assume that the real interest equals zero, so that lifetime earnings are given by the sum over all T annual earnings.

Second, I repeat this simulation procedure for quadratic log earning trajectories as generated by $y_{it} = \eta_i + \gamma_i t + \delta_i t^2$. For simplicity I hold η_i and γ_i constant while individuals' quadratic earning growth δ_i correlates within families according to

$$\begin{pmatrix} \delta_i^{son} \\ \delta_i^{father} \end{pmatrix} \overset{d}{\sim} N\left(\begin{pmatrix} \mu_\delta \\ \mu_\delta \end{pmatrix}, \begin{pmatrix} \sigma_\delta^2 & \sigma_\delta^2 \rho \\ \sigma_\delta^2 \rho & \sigma_\delta^2 \end{pmatrix} \right)$$

While earnings differ as as of variation in earning growth rates they are not affected by transitory shocks in order to abstract from attenuation bias. Following Haider and Solon's proposed estimation procedure, I first determine the age t for which the slope coefficient λ_t of the projection of log annual earnings y_{it} on log lifetime earnings is closest to one. I then apply OLS regression of $y_{s,it}$ at this age t on $y_{f,i}^*$ to derive an estimate $\hat{\beta}$.

The size of the life-cycle bias (the difference between the estimate $\hat{\beta}$ and the true intergenerational elasticity of earnings parameter β) is plotted in figure 2 and figure 3 against the range of possible values for ρ, separately for various choices for the variance of earning growth rates σ_γ^2 and σ_δ^2.

As implied by the theoretical discussion, the results indicate that the remaining life-cycle bias in the intergenerational elasticity estimate will be small for variation in log-linear and more substantial for variation in log-quadratic earning trajectories. The size of the bias will be smaller for positive intergenerational elasticity of earnings since in this case the difference between current log earnings and log annuitized lifetime earnings will tend to have the same sign within families.[8] Furthermore, the size of the life-cycle

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[8] The underlying mechanism is easily understood if one regards a scenario in which

Figure 2: Size of the Life-Cycle Bias for Linear Log Earnings Trajectories
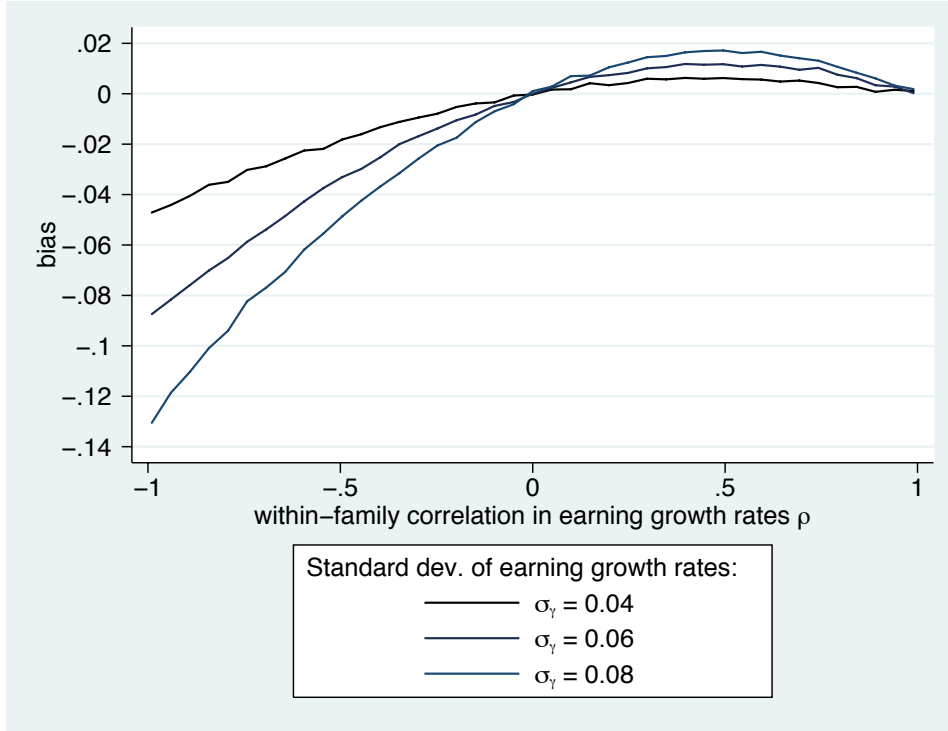
Notes: Simulation results, linear log earnings formation, $T = 50$, $\bar{\eta} = 20$, $\mu_\gamma = 0.03$. Bias denotes $\hat{\beta} - \beta$, plotted across $\rho = Corr(\gamma_i^{son}, \gamma_i^{father})$, with $\rho \in [-1, 1]$, for three different specifications of σ_γ^2.

bias generally increases with the variance of earning growth rates in the population, since an increase in the variance magnifies differences between log annual earnings and log annuitized lifetime earnings at age t.

Note that the bias can not be corrected for by accounting for the classical attenuation bias arising from variance in fathers' and sons' annual earnings. The variance of earnings is constant along each plotted line in figure 2 and figure 3, but the life-cycle bias is not as it depends on the correlation of earning growth rates within families. Furthermore, for positive values of within-family correlation of earning growth rates the estimator *overstates* the true intergenerational elasticity of earnings. Attempts to account for an attenuation bias could therefore increase the bias further in some cases. These results are perhaps interesting given that estimates of β are remarkably

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earning growth rates are perfectly negative correlated within families. In this case, if low earning growth rates result in current log earnings to understate log annuitized lifetime earnings for some father, then his son will have high earning growth over age and his current log earnings will overstate his log annuitized lifetime earnings.

Figure 3: Size of the Life-Cycle Bias for Quadratic Log Earnings Trajectories
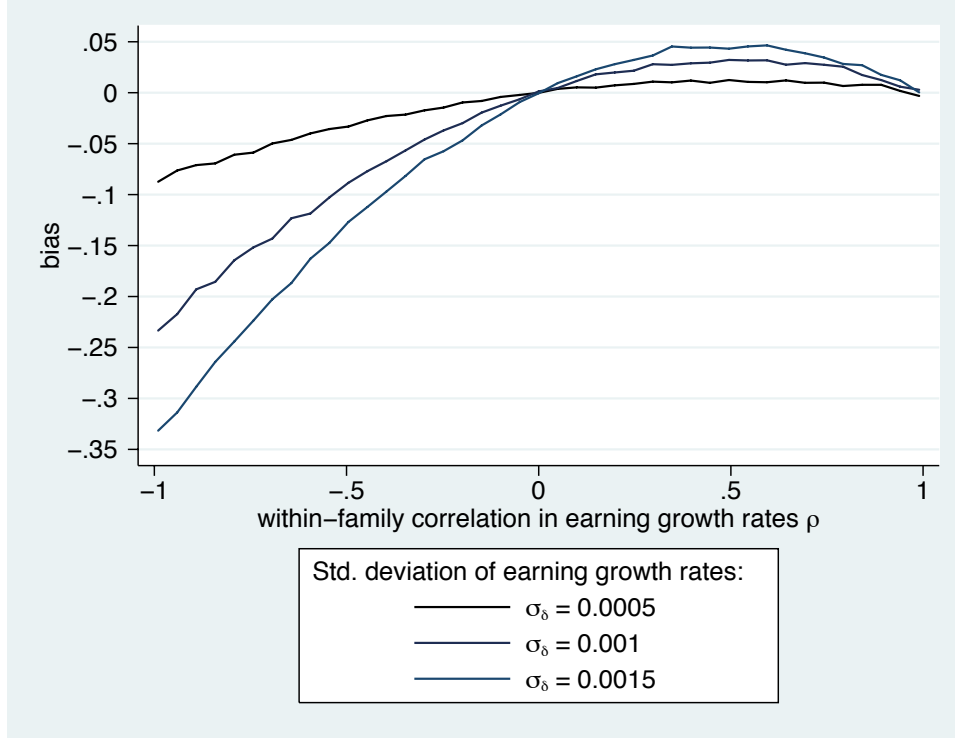
Notes: Simulation results, quadratic log earnings formation, $T = 50$, $\bar{\eta} = 20, \bar{\gamma} = 0.03$, $\mu_\delta = -0.001$. Bias denotes $\hat{\beta} - \beta$, plotted across $\rho = Corr(\delta_i^{son}, \delta_i^{father})$, with $\rho \in [-1, 1]$, for three different specifications of σ_δ^2.

large in some recent empirical studies.

Since the exact size of the bias is very sensitive to parameter choices such simulations only allow for a rough assessment of the magnitude of the life-cycle bias, but the results indicate that the remaining life-cycle bias might be relatively small in comparison to other empirical strategies. In fact, the life-cycle bias becomes negligible if I allow linear and quadratic earning growth rates to correlate and calibrate all moments according to sample moments derived from German employee history data.[9] However, the chosen earnings formation models deliver relatively smooth simulated earning trajectories with C^1 or C^2 continuity and the assumed form of intergenerational interdependence is very simplistic. Actual earning trajectories might feature more

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[9]Specifically I use IAB employee history data provided by the Institute for Employment Research (IAB) and sample 20000 individuals for whom I observe earning observations in at least 25 years between 1975 and 2007. OLS regressions of (real) log annual earnings on a polynomial in age yield estimates for individual-specific earning growth rates γ_i and δ_i. Sample moments are $\mu_\gamma = .078$, $\mu_\delta = -.00076$, $\sigma_\gamma = .082$, $\sigma_\delta = .001$ and $Corr(\gamma, \delta) = -0.98$.

abrupt changes over age and the correlation within families might follow
more complicated patterns. The simulation results nevertheless make the
point that the size of the life-cycle bias could be large in some cases, and that
the size of the bias can be expected to differ across countries as it depends
on the nature of variation in individuals' life-cycle earnings trajectories.

Additional Notes

In comparison to other empirical strategies, the sample selection procedure
of measuring earnings around a specific age typically calls for selection of
specific sub-samples from a given sample. The potential for sample selec-
tion from sample attrition in survey data might become more problematic.
Furthermore, if the intergenerational elasticity of earnings changes over time
or over subgroups of the population then estimates might reflect a differ-
ent population parameter than estimates stemming from previously applied
methods that followed a less strict sampling procedure.

Finally, while previously applied estimation methods are also biased and
inconsistent in the number of sampled individuals, they can be consistent in
the number of sampled individuals and the number of annual earning obser-
vations per individual. In contrast, the proposed sample selection procedure
prohibits using earning observations that are not in a specific age interval,
even if they would be available in the sample. It follows that consistency of
the estimator cannot be reached in the number of available annual earning
observations per individual either.

The Generalized Errors-in-Variables Model: Summary

In comparison to previously applied methods, the sample selection procedure
of measuring earnings around a specific age shifts the source of potential
inconsistencies in intergenerational elasticity of lifetime earnings estimates
from the assumption of a uniform earning growth rate in the population
(previously employed methods controlled for the central tendency of earn-
ings growth in the population) to the assumption of a uniform age at which
differences in current earnings truly reflect differences in lifetime earnings in
the population. Estimates might suffer from life-cycle bias arising through
heterogeneous variation in the shape of individuals' life-cycle earning tra-
jectories that correlates within families, in a similar way as previously em-
ployed methods suffered from life-cycle bias arising through heterogeneity
in individuals' earnings growth rates. Attempts to account for a perceived
attenuation bias might bias estimates further. Simulation results indicate
that the remaining life-cycle bias might be relatively small in comparison to
previously applied methods, but the exact size depends on the correlation in
the shape of earning trajectories within families, which is unknown. Empir-
ical analysis of long panel data that allows to compare estimates based on

annual data and estimates based on lifetime earnings might help to quantify the remaining bias.

3   Structural Estimation of Life-cycle Patterns in Earnings

Many other empirical strategies employed in the literature attempt to structurally model the life-cycle earning growth process. Exemplary for this part of the literature I will discuss the identification strategy described in Vogel (2006). In line with Haider and Solon's analysis, Vogel focuses on the problem of heterogeneous variation in earnings growth resulting from variation in education levels. Vogel proposes to explicitly model the life-cycle patterns in earnings across different skill groups instead of employing the classic assumption of an uniform rate of earning growth in the population.

Earning growth patterns can be estimated by panel regressions of observed log earnings y_{it} of individuals i at time t on a polynomial of individuals' age A_{it} and a time effect t. A random effects model[10] can be assumed for estimation of

$$y_{it} = \bar{\alpha}_E + \alpha_i + \gamma_{E,1} A_{it} + \gamma_{E,2} A_{it}^2 + \gamma_{E,3} A_{it}^3 + \gamma_{E,4} A_{it}^4 + \phi_E t + \nu_{it} \qquad (6)$$

Separate regressions for a number of educational groups E yield estimates for education-specific earning growth rates, as depicted in figure 4 for a sample from German data.[11] Estimated individual fixed effects and educational-specific earning growth rates can then be used to construct approximated yearly earnings for the complete life-cycle of an individual. An estimate for lifetime earnings of person i is given by the present value of the sum of all predicted yearly real earnings between the year of assumed labor market entry and labor market exit.

Vogel argues that while such simulated log lifetime earnings y_i might still differ largely from true log unobserved lifetime earnings y_i^*, they will not provoke a life-cycle bias in intergenerational elasticity estimates by eliminating the systematic age-dependent bias in lifetime earnings estimates for young and old workers.

Unobservable Determinants of Earning Growth Rates

Arguably, educational achievement is an important determinant of individual earning growth rates. Accounting for differential earning growth rates across educational groups can therefore lead to more precise estimates of lifetime

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[10]The time-invariant relation of year t and age A_{it} corresponds to the presence of a time-invariant regressor.

[11]See Wagner, Frick, and Schupp (2007) for a description of the German Socioeconomic Panel. I thank Thorsten Vogel for helpful advice on sample selection and preparation.

Figure 4: Life-cycle Earning Profiles Across Educational Groups in Germany
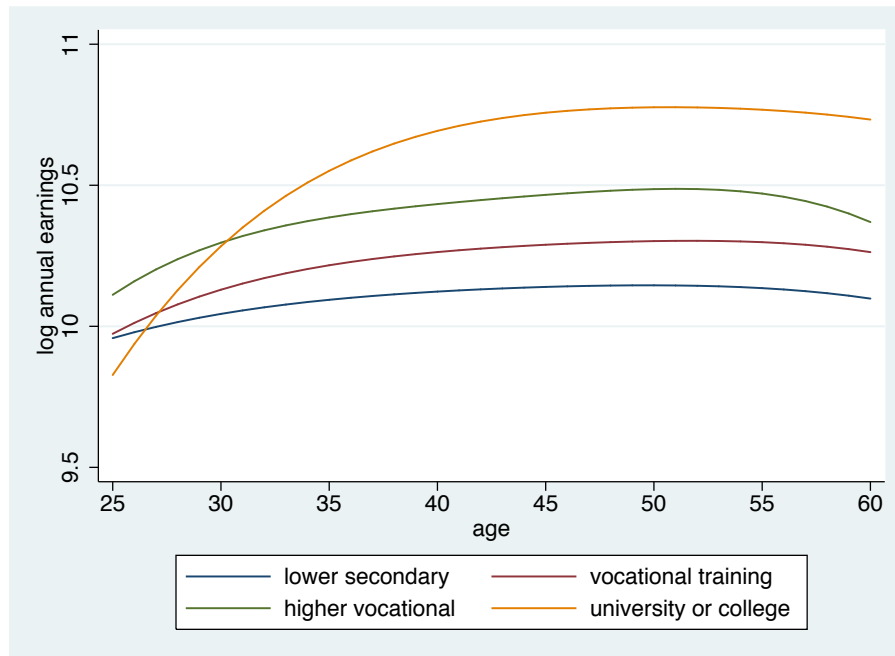
Notes: life-cycle earning profiles of men across skill groups in Germany as estimated by equation (6), plotted for t=1984 in year 2000 euros. Skill level I-IV denote men (I) without vocational training, (II) with vocational training, (III) with further vocational education and (IV) with a degree from technical college or university. Data: German Socioeconomic Panel (1984-2006). Number of earning observations per skill group: (I) 10962; (II) 37462; (III) 8255; (IV) 14274. Sample selection rules as reported in Vogel (2006).

earnings compared to the assumption of an uniform earning growth rate in the population, as demonstrated by Vogel (2006). However, the problem remains in that it is unknown which share of within-family correlation of earning growth rates remains unexplained. After accounting for differential earning growth rates across educational groups, other determinants (that might be shared within families) of earnings than education will lead to heterogeneous variation of earning growth rates around the mean earning growth rate within any given educational group. Measurement errors in earning growth rates of fathers and sons are therefore likely to be correlated even once growth rates have been differentiated by educational levels or other characteristics.[12] For example, when a father's earning growth is steeper than the average in his educational group (e.g. because the father is more capable than the average individual in his educational group, or because he

---

[12]For example, Dustmann (2008) differentiates earning growth rates among foreign- and native-born individuals.

chose some specific occupation that typically leads to steeper earning growth rates), then the earnings growth rate of his son might also be steeper than the average growth rate (e.g. because the son inherited at least partially his father's high capability, or because the son is more likely to enter the same profession as his father relative to non-family members[13]).

These specific examples indicate that estimates of individual earnings growth rates could be improved by inclusion of further characteristics of fathers or sons in equation (6). But the crucial insight is that we will not be able to sufficiently project individuals' life-cycle trajectories of earnings, since individual growth rates are determined by both potentially observable (e.g. schooling or choice of profession) and unobservable (e.g. ability or motivation) characteristics that might correlate within families. Studies on the determinants of wages, which find that a large share of wage differentials remains unexplained after controlling for observable characteristics like education and experience (see for example Autor and Katz (1999)), indicate that such differences in unobservables will be important. Inconsistencies in estimates can therefore be expected to be large as of within-family correlation of the unexplained part of individual earning growth rates. Furthermore, the direction of the bias will be unknown since the unexplained part of individual earning growth rates could be either positively or negatively correlated within families.[14]

Empirical Evidence

An ad-hoc empirical verification of these arguments can be made by iteration of the earning growth estimation procedure separately for groups of individuals who have the same educational level but who differ in other characteristics. Using the German Socioeconomic Panel (GSOEP) to construct a sample according to sample selection rules reported in Vogel (2006) I estimate in a first step equation (6) for all individuals that attained higher vocational education, thereby deriving an estimate of individual fixed effects α_i. Conditional on the estimated individual fixed effects I sort individuals then into two groups for high ($\hat{\alpha}_i > 0$) and low ($\hat{\alpha}_i < 0$) lifetime earnings, respectively. In a second step I re-estimate equation (6) separately for both groups in order to derive average life-cycle trajectories of earnings. The result, plotted in figure 5, indicates that individuals with higher lifetime earnings tend to have a steeper earnings growth rate (especially at the begin-

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[13]For example, within-family correlation in the choice of profession can be observed in the likelihood of becoming president of the United States. More comprehensive evidence based on a somewhat larger sample is given in Corak and Piraino (2010).

[14]A simple example for negative correlation: sons might attempt to reach at least the same educational level as their fathers, even when their ability is lower. If lower ability leads to lower earning growth relative to the central tendency of earning growth in this education group then earning growth patterns conditional on education are negatively correlated within families.

Figure 5: Life-cycle Earning Profiles by Level of Lifetime Earnings
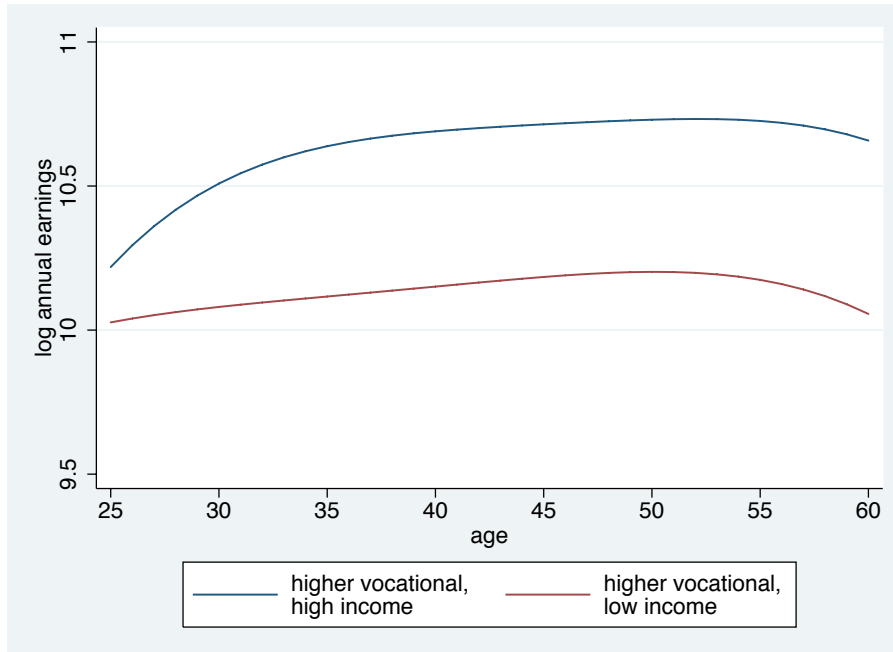
Notes: life-cycle earning profiles of men with higher vocational education in Germany as estimated by equation (6), plotted for t=1984 in year 2000 euros. Data: German Socioeconomic Panel (1984-2006). Number of sampled earning observations: 8255. Sample selection rules as reported in Vogel (2006).

ning of their career), even once education is controlled for. Differentiation of earning growth rates by educational levels only would lead to underestimation (overestimation) of lifetime earnings for such sons (fathers) if we employ typical panel data that covers sons at young and fathers at later age. If lifetime earnings correlate within families we will therefore underestimate the intergenerational elasticity of earnings.

4 Instrumental Variable Methods

An alternative procedure to address measurement error under the assumption that the classical errors-in variables model holds is given by instrumental variable (IV) methods. For example, unobserved lifetime earnings of fathers can be instrumented by fathers' education; Solon (1992) derives that under certain assumptions on the covariance between fathers' education and measurement errors in earnings the probability limit of this IV estimator will be an upper bound of the population parameter β. Zimmerman (1992) gives additional examples for IV identification strategies. Extending on the argu-

ments given in the previous two chapters I will argue that the IV estimator might not be an upper bound for the population parameter β.

Random Measurement Error

Following the discussion in Solon (1992), note that since father's education $E_{f,i}$ can be expected to have a direct effect on son's earnings in addition to the indirect relation through correlation with father's income, the true data generating process can be written as

$$y_{s,i}^* = \alpha + \eta_1 y_{f,i}^* + \eta_2 E_{f,i} + \varepsilon_i$$

If we estimate the regression of the observed approximation of lifetime earnings of sons $y_{s,i}$ (with $y_{s,i} = y_{s,i}^* + u_{s,i}$) on the observed approximation of lifetime earnings of fathers $y_{f,i}$ (with $y_{f,i} = y_{f,i}^* + u_{f,i}$) by IV with father's education as instrument, then the probability limit of the estimated coefficient is

$$
\begin{aligned}
plim\,\hat{\beta}_{IV} &= \frac{Cov(E_f, y_s)}{Cov(E_f, y_f)} \\
&= \frac{Cov(E_f, \alpha + \eta_1 y_f^* + \eta_2 E_f + \varepsilon + u_s)}{Cov(E_f, y_f)} \\
&= \frac{Cov(E_f, \eta_1 y_f - \eta_1 u_f + \eta_2 E_f + u_s)}{Cov(E_f, y_f)}
\end{aligned}
$$

where the last step follows since ε is assumed to be uncorrelated to E_f. Solon (1992) proceeds to show that the probability limit of $\hat{\beta}_{IV}$ is an upper bound for β under the assumption that E_f is uncorrelated with u_f and u_s,

$$
\begin{aligned}
plim\,\hat{\beta}_{IV} &= \frac{Cov(E_f, \eta_1 y_f + \eta_2 E_f)}{Cov(E_f, y_f)} \\
&= \eta_1 + \eta_2 \frac{Var(E_f)}{Cov(E_f, y_f^*)} \\
&= \beta + \eta_2 \left(\frac{Var(E_f)}{Cov(E_f, y_f^*)} - \frac{Cov(E_f, y_f^*)}{Var(y_f^*)} \right) \\
&= \beta + \eta_2 \frac{\sigma_{E_f}}{Corr(E_f, y_f^*)\sigma_{y_f^*}} \left(1 - Corr(E_f, y_f^*)^2 \right)
\end{aligned}
$$

in which the next to last step follows from the omitted variable formula.[15] We have that the IV estimator is an upper bound for the population parameter

---

[15]The relationship between the population parameter β in equation (1) and η_1 and η_2 is given by

$$
\beta = \frac{Cov(y_s^*, y_f^*)}{Var(y_f^*)} = \frac{Cov(\alpha + \eta_1 y_f^* + \eta_2 E_f + \epsilon, y_f^*)}{Var(y_f^*)} = \eta_1 + \eta_2 \frac{Cov(E_f, y_f^*)}{Var(y_f^*)}
$$

β if (i) father's education has a non-negative effect on son's earnings ($\eta_2 \geq 0$) and (ii) the correlation between father's education and earnings is bounded between zero and one.

Non-Random Measurement Error

Such assumptions on the correlation between education and earnings are reasonable, but I argue that the required assumption that E_f is uncorrelated with u_f and u_s might generally not hold.

For example, the fact that highly educated individuals will tend to have a steeper earnings growth over the life-cycle (see Vogel (2006)) hinders interpretation of the IV estimator in the same way as interpretation of the OLS estimator. If we employ typical panel data that covers annual earnings observations for sons at young and for fathers at later age to proxy for unobserved lifetime earnings y_i^* then lifetime earnings of highly educated sons will be understated and lifetime earnings of highly educated fathers will be overstated, even if we control for the central tendency of earnings growth in the population. If educational achievement is correlated across generations we therefore have that $E[u_f | E_f = high] > 0$ and $E[u_s | E_f = high] < 0$. In combination with the corresponding argument for lowly educated individuals it follows that $Cov(E_f, u_f) > 0$ and $Cov(E_f, u_s) < 0$.

The probability limit of the IV estimator is therefore more generally given by

$$
\begin{aligned}
plim \, \hat{\beta}_{IV} &= \frac{Cov(E_f, \eta_1 y_f - \eta_1 u_f + \eta_2 E_f + u_s)}{Cov(E_f, y_f)} \\
&= \frac{Cov(E_f, \eta_1 y_f + \eta_2 E_f)}{Cov(E_f, y_f)} + \frac{Cov(E_f, u_s) - \eta_1 Cov(E_f, u_f)}{Cov(E_f, y_f)} \\
&= \beta + \eta_2 \left(\frac{Var(E_f)}{Cov(E_f, y_f)} - \frac{Cov(E_f, y_f^*)}{Var(y_f^*)} \right) \\
&\quad + \frac{Cov(E_f, u_s)}{Cov(E_f, y_f)} - \eta_1 \frac{Cov(E_f, u_f)}{Cov(E_f, y_f)}
\end{aligned}
$$

Since the example demonstrates that the latter two terms can be negative it follows that $\hat{\beta}_{IV}$ will not necessarily be an upper bound to the true population parameter β.

The analysis by Haider and Solon (2006) or Grawe (2006) might be understood in that the IV estimate would bound the true population parameter if we would observe annual earnings of fathers and sons at the age t for which differences in log annual earnings in the population correspond to differences in log lifetime earnings (so that the population parameter $\lambda_t = 1$). But the discussion in section 2 shows that there exists no age at which measurement error in lifetime earnings can be expected to be uncorrelated to parental

characteristics. Generally, while at some age t differences in log annual earnings might truly represent differences in log lifetime earnings for the *whole* population, log lifetime earnings of subgroups of individuals (e.g. defined by education) might nevertheless be systematically over- or underrepresented by log annual earnings at this age.

5 Conclusions

The methodology for estimation of the intergenerational mobility in lifetime earnings has been repeatedly revised over the last decades, each time exposing flaws in previously employed methods and challenging results that were based on them. But the problems stemming from variation of earning growth rates around the population mean rate had been finally much better understood with the work of Haider and Solon (2006), Grawe (2006) and Vogel (2006), and it seemed that simple workarounds like restricting the age at which sampled individuals' earnings are to be observed would suffice to derive consistent estimates. As with previous methodological improvements, researchers were enthusiastic to employ the new methodology for re-estimation of intergenerational mobility parameters across countries. The generalized errors-in-variable model described in Haider and Solon (2006) has been widely adopted as motivation for empirical strategies and the proposed procedure of measuring earnings around mid-life has since been frequently applied.[16]

But these methodological improvements might still not fully eliminate the bias arising from life-cycle effects. I demonstrated in previous sections that estimates based on recent empirical identification strategies might be biased and inconsistent as of correlation in measurement errors of fathers' and sons' lifetime earnings in the same manner as estimates based on the empirical methods of the generations before, albeit probably to a lesser degree.

One might argue that remaining smaller scale life-cycle biases in estimates do not pose major problems for most of the applied literature, since often interest lies on the relative intergenerational mobility across populations (e.g. in cross-country studies). Estimates might be sufficiently precise if we believe that the magnitude of the remaining life-cycle bias in estimates is similar across countries. Theoretically, such belief would correspond to the

---

[16]Among others, in Dahl and DeLeire (2008) and Gouskova, Chiteji, and Stafford (2009) for the United States; in Eisenhauer and Pfeiffer (2008) and Schnitzlein (2008) for Germany; in Bohlmark and Lindquist (2006), Björklund, Lindahl, and Plug (2006), Björklund, Jäntti, and Solon (2007) and Björklund, Jäntti, and Lindquist (2009) for Sweden; in Nilsen, Vaage, Aakvik, and Jacobsen (2008) for Norway; in Raaum, Bratsberg, Røed, Österbacka, Eriksson, Jäntti, and Naylor (2007) for Denmark, Finland, Norway, the UK and the US; in Nicoletti and Ermisch (2007) for the UK; in Cervini Pla (2009) for Spain; in Piraino (2007) and Mocetti (2007) for Italy; in Lefranc, Ojima, and Yoshida (2008) for Japan and France.

assumption that idiosyncratic deviations from the population mean relationship between current and lifetime earnings are either not notably correlated within families, or that this correlation is of the same nature across countries. It is therefore a direct restriction on a potential cause for cross-country differences in intergenerational mobility. Since we expect that countries differ with respect to the extent of intergenerational mobility of earnings we might also suspect that they differ in this specific aspect of intergenerational mobility. Cross-country comparisons of estimates could thus have limited reliability if differences in estimates are small.

The result that approximation of lifetime earnings by annual earnings at a specific age might not lead to unbiased estimates implies that empirical strategies should make use of all earnings information available in the data in order to push back the assumptions that have to be made on the shape of earning trajectories in the population as far as possible. Furthermore, the argument that the projection of earning growth rates by means of observable determinants will be insufficient suggests that further research could benefit from a more comprehensive usage of partially observed earning growth patterns. In the literature on intergenerational earnings mobility it is standard procedure to base estimates on a sample that only includes individuals for which a minimum number of yearly earning observations are available. Researchers usually disregard, however, the actual observed earning growth across these observations. Such partially observed earning growth patterns have the invaluable advantage of being determined by all relevant observable and unobservable characteristics of the individual, thereby containing more information on the likely path of individual earning growth than estimates that are only based on observable individual characteristics. Accounting for partially observed earning growth rates might therefore help to reach faster rates of convergence in the number of earnings observation per individual of lifetime earning estimates to true lifetime earnings, and thereby more precise estimates of intergenerational mobility measures.

6 Appendix

A.1 Correlation between D_{it} and γ_i

Proposition. *(i) For all t, the difference between log annual earnings y_{it} and the log of the annuitized value of the present discounted value of lifetime earnings varies with respect to the individuals' earnings growth rate γ_i. (ii) For any given t, the difference will be equal for at most two different realizations of γ_i.*

Proof. Suppose that log annual earnings of worker i at age t are given by

$$y_{it} = \eta_i + \gamma_i t$$

For simplicity assume infinite lifetimes and a constant real interest rate $r > \gamma_i$. The annuitized value of the present discounted value of lifetime earnings, denoted by B_i, is then given by

$$\sum_{s=0}^{\infty} exp(\eta_i + \gamma_i s)(1+r)^{-s} = \sum_{s=0}^{\infty} B_i(1+r)^{-s} = \frac{1+r}{r}B_i$$

It follows that the log of the annuitized value of the worker's present discounted value of lifetime earnings is given by

$$
\begin{aligned}
log\, B_i &= log\left(\frac{r}{1+r}\sum_{s=0}^{\infty} exp(\eta_i + \gamma_i s)(1+r)^{-s}\right) \\
&\cong log\left(\frac{r}{1+r} exp(\eta_i)(1+r)/(r - \gamma_i)\right) \\
&= log\, r + \eta_i - log(r - \gamma_i)
\end{aligned}
$$

The difference D_{it} between log annual earnings y_{it} and the log of the annuitized value of the present discounted value of lifetime earnings $log\, B_i$ is therefore

$$
\begin{aligned}
D_{it} &= y_{it} - log\, B_i \\
&= \eta_i + \gamma_i t - (log\, r + \eta_i - log(r - \gamma_i)) \\
&= \gamma_i t - log\, r + log(r - \gamma_i)
\end{aligned}
$$

Depending on t, D_{it} decreases or increases in individuals' earning growth rates γ_i,

$$\frac{\partial D_{it}}{\partial \gamma_i} = t - \frac{1}{r - \gamma_i}$$

The second derivative with respect to γ_i is negative,

$$\frac{\partial^2 D_{it}}{\partial^2 \gamma_i} = -(r - \gamma_i)^{-2} < 0$$

D_{it} is therefore a strictly concave function of γ_i conditional on t given, and a specific value of D_{it} can stem from at most two different values of γ_i. $\qquad\square$

A.2 Correlation between λ_{it} and γ_i

Proposition. *For all t, the ratio of annual log earnings y_{it} to log lifetime earnings y_i^* varies with respect to individuals' earnings growth rate γ_i.*

Proof. Let

$$
\begin{aligned}
\lambda_{it} &= \frac{y_{it}}{y_i^*} \\
&= \frac{\eta_i + \gamma_i t}{log \sum_{s=0}^{\infty} exp(\eta_i + \gamma_i s)(1+r)^{-s}} \\
&\cong \frac{\eta_i + \gamma_i t}{\eta_i + log(1+r) - log(r - \gamma_i)}
\end{aligned}
$$

Assume that λ_{it} does not depend on γ_i and equals a constant C. Thus assume the equality

$$
C = \frac{\eta_i + \gamma_i t}{\eta_i + log(1+r) - log(r - \gamma_i)} \tag{7}
$$

holds for some t. Transformation

$$
\left(\eta_i + log(1+r) - log(r - \gamma_i)\right) C = \eta_i + \gamma_i t
$$

and derivation with respect to γ_i yields

$$
\frac{1}{r - \gamma_i} C = t
$$

While the right hand side of the equation is constant, the left hand side varies with γ_i; the equality (7) does not hold (proof of contradiction). It follows that λ_{it} is not constant but varies over γ_i for all t. $\square$

A.3 Life-cycle Bias: Left Hand Side Measurement Error

Proposition. *The error term $u_{s,it}$ correlates with the regressor $y_{f,i}^*$, causing a life-cycle bias in the OLS estimate of β.*

Proof. As shown, log annual earnings of sons are given by

$$
y_{s,it} = \lambda_t \alpha + \lambda_t \beta y_{f,i}^* + \lambda_t \epsilon_i + u_{s,it}
$$

Using the linear log earnings formation model given in equations (3) and (4) to rewrite the error $u_{s,it}$

$$
\begin{aligned}
u_{s,it} &= y_{s,it} - \lambda_t y_{s,i}^* \\
&= \eta_i + \gamma_i^s t - \lambda_t \left(\eta_i + log(1+r) - log(1+r - exp(\gamma_i))\right)
\end{aligned}
$$

where γ_i^f, γ_i^s denote earning growth rates of fathers and sons. For simplicity assume that the intercept of log earnings trajectories is constant, so that $\eta_i = \bar{\eta}$, and that the distribution of earning growth rates is stable across generations, so that $Var(\gamma_i^f) = Var(\gamma_i^s) = \sigma_\gamma^2$. The probability limit of the least square estimator of a linear regression of annual log earnings of sons $y_{s,it}$ on log lifetime earnings of fathers $y_{f,i}^*$ is then given by

$$
\begin{aligned}
plim\,\hat{\beta} &= \frac{Cov\left(y_{f,i}^*, y_{s,it}\right)}{Var\left(y_{f,i}^*\right)} \\
&= \frac{Cov\left(y_{f,i}^*, \lambda_t\alpha + \lambda_t\beta y_{f,i}^* + \lambda_t\epsilon_i + u_{s,it}\right)}{Var\left(y_{f,i}^*\right)} \\
&= \lambda_t\beta - \frac{Cov\left(log(1+r-exp(\gamma_i^f)), \gamma_i^s t + \lambda_t log(1+r-exp(\gamma_i^s))\right)}{Var\left(log(1+r-exp(\gamma_i^f))\right)}
\end{aligned}
$$

The probability limit of the least square estimator at age t^* for which $\lambda_t = 1$ is therefore given by[17]

$$
plim\,\hat{\beta} = \beta - \frac{Cov\left(log(1+r-exp(\gamma_i^f)), \gamma_i^s t^* + log(1+r-exp(\gamma_i^s))\right)}{Var\left(log(1+r-exp(\gamma_i^f))\right)}
$$

If we suspect that lifetime earnings correlate within families we should also expect correlation of earning growth rates within families. The second term in the previous expression is therefore non-zero for a general class of joint distribution functions for γ_i^f, γ_i^s and represents the life-cycle bias. For example, assume that earning growth rates are joint normally distributed

$$
\gamma = \left(\begin{array}{c}\gamma_i^f \\ \gamma_i^s\end{array}\right) \stackrel{d}{\sim} N\left(\left(\begin{array}{c}\mu_\gamma \\ \mu_\gamma\end{array}\right), \left(\begin{array}{cc}\sigma_\gamma^2 & \sigma_\gamma^2\rho \\ \sigma_\gamma^2\rho & \sigma_\gamma^2\end{array}\right)\right)
$$

Application of the Delta method on the non-linear transformation (assuming that r is large relative to μ_γ and σ_γ so that the logarithm is always defined)

$$
G(\gamma_f, \gamma_s) = \left(\begin{array}{c}g_1(\gamma^f, \gamma^s) \\ g_2(\gamma^f, \gamma^s)\end{array}\right) = \left(\begin{array}{c}log(1+r-exp(\gamma_i^f)) \\ \gamma_i^s t^* + log(1+r-exp(\gamma_i^s))\end{array}\right)
$$

---

[17]From

$$
\lambda_t = \frac{Cov(y_i^*, y_{it})}{Var(y_i^*)} \cong \frac{\sigma_\eta^2 + t\sigma_\gamma^2/r}{\sigma_\eta^2 + \sigma_\gamma^2/r^2}
$$

it follows that t^* can be approximated by $1/r$.

with

$$\frac{\partial G(\gamma_f, \gamma_s)}{\partial \gamma'} = \begin{pmatrix} \frac{-exp(\gamma^f)}{1+r-exp(\gamma^f)} & 0 \\ 0 & t^* + \frac{-exp(\gamma^s)}{1+r-exp(\gamma^s)} \end{pmatrix}$$

yields approximation of the covariance matrix

$$Var\left(G(\gamma_f, \gamma_s)\right) \approx \left(\frac{\partial G(\gamma_f, \gamma_s)}{\partial \gamma'}\right) \begin{pmatrix} \sigma_\gamma^2 & \sigma_\gamma^2 \rho \\ \sigma_\gamma^2 \rho & \sigma_\gamma^2 \end{pmatrix} \left(\frac{\partial G(\gamma_f, \gamma_s)}{\partial \gamma'}\right)'$$

$$= \begin{pmatrix} \left(\frac{-exp(\mu_\gamma)}{1+r-exp(\mu_\gamma)}\right)^2 \sigma_\gamma^2 & \frac{-exp(\mu_\gamma)\sigma_\gamma^2 \rho}{1+r-exp(\mu_\gamma)}\left(t^* + \frac{-exp(\mu_\gamma)}{1+r-exp(\mu_\gamma)}\right) \\ \frac{-exp(\mu_\gamma)\sigma_\gamma^2 \rho}{1+r-exp(\mu_\gamma)}\left(t^* + \frac{-exp(\mu_\gamma)}{1+r-exp(\mu_\gamma)}\right) & \left(t^* + \frac{-exp(\mu_\gamma)}{1+r-exp(\mu_\gamma)}\right)^2 \sigma_\gamma^2 \end{pmatrix}$$

The covariance can be evaluated for specific choices of σ_γ^2, ρ and r. Since the underlying earnings formation model is quite unrealistic (infinite life-times) one should not use this covariances to compute the potential size of the life-cycle bias. Simulation results on the size of the life-cycle bias reported in section 2 are based on a more realistic earnings formation model. □

A.4 Life-cycle Bias: Right Hand Side Measurement Error

Suppose that we wish to estimate the regression model

$$y_{s,i}^* = \alpha + \beta y_{f,i}^* + \epsilon_i$$

Assume now that because log lifetime earnings of fathers $y_{f,i}^*$ are not observed they are proxied by $y_{f,it}$, log annual earnings at age t, while sons' log lifetime earnings $y_{s,i}^*$ are observed. We express the linear projection of $y_{f,it}$ on $y_{f,i}^*$ as

$$y_{f,it} = \lambda_t y_{f,i}^* + u_{f,it}$$

The probability limit of the least square estimator of a linear regression of log lifetime earnings of sons $y_{s,i}^*$ on log annual earnings of fathers $y_{f,it}$ at year t^* such that $\lambda_t = 1$ is then given by

$$\begin{aligned} plim\, \hat{\beta} &= \frac{Cov(y_{f,it}, y_{s,i}^*)}{Var(y_{f,it})} \\ &= \frac{Cov(y_{f,i}^*, \alpha + \beta y_{f,i}^* + \epsilon_i) + Cov(u_{f,it}, y_{s,i}^*)}{Var(y_{f,i}^* + u_{f,it})} \\ &= \frac{\beta Var(y_{f,i}^*) + Cov(u_{f,it}, y_{s,i}^*)}{Var(y_{f,i}^*) + Var(u_{f,it}) + 2Cov(u_{f,it}, y_{f,i}^*)} \\ &= \beta\theta + \frac{Cov(y_{f,it} - y_{f,i}^*, y_{s,i}^*)}{Var(y_{f,i}^*) + Var(u_{f,it})} \\ &= \beta\theta + \theta\frac{Cov(\gamma_i^f t^* + log(1+r-exp(\gamma_i^f)), log(1+r-exp(\gamma_i^s)))}{Var(log(1+r-exp(\gamma_i^f)))} \end{aligned}$$

where the third and fourth line follow since the errors ϵ_i and $u_{f,it}$ are by construction uncorrelated with $y_{f,i}^*$, and where

$$\theta = \frac{Var(y_{f,i}^*)}{Var(y_{f,i}^*) + Var(u_{f,it})}$$

is the attenuation factor. The second term in the last line contains the life-cycle bias for log-linear earnings formation models as given by equation (3) and (4). If father's lifetime earnings are approximated by annual earnings measured at age t^* such that $\lambda_t = 1$ then estimates might be biased by attenuation bias entering trough $Var(u_{f,it})$ and by life-cycle bias entering through the correlation in earning growth rates within families that manifests in correlation between the errors $u_{f,it}$ and ϵ_i.

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