

JEANINE KIPPERS

# Empirical Studies on Cash Payments



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# Empirical Studies on Cash Payments

Empirische studies naar contante betalingen

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aan de Erasmus Universiteit Rotterdam  
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# Preface

In August 1999 I started the research for this thesis, inspired by the subject of currency use through my work at the Currency Policy Department of De Nederlandsche Bank. I discovered that bank notes and coins, commonly used by everyone, still hold many secrets. Little is known about their use in practice, while they are the most popular means of payment. To the best of my knowledge, De Nederlandsche Bank was the first to initiate a survey on cash payments in 1998. This survey produced a unique data set of observations on the use of bank notes and coins over the counter. This data set has proven to be an essential source of information in the preparation of the euro cash changeover in 2002, but has also been the major inspiration for the research presented in this thesis.

During the last four and a half years I have combined this research with my regular work. De Nederlandsche Bank promotes its employees to actively participate in scientific research in the field of currency, or other subjects related to the financial sector. I am thankful to De Nederlandsche Bank, and in particular to Henny van der Wielen, Ton Dongelmans, Bram Scholten, Ton Roos and Wiebe Ruttenberg, for giving me the resources and the opportunity to work on this thesis, which I did partly during office hours. If not, I would not have been able to finish this thesis now. Nonetheless, at times it has been difficult to make the necessary time available to make progress, especially around the time of the changeover to euro cash, a major project I was involved in through my work for the Currency Policy Department.

Philip Hans Franses has shown great flexibility as my promotor. Although he had no control over the time I could spend on the research, he was helpful in trying to find a way to get the work done within a reasonable timeframe. His creative thinking has been inspiring. The highlights were most definitely the interviews Philip Hans gave on radio and local television in reaction to an article we published in ESB. In this article we stated that the payment system would theoretically be better off without a 1 and 2 euro cent. The media attention evidenced the public interest in the subject of currency. Many thanks to Philip Hans for his guidance and enthusiasm in bringing about this thesis. I would also

like to thank the members of the small committee, consisting of Herman van Dijk, Lex Hoogduin and Kees Koedijk for reading the manuscript and giving useful comments.

Richard Paap coauthored our first paper, but more generally has been of great help. He was always willing to advise me on whatever econometric problem was bugging me, often through clear suggestions by email. Erjen van Nierop, another coauthor of the first paper, did much of the work that lies at the basis of the research presented in this thesis. Marielle Non, as a students assistant, has done great work on the experiments we conducted. And Merel van Diepen, Niels Tromp, Claudia Laheij, all econometric students, have also contributed substantially. They thought of an effective method to conduct interviews and observe cash payments, for which they spent many hours in retail facilities. My thanks to all.

I have several colleagues to thank for, as they helped in many ways: Ger Bom, Gerbert Hebbink, Wilko Bolt, Carsten Folkertsma, Maria Demertzis and Joke Mooij. Friends and family, I am grateful for all your mental support.

Lastly, but most importantly, I want to thank Jan Berndsens. He motivated me to start on this thesis, he urged me to work when work needed to be done, he made me relax when I needed it, and he has relieved me of many burdens to be able to work on this thesis. We have had long discussions that inspired me to go on with the research. He has contributed to this thesis in many ways. I dedicate it to him.

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# Chapter 1

## Introduction

### 1.1 Motivation

Individuals nowadays have the choice between several modern means of payment in retail transactions. In the Netherlands, the amount of electronic payments has increased heavily in recent years. The debit card is now frequently used by most, and the more recently introduced chipcard currently experiences an explosive growth of use. In fact, the amount of chipcard payments has almost tripled in 2002, and the amount of debit card payments continues to grow by more than 10% yearly<sup>1</sup>. The Dutch retail sector also reports an increase in turnover through electronic means of payment. Where in 1998 still 61% of its turnover was generated by cash payments, the majority of its turnover now settles electronically<sup>2</sup>.

In the same report of the retail sector, however, it is written that more than 80% of all retail transactions are still paid in cash. This means that cash is largely used for small payment amounts and that it still constitutes the most common means of payment, when considered over transactions. It is estimated that some 7 billion cash transactions take place in the Netherlands yearly<sup>3</sup>. This means that an individual pays with cash on average 1 to  $1\frac{1}{2}$  times a day. So most individuals deal with cash on a daily basis. If one could consider cash to be a commodity, no marketing efforts would be needed to increase its familiarity among the public. It would be the product most known of all.

To provide for the day-to-day exchange of notes and coins in cash payments, banks and retailers participate in a large and costly distribution system. The social costs for

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<sup>1</sup>De Nederlandsche Bank Annual Report 2002, page 132.

<sup>2</sup>'Afrekenen in winkels 2002', Hoofdbedrijfschap Detailhandel, oktober 2002, page 35-36.

<sup>3</sup>Bolt (2003), page 10.

maintaining this system have recently been estimated to be around 2 billion euro yearly in the Netherlands<sup>4</sup>. Retailers make costs for facilitating cash payments at their points of sale, but also for activities in preparation of lodging their cash turnover at the bank, such as counting and packaging bank notes and coins. Furthermore, they need to maintain stock of small money to provide their customers with change. Banks, at their turn, operate a large ATM- and branch network to provide their customers with cash. In the Netherlands, banks have set up centralized cash centres, where notes and coins are counted and processed on a large scale. Some of these cash centers are operated by third parties specialized in cash handling. For the exchange of bank notes and coins, both retailers and banks hire in-transit companies for secure transportation.

The issuing authority is the closing entry of the cash cycle. As the central bank of the Netherlands, De Nederlandsche Bank (DNB or later the Bank) issues bank notes. DNB accommodates fluctuations in demand. In general, central banks make sure that they have enough cash in stock to meet demand at all times. Furthermore, for maintaining the quality of bank notes in circulation, central banks perform quality checks on the bank notes they receive from banks. Counterfeits and damaged or soiled bank notes, detected by highly sophisticated equipment, are replaced by new ones. Hence, a central bank faces production and processing costs for executing its issuing task.

Figure 1.1 graphically summarizes the cash cycle in the Netherlands. Through this distribution system millions of notes and coins are processed on a daily basis. Considering the costs involved for all parties, a society has a great interest in achieving the most efficient payment system possible.

In addition to managing the continuous logistic process of distributing bank notes and coins, issuing authorities face long-term policy decisions concerning the denominational structure of the currency they issue. Cash is issued in a range of different denominations, such that any transaction amount can be paid in a convenient manner. For example, if payment amounts between 1 and 100 unit values (euro or dollar, or any other currency) occur frequently, then it would be convenient to have several denominations available between a lowest denomination of 1 and the highest denomination of 100 to make payments. Otherwise, it would take too many bank notes and coins to make most payments. The composition of denominations that form a currency range allow for payment and change to make cash transactions more efficient.

In general central banks issue bank notes, and governments issue coins. This split

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<sup>4</sup> Report 'Betalen kost geld', March 2004, Werkgroep Kostenonderzoek Toonbankbetaalproducten, Maatschappelijk Overleg Betalingsverkeer, page 10.

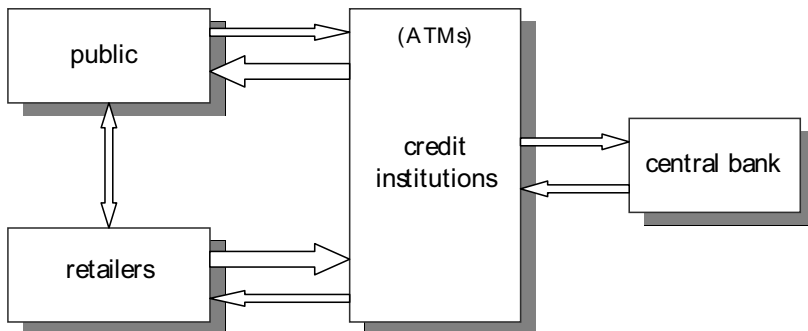


Figure 1.1: The cash cycle in the Netherlands.

in responsibilities has a historical background. Coins have been the common means of payment for the general public for thousands of years. Bank notes started out as trade paper issued by banks, and have only become a common means of payment in more recent history. Nowadays, central banks are the sole issuers of bank notes. In addition, they generally operate the distributional task of the government where it concerns coins. The denominational structure of coins, however, is still decided by the government.

On the one hand, the range of denominations should best meet the needs of the public. For this reason it might be needed to change the denominational structure regularly. Inflation, for example, will create a natural demand for a new highest denomination. On the other hand, changes to the denominational structure involve considerable expenses for professional cash handling parties, banks, retailers and central banks. One can think of the adaptation of cash registers, ATMs, sorting machines, and the origination and production of new to be introduced bank notes and coins. Issuing authorities are therefore careful not to take such decisions too frequently. However, how can it be determined what denominational range meets the needs of the public? And by what means does an issuing authority know if a denomination is successfully introduced? These are the type of questions we concern ourselves with in this thesis. By the following example, we aim to illustrate the dynamics of denominational ranges over time, and the influence of public opinion thereupon, to arrive at the conclusion that research in the area of denominational ranges should thus be of an empirical nature.

By taking a journey into history, we can give some background information on the denominational changes the Netherlands have experienced in the 20th century. Figure

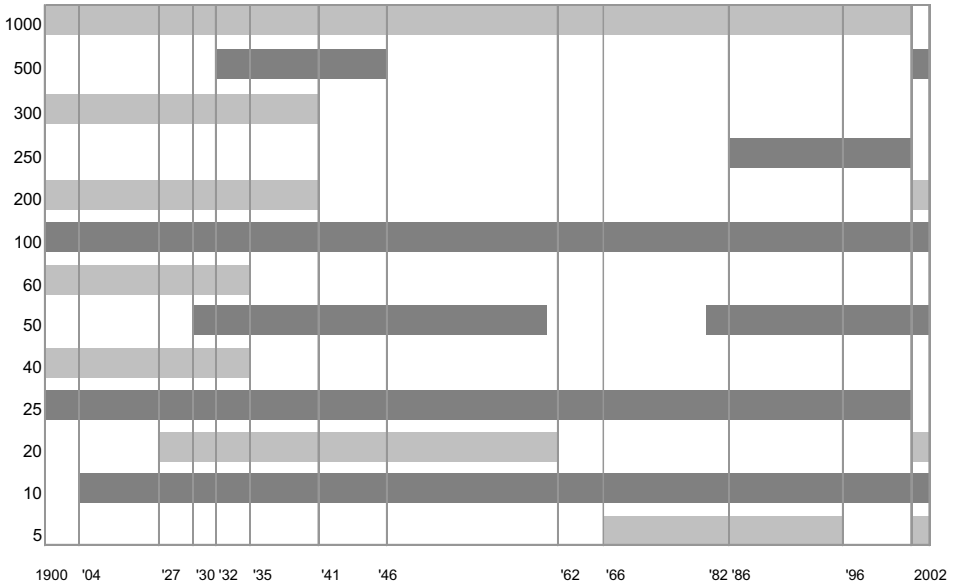


Figure 1.2: Bank note denominations issued by De Nederlandsche Bank since 1900 (source: Bolten (1999)).

1.2 presents the various bank note denominations issued by De Nederlandsche Bank over time. It should be noted that it abstracts from the variety of designs that circulated for most of the bank note denominations. Some of the most remarkable features in the figure are highlighted in the following.

In 1900, DNB issued bank notes in denominations such as 40, 60 and 300. Going back further in history, we find the origin of these bank notes as early as 1814. Just after the French occupation King William I established DNB and the state granted the Bank the right to issue bank notes in the following denominations: 25, 40, 60, 80, 100, 200, 300, 500 and 1000 guilder<sup>5</sup>.

The rationale for this bank note structure is unknown to us. In the 19th century bank notes were primarily used in trade and maybe there was some logic for using these nominal values then. Only from 1863 onward the Bank was given the freedom of choice in

<sup>5</sup>De Jong (1967), page 61.

denominations, although initially 25 guilders remained the lowest denomination allowed<sup>6</sup>. In 1904, when bank notes became legal tender in the Netherlands, a 10-guilder bank note was issued<sup>7</sup>, followed by a 5-guilder bank note some 60 years later. This development indicates the increasing use of bank notes as a common means of payment by the public in the 20th century.

So already in 1814, when a 25-guilder bank note was first issued, the foundation was laid for the 10-25-50 range, which is so typical for the range of Dutch guilder bank notes (and coins). This in contrast to the 10-20-50 range, or similarly the 1-2-5 range, that is more commonly applied to currency in other countries (see [www.banknotes.com](http://www.banknotes.com) which suggests that almost all countries rely on the 1-2-5 range). Interestingly, in an earlier draft of the patent, in which the right for DNB to issue bank notes was laid down, the values 10, 20, 40, 60, 80, 100 were suggested for the bank note range<sup>8</sup>. It would be interesting to examine why the originally suggested - and more common - 20 guilder bank note was changed to a 25 guilder bank note, but that is beyond the scope of this book.

From Figure 1.2 we can tell that a 20-guilder bank note was nevertheless introduced, although more than a century later in 1927. Within a few years, the introduction of a 50-guilder bank note followed. These introductions were decided upon in 1924, together with the decision to cancel denominations 40, 60, 200 and 300 in due time<sup>9</sup>. This suggests that DNB aimed to gradually change to the common 10-20-50 range. We find some support for this conjecture in the 1925 report of DNB to the Board of Commissioners. There it can be read that the 25-guilder note was indeed intended to be cancelled, while a 500-guilder bank note was planned to be introduced. Both decisions were delayed until the need for these bank notes could be judged in connection with the other changes in the denominational range. As we can tell from Figure 1.2, the 500-guilder bank note was eventually introduced, but the 25-guilder bank note was not withdrawn until the Netherlands changed over to euro bank notes and coins in 2002. This means that both a 20 and 25 denomination circulated simultaneously. Figures 1.3 and 1.4 show images of these bank notes around 1940.

Both the 20- and 50-guilder bank note were withdrawn from circulation in the early 1960s. From this event, we can infer that their introduction had not been very successful. Indeed, their issuance was discontinued because their circulation had dropped to very

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<sup>6</sup>De Jong (1960), page 41.

<sup>7</sup>This banknote substituted the 10-guilder currency note. In the past currency notes were issued by the state as replacements for coins to prevent or overcome a shortage of coin silver.

<sup>8</sup>De Jong (1967), page 62.

<sup>9</sup>Grolle (1991), page 55.





Figure 1.3: This bank note with the image of Queen Emma circulated in the Netherlands from 1940 until 1945.

low levels<sup>10</sup>. Apparently, they were not as popular with the public as their neighboring bank notes of 25- and 100-guilder. This is confirmed by the relatively high circulation figures for these denominations, listed in the Annual Reports of DNB around that time. In summary, the supposedly intended change to a 10-20-50 structure seemed to have been rejected by the public.

The 50-guilder bank note, however, was re-introduced in 1982. Its introduction was meant to relieve the 100-guilder bank note, as it took up more than 60% of total circulation in the late 1970s and became too critical for an undisturbed circulation of bank notes. For similar reasons, also a higher denomination of 250-guilders was introduced in 1986<sup>11</sup>. With this introduction the 10-25-50 bank note range was almost complete. The only value missing was 500 guilders. This bank note had never been reissued after its withdrawal during World War II.

With the recent introduction of the euro bank notes and coins in 2002, the Netherlands has shifted from its cherished 1-2 $\frac{1}{2}$ -5 range to the more common 1-2-5 range, formerly employed by the other 11 countries that made the transition to the euro. Against the historical background described above, the question arises how the Dutch public has

<sup>10</sup>Grolle (1991), page 84 and 99.

<sup>11</sup>Grolle (1991), page 117.



Figure 1.4: This bank note with the image of Mr. Mees, former president of De Nederlandsche Bank, circulated in the Netherlands from 1934 until 1945.

handled this transition. We chose to consider the Dutch public as a research object exactly for this reason. The type of empirical research we present in this thesis can be done for each country in the world, but the fact that the Dutch public underwent a transition from  $1-2\frac{1}{2}-5$  to  $1-2-5$ , gave us the unique opportunity to consider empirical data from a natural experiment.

In this section we described the cash cycle and we showed some examples of the changes in denominational structure experienced in the Netherlands. We arrive at the following general notions concerning currency. First, cash distribution is costly and many parties are involved. Second, given the intensive use of cash throughout the cycle, decisions of the issuing authorities on its denominational structure have a major impact on society. Third, the behavior of the public is leading in the composition of the currency range. On the basis of these notions we conclude that empirical research into currency issues can be of substantial interest.

## 1.2 Empirical currency research

Currency use appears to be an extensive area of research. Over the years, many studies were published in the (economic) literature dealing with several currency related issues. Economists like Hentsch, Fase, Cramer, Boeschoten and Van Hove have carried out im-

portant work in this area. In this section we intend to give an overview of the existing literature on empirical research of currency issues. Our focus is on the questions that currency research has attempted to answer, and the data and the methods that were used for these questions. We limit ourselves to the studies we consider to be relevant as an introduction to the research presented in this thesis. We summarize the most prominent and influential publications, but by no means do we claim to be exhaustive. Our goal is to provide a framework to position our own approach to empirical currency research, which we put forward in this thesis.

We roughly distinguish between three groups of currency research that we subsequently discuss. Numerous studies in the first group deal with the *modeling of aggregate currency demand by denomination* for forecasting purposes. Forecasting denomination-specific demand is essential for inventory management and production planning of central banks in their issuing task.

Most econometric models for this purpose are regression models that explain denomination-specific demand for bank notes by variables that relate to the two functions of currency, that is, bank notes held for transaction purposes and bank notes held as a store of value (hoarding), see Fase and Van Nieuwkerk (1976, 1977), Browne (1981), Kimball (1981), Fase (1981*a*), among others. Common explanatory variables are national income, consumption and retail sales to explain transactions demand, and interest rates, tax rates and household wealth to explain hoarding demand. The interest rate is thereby considered as the opportunity cost of hoarding currency. Other explanatory variables usually account for (structural) changes in the payment environment that influence currency demand, such as the introduction of a new means of payment, or increased use of ATMs.

Alternative models are considered by Fase (1981*a*), Cramer (1983), Manski and Goldin (1987) and Kohli (1988). Fase (1981*a*) finds that ARIMA-models forecast seasonal fluctuations in currency demand particularly accurate, while the regression model is a valuable forecasting tool for annual or medium-term prediction. Cramer (1983) assumes that the demand of the public for denominations reflects the needs of efficient payments. He defines an efficient payment as a payment involving a minimum number of coins and bank notes, including overpayment and change. Cramer (1983) simulates the average frequency of use for all denominations in the Dutch range in 1981, by computing the efficient payment schemes for a range of guilder amounts. The number of denominations in circulation, corrected for their frequency of use, then appears to decline as the face value of the denomination increases. Cramer argues that this relation can be regarded as a demand function with a negative elasticity. The Cramer (1983) article provides useful insights

also for the approach we take in this thesis, so we will return to this paper in more detail in later chapters.

Manski and Goldin (1987) argue that a high-inflation setting where the denominational range changes more often than in low-inflation settings, requires alternative demand functions for currency. Their model entails denomination-specific demand functions based on the concept that the number of tokens in circulation is equal to the total expected use of a denomination in cash payments divided by its velocity of circulation. The general assumption that the aggregate demand for denominations reflect their need in cash payments, is taken from Cramer (1983). The expected use is specified as the product of a function indicating the expected use for a certain transaction amount and a density function expressing the distribution of transaction amounts, integrated over all possible amounts and multiplied by the total number of cash transactions. Both functions are specified according to chosen functional forms, after which the demand functions are turned into econometric models. The resulting model is flexible to changes in denominational ranges and the real value of a denomination. Manski and Goldin (1987) use aggregate time series of circulation figures, GDP and price levels in Israel to estimate the parameters of the model. We note that the chosen flexible functional form for the expected use of denominations in cash transactions is not based on any (empirically tested) theory of how denominations are used in cash payments.

Kohli (1988) takes a different approach to modeling bank note demand. He assumes that the demand for the different denominations is determined through utility maximization of the currency mix by economic agents, given their total demand for currency. Kohli (1988) then models the share of denomination-specific demand in total currency demand, rather than the demand by denomination in absolute terms. Utility is assumed to be determined by the characteristics of the currency mix, such as the physical size of denominations, their purchasing power, spread between denominations and their neighboring denominations, and interest rate weighted by denomination. In the specified system of demand functions, the characteristics parameters are equal across denominations. The estimation results for yearly data of Swiss bank notes covering the period 1950-1984 show significant parameters for the characteristics. For instance, the positive sign of the parameter of physical size indicates that the public seems to have a preference for large bills, as the share in demand increases with the size. The most important feature of Kohli's model is that it enables to predict the effect of changes in bank note characteristics, such as the demand for a denomination that does not yet exist and the effect it will have on the demand for neighboring denominations.

These studies, with the exception of Cramer (1983), only deal with bank notes. Coins have relatively little value, but their volume in circulation is usually very high. Given the difference in use between coins and bank notes, forecasting coin demand requires separate modeling, see for example Fase (1981*b*), Van der Knoop and Hooijmans (1989), Croushore and Stark (2002) and Croushore (2003). The known phenomenon associated with (small) coins, is that they get lost in circulation. This loss determines to a great extent the growth rate of total demand for coins. Goldin (1985) estimates loss rates for Israel coins. He develops a model for the probability that a coin surviving at time  $t$  was issued in year  $T$ . This model uses the distribution of the date-of-issue for coins, and includes some explanatory variables that relate to the characteristics of coins, such as real value and diameter. The parameters of the model are estimated for a sample of coins taken from circulation. The results showed that the loss rate increases with a decline in size and real value. In other words, low-denomination coins of small size have the highest loss rate. Bos (1994) applies this model to the Netherlands, and finds that the cumulative loss of Dutch guilder coins amounts to half of the number registered in circulation. This result indicates that circulation figures for coins should be handled with care, as they do not fully represent active circulation.

The second group of currency research concerns all studies of *currency hoarding*. Hoarding is generally regarded as all holdings of currency that are not intended for transaction purposes. Researchers are interested in the extent to which currency is hoarded, and the reason why individuals seem to be hoarding rather high amounts.

In an American line of studies this problem is referred to as 'the mystery of the missing currency', see Anderson (1977), Kimball (1981), Sumner (1990), Sprenkle (1993), Porter and Judson (1996). In these studies transactions demand is estimated to be less than half of the total US currency outstanding. The estimates vary from 20% to 50%. We discuss a few of the methods used. Sumner (1990) and Kimball (1981) apply the principle that transactions demand is determined by retail sales, similar to the approach taken in modeling currency demand. Based on this assumption, the proportion held for transaction purposes is estimated to be around 40%. Kimball (1981) finds that this proportion has been declining since the 1960s and is higher for large denominations. Anderson (1977) claims that only one third of total currency outstanding is in active use. This statement is based on the observed annual redemptions of worn-out bank notes and the assumption that these concern bank notes in active use at the end of their average life of  $1\frac{1}{2}$  years, like the USD 1 note. The Federal Reserve conducted household surveys in the mid-1980s to gain insight into the amount of currency in the hands of the public. The results indicated

that less than 20% of all currency was kept by households, see for example Sprenkle (1993). Haughton (1995) notes that these surveys probably underestimate currency holdings of households. Respondents of a survey are likely to underreport their holdings if part of these holdings are meant to hide wealth from the authorities, or because they are unaware of the total amount held by the household.

Although the various estimates differ, these studies all indicate that more than half of the currency outstanding is not held for transaction purposes. Various explanations for the missing currency are considered. First, large amounts of cash are indeed held by individuals as a store of value, either to hide wealth from tax authorities or because of lack of trust in banks. Second, currency is used in the so-called underground economy, for example for illegal transactions. Third, the missing currency is held by foreigners. Regarding this latter aspect, the views expressed in various studies differ. Anderson (1977) and Sumner (1990) assume that large amounts of currency are hoarded by a minority of the public, while Sprenkle (1993) and Porter and Judson (1996) argue that the larger part of the missing currency is in foreign holdings. Porter and Judson (1996) study differences between seasonal variation of demand between the US dollar and Canadian dollars, to arrive at an estimated percentage of foreign holdings of around 70%. The underlying assumptions are that domestic demand for US dollars has the same seasonal pattern as domestic demand in Canada, that there is no foreign demand for Canadian dollars, and that foreign demand has no seasonal pattern. Kimball (1981) observes that the denominational composition of cash flows in and out of the Florida Federal Reserve Bank differs from that of Federal Reserve Banks in general, indicating greater use of large denomination notes in Florida than in the US on average. As Florida is generally known as the drug center of the US, he concludes that illegal activities are affecting the demand for currency to some extent, but is careful to draw definite conclusions on the basis of these figures. In all studies mentioned, it is however commonly agreed that currency used in the underground economy could not explain more than a small part of the missing currency. Given the variety of views, and the assumptions made, it seems that the American mystery remains.

Boeschoten and Fase (1992) and Boeschoten (1992), report on several studies regarding the hoarding of bank notes. They estimate hoarding percentages of guilder bank notes in the Netherlands to be around 50%, and even 70-80% for the highest denomination (1000-guilder bank note). They use two methods to arrive at these estimates. They compare the return rates of bank notes to return rates of bank notes that are mainly used for transaction purposes. Also, they compare the average life of bank notes to a

benchmark life of non-hoarded bank notes. Boeschoten (1992) extends this analysis to 13 other countries. The highest hoarding percentages were found in the countries with the highest denominations (the Netherlands, Switzerland and Germany). In another, rather unique study, Boeschoten and Fase (1992) conduct a specially designed survey to try and locate all 1000-guilder bank notes in the Netherlands. The survey dealt with the delicate subject by asking respondents about the use of 1000-guilder bank notes in their sector as a whole, rather than their own personal holdings. Through several interview rounds with cash experts, respondents from different business sectors, and insiders in the underground economy, they obtain some interesting results. Boeschoten and Fase (1992) were able to trace 60% of the 1000-guilder bank notes outstanding. An untraceable 40% thus remained. Households held some 10% of the notes, slightly less than notes held for drug trades. The holdings of foreigners were limited to an estimated 1% of all 1000-guilder notes outstanding.

Finally, the third group of currency research we discuss, concerns studies that attempt to *evaluate the use of denominational ranges in practice*. The purpose of such an evaluation is to investigate whether the existing composition of denominations meets the need of the public, or receive indications that some changes are called for. The approaches taken to this problem are rather diverse.

Fase (1981*a*) asks various relevant questions to the public directly. In a national survey among Dutch households in 1978, the respondents were polled about their satisfaction with the bank note range in the Netherlands at that time. They were asked to give their opinion on possible withdrawals of existing bank notes, or introductions of new bank note denominations. About 80% did not want any of the bank notes withdrawn. Some 15% of the respondents found the highest 1000-guilder note to be unnecessary, and 6% would not miss the lowest 5-guilder bank note. On the other hand, some of the respondents wished to have a new bank note introduced, namely a new bank note of  $2\frac{1}{2}$  guilder (6%), a bank note of 50-guilders (6%), and a bank note of 500 guilders (3%). As we showed in Section 1.1, a new 50-guilder note was indeed issued in 1982.

Payne and Morgan (1981) introduce the D-metric approach, consisting of simple rules that may help issuing authorities to compose their currency range, see also Barry (1994) and Mushin (1998). The main variable for this approach is average day's pay ( $D$ ). The method is presented as a diagram that shows six coin slots and six note slots, and a note-coin slot in the middle. The slots represent the range of values in which a denomination should lie. For example, the 4th bank note denomination should have a value between  $\frac{1}{2}D$  and  $D$ . The nominal value of the 5th bank note denomination is between  $D$  and  $2D$ .

The lowest bank note denomination, or the highest coin denomination should occupy the note-coin boundary slot that ranges from  $\frac{D}{50}$  to  $\frac{D}{20}$ . The lowest coin denomination should not have a value lower than  $\frac{D}{5000}$  and the highest bank note denomination has a maximum value of  $5D$ . According to Payne and Morgan (1981) these rules can be used to compose a new denominational range, or to indicate what adjustments should be made to the range in response to inflation. For example, if the value of the lowest coin drops below  $\frac{D}{5000}$ , it should be withdrawn from circulation. Unfortunately, Payne and Morgan (1981) do not present a theoretical basis for their approach.

Mushin (1998), in reaction to Barry (1994), rejects the D-metric model on the basis of several arguments. For instance, he states that payment habits have an impact on denominational structures, and differ between countries and over time. This factor is not considered in the D-metric approach. Also, he expresses his doubts about the note-coin boundary in the model. Mushin (1998) correctly notes that costs and durability of bank notes and coins are the main considerations for issuing authorities to choose between notes or coins. He furthermore adds that the public generally has a preference for either bank notes or coins. These are factors of influence in determining the note-coin boundary, but in both cases  $D$  is irrelevant.

Hentsch (1973, 1983, 1985) develops a benchmarking method to evaluate denominational ranges in use with circulation figures by denomination. He discovers that the total value of bank notes and coins in circulation appears to obey a simple rule. Under this empirically found law, the number of notes and coins in circulation is proportional to the square root of their respective face value. In a graphical representation, Hentsch relates actual circulation numbers to the numbers calculated according to this rule. Differences between the two corresponding lines are helpful to reveal anomalies in the currency range. For example, relatively high circulations might indicate that a new denomination should be introduced. Relatively low levels may result if a denomination is not accepted by the public. The proposed diagram can be used to compare currency ranges across countries, or currency ranges in one country over time. Van Hove and Vuchelen (1996) argue that the number of notes and coins outstanding should be corrected for hoarding before applying Hentsch's method. Their argument is that currency is meant to provide for cash payments and Hentsch's method only evaluates the currency system on this basis. They suggest to use the estimation methods provided by Anderson (1977) and Boeschoten and Fase (1992) to estimate hoarding percentages for each denomination. Their results for Belgium and France indicate that the corrected number of notes and coins outstanding better fit the number according to Hentsch's law.



Boeschoten and Fase (1989) apply a theoretical model developed by Cramer (1983) to draw conclusions on the use of the Dutch guilder range. Cramer's model has been a guiding principle in various studies. His basic assumption is that the demand of the public for denominations reflects the needs of efficient payments. He defines an efficient payment as a payment involving a minimum number of coins and bank notes, including overpayment and change. Furthermore, he develops an algorithm by which efficient payment schemes can be computed for any range of payment amounts. By means of this algorithm, Boeschoten and Fase (1989) simulate efficient payments for a sample of payment amounts resulting from household surveys in the Netherlands in 1984-1986. They conclude that a higher proportion of large denominations is in actual circulation in the Netherlands than proportionally used for cash payments according to the Cramer principle. This is conceived to be a confirmation of the use of large denomination for hoarding. Furthermore, they analyze the effects of non-efficient payment behavior on aggregate demand by means of simulation. They compute the number and value of notes and coins needed if all payments in the sample were paid in exact amounts (no change) and if all payments were made with the next higher denomination. The results indicate that the number of notes and coins used for inefficient payments increases by 30% to 70% in comparison to those needed for efficient payments. Their value, however, decreases in case of exact payments. Boeschoten and Fase (1989) present another interesting scenario analysis when computing the number and value of bank notes used in efficient payments, while changing the bank note range. This theoretical exercise indicates that the number of notes and coins needed in efficient cash payments decreases if the 25-guilder and 250-guilder note were replaced by a 20-guilder and 200-guilder note. They finally conclude that the Dutch denominational range in the late 1980s seemed to meet the payment needs of households rather effectively, and only a transition from the  $1-2\frac{1}{2}-5$  range to a more regularly found  $1-2-5$  range would be an improvement.

In the context of denominational ranges, two other studies are worth mentioning. Chen (1976), and in a continuation of this article Chen and Tsaur (1983), analyze the macroeconomic effects of changes in the denominational structure. In both articles, it is concluded that if the denominational structure improves, the price level will decrease. Differently stated, the absence of a desirable denomination in the currency mix offered by issuing authorities, is inflationary. Their line of reasoning is the following. An improvement of the currency mix has an efficiency effect, that is, the cost of using currency decreases because less notes and coins are needed. On the other hand, there is an opposite substitution effect. If the denominational range improves, currency becomes more attrac-

tive and its demand will increase. They argue that the substitution effect is stronger than the efficiency effect if the denominational range is extended with a desirable denomination. Reversely, if the denominational range is lacking a desirable denomination, currency becomes less attractive, and people will decrease their money balances. Chen (1976) concludes that the absence of a desirable denomination would be inflationary as this effect is stronger than the effect that more notes and coins are needed to hold that amount.

To summarize this section, the following three issues have been empirically analyzed to a great extent. The first issue concerns the modeling and forecasting of aggregate demand for currency denominations. Secondly, studies attempt to estimate currency hoarding, especially where it concerns high-valued bank notes. Third, methods are presented to evaluate denominational ranges in use, with the purpose of determining whether they still meet the needs of the public.

In most studies, intensive use has been made of the available data on aggregate demand for bank notes and coins. Other sources of data are familiar macroeconomic variables such as retail sales and interest rates, or central bank statistics such as the issuance and receipts of currency. In some cases, household surveys have been conducted to gain information on the use of currency by the public. It can be concluded that, so far, empirical currency research has predominantly entailed analysis of currency use at the aggregate level, involving macro data.

From the existing literature we can draw the following conclusions. A significant part of currency outstanding is apparently not held for transaction purposes. Hence, information on actual currency use for cash transactions must be sought at the individual level, as aggregate level information does not provide conclusive statements. Furthermore, the theoretical concepts that have been used regarding individual cash payments, have never been tested empirically. At present there is no empirical study on how individuals handle cash payments. We consider this as an omission. Indeed, the way cash payments are done determines to a great extent the functioning of the currency system.

The research we present in this thesis therefore focuses on the empirical analysis of currency use at the individual level.

### **1.3 Outline of this thesis**

Our aim is to contribute to empirical currency research by analyzing the use of currency in cash payments. From such analysis we want to gain insights into the use of individual

denominations in order to draw conclusions on the overall performance of a currency range in practice.

Our research subject is currency use in the Netherlands in 1998-2003. During this period, the Dutch public used two different denominational ranges. That is, guilder notes and coins were used before the introduction of euro cash in 2002. The euro range follows the commonly applied 1-2-5 range perfectly, while the guilder range was based on the less common  $1-2\frac{1}{2}$ -5 range. This transition allows us to compare the use of both currency ranges.

The research presented in subsequent chapters of this thesis, is also captured in four papers. These papers are Kippers, Nierop, Paap and Franses (2003), Kippers and Franses (2003*a,b*) and Franses and Kippers (2003). For the composition of this thesis, the relevant text has been rearranged and at times slightly rewritten with the purpose of presenting a coherent structure. At times, the discussion therefore extends the papers mentioned above.

The outline of this thesis is as follows. Chapter 2 discusses theoretical issues concerning denominational structures and individual cash payments. The theoretical conclusions drawn in this chapter are used in subsequent chapters that describe the empirical analysis of cash payments and currency use. Firstly, we highlight the ongoing discussion on the theoretically optimal range of denominations. As far as we know, no single solution has been presented to this mathematical optimization problem. In particular, we discuss whether this discussion can shed light on the question if the 1-2-5 range is better than the  $1-2\frac{1}{2}$ -5 range. Secondly, we answer this question by calculating the efficiency of both systems, using the theoretical concept of efficient cash payments, developed by Cramer (1983). We also use this concept to measure the theoretical effects on efficiency in cash payments in case some of the euro denominations would be left out. The underlying notion is that the current euro range contains more coin and bank note denominations than the former guilder range. In the continuance of Chapter 2, we move from the restrictive theory of efficient cash payments to a more general approach to cash payment behavior. We use concepts from behavioral science and marketing research to develop a theoretical framework for cash payment behavior in which a cash payment is considered to be a choice process from the perspective of the paying party. By theoretically analyzing the payment process in detail, we are able to further specify the research question we want to answer through empirical analysis in the sequel of this thesis.

In Chapter 3 we develop a statistical model that describes the probability that an individual chooses a combination of denominations when s/he faces a cash payment. We

take into account the importance of the payment amount and the wallet content in the payment process, which follows from the theoretical analysis of Chapter 2. Furthermore, we show how this model can be estimated, and how its results can be interpreted in light of the research question posed.

Chapter 4 presents the first empirical analysis of cash payments, using a data set of observed cash transactions in the Dutch retail sector in 1998. First, we test whether actual cash payments were made according to the theoretical principle of efficient behavior, as is generally assumed in currency research. Second, we estimate the parameters of our cash payment model. As the available data of cash payments do not include the corresponding wallet contents, we utilize a second data set of wallet contents obtained through an email survey. A graphical representation of the estimation results is used to draw inference on the use of denominations. We find that a certain preference ranking was present for Dutch guilder denominations in cash transactions, indicating that the guilder range was suboptimally used.

The results of a second empirical analysis of cash payments in euro are presented in Chapter 5. As the data were especially collected for analysis using our model, special attention was paid to the registration of the wallets that matched the observed cash payments at two different retail stores in the Netherlands. The estimation results are analyzed with the use of likelihood ratio testing. We find that the preference ranking we spotted for guilder denominations, cannot be detected for euro denominations in cash payments in the Netherlands.

Chapter 6 then presents the results of observed cash payment behavior in an experimental setting that simulates euro payment situations. The experiments give us the opportunity to study the effect of changes in the euro bank note range on payment behavior. We conduct six different experiments. The first experiment contains all bank note denominations, while in subsequent experiments each time one bank note denomination is removed. A comparison of the estimation results of our model for each of the experiments indicates that payment behavior can become more aberrant if certain denominations are removed from the euro bank note range.

Chapter 7 summarizes the research presented in this thesis, and draws some final conclusions on the use of denominations in cash payments. Furthermore, we discuss several opportunities we see for future empirical research into currency use.



# Chapter 2

## Theoretical issues

In this chapter we discuss theoretical issues that are relevant to the empirical research presented in this thesis.

### 2.1 Optimal range of denominations

Although bank notes and coins have been issued for centuries, the question what denominations make up the optimal currency range is still topical, evidenced by a recent line of articles that aims to answer this question theoretically. The evolution of currency systems is a research subject of its own, and would include studying historical and cultural developments as these seem to have influenced currency systems to a great extent, see for example Tschoegl (2001). Obviously this is beyond the scope of this thesis. We are rather interested in the currency systems we observe today, that are generally based on the 1-2-5 structure, like the euro bank notes and notes introduced in 2002. Recent articles do not present a clear solution on the optimal range of denominations. Still, the 1-2-5 structure commonly applied to currency worldwide, is generally accepted as the optimal one. This suggests that the current euro range is more suitable than the former guilder range. If so, then the literature on this subject should provide the necessary theoretical grounds. We shortly present the relevant literature in this section. See also Van Hove (2001) for an excellent overview.

Two approaches are followed in solving the problem of the optimal range of denominations. The first approach optimizes the currency range according to the “principle of least effort” (Caianiello, Scarpetta and Simoncelli (1982), Sumner (1993), and Van Hove and Heyndels (1996)) That is, in their view, the optimal range of denominations results in cash payments in which, on average, the smallest number of tokens are involved.

The second approach starts from the idea that the number of denominations should be minimized (Telser (1995), Wynne (1997) and Tschoegl (1997)). The smaller the number of denominations in a range, the lower the issuing costs for issuing authorities, and the easier it is for the public to recognize the different notes and coins for cash transactions.

The above two optimizing criteria, that is, the “principle of least effort” and the limitation of denominations, work in opposite directions. By means of some simple counting exercises with different denominational ranges, Hentsch (1973, 1975) shows that the higher the density of the range, the less bank notes and coins are needed to make a payment. For clarification we present an example taken from Caianiello et al. (1982). Suppose that for each occurring payment amount a denomination would exist with the same face value, then no more than one bank note or coin would be needed for payments. Although efficient in this sense, anyone could see it would be impossible to develop, carry and handle so many different denominations. At the other end of the spectrum, we put the situation where only one denomination is issued, e.g., a bank note with unit value 1. On average 50 bank notes would be needed to make payments between 1 and 100 units. As this example shows, issuing authorities face a trade-off when deciding on a denominational structure.

Caianiello et al. (1982) show that a currency range is optimal according to the “principle of least effort”, if the values of subsequent denominations are spaced apart by a constant factor. They also demonstrate that the number of notes and coins needed in payments grows with an increasing spacing factor. As the spacing factor cannot be 1, a spacing factor 2 would be most convenient. However, for a currency range to be compatible with the decimal system, it is suggested that the spacing factor should be a power of 10. Then the values 1, 10, 100 and so forth are always included in the currency range. If the power is equal to  $\frac{1}{2}$ , then an extra denomination is added between two consecutive 10-powers. If the power is equal to  $\frac{1}{3}$ , then 2 values are added between 1 and 10. In this case the range would be 1 - 2.2 - 4.6 - 10 - 21 - ... They conclude that the Italian lira range, with its 1-2-5 structure, corresponds well to a spacing factor of the cube root of 10 and also fits the condition that a currency range should not contain too many denominations.

Sumner (1993) argues that the spacing factor between subsequent denominations should be equal to 3. As opposed to Caianiello et al. (1982), he allows for overpayment and change. Sumner’s statement is not supported by Van Hove and Heyndels (1996). They reject Sumner’s proof. Furthermore, by means of simulation, using the concept of efficient payments introduced by Cramer (1983), they compare the number of notes and coins exchanged for the two currency systems. They clearly show that a currency range with spacing factor 2 outperforms the currency range with spacing factor 3 according to

the "principle of least effort". As the denominational mix should be compatible with the decimal system, and the number of denominations should be limited, they regard the 1-2-4-8-... system to be inconvenient. The 1-2-5 range, or the  $1-2\frac{1}{2}$ -5 range, in their view are acceptable alternatives.

Telser (1995), followed by Wynne (1997) and Tschoegl (1997), draws a parallel between the problem of finding the optimal range of denominations and the optimal set of weights. In other words, what is the smallest number of standard weights needed for weighting any quantity within a given interval on a two-pan balance. This is also known as the "problem of Bachet", a French mathematician. As the standard weights correspond to denominations, the number of denominations is minimized in this problem, rather than the number of tokens exchanged. Telser (1995) finds that the solution is a range with denominations spaced apart by a factor 2 in case of exact payment (standard weights in one pan), and 3 in case of overpayment (standard weights in both pans). As overpayment should be allowed for, Telser finds that the optimal range of denominations has a spacing factor of three, that is, 1-3-9-27-...

In a reaction to all previous studies, Van Hove (2001) then concludes that finding the optimal range of denominations should be considered as a multicriteria optimization problem. In Van Hove's view the "principle of least effort" should still be given greater weight as a criterion, but he agrees with Telser that the number of denominations should be limited. He furthermore adds the requirement that the denominational range should be compatible with the decimal system for easy mental calculations. Against this background, Van Hove also finds the 1-2-5 range to be an adequate compromise. He further argues that the "problem of Bachet" is not applicable to currency systems. As Van Hove (2001) argues, in the weighting problem only one standard weight is used, while in cash transactions one can use multiple units of one denomination. Although the analogy is appreciated, the problems unfortunately cannot be compared.

Starting from the idea that the "principle of least effort" should be the main optimizing criterion and assuming that the decimal system should be applied for easy mental calculation, the 1-2-5 range is not the single optimum. Like Van Hove and Heyndels (1996) and also Mushin (1998), we argue that the  $1-2\frac{1}{2}$ -5 range fits these criteria equally well. So it cannot simply be concluded that the euro range, that fits the 1-2-5 structure perfectly, is theoretically better than the former guilder range with its  $1-2\frac{1}{2}$ -5 structure. In the next section, we evaluate both ranges on the basis of the "principle of least effort" to see which of the two is theoretically better.



## 2.2 Efficiency in cash payments

Boeschoten and Fase (1989) already concluded that the Dutch guilder range was efficient to some extent, but that a transition to the more common 1-2-5 range would give additional benefits. This suggests that the structure of the euro range could decrease the number of notes and coins needed in everyday cash payments as compared to the less common 1-2 $\frac{1}{2}$ -5 range that was applied in the guilder era.

The guilder bank notes 1000, 250, 100, 50, 25 and 10 were replaced by the euro bank notes 500, 200, 100, 50, 20, 10 and 5 which are rather similar in value (EUR 1 = NLG 2.20371), except for the 200-euro note. This bank note is new, as the guilder range did not include any bank note with a comparable value. Furthermore, the euro coins consist of denominations 2, 1, 0.50, 0.20, 0.10, 0.05, 0.02 and 0.01. This range involves two more coins than the guilder range used to have, which were 5, 2.5, 1, 0.25, 0.10 and 0.05 guilders. The 20-eurocent coin is new as it amounts to about 50 guilder cents, as well as the 1-eurocent coin, which is about 2 guilder cents. It may be that the re-introduction of a 1-cent coin, though its value in euros is higher than 1 guilder cent, could cause an increase in the use of coins in cash payments as all amounts to be paid are no longer rounded off to 5 cent amounts (at least not at the time of writing this thesis).

Given the changes in value and in the amount of coins and notes, it is of interest to see what the implications are of the transition from guilders to euros. In this section we address this question by examining if the new euro range is more efficient than the old guilder range. Like Boeschoten and Fase (1989) and Van Hove and Heyndels (1996), we make use of the theoretical concept of efficient payments introduced in Cramer (1983). See Table 2.1 for an overview of both currency ranges.

### 2.2.1 Cramer's model

Cramer (1983) introduces the concept of efficient payments. If individuals behaved according to this principle, each amount would be paid such that the number of notes and coins exchanged is minimized. In practice, it is unlikely that all payments are efficient because individuals might not behave according to the "principle of least effort", nor will they all have the necessary denominations in their wallet. However, the strength of the Cramer model is that it provides a simple way to illustrate basic differences between denominational ranges as it can be applied to any denominational range.

All payment schemes that minimize the number of notes and coins exchanged are efficient payments. Each amount has one or more efficient payment schemes, as can be

Table 2.1: Denominations in circulation in the Netherlands in 2001 and 2002

	NLG in 2001	EUR in 2002 <sup>a</sup>
banknote denominations	1,000	500
		200
	250	100
	100	50
	50	25
	25	10
	10	5
coin denominations	5	2
	2.5	1
	1	0.50
		0.20
	0.25	0.10
	0.10	0.05
	0.05	0.02
		0.01

<sup>a</sup> Comparable values are those with either factor 2 or 2.5 (conversion rate is EUR 1 = NLG 2.20371).

seen from the following example. The amount EUR 11.30 has three efficient payment schemes in which 4 tokens (euro bank notes and coins combined) are used. Any other combination will need more than 4 tokens to make up the amount EUR 11.30.

- 10, 1, 0.20 and 0.10 make an exact payment
- 10, 1 and 0.50 sum up to 11.50, and 0.20 is given as change
- 10 and 2 sum up to 12.00, and 0.50 and 0.20 are given as change

Cramer (1983) formulates efficient payments in mathematical terms as the solution to an optimizing problem. For completeness, we present it below.

Consider  $A$  to be the amount to be paid, and  $n(A)$  the combination of the different notes and coins used in the cash payment. If the different denominations in an arbitrary currency range are numbered as  $d = 1, \dots, D$ , then  $n(A, d)$  denotes the number of tokens of denomination  $d$  used for paying amount  $A$ . A positive  $n(A, d)$  refers to use as a payment, while a negative  $n(A, d)$  means that the  $n$  tokens of denomination  $d$  are given as change. We denote the value of denomination  $d$  by  $v_d$ . Efficient payments  $n(A, d)$  are then the

solution to the following problem:

Minimize

$$n(A) = \sum_d |n(A, d)| \quad (2.1)$$

subject to

$$\sum_d n(A, d)v_d = A. \quad (2.2)$$

Cramer develops an algorithm for determining all efficient payment schemes for a range of payment amounts. This algorithm is described in full detail in appendix A.1.

### 2.2.2 Euro versus Dutch guilder

We use the Cramer algorithm described in appendix A.1 to generate efficient payment schemes for all amounts between NLG 0.05 and NLG 220.35 for the guilder range. For the euro range, efficient payment schemes are generated using the same amounts, converted to euro, with EUR 99.99 being the highest amount. This enables us to compare the difference between the two denominational ranges in terms of efficiency. As we limited the range of amounts to EUR 99.99 (or NLG 220.35), the highest denomination bank notes, 1000 guilder and 500 and 200 euro bank notes, are excluded from this exercise. We chose this limit as we want to focus on the most commonly used bank notes and coins in cash payments.

In comparing the efficient payment schemes for different currency ranges, two aspects of efficiency can be distinguished. First, one can adopt the notion that the smaller the number of tokens exchanged on average, the more efficient is the range. Second, we can assume that the more efficient payment schemes exist for a certain amount, the more opportunities individuals have to make an efficient payment. Otherwise stated, the higher the probability that an efficient payment is made, the more efficient is the range. If we look at all efficient payment schemes and at the number of tokens used in each payment, we combine these two aspects of efficiency. In other words, we assume that the more efficient payment schemes there are with a small amount of tokens, the higher the efficiency of the range.

Table 2.2 shows the relevant statistics for all payment schemes for the guilder and euro range. The euro range provides more efficient payment schemes, 16,151 versus 14,928 for the guilder range. For some euro amounts, EUR 33.33 for example, 54 different efficient payments exist. This maximum is 30 efficient payment schemes for guilder amounts in the considered range, such as NLG 166.65. For both ranges, however, on average 5 tokens

are needed to pay an amount efficiently, with a slight efficiency advantage for guilder range. Differences between the averages of tokens exchanged per payment scheme are also negligible. This is confirmed by the equality of median, minimum and maximum. Overall, if we interpret these statistics for all efficient payment schemes as the combined aspects of efficiency for a denominational range, the payment process seems to have improved nor deteriorated in the Netherlands with the introduction of the euro.

Table 2.2: Statistics on all efficient payment schemes for amounts between NLG 0.05 and NLG 100.00

	guilder range	euro range <sup>a</sup>
Payment schemes		
number	14,928	16,151
maximum	30	54
Tokens used for each scheme		
average	5.61	5.84
minimum	1	1
median	6	6
maximum	8	8
Tokens used for each amount		
average	4.92	5.05

<sup>a</sup> Amounts in guilder are converted to amounts in euro (EUR 1 = NLG 2.20371).

### 2.2.3 Changing coin denominations

As mentioned, the euro range contains more coins than the guilder range, and especially the contribution of the coins of 1- and 2-cent to the payment efficiency is unclear. In Finland all amounts to be paid in cash are rounded to the nearest multiple of EUR 0.05. Although the 1 and 2 euro cent are still legal tender in Finland, their need is abolished due to rounding. We can investigate the theoretical effect of the Finnish example by applying Cramer's algorithm to all amounts between EUR 0.01 and EUR 100 to two different cases. In one case (general EMU) the amounts are multiples of EUR 0.01. In another case (Finland), the amounts are multiples of EUR 0.05, thus starting with the amount EUR 0.05. This second exercise will show the effect of abolishing the need for 1 and 2 euro cent coins on efficiency.

The results shown in Table 2.3 are quite striking. The number of tokens used for paying an arbitrary amount, decreases from 5.05 to 4.25 on average. Although there are much less efficient payment schemes, the improvement in efficiency is distinctly shown by the differences regarding the number of tokens exchanged in each payment scheme, in which both aspects of efficiency are combined. On average, the average number of tokens per payment scheme decreases from 5.83 to 4.93. Also, the maximum number of required tokens decreases from 8 to 7. This theoretical exercise tells us that payments can be done considerably more efficiently without the 1-cent and 2-cent coins.

Table 2.3: Statistics on all efficient payment schemes for amounts between EUR 0.01 and EUR 100.00

	all euro denominations <sup>a</sup>	without 1 and 2 cents <sup>b</sup>
Payment schemes		
number	36,591	5,957
maximum	54	36
Tokens used for each scheme		
average	5.83	4.93
minimum	1	1
median	6	5
maximum	8	7
Tokens used for each payment		
average	5.05	4.25

<sup>a</sup> Amounts are multiples of EUR 0.01.

<sup>b</sup> Amounts are multiples of EUR 0.05.

This means that in case the 1 and 2 euro cent coins are eliminated from the Dutch payment system, the payments would theoretically be done much more efficiently with euros than before with guilders. The question is whether this improvement results from the transition to a 1-2-5 structure. As we stated before, the higher the density of a denominational range, the lower the average number of tokens needed in cash payments. The 1-2 $\frac{1}{2}$ -5 structure of the Dutch guilder was not complete as the range lacked a 50-cent coin. To be conclusive on this issue, we repeat the exercise done above with the Dutch guilder range, but now add a hypothetical 50-cent coin. Then we compare the efficiency of the euro range without 1 and 2 cents with the efficiency of a guilder range with a 50-cent coin added. In fact, this is a direct comparison of the efficiency of a 1-2-5 range and a 1-2 $\frac{1}{2}$ -5 range as the ranges are similar in number of denominations, but differ only

in structure. So, this exercise enables us to make a final judgement on which of the two ranges is superior in terms of efficiency. For both ranges the efficient payment schemes are generated for amounts between 0.05 and 100 units. Note that we disregard the difference in value between 1 euro and 1 guilder, but rather concentrate on the difference in denominational structure. Table 2.4 presents the results.

Table 2.4: Statistics on all efficient payment schemes for amounts between EUR 0.05 and 100.00

	1-2-5 range <sup>a</sup>	1-2 $\frac{1}{2}$ -5 range <sup>b</sup>
Payment schemes		
number	5,957	6,333
maximum	36	27
Tokens used for each scheme		
average	4.93	4.98
minimum	1	1
median	5	5
maximum	7	7
Tokens used for each payment		
average	4.25	4.25

<sup>a</sup> For example, the euro range without 1 and 2 cent coins.

<sup>b</sup> For example, the guilder range with an added 50 cent coin.

The differences in statistics are minor. In fact, assuming that each payment amount has equal probability of occurring, then theoretically the exact same number of tokens is exchanged on average for both ranges, namely 4.25. So we have to conclude that the 1-2-5 and 1-2 $\frac{1}{2}$ -5 range are equally efficient, theoretically at least. These calculations confirm the conclusions we drew from the theoretical studies on optimal ranges we reviewed in Section 2.1, that both ranges have the same characteristics with regard to optimality.

### 2.2.4 Changing note denominations

The current euro bank note range also contains one denomination more than the previous guilder range. We now use the concept of efficient cash payments to analyze the effects of removing one bank note denomination from the range. We are interested in this issue for several reasons. First, as we discussed in Section 2.1, a bank note range should not contain too many different denominations. This might cause confusion, with inefficient

payment behavior as a consequence. Second, in emergency situations, it might be necessary to temporarily put one bank note denomination out of use (for example due to a counterfeiting attack, strikes, or delivery failure). With the concept of efficient payments, we can understand the theoretical effects of removing one denomination from the current euro bank note range.

The starting point for our calculations is the complete euro bank note range complemented by a lowest denomination of 1 euro. The resulting (virtual) range consists of the following denominations: 500, 200, 100, 50, 20, 10, 5 and 1 euro. Our focus is on bank note denominations, and by limiting the range to these denominations, we reduce computational efforts. We only add a 1 euro denomination for its use as the unit value. We apply Cramer's algorithm again to compute efficient payment schemes for all amounts between 1 and 1000 euro, for six different denominational ranges. The first is our basic range which includes all denominations listed above. We subsequently remove 200, 100, 50, 20 and 10 euro from the denominational range. Therefore, these five ranges have one denomination less than the basic range. Note that we do not remove the 500-euro or 5-euro bank note denomination, as they are the highest and lowest bank note denominations respectively. In this case we do not limit the calculations to amounts that include only commonly used note denominations. We chose a wider range of amounts, up to 1000 euro, to include the 200 euro note in the analysis. As compared to the guilder range, that has no bank note with comparable value, it can be regarded as an additional bank note to the range (see Table 2.1). In Chapter 6 we will come back to the results of this theoretical exercise for each of these five ranges.

Table 2.5 shows some characteristics of the resulting efficient payment schemes for each of the denominational ranges. The first row of the table shows that the number of efficient payment schemes decreases rapidly if a single bank note denomination is removed from the range. For example, in the full range, amounts can be paid efficiently with up to 18 different payment schemes, while this maximum decreases to 10 or less if a single denomination is left out of the range. This means that individuals have less opportunities to make an efficient payment. As can be expected, the average number (last row) of tokens exchanged in an arbitrary amount that is paid efficiently, increases when the denominational range becomes smaller. This effect is largest when the 200- or 20-euro note is removed from the range, with an average of 4.8 tokens required to pay an amount efficiently, in contrast to 4.5 tokens with the full range available.

If we consider the tokens exchanged per payment scheme in which both aspects of efficiency are combined, we find the differences to be small. In all cases the maximum

Table 2.5: Statistics on efficient payment schemes for all ranges. The amounts are in between 1 and 1000 euro with multiples of 1 euro

number	all	leaving out one note				
		200	100	50	20	10
Payment schemes						
number	2,553	1,766	1,873	2,034	1,862	1,964
maximum	18	6	9	8	8	10
Tokens used for each scheme						
average	5.18	5.26	5.10	5.34	5.27	5.12
median	5	5	5	5	5	5
minimum	1	1	1	1	1	1
maximum	8	8	8	8	8	8
Tokens used for each amount						
average	4.52	4.83	4.62	4.84	4.82	4.65

number of tokens used in a payment scheme is 8. The average number of tokens exchanged in a payment scheme even decreases if the 100- or 10-euro note is removed from the full range. This is explained by the fact that the reduction in efficient payment schemes, when the 100-euro or 10-euro note is removed from the range, mainly concerns those efficient payment schemes that involve many tokens (7 or 8).

We can conclude from this theoretical analysis that the removal of one bank note denomination has a negative effect on the payment system, but the effect is not as substantive as one might intuitively expect. If we compare across the different denominational ranges, we can conclude that the withdrawal of the 100-euro or 10-euro bank notes has the smallest negative effects. It will reduce the number of efficient payment schemes by 25%, and the average number of tokens exchanged per amount increases by only 2-2.5%. The 50-euro bank note, on the other hand, seems to be more important. Removing this bank note from the currency range will increase the average number of tokens exchanged per amount by 7%.

### 2.2.5 Discussion

The calculations done so far, tell us that the current euro range would become much more efficient with the abolishment of 1- en 2-cent coins from the payment system. In fact, with the 1- en 2- cent coins, the euro range has not brought any improvement in the



Netherlands in terms of efficiency as compared to the pre-euro era. By the same token, one could argue that the former guilder range would have been more efficient, had a 50 cent coin been introduced. Ultimately, we can conclude that the 1-2-5 structure, and the  $1-2\frac{1}{2}$ -5 structure do not differ significantly in terms of efficiency.

The conclusions drawn are only valid in case (i) individuals use notes and coins in an efficient way, (ii) individuals are indifferent towards use of specific notes and coins, (iii) individuals have access to all necessary notes and coins, and (iv) payment amounts are equally likely to occur.

Premise (i) and (ii) concern behavioral assumptions that have not yet been empirically tested. The concept of efficiency is an appealing model of cash payment behavior. It reflects rational behavior as time and effort are minimized. Furthermore, it provides a means to simulate the use of bank notes and coins in cash payments. However, it is unknown, whether individuals generally strive for efficiency in cash payments. This should be investigated empirically, as we will pursue in later chapters. As was noted in the discussion on optimal ranges (see Section 2.1), efficiency has a limit in the number of different denominations. If too many denominations are issued, some may fall out of use, thereby eliminating the potential efficiency gain of more denominations. In general, there might be reasons for the public to reject a denomination, or use it less than others. The public might not appreciate the size, whether too small or too large. They might prefer bank notes to coins, or vice versa. Or they simply reject the design. This lack of indifference, as we will discuss below, can be tested empirically.

Premises (iii) and (iv) are obviously false in practice. Individuals are restricted to the content of their wallet for cash payments, and it is known that the distribution of payment amounts is concentrated around small amounts. Empirical analysis of cash payments should take these limitations into account.

## 2.3 Behavioral aspects of cash payments

In the previous section we highlighted the theoretical differences in efficiency between the euro range and the former guilder range. We argued that the conclusions only hold if individuals behave according to the theory and its underlying assumptions. For various reasons it may be implausible to make these assumptions. This calls for perhaps an alternative theoretical view on payment behavior, and this is what we pursue in this section.

In this section we study behavioral aspects of cash payments to arrive at an alternative,

less restrictive, theoretical framework of individual cash payment behavior. With this framework, we are able to formulate the research question we aim to answer by means of empirical analysis at the individual level.

How does a paying individual arrive at a choice of notes and coins to make a cash payment? In our view this decision is the outcome of a choice process. And it is this view that lies at the basis of the micro-level analysis of cash payments in the sequel of this thesis. We therefore elaborate on this view in this section, enforcing arguments by using relevant research in behavioral science on consumer decision-making.

Let us take a closer look at a cash payment. It involves an exchange of cash between two parties, that is, a payer and a receiver. In case the payment amount exceeds the amount due, the payer receives change from the receiving party. From a rational point of view, one can imagine that the time and effort needed to make the payment would be minimized if the paying and receiving party cooperate in such a way that the payment involves a minimum number of bank note and coins, given the amount due. But the availability of denominations is limited, at least to the paying party. He or she may therefore not be able to make the payment that is optimal in the sense mentioned above. Also, rationality in cash payment assumes that payers will have time, opportunity, ability and motive to find the choice that will minimize the total number of notes and coins exchanged in the payment. It is unlikely that all payers make the sometimes complex calculations needed to identify the “rational” choice. Furthermore, although individuals will most likely seek to optimize the payment in some sense, the “rational” payment might not be perceived as the optimal choice at each payment occasion. For instance, while one payer might seek to pay with as many coins as possible to empty his or her wallet, the other might want to “break” a just received high-value note to receive change.

Behavioral scientists often study individual decision-making. Their theories suggest an alternative approach to rational behavior if cash payments are regarded from the viewpoint of the individual payer. We will next explore how some theories developed in behavioral science can be applied to the behavior of paying parties in cash payments. Notice that we focus on the behavioral aspect of cash payments and that we limit the analysis of cash payments to the part of the paying party. After all, the paying party has the active part of the payment, as the choice for a payment determines the change amount. It is assumed that the payer anticipates on the receiver’s behavior and includes an expectation of change. It is therefore worthwhile and sufficient to study the behavior of the paying party.

Behavioral scientists are, among other things, interested in the decision-making pro-

cess by which consumers make a choice between the alternatives that are available for a particular choice-task. The consumer engages in a process of gathering and processing information of the different alternatives and uses this information to compare the alternatives and finally choose the “best” among them, that is, the alternative that satisfies best the goals of the consumer. Consumers can select different strategies for this choice process, ranging from choosing the first alternative that satisfies a simple criterion to the comparison of each alternative against another. Several approaches have been followed to understand the consumer’s selection of a strategy. We will discuss two approaches from behavioral science that are relevant to the payer’s choice task in cash payments and are often used in consumer choice.

Bettman (1979) develops an information-processing theory of choice. The central proposition in his theory is that consumers have a limited capacity for processing incoming information and therefore simplify their choice process if the task is complex in relation to the processing capacity available. Simplification of the process is done by the use of heuristics or rules of thumb to make a choice among alternatives. The amount of simplification of a choice process can be seen to depend upon the amount of limited processing capacity the consumer allocates to the choice task and the complexity of the choice task relative to the capacity available. See for example Swait and Adamowicz (2001) for a detailed study on the influence of task complexity on consumer choice and related modeling issues.

In cost-benefit frameworks, see for example Wright (1975) and Payne (1982), consumers are assumed to make a trade-off between the effort of finding the optimal choice and the error of choosing a suboptimal choice when selecting a choice strategy. In other words, the consumer compromises between the desire to make a correct decision and the desire to minimize effort. Choosing a strategy with reduced strain, which corresponds with a simplifying strategy, can reduce effort.

Both approaches suggest that a consumer in general will simplify his or her choice process. Furthermore, certain properties of the choice task and the choice environment are discussed, which contribute to the amount of simplification of a choice process. We describe several of those aspects relevant to the process the payer engages in when choosing the notes and coins for a cash payment. We will refer to this as the payment process.

### **2.3.1 Simplification of payment process**

In behavioral science it is suggested that the extent to which the consumer simplifies the choice process, depends on choice-task and choice environment properties. We describe

these properties in the context of cash payments and find arguments to suggest that payers simplify the payment process.

A payment can be regarded as the finalizing stage of a purchasing process. It is thus a subtask of secondary importance to the payer who has a higher order goal to purchase a product or service. The payer will probably assign limited processing capacity or effort to the payment process. This is called the motivation component in Bettman's theory. Furthermore, there could be competing activities during the payment process, like putting the purchased goods on a register. There could also be events in the payment environment that draw attention away from the payment process, like promotional activity in a supermarket. Finally, time pressure could play a role of importance, for example a cashier who is waiting for the payment to be made and cues that are lining up at the register. These are choice-environment properties that decrease the amount of processing capacity, or the effort a consumer is willing or able to put into a choice process.

Payne (1982) states that the probability of an error in choice making and the size of the possible error are factors of influence to the effort chosen to make choice. Which errors could occur in a payment? One might make a "wrong" choice of notes and coins, which will lead to more coins returned as change than was originally envisaged. This type of error is likely to have relatively low significance to the payer. Therefore, he or she will put limited effort into the payment process.

In order to identify possible options to make the payment, a payer must know which notes and coins are available to him or her. If this information can be retrieved from memory, only little processing capacity or effort is needed. If not, the payer will literally have to search through wallets and pockets to retrieve the information needed. The more the payer wants to learn of the notes and coins available to him or her, the larger the effort (cost) needed.

Bettman (1979) and Payne (1982) also describe choice-task properties that will increase task-complexity, such as the number of available alternatives to the choice-task and the number of dimensions of the alternatives that need to be evaluated to make an optimal choice. In case of a payment choice, a payer has to make combinations of notes and coins and calculate the combined value to identify if they are useful for payment. In other words, is the combined value at least equal to the amount due? Furthermore, the payer will have to evaluate and compare the options by estimating the consequences they will have as a payment. For example, how many coins and notes will be returned as change? The number of options for the payment increases heavily as the number of available denominations increases. The payer will also have to make calculations to iden-

tify and evaluate each of these options. According to Bettman, consumers have limited computational skills or time, and the complexity of the task increases with computational effort needed to make a choice.

To summarize, as the payment task has a low level of importance to the payer, activities in the environment may compete for the payer's attention. The payer might feel time-pressure during the payment process, and may perceive decision errors to have low penalty. Furthermore, the payer is likely to use a limited processing capacity or effort for the choice process. Also, the information needed to make the decision could involve a time-consuming search effort and a complex computational effort. These arguments suggest that consumers most likely simplify the payment process. Next we discuss ways in which this can be done.

### **2.3.2 Consideration sets in cash payments**

Bettman (1979) describes, among others, the two-stage strategy as one way for consumers to simplify the choice process. In the initial phase, the elimination phase, the total number of available alternatives is reduced to a smaller set of alternatives. These alternatives in the reduced set are seriously evaluated in the second phase, which is called the evaluation phase. It is assumed that consumers use simple criteria to eliminate the alternatives in the first phase, while in the second phase alternatives are compared more intensively. Although Howard and Sheth (1969) do not specifically mention an elimination phase, they also state in their theory of buyer behavior that a consumer only considers a small number of alternatives relative to all goal-satisfying alternatives available. They call this the "evoked" set of alternatives. The concept of a smaller set of alternatives that is seriously evaluated, has been adopted in many studies of brand choice in marketing research. It is also called the "consideration set".

In marketing research one tends to be interested in the consideration set itself. The concept of a consideration set in consumer brand choice has some appealing implications for marketing managers in designing their marketing strategy. If consumers only consider a small number of brands, a brand can only be chosen if it is represented in the consideration set. A brand will have advantages to competitive brands if it enters into the consideration set of more consumers. Marketing managers therefore have several interests. They want to know how well the brand is represented in the consideration set. Furthermore, they want to develop a marketing strategy focused on increasing the probability of entering the consideration set or being chosen from the consideration. The first should be applied to a brand that is poorly represented in the consideration set, while the latter

is only relevant when the brand is well represented in the consideration set. The effect of promotional activity on each of these probabilities, which will probably also differ, is essential information to the marketing manager. Marketing research is generally focused on quantifying these effects by estimating a brand choice model.

The concept of consideration sets is intuitively plausible, but there is also empirical evidence to support the existence of consideration sets in brand choice. For example, the exhibit in Hauser and Wernerfelt (1990) lists consideration set sizes from published studies, and shows that the considered brands are relatively small to the total brands available in the product class. Moreover, a multistage approach to brand choice where the final choice is made from a small set of alternatives, has shown to be useful in modeling brand choice. Various studies show that incorporating consideration sets in brand choice models results in more accurate predictions of brand choice, see Shocker, Ben-Akiva, B. Boccara and Nedungadi (1991) for an overview of research in the area of consideration sets.

The above notions can be taken aboard as follows. Through insights given by behavioral science we can safely assume that individuals will simplify the payment process by making a choice among notes and coins available in order to make a cash payment. Furthermore, this process can be viewed as bearing similarities with the brand choice process in marketing research. In the next section we will explore the possibility of using the theoretical results for consumer brand choice to construct a useful theoretical framework for the payment process.

### **2.3.3 Towards a behavioral theory of currency use**

We want to construct a theoretical framework consistent with the decision-making process by which individuals choose bank notes and coins to make a cash payment. As a starting point we use the framework described by Shocker et al. (1991) for the process involved in brand choice.

According to Shocker et al. (1991), the consumer processes hierarchical or nested sets of alternatives. In this process of nesting, they define four sets of alternatives, ranging from large to small. The universal set refers to the total set of all alternatives that could be obtained by any consumer under any circumstance. The awareness set (or knowledge set) is a subset of the universal set, containing those alternatives the consumer is "aware" of and are believed to be appropriate for the consumer's goal. The consideration set evolves from the awareness set and consists of "those goal-satisfying alternatives salient or accessible on a particular occasion". The smallest set in the process is the consumer's

final choice set, a subset of the consideration set. The consideration set can change during the choice process. Shocker et al. (1991) therefore define a choice set as 'being the final consideration set, that is, the set of alternatives considered immediately prior to choice'. Most authors in academic marketing research regard these last two sets to constitute one and the same. Hence, we will also only use the term consideration set as being the final consideration set prior to choice.

Analogous to Shocker et al. (1991), we explore the idea of hierarchical sets in the payment process. We describe a process in which the payer moves from the largest to a smaller set of alternatives from which to make a final choice. As a starting point, the universal set would be the largest set of notes and coins a payer potentially brings to a payment environment. Typically, a payer carries cash in a wallet, which cannot contain an unlimited amount of bank notes and coins. This largest set is defined as the feasible set of denominations, that contains the largest number of notes and coins a payer reasonably could carry (in his wallet). The available set, as a subset of the feasible set, then contains those notes and coins that a payer actually brings to a specific payment environment. A practical example of the available set is the actual content of a payer's wallet. Note that the available set will vary with each payment. The awareness set subsequently consists of those notes and coins in the available set that the payer is aware of at the time of the cash payment. This awareness is retrieved from memory or the result of a physical search process. Even if the payer does decide to engage in such a process, not all notes and coins might be noticed or the process might not be fully executed. It can be expected that the awareness set will be smaller than the available set.

At this point, the payment process starts to deviate from the typical consumer brand choice process. A payer could choose one note or coin to serve as payment, but can also form combinations of notes and coins for payment, of which the joint value exceeds or equals the amount due. This observation introduces a change in type of elements in the hierarchical sets at this stage of the payment process. The alternatives are now defined as combinations of notes and coins. The payer will start to explore combinations of the awareness set of notes and coins. It is unlikely that all possible combinations of these notes and coins will be considered. The paying individual might not fully engage in the computational processing needed to calculate all possible combinations, especially as the processing effort increases with the number of elements in the combination.

The paying individual makes a final choice from the consideration set. Notice that this set also will contain choice alternatives that are combinations of notes and coins. The consideration set is defined to contain goal-satisfying alternatives evolving from the

awareness set. In the payment process individuals may have different goals. Breaking a high denominational note, emptying the wallet by paying with many coins, minimizing the total number of tokens exchanged, minimize change received, are all examples of such goals. The goals may vary by individual or even by payment.

The payer might engage in extra processing to change the consideration set if it is not satisfactory. This might occur if no combination meets the criteria of entering the consideration process, or if events occur that change the consideration criteria. If so, the payer could (a) engage in a new search process to expand the awareness set of denominations, (b) expand the awareness set of combinations by extra computational effort to form combinations of denominations, or (c) reconsider the elements of the awareness set of combinations. The described framework for the payment process is schematically summarized in Figure 2.1. We intend to model the payment process, and although we will not be able to model the full framework described above, our model will be a sufficiently good approximation to answer the central research question of this thesis we arrive to in the next section.

## 2.4 Central research question

In the section above we discussed a theoretical framework for the choice process an individual engages in to make a cash payment. This framework is based on findings from behavioral science and consumer brand choice research. One could argue why so much attention is paid to the payment process. After all, payments are very common, seem to be made almost automatically, and usually take no more than a few seconds. By zooming in on the payment process, as we did in Section 2.3, we have arrived at the following two conclusions.

Firstly, an individual probably forms a consideration set of notes and coins before making a payment. This is an important observation in light of the theoretical discussions of the optimal denominational structure (see Section 2.1) and efficiency of currency ranges. It could mean that there is a possibility that denominations are not considered for payment, even though they might be the most useful ones to include in a payment. We are not (yet) interested in the reasons why, nor do we intend to test the theoretical framework as presented above. This would be too ambitious. At this point in time, we are merely interested in the question whether some denominations in general are less present in the consideration set than others, and therefore are less likely to be selected for payment. From the theory we explored, we can assume that this event can happen. This



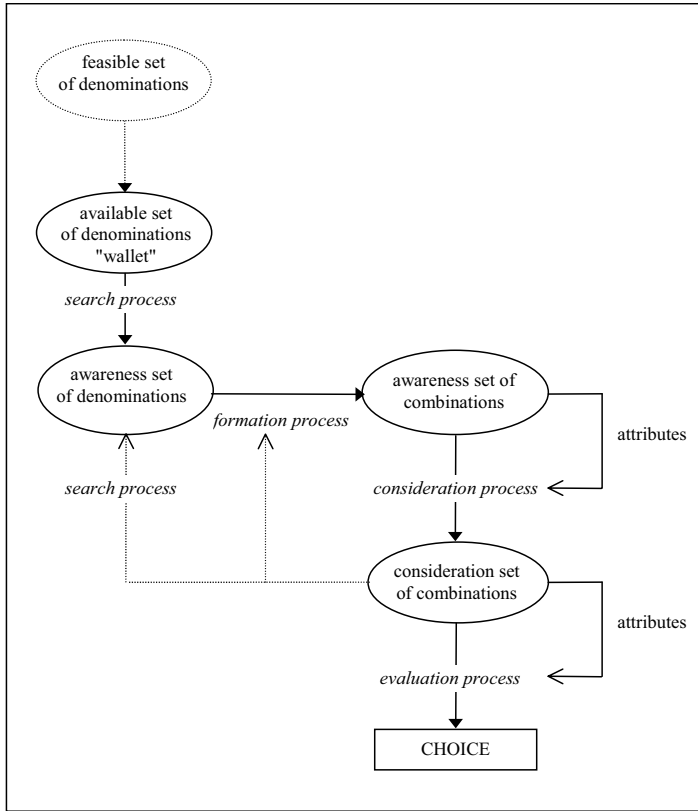


Figure 2.1: Theoretical framework for cash payment behavior.

implies that one of the basic assumptions of efficient behavior, namely that individuals are indifferent towards the use of specific coins and notes, might not hold. Also, we concluded in Chapter 1 by stating that individual cash payment behavior should be studied in order to evaluate the benefit of denominations in use. The concept of consideration sets supports this conclusion. Central banks have a natural interest to issue a currency range that ensures a most efficient payment system. The prerequisite for such a payment system to be efficient, is that the public uses the range of denominations to its full potential. This is not the case if one or more of the denominations fall out of use or become unpopular. Thus, the minimum requirement for an efficient payment system is that all

denominations are equally considered for payment. Or differently stated, individuals are indifferent towards the use of specific notes and coins. We aim to search for a means to analyze deviations from this requirement, as this would be a sign that the denominational range concerned is not utilized optimally.

Secondly, from the theoretical analysis we can conclude that the wallet content and the payment amount influence the payment process to a great extent. In an empirical analysis of individual cash payments, we need to include these crucial factors of influence.

To summarize, the research question we intend to answer in this thesis can thus be specified as follows. How can we determine whether all included denominations are considered equally for cash payments? Otherwise stated, by what means can potential preferences for denominations be identified?

To this end, we first develop a statistical model that could be used in case we would have actual transactions data, see Chapter 3. Next, we will apply this model to various data sets concerning the Dutch guilder and the euro in Chapters 4 through 6.



# Chapter 3

## An econometric model

We intend to study individual cash payment behavior. In Section 2.3 we analyzed cash payments as a choice process from the viewpoint of the paying individual. This theoretical analysis suggests that individuals consider only a selected set of the available notes and coins for payment. It would be of interest to examine whether some notes or coins are more or less considered than others in otherwise comparable situations. We formulated a research question accordingly. Even if one had observations on cash payments, such an examination of the use of denominations should go beyond descriptive statistics. As we outlined in Section 2.3 a payment can be seen as a choice process, in which the outcome is dependent on several payment-specific factors. This suggests a modeling approach. From a simple count of the denominations used, it cannot be seen if all denominations are equally relevant for the payment amounts concerned. Nor does it take into the account that the wallet of the paying individual does not always contain the relevant denominations. A model for cash payments should therefore allow for the specific circumstances at the time of payment. This chapter outlines an econometric model that is consistent with the characteristics of cash payments. In developing our model we do not account for consideration sets specifically, although we analyzed in Section 2.3 that paying individuals are likely to limit their choice set in making payments. We rather focus on the general idea that denominations might not be equally considered for payment. We aim at specifying a model that allows for the identification of preferences for certain denominations. For reasons of convenience, we adopt a structure that assumes a sequential choice by denomination, which turns out to function well in describing cash payments. This model lies at the basis of three empirical studies of cash payments. These studies are described in full detail in Chapters 4 to 6.

In this chapter we first outline relevant preliminaries. Next, in Section 3.2, we put

together the statistical model. Finally, in Sections 3.3 and 3.4 we discuss parameter estimation and interpretation of estimation results. In building this model, we assume that all relevant data are available. In Chapters 4 through 6 we apply the model and discuss the issue of data collection.

### 3.1 Preliminaries

The focus of this study is on individual cash payments from the viewpoint of a payer  $i$ . In this sense the payer faces a choice problem. A certain amount  $A_i$  has to be paid and the paying individual disposes of a set of bank notes and coins in his wallet. Which of these bank notes and/or coins will he or she select to fulfill the payment? We aim at modeling this choice.

The choice is described by the  $D$ -dimensional discrete random variable  $Y_i$ , where  $D$  corresponds to the number of denominations in a range. This random variable describes the number and combination of coins and bank notes selected by the individual  $i$  to pay amount  $A_i$ . The realizations of this random variable are denoted by  $y_i$ . Our goal is to describe the probability process for  $Y_i$ .

The possible values that the random variable  $Y_i$  can take depend of course on the content of the wallet of individual  $i$ . Let  $W_i$  be a similar  $D$ -dimensional random variable describing the contents of the wallet of payer  $i$ . That is,  $W_i$  represents the number of notes and coins that are available to the paying individual at the moment of payment. Suppose for now that the value of the random variable  $W_i$  is known and denoted by  $w_i$ . We aim at modeling the probability of outcome  $y_i$  if amount  $A_i$  has to be paid, given the content of the wallet  $w_i$ , that is,

$$\Pr [Y_i = y_i | w_i]. \quad (3.1)$$

The probability model describing  $Y_i$  has to fulfill certain requirements. For example, it is not possible that payer  $i$  pays with denominations which are not in the wallet. This imposes the restriction on  $Y_i$  that  $Y_{d,i}$  is smaller than or equal to  $w_{d,i}$  for each denomination  $d = 1, \dots, D$ . Furthermore, we have to impose that the monetary value of  $Y_i$  has to be larger than or equal to  $A_i$ . The conditions for an appropriate probability model can be summarized as follows:

- (a) The value of  $Y_{d,i} \leq w_{d,i}$  for  $d = 1, \dots, D$ .
- (b) The probability of choosing  $y_i$  when the monetary value of  $y_i$  is lower than  $A_i$  is zero.

- (c) The probability of choosing  $y_i$  when elements of  $y_i$  are expected to be returned as change is zero.
- (d) If there is only one possibility of paying amount  $A_i$  given  $w_i$ , then the probability of choosing this  $y_i$  is 1.
- (e) The probabilities of all feasible values of  $Y_i$  given  $w_i$  sum up to 1.

We present an example for illustration. Assume a virtual denominational range consisting of bank notes with nominal values (100, 25, 10, 5). Furthermore, assume that the paying individual has one bank note of 25, 10 and 5 in his wallet, that is,  $w_i = (0,1,1,1)$ , and that an amount of 15 has to be paid. In this situation, the paying individual has the following options that are consistent with conditions (a) and (b), that is, the wallet contains the relevant bank notes and the value is higher than or equal to 15.

- (0,1,0,0) with value 25
- (0,1,1,0) with value 35
- (0,1,1,1) with value 40
- (0,0,1,1) with value 15
- (0,1,0,1) with value 30

From this list it is obvious that the individual will not choose (0,1,1,0), (0,1,1,1) or (0,1,0,1) as these options include “unnecessary” notes, consistent with condition (c). That is, when paying with (0,1,1,0) for amount 15, the note of value 10 will also be returned as change. A payment of (0,1,0,0) will suffice. In this example the only remaining payment options are (0,1,0,0) and (0,0,1,1). The paying individual will choose either one or the other, consistent with condition (e). If the wallet had not contained a bank note of 25, that is  $w_i = (0,0,1,1)$ , then only one payment option would remain, that is, (0,0,1,1). The payer would have no choice and use this combination, as stated by condition (d).

Given these conditions for an appropriate probability model for cash payments, we have to conclude that standard discrete choice models do not apply. As a denominational range usually consists of 12 or more denominations, possible outcomes of  $Y_i$  can easily total up to a large number. Also, each payment amount  $A_i$  and each wallet  $w_i$  generates a different choice set for the payment choice. It would therefore be inconceivable to assign a probability to each possible outcome of  $Y_i$ .

We therefore suggest to decompose  $\Pr[Y_i = y_i | w_i]$  into a sequence of conditional probabilities describing the probability of the individual elements of  $Y_i$ . For each of the conditional probabilities we can formulate an appropriate probability model. We use the

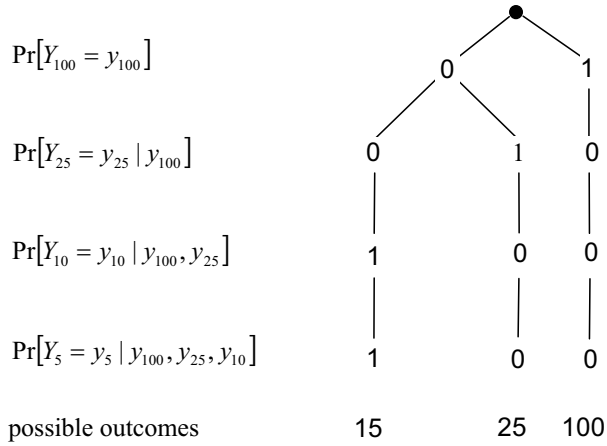


Figure 3.1: Example of nesting structure for probability model of cash payments.

following decomposition to allow for the interdependency of the different elements of  $Y_i$

$$\Pr[Y_i = y_i | w_i] = \Pr[Y_{D,i} = y_{D,i} | w_i] \Pr[Y_{D-1,i} = y_{D-1,i} | y_{D,i}, w_i] \Pr[Y_{D-2,i} = y_{D-2,i} | y_{D,i}, y_{D-1,i}, w_i, ] \cdots \Pr[Y_{1,i} = y_{1,i} | y_{D,i}, \dots, y_{2,i}, w_i]. \quad (3.2)$$

Thus, we start to model  $Y_{D,i}$  given  $w_i$ . Next, we model  $Y_{D-1,i}$  given  $w_i$  and  $y_{D,i}$ , and so forth. This modeling structure can be viewed as a sequential choice process denomination by denomination. That is, an individual first selects the number of bank notes of the highest denomination for payment, given the content of his or her wallet. Then he or she moves on to the next denomination and chooses how many of these to include in the payment, given the remaining content of the wallet. This process continues until all denominations are considered, from high to low.

Using this decomposition, it is easier to incorporate the restrictions on the  $Y_i$  variable and on the probabilities. There are perhaps other options, but the option chosen seems appropriate for the purpose of the research presented in this thesis. The approach taken is illustrated in Figure 3.1, based on the example given above.

The (virtual) denominational range consists of four bank notes with nominal values 100, 25, 10 and 5 and the amount to be paid is 15. But now, we assume that the paying individual has one bank note of each in his or her wallet, that is,  $w_i = (1, 1, 1, 1)$ . The

nesting structure as depicted in Figure 3.1 would satisfy the above mentioned conditions and result in three possible outcomes.

If the bank note of 100 is already chosen by the individual, the probability of choosing 25, 10 and 5 is zero as 100 already exceeds the payment amount 15. If neither bank note 100 nor 25 is chosen, the probability of choosing both 10 and 5 is 1 as this is the only remaining option to form the payment amount.

## 3.2 The statistical model

The next step of our modeling strategy concerns the specification of the individual probabilities in (3.2). Let us first consider modeling  $Y_{D,i}|w_i$ . The maximum number of notes of type  $D$  that can be used for paying  $A_i$  is  $w_{D,i}$ , consistent with condition (a) for an appropriate payment model. However, as condition (c) states, bank notes and coins will not be chosen if they are expected to be returned as change. Thus, if the nominal value of the  $d$ -th denomination is denoted by  $v_d$ , the number of notes of type  $D$  an individual can use for paying  $A_i$  is bounded from above by

$$ub_{D,i} = \min\left(\text{ceil}\left(\frac{A_i}{v_D}\right), w_{D,i}\right). \quad (3.3)$$

Although the natural lower bound for  $Y_{D,i}$  would be zero, it might be needed to use at least one note of type  $D$  to be able to meet condition (b). For example, consider the situation where an individual has to pay an amount of 130, and his wallet contains three notes, two with a nominal value of 100 and the other with a nominal value of 50. Then the individual has no choice but to pay at least with one note of 100 to be able to make the payment. In general, an individual  $i$  has to pay with  $k$  denominations  $D$  if the difference between  $A_i$  and the monetary value of the denominations  $D - 1$  to 1 is more than  $k$  times the monetary value of note  $D$ . Hence, the choice range of individual  $i$  is bounded from below by

$$lb_{D,i} = \max\left(\text{ceil}\left(\frac{A_i - \text{amount}(w_{1,i}, \dots, w_{D-1,i})}{v_D}\right), 0\right), \quad (3.4)$$

where the function  $\text{amount}(x_p, \dots, x_q)$  is defined as the monetary value of the denominations  $x_p$  through  $x_q$ , that is,

$$\text{amount}(x_p, \dots, x_q) = \sum_{k=p}^q v_k x_k. \quad (3.5)$$



For denominations  $D - 1$  through 1, choice ranges apply in a similar manner. To construct the upper bound and lower bounds for denomination  $d$ , we have to correct for the monetary values of  $y_{D,i}$  to  $y_{d+1,i}$  as we condition on these values. The upper and lower bounds are given by

$$lb_{d,i} = \max\left(\text{ceil}\left(\frac{A_i - \text{amount}(y_{d+1,i}, \dots, y_{D,i}) - \text{amount}(w_{1,i}, \dots, w_{d-1,i})}{v_d}\right), 0\right) \quad (3.6)$$

and

$$ub_{d,i} = \min\left(\text{ceil}\left(\frac{A_i - \text{amount}(y_{d+1,i}, \dots, y_{D,i})}{v_d}\right), w_{d,i}\right) \quad (3.7)$$

for  $d = D - 1, \dots, 1$ .

Given the values of the upper bounds and lower bounds on each element of  $Y_i$  we have to specify probabilities for the choice options of the individuals. We can distinguish two cases. If  $lb_{d,i} = ub_{d,i}$ , then the individual has no freedom of choice and hence

$$\Pr[Y_{d,i} = y_{d,i} = lb_{d,i} = ub_{d,i} | y_{d+1,i}, \dots, y_{D,i}, w_i] = 1. \quad (3.8)$$

If  $lb_{d,i} < ub_{d,i}$ , we assume that  $Y_{d,i}$  given  $y_{d+1,i}, \dots, y_{D,i}$  and  $w_i$  is truncated Poisson distributed on the region  $[lb_{d,i}, ub_{d,i}]$  denoted by

$$Y_{d,i} | y_{d+1,i}, \dots, y_{D,i}, w_i \sim \text{POI}(\exp(x'_{d,i}\beta_d)) \times I[lb_{d,i}, ub_{d,i}], \quad (3.9)$$

where  $\beta_d$  is a parameter vector and  $x_{d,i}$  contains explanatory variables for denomination  $d$ . Hence, values of  $Y_{d,i}$  outside the permitted region  $[lb_{d,i}, ub_{d,i}]$  correspond to zero probability as they violate one or more of the conditions (a)-(e). In sum, the conditional probabilities are defined as

$$\Pr[Y_{d,i} = y_{d,i} | y_{D,i}, \dots, y_{d+1,i}, w_i] = \begin{cases} \frac{\exp(-\exp(x'_{d,i}\beta_d)) \exp(x'_{d,i}\beta_d)^{y_{d,i}}}{c_{d,i} y_{d,i}!} & \text{if } lb_{d,i} \leq y_{d,i} \leq ub_{d,i} \\ 0 & \text{otherwise,} \end{cases} \quad (3.10)$$

for  $d = 1, \dots, D$ , where  $c_{d,i}$  corrects for the truncation and is given by

$$c_{d,i} = \sum_{z=lb_{d,i}}^{ub_{d,i}} \frac{\exp(-\exp(x'_{d,i}\beta_d)) \exp(x'_{d,i}\beta_d)^z}{z!}. \quad (3.11)$$

The main explanatory value in (3.9) is expected to be given by the payment amount.

In our nested approach we define  $ACORR$  as the amount to be paid minus the value of the payments chosen for higher denominations, scaled to its face value (hence CORR of correction), that is,

$$ACORR_{D,i} = \ln \left( \frac{A_i}{v_D} \right)$$

$$ACORR_{d,i} = \ln \left( \frac{A_i - \text{amount}(y_{D,i}, \dots, y_{d+1,i})}{v_d} \right) \text{ for } d = 1, \dots, D-1. \quad (3.12)$$

This variable can be seen as a scaled distance measure between the amount  $A_i$  and the amount already paid with higher denominations. If the value of  $ACORR_{d,i}$  is high, this distance is large. The scaling with  $v_d$  and the correction for the monetary value of the higher denominations allows us to compare the effect of the values of  $ACORR$  across denominations. Additionally, we include a constant. In a way this constant can be seen as a basic level of the expected value of  $Y_d$ . Consider the situation where the amount to be paid is equal to the nominal value of the  $d$ -th denomination, that is, when  $ACORR$  equals 0. The following holds. The expected value for  $Y_d$  equals 0.5 when the constant is 0. The more the constant is below 0, the closer the expected value for  $Y_d$  will be to the lower bound 0. The more the constant is above 0, the more the expected value of  $Y_d$  will lie in the neighborhood of the upper bound 1.

### 3.3 Parameter estimation

The likelihood function of our model equals the product of the choice probabilities over the  $I$  individuals, that is,

$$L(y; \beta) = \prod_{i=1}^I \sum_{w_m \in W_i} \Pr [Y_{D,i} = y_{D,i} | w_m] \Pr [Y_{D-1,i} = y_{D-1,i} | y_{D,i}, w_m]$$

$$\dots \Pr [Y_{1,i} = y_{1,i} | y_{D,i}, \dots, y_{2,i}, w_m], \quad (3.13)$$

where  $\beta = (\beta_1, \dots, \beta_D)$  and  $y = (y_1, \dots, y_I)$ . The parameters of this likelihood function can be estimated using standard optimization algorithms. Under the usual regularity conditions, the ML estimator  $\hat{\beta}$  is asymptotically normal distributed with as mean the true value of  $\beta$  and covariance matrix equal to the inverse of the information matrix.

It should be noted that parameters for the lowest denomination, that is,  $d = 1$ , cannot be estimated due to our model specification, which imposes that the lower and upper bound are always equal for the lowest denomination. Note furthermore that our model, due to its hierarchical structure, cannot use so-called added payments for estimation. These are payments in which small change is added to the payment to simplify the change returned by the cashier. As it mainly concerns the use of small change, it is not considered the central issue of this research.

We have written a program in Gauss for parameter estimation. However, one can also use Eviews. As a courtesy to the reader, we give the Eviews code in Appendix B.

### 3.4 Interpretation

In this section we discuss how estimation results of the model presented in this chapter can be used to draw inference.

The parameter estimates of  $ACORR_d$  are expected to be significant and positive for all denominations, except the lowest denomination for which no parameters can be estimated. A positive parameter indicates that an increase in the payment amount will lead to an increase in the expected number of denominations  $d$  chosen.

We use an example of guilder payments to illustrate the logic of this. Let us consider two situations. In the first an amount of NLG 155.00 has to be paid and the paying individual has already used a bank note of NLG 100 to pay for this amount.  $ACORR$  would be equal to 0.09. In the second situation an amount of NLG 195.00 has to be paid, and again the paying individual has already used a bank note of NLG 100.  $ACORR$  would take on a higher value of 0.64. The paying individual would have to pay a remaining amount of NLG 55.00 in the first case and a remaining amount of NLG 95.00 in the second case, with all the denominations lower than NLG 100 that are available to him or her. Given that the paying individual has the choice of paying with up to 2 bank notes of NLG 50, then a positive sign of the parameters of  $ACORR$  indicates that the paying individual would be more inclined to pay with two bank notes of NLG 50 in the second situation than in the first.

Ultimately we aim to test whether preferences can be observed in the use of certain denominations, or otherwise stated, if indifference towards denominations in cash payments can be tested. We argued at the start of this chapter that one could only investigate possible preferences for denominations in cash payments by means of statistical modeling. This is because the use of denominations should be studied under equal payment

circumstances for proper comparison. This is exactly what our cash payment model does. We assume that the payment circumstances are determined by the wallet content and the payment amount. In our model these are represented by the truncation and the value of explanatory variable  $ACORR$ . As  $ACORR_d$  is scaled to the face value of denomination  $d$ , its value can be directly compared across denominations. Therefore, payment circumstances are equal when truncation and  $ACORR$  are equal, and differences in use of denominations under equal circumstances are thus reflected in differences in estimated parameter values. This means that we are interested in the equality of parameters across denominations, rather than the values themselves.

To formally test if there are differences in preference, we can use a likelihood ratio (LR) test. Indifference implies that intercept parameters are the same across denominations, and also that the  $ACORR$  parameters are the same across denominations. This is the null hypothesis of the LR-test. Two models are compared, one with the restriction that the parameters are equal across denominations, and a second with unrestricted parameters. The LR statistic is defined as  $-2\ln(\frac{L_R}{L_{UR}})$ , where  $L_{UR}$  represents the likelihood of the unrestricted model and  $L_R$  represents the likelihood of the restricted model. It follows a chi-square distribution with the number of restrictions as degrees of freedom. Rejection of the null hypothesis suggests that the parameters significantly differ across denominations. In that case, there is no indifference towards denominations by individuals, and certain denominations are (more or less) preferred. See also Appendix B for the relevant Eviews code.

The distributional result of the LR-test is verified using simulation techniques. Starting point for our simulations is the set of pure efficient payment schemes for amounts 0.01 to 100 euro as generated using the Cramer algorithm as described in Appendix A.1. Pure efficient payments, in the Cramer sense, are by definition based only on the face value of denominations and therefore constitute examples of payments with indifference towards denominations. If the cash payment model is estimated using a data set of pure efficient payments, the LR test of equal parameters across denominations should follow the asymptotic distribution. The following simulation scheme is executed.

**Step 1** For all payments, matching wallets are generated by assuming that the wallet contains the payment, with three tokens of each denomination added such that the wallet content is large enough to allow a wide choice range.

**Step 2** A sample of size 1,000 is randomly drawn. The cash payment model is estimated with the restriction the parameters  $\beta_d$  are equal across six denominations, that is, 1 through 50 euro, as well as without restrictions. The resulting likelihood

values for the restricted and unrestricted models are used for a LR test with a null hypothesis of equal parameters across the relevant denominations, which is assumed to be chi-square distributed with 10 degrees of freedom (5 intercepts and 5 ACORR parameters).

Step 2 is repeated 200 times. The resulting empirical size turns out to be 6%, which is rather close to the nominal size of 5%. Given this finding, we can safely rely on the asymptotic chi-square distribution under the null hypothesis.

Next, we examine the power of this test, also to assess what the sample size of a data set would need to be to conclude on significant differences between parameters across denominations. To estimate the power of the LR test for small samples, we also use simulation techniques.

The starting point for this simulation is again the set of pure efficient payment schemes for amounts 0.01 to 100 euro as generated using the Cramer algorithm, as described in Appendix A.1. The following simulation scheme is executed to examine the power of the LR test for small samples, that is,

**Step 1** For a fraction  $\alpha$  of these schemes, payments with one token of denomination  $d$  are replaced by payment with two tokens of denomination  $d + 1$ , provided that denomination  $d$  has a face value of two times the denomination  $d + 1$ .

**Step 2** For all payments, matching wallets are generated by assuming that the wallet contains the payment, with three tokens of each denomination added.

**Step 3** A sample size of  $n$  is randomly drawn. The cash payment model parameters are estimated with the restriction that the parameters are equal across denominations 1 through 50 euro, as well as unrestricted. The resulting likelihood values are used for the LR test.

**Step 4** Step 2 is repeated  $N$  times. The resulting percentage of rejected tests measures the empirical power for sample size  $n$ .

By the replacement of payments, as executed in step 1 of the simulation scheme, we introduce a preference in the data. We use a replacement of a 20-euro note by two 10-euro notes for simulating a preference for the 10-euro note. We find that when  $n$  is set to 75, the empirical power of the LR-test already increases rapidly with larger replacement rates, as can be seen from the simulation results presented in Table 3.1. That is, with this sample size a strong preference can be detected by means of the LR-test.

Table 3.1: Empirical power of LR test for varying replacement rates

Replacement rate <sup>a</sup>	25%	50%	75%
Rejection frequency	0.01	0.37	0.96
Number of replications	500	200	200

<sup>a</sup> The replacement rate represents a preference. It is defined as the percentage of efficient payments in a sample in which payment with one 20-euro note is randomly replaced with two 10-euro notes. The empirical power is defined as the percentage of runs in which the null hypothesis of equal parameters is rejected.

Now, we should say a few words about the effective sample size. It cannot be concluded that a data set of 75 observations will be sufficiently large for drawing inference on the use of denominations in cash payments. The number of observations will usually be higher than the number of effective observations with 'free choice', that is, for which the upper bound exceeds the lower bound. This is partly dependent on the wallet content. As in the simulation schemes the wallet content is chosen in such a manner it is not very restrictive, one can assume that observations of cash payments and wallets in real life will result in less effective observations. Therefore, if a data set of observations is to be used for statistical testing, it must be established that its sample size is about comparable or larger than the sample size of 75 in the simulations scheme, where it concerns observations of free choice. Then the empirical power of the LR-test is sufficiently high to draw conclusions on payment behavior.

In Chapters 4 to 6 we present applications of the model to three unique empirical studies of cash payments in the Netherlands. In each of the applications we use the estimation results to draw inference on the indifference towards denominations in cash payments.



# Chapter 4

## Empirical results I: the Dutch guilder

### 4.1 Introduction

This chapter presents the first empirical study of cash payments in the Netherlands in 1998, when bank notes were issued in 6 denominations with values 1000, 250, 100, 50, 25 or 10 guilders. Coins were issued in 6 denominations with values 5, 2.5, 1, 0.25, 0.10 and 0.05 guilders. The analysis centers around two data sets. These data sets are described in full detail in Section 4.2. We use descriptive techniques and statistical modeling to analyze these data sets. In Section 4.3 we examine if individuals indeed have a tendency to pay efficiently, one of the basic premises on individual cash payment behavior in economic literature. We subsequently move on to a more elaborate analysis of cash payment behavior, by estimating the parameters of our cash payment model. Section 4.4 discusses the specifics of estimating the model with the two data sets used. Section 4.5. shows how the estimation results can be used to determine whether some denominations in the Dutch guilder range are more or less considered for payment than others. Section 4.6 concludes.

### 4.2 Data

For this study we make use of two main data sets. The first data set consists of cash transactions. It contains a random sample of a large number of cash payments registered on the authority of De Nederlandsche Bank in 1998. The survey was conducted to gain insight into the number of bank notes and coins that are used daily in over-the-counter



cash payments. Such information was relevant for a thorough preparation of the euro changeover in the Netherlands. For this specific purpose two methods of data collection were originally considered. The first method would have minimized effort and cost. It was considered to select a representative group of citizens and ask them to keep a diary of their cash payments during a week. The second method concerned the actual observation of cash payments at the counters of a representative sample of points of sale. For reasons of precision the second method of data collection was chosen. It was feared that data set resulting from the first method would be biased towards cash payments with higher amounts. Individuals are expected to be more oblivious to smaller amounts under, say, NLG 10 (equivalent to EUR 4.54). As these amounts were considered to be equally relevant, the first method was found to be less favorable.

A representative sample of retail outlets was selected in municipal and rural areas spread over the country. Different types of retail outlets were included, ranging from supermarkets and warehouses to bakeries and shoe stores. Cash payments were registered during a week to account for daily fluctuations. Furthermore, the number of cash payments resulting from a week of surveying was expected to be sufficient to give judgement on the use of notes and coins in cash payments for each of four retail categories, that is, supermarkets, department stores, specialized food stores and non-food stores. Special permission was given by the management of 69 retail outlets to observe payments at one of their cash registers. The retail outlets were anonymized, in such a way that only the retail category was registered. The survey resulted in a data set of 40,700 payments, of which almost 35,000 were cash payments. We randomly selected 2,000 cash payments from the data set, as we expected this to constitute a large enough data set to estimate the parameters of our cash payment model. These cash payments constitute the main data set of our study. For each payment, the data set contains the payment amount, the notes and coins that were used to pay the amount, and the notes and coins received back as change in case of overpayment. Figures 4.1 and 4.2 present some characteristics of the cash payment data set.

Figure 4.1 clearly demonstrates that the distribution of retail cash payments is skewed to the right. So, as we already mentioned in previous chapters, cash is mainly used for small amounts. In fact, the cumulative percentage line indicates that in the sample over 40% of the cash payments concerned amounts lower than 10 guilders (= EUR 4.54), just below 70% had payment amounts of less than 20 guilders, and more than 93% of the payments were lower than 50 guilders (=EUR 22.69). Indeed, Figure 4.2 shows that the bank note most often used in the sample was the 10-guilder bank note.

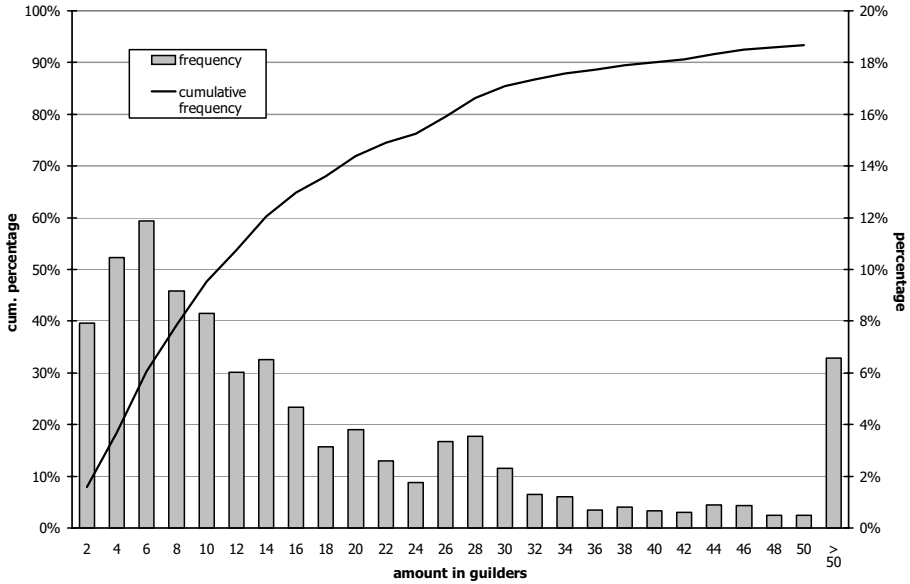


Figure 4.1: Distribution of amounts in sample of 2,000 payments in guilders

Unfortunately, no wallets were registered at the time of the cash payment, so the cash payment data set does not contain matching wallets. As we need those for the analysis we present in this chapter, we additionally collected data on individuals' wallets. Data were collected through an e-mail survey in July 2001. Out of 1,500 surveys, 840 were returned. The respondents were asked to register the number of bank notes and coins in their wallet at the time of registration. The highest bank note denominations were excluded from the survey to attain a higher response. As these are held as store of value rather than for transaction purposes, respondents could feel their privacy to be invaded, while data on these denominations were not considered to be relevant to the study anyhow. This is confirmed by their low use in retail cash payments, as is shown in Figure 4.2. Analysis of the wallet contents, in connection to individual characteristics like age and gender, showed no strong sign of underrepresentativeness. The survey resulted in a data set of 840 wallets, in the following referred to as the wallet contents data set. Figure 4.3 shows the average number of bank notes and coins in the wallets of the respondents. On average, respondents had 3 bank notes and almost 14 coins in their wallet with a total value of

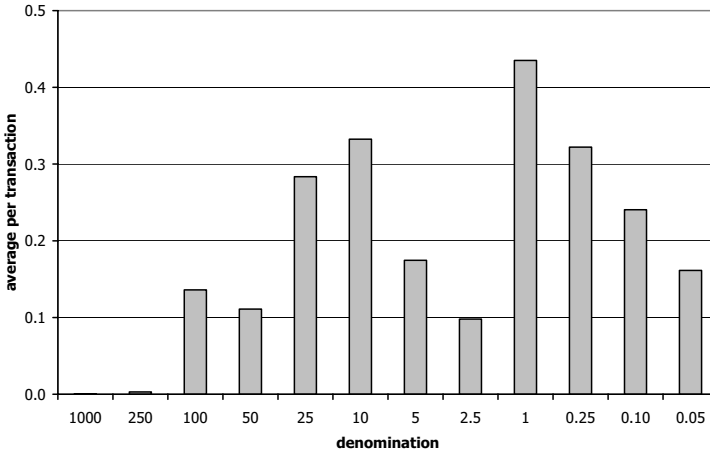


Figure 4.2: Average number of tokens used for payment in sample of 2,000 cash transactions in guilders

NLG 118.00 (= EUR 53.55).

### 4.3 Efficiency

Our cash payment data set has actual transactions, and it is tempting to put the basic premise of efficiency under scrutiny. This premise has been widely used in theoretical currency research. As we discussed in Section 2.1, various articles are concerned with deriving the optimal denominational structure as a solution to a theoretical optimizing problem in which the optimizing criterion is efficiency, or payment according to the “principle of least effort”. According to this view, the optimal currency system would allow efficient payments with the smallest number of bank notes and coins, on average. For solving this problem it is needed to assume that individuals behave accordingly. In other words, individuals, when facing cash payments, are assumed to make the necessary computations to pay an amount with the smallest number of bank notes and coins. In addition, for theoretical arguments, it is implicitly assumed that individuals would have access to all the relevant possible combinations. In Section 2.2, we used the concept of efficient payments to draw conclusions on which of two currency ranges is theoretically better. We concluded

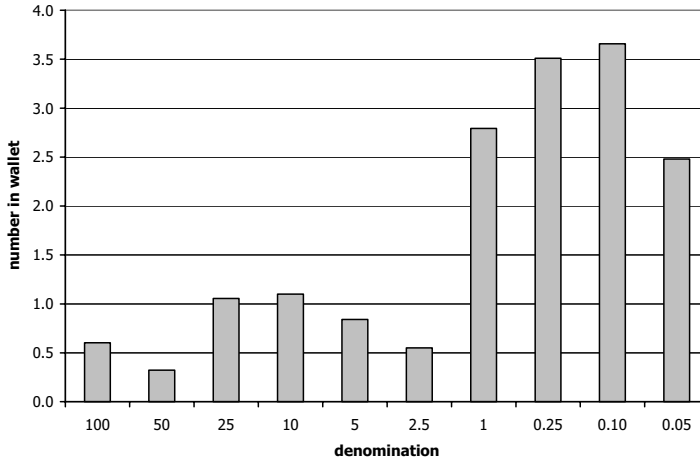


Figure 4.3: Average number of guilder banknotes and coins in sample of 840 wallets

that the new range of euro bank notes and coins would have outperformed the former guilder range, had the 1- and 2- cent euro coins not been introduced. The “principle of least effort” concurs with rational behavior and it can be assumed that individuals would at least strive for some degree of efficiency. If not, the conclusions drawn in Chapter 2 do not hold. In this section we explore payment behavior as regards to efficiency, while accounting for a limited access to notes and coins, making use of our main data sets with actual cash payments and wallet contents.

Again, the theoretical concepts introduced by Cramer (1983) are a valuable starting point in our analysis. We refer to Section 2.2.1 for a description of Cramer’s model. We aim to draw conclusions on efficient payment behavior by comparing the number of notes and coins used in actual payments to the efficient number for the relevant payment amount. Analogous to Cramer, we define efficient payments as all payment schemes in which the number of notes and coins that form a payment amount, including change, is minimized. However, we do not assume that individuals have access to all the necessary denominations. If we relax this assumption, that is, if we include the restriction of the wallet contents of the individual, the following optimization problem, which we might call a wallet-specific algorithm, results. A restricted efficient payment for a given amount  $A$  and wallet  $w$  is the solution to:

Minimize

$$n(A) = \sum_d |n(A, d)| \quad (4.1)$$

subject to

$$\begin{aligned} \sum_d n(A, d) v_d &= A \\ n(A, d) &\leq w_d \quad \forall d, \end{aligned} \quad (4.2)$$

where  $w_d$  represents the number of tokens of denomination  $d$  that is available to the individual in his or her wallet. This concept is somewhat different than the one in Cramer (1983) in the sense that restricted efficient payment schemes vary across wallets. This means that for each wallet, different efficient payment schemes hold. For that reason, we adjust the algorithm in Cramer (1983) to compute the efficient payment scheme to meet the extra restriction of the wallet. This is described in full detail in appendix A.2

Due to the complexity and time-consuming character of the algorithm, we limit ourselves to 192 cash transactions from the 2,000 transactions in the cash payment data set. We generate efficient payment schemes for these transactions, for ten different wallets each. We randomly select ten wallets that are feasible for each payment from the wallet content data set. We compare actual payments with the efficient payment schemes for each wallet and compute the difference in numbers of notes and coins used, averaged over ten wallets for each payment amount. We concentrate on the payment intention of the paying individual. If his payment scheme coincides with the efficient payment scheme for the same amount, we consider the payment to be efficient, even though inefficient change would result in a payment with more tokens exchanged than in the efficient payment. The basic idea is that the paying individual did intend to make the payment as efficiently as possible, which is exactly what we want to examine. In practice, this is achieved by correcting inefficient change in the data set prior to comparison with efficient payments. In 9% of the cases we corrected change given. For example, if the cashier returned two coins of 5 guilders, we changed it to one bank note of 10 guilders. Table 4.1 presents the frequency of differences between tokens used in actual payments, and tokens needed in efficient payments, given the content of the wallet.

Table 4.1 shows that 65 individuals, nearly 34 percent of the sample, pay with no

Table 4.1: Frequency distribution of inefficiencies defined as the number of notes and coins used more than is needed for wallet-specific efficient payment

Range	amount of individuals	cumulative amount of individuals	$p$ -value <sup>a</sup>
0.0	65	65	1.00
0.0 - 0.5	23	88	0.89
0.5 - 1.0	29	117	0.00
1.0 - 1.5	17	134	0.00
1.5 - 2.0	13	147	0.00
2.0 - 2.5	9	156	0.00
2.5 - 3.0	13	169	0.00
> 3.0	23	192	0.00

<sup>a</sup> To check whether the cumulative amount of individuals constitutes a significant fraction of the sample of 192 individuals, we use the binomial sign test. Under the null hypothesis  $H_0$ , the probability of paying efficiently is the same as paying inefficiently, that is  $\Pr[\text{efficient}] = \Pr[\text{inefficient}] = 0.5$ . The alternative hypothesis  $H_1$  states that the probability of paying efficiently is higher than the probability of paying inefficiently.

inefficiency for all 10 different wallets. To check whether this percentage constitutes a significant fraction of the sample, we use the binomial sign test. Under the null hypothesis  $H_0$  the probability of paying efficiently is the same as paying inefficiently, that is  $\Pr[\text{efficient}] = \Pr[\text{inefficient}] = 0.5$ . The alternative hypothesis  $H_1$  states that the probability of paying efficiently is higher than the probability of paying inefficiently. Under  $H_0$  the number of efficient payments has a binomial distribution with parameters  $n = 192$  (number of transactions) and  $P = 0.5$ .

For the 65 efficient transactions out of 192 individuals mentioned above, the p-value of the sign test equals 1. This tells us that this number does not constitute a significant proportion of the sample, and thus we cannot reject  $H_0$ . So, if we were to test whether individuals pay perfectly efficiently, the data tell us this is not the case. Then, the figure in the first row should have been closer to 192. When applying a less strict definition of efficiency, for example, allowing an inefficiency of 0.5 tokens, the number increases to 88 individuals, which is still insignificant. The p-value of the sign test becomes smaller than 0.05 at an inefficiency of 0.9, when 111 individuals pay with an inefficiency of 0.9 at maximum. This means that there is a significant number of individuals that use only 0.9 or less redundant denominations when making payments, taking into account the content of the wallet. So, out of at least 1 of the 10 possible wallets, an efficient payment was done.

These results can be interpreted as an indication that individuals do aim for efficiency,

and thus behave in an economically rational manner. But a large number of payments clearly is not done efficiently. In the next section we analyze payment behavior by estimating our cash payment model.

## 4.4 Inference

From the previous section it can be concluded that although efficiency explains payment behavior to some degree, it does not fully explain the choice of notes and coins in cash payments. It seems that influences other than efficiency account for the observed cash payments. Furthermore, so far we have not paid attention to differences in the use of bank note and coins, apart from their most apparent characteristic. The face value is the only feature that determines the utility of a note or coin in efficient payment schemes. The question now is whether such an indifference towards denominations, apart from their nominal value, is plausible? One would expect equal treatment of notes and coins in equal circumstances. Given the availability of the database of cash payments, we are now challenged to explore the indifference assumption further. Specifically, we have the following question: were some denominations more or less preferred than others in the Netherlands in 1998? We argued in Chapter 3, that even if one has observations of cash payments available, analysis of these data should go beyond descriptive statistics to be able to answer such a question.

We elaborate some more on this issue to make our point. If the number of times denominations are used in cash payments is simply counted, the resulting frequency of use does not lead to any meaningful conclusions. We illustrate this by regarding the frequency of use for the Dutch guilder denominations, calculated from our cash payment data set, as presented in Figure 4.2 above. From this figure it is obvious that the NLG 1000 and NLG 250 bank notes are rarely used in cash payments. As the average payment amount in the sample was NLG 18, this is not a surprising result. Note that we now only look at their use in cash transactions. As we learned from the literature on high denominations in Section 1.2, see for example Boeschoten (1992), these bank notes are used for hoarding mainly. This means, that although they are hardly used in cash transactions, their overall demand can take up a large part of the aggregate demand for bank notes, given their high value. This observation emphasizes the point we made in Chapter 1, that if one wants to examine the use of denominations in cash transactions, one should study their use at the individual level, rather than their aggregate demand. From Figure 4.2 it is also apparent that three denominations can be distinguished as having a low frequency of use

as compared to their neighboring denominations. First, the NLG 50 has a lower frequency than both NLG 100 and NLG 25. Second, in comparison with the bank note with the lowest denomination, the highest coin NLG 5 shows a low frequency, while the use of the NLG 2.50 is even lower. In all, one could conclude that three denominations seem to be less preferred from these descriptive statistics.

However, such firm conclusions cannot be drawn on the basis of these simple statistics. Different situations lead to different uses of denominations. For example, the use of a NLG 100 note is lower for a payment amount of NLG 5.00 than for NLG 90.00. Next, if the NLG 5 is better represented in the wallets of individuals than NLG 2.50, which seems to be confirmed by the average wallet content presented in Figure 4.3, it cannot be concluded that NLG 2.50 is less popular in cash payments. The individual simply has less opportunity to choose the NLG 2.50.

When such specific circumstances at the time of payment need to be taken into account for truly judging the use of denominations, one needs to conclude that this can only be done by means of statistical modeling. In this section we estimate the parameters of our cash payment model we developed in Chapter 3 to answer the question if there were any preferences for Dutch guilder denominations in cash payments in 1998.

#### 4.4.1 Unobserved wallets

In Chapter 3, we modeled the cash payment probability conditional on the content of the wallet  $w_i$ . In our main data set, however, the wallet content of the individual at the time of the transaction is unknown. Therefore, we consider the wallet content as an unobserved random variable. The probability that the wallet of individual  $i$  equals  $w_i$  is denoted by  $\Pr[W_i = w_i]$ . The probability that individual  $i$  chooses  $y_i$  unconditional on the wallet content is the sum over all possible wallet contents times the probability that this wallet was the actual wallet, that is,

$$\Pr[Y_i = y_i] = \sum_{w_m \in \mathcal{W}_i} \Pr[Y_i = y_i | w_m] \Pr[W_i = w_m], \quad (4.3)$$

where  $\mathcal{W}_i$  denotes the set of feasible wallet contents for  $y_i$  and  $\Pr[Y_i = y_i | w_m]$  is specified as in (3.9).

It is difficult to specify a plausible probability model for the wallet contents. To obtain a plausible distribution we use a representative sample of feasible wallet contents consisting of  $M$  observations, labeled as  $r_m$  for  $m = 1, \dots, M$ . We use the empirical



distribution of this aselect sample to model the distribution of  $W_i$  in the following way

$$\Pr[W_i = r_m] = \frac{1}{M} \text{ for } m = 1, \dots, M. \quad (4.4)$$

#### 4.4.2 Explanatory variables

On the basis of the data available to us, we can specify a number of explanatory variables for (3.9) additional to our main explanatory variable  $ACORR$ , as specified in (3.12), and a constant. We define dummy variables that relate to the retail facility where the cash transactions took place. The transactions are observed in four identifiable retail branches. We define  $DUM_{j,i} = 1$  if the transaction of individual  $i$  took place in facility  $j$  and 0 otherwise.

#### 4.4.3 Parameter estimation

The likelihood function of our model equals the product of the choice probabilities over the  $I$  individuals, that is,

$$L(y; \beta) = \prod_{i=1}^I \sum_{w_m \in W_i} \Pr[Y_{D,i} = y_{D,i} | w_m] \Pr[Y_{D-1,i} = y_{D-1,i} | y_{D,i}, w_m] \dots \Pr[Y_{1,i} = y_{1,i} | y_{D,i}, \dots, y_{2,i}, w_m], \quad (4.5)$$

where  $\beta = (\beta_1, \dots, \beta_D)$  and  $y = (y_1, \dots, y_I)$ . To estimate the model parameters we combine the likelihood function (3.13) with the wallet content probabilities (4.4) to the log likelihood function (4.6).

The parameters of this likelihood function can be estimated using standard optimization algorithms. For computational reasons, we average over a random sample of  $K$  instead of over all  $M$  wallets. Experiments with different values of  $K$  showed that estimation results do not change much for values of  $K \geq 10$ . Under the usual regularity conditions, the ML estimator  $\hat{\beta}$  is asymptotically normal distributed with as mean the true value of  $\beta$  and covariance matrix equal to the inverse of the information matrix. To compute the covariance matrix we use the sandwich covariance estimator, which provides heteroskedastic-consistent standard errors, see White (1980).

$$\begin{aligned}
l(y; \beta) &= \sum_{i=1}^I \ln \left( \sum_{m=1}^M \Pr [Y_{D,i} = y_{D,i} | r_m] \right. \\
&\quad \left. \dots \Pr [Y_{1,i} = y_{1,i} | y_{D,i}, \dots, y_{2,i}, r_m] \Pr [W_i = r_m] \right) \\
&= \sum_{i=1}^I \ln \left( \frac{1}{M} \sum_{m=1}^M \Pr [Y_{D,i} = y_{D,i} | r_m] \right. \\
&\quad \left. \dots \Pr [Y_{1,i} = y_{1,i} | y_{D,i}, \dots, y_{2,i}, r_m] \right) \tag{4.6}
\end{aligned}$$

## 4.5 Estimation results

We estimate the cash payment model, as specified by (3.2)-(3.9) and (4.3) and (4.4), for the data set with guilder payments and wallets. We refer to Chapter 3 for a detailed description of the cash payment model. We exclude the highest denominations NLG 1000 and NLG 250 as data on wallet contents were collected without these denominations to avoid non-response. As these denominations are not primarily used as means of payment in retail transaction, they are also of less interest to our study. A cash payment data set containing 1980 observations results for parameter estimation. From the cash payment data set we also exclude the so-called added payments. As mentioned in Section 3.3, the cash payment model cannot use these for estimation. These payments are not considered to be the central issue of this research as they mainly concern payments to which the paying individual adds small coins to simplify change given by the cashier. Furthermore, excluding these payments still leaves a sufficiently large data set of 1,617 cash payments. For each of these observations, 10 feasible wallet contents were randomly selected from the separate data set with wallet contents. We estimate two models, a basic model with only a constant and a second model including  $ACORR_d$  as explanatory variables. Comparing the two models by means of a likelihood ratio test leads to a LR-statistic of 1312 with 9 degrees of freedom. We therefore conclude that the second model represents the data better than the basic model. This confirms our expectation that the payment amount adds to the explanatory power of the model as compared to the basic Poisson model in which only a constant is included. The second model is therefore used for interpretation of our results. Its estimation results are presented in Table 4.2. The subsequent inclusion of dummy variables, indicating the retail facility where the payment took place, did not

give further improvement to the model.

Table 4.2: Parameter estimates with heteroskedastic-consistent standard errors in parentheses for the cash payment model

denomination	constant		<i>ACORR</i>	
	parameter	standard error	parameter	standard error
100	0.648 <sup>a</sup>	(0.061)	1.188 <sup>a</sup>	(0.034)
50	-0.325 <sup>a</sup>	(0.097)	0.767 <sup>a</sup>	(0.051)
25	0.341 <sup>a</sup>	(0.037)	1.268 <sup>a</sup>	(0.031)
10	0.472 <sup>a</sup>	(0.025)	1.625 <sup>a</sup>	(0.036)
5	0.148 <sup>a</sup>	(0.023)	2.043 <sup>a</sup>	(0.053)
2.5	-0.288 <sup>a</sup>	(0.039)	1.301 <sup>a</sup>	(0.050)
1	0.474 <sup>a</sup>	(0.027)	1.203 <sup>a</sup>	(0.032)
0.25	0.223 <sup>a</sup>	(0.036)	1.234 <sup>a</sup>	(0.048)
0.10	-0.090 <sup>a</sup>	(0.007)	1.613 <sup>a</sup>	(0.036)

<sup>a</sup> Significantly different from zero at the 5 per cent level.

The parameters for NLG 0.05 cannot be estimated due to our model specification, which imposes that the lower and upper bound are always equal for the lowest denomination. The parameter estimates of  $ACORR_d$  variables are significant and positive for all other denominations. A positive parameter estimate for  $ACORR$  was expected, indicating that an increase in the payment amount will lead to an increase in the expected number of denominations  $d$  chosen. We refer to Section 3.4 for elaboration.

To illustrate the effect of the  $ACORR_d$  variable on the expected value of  $Y_d$ , we plot in Figures 4.4-4.5 the expected value as a function of the  $ACORR_d$  variable for the different bank notes and coins. The lower bound is set equal to 0, while the upper bound is set according to the value of the  $ACORR_d$  variable. Note that if we assume no wallet limitations, the upper bound is a function of the  $\exp(ACORR_d)$  variable as can be seen from (3.7) and (3.12). The probability that  $Y_d$  is larger than  $\exp(ACORR_d)$  is always zero by definition. Hence, for  $\exp(ACORR_d) \leq 1$ , the upper bound  $ub_d$  equals 1, for  $1 < \exp(ACORR_d) \leq 2$  the upper bound  $ub_d$  equals 2, and so forth.

To illustrate the parameter uncertainty in the expected values, we also plot a point-wise 95% confidence interval based on the asymptotic distribution of the parameter estimates. This confidence interval is computed using simulation. For each denomination  $d$ , we simulate  $\beta_d^{(n)}$  parameter from a normal distribution with as mean  $\hat{\beta}_d$  and as covariance

matrix the estimated heteroskedastic-consistent covariance matrix for  $n = 1, \dots, N$ , see also Table 4.2. For a given value of the  $ACORR_d$  we compute the choice probabilities between the upper bounds  $ub_d$  and lower bounds 0 and construct the expected value of  $Y_d$  as follows

$$E[Y_d; \beta_d^{(n)}] = \sum_{z=0}^{ub_d} z \Pr[Y_d = z | ACORR_d], \quad (4.7)$$

where the  $ACORR_d$  summarizes  $y_{d+1}, \dots, y_D$  and we assume that there are no wallet limitations. The average value of these simulated expected values over  $n$  provide the expected value of the expectation of  $Y_d$ . The 95% confidence intervals correspond to the 2.5% and 97.5% percentile of the  $N$  simulated expected values.

Figure 4.4 shows the expected value for the bank notes. The expected values are increasing for all denominations. The jumps in the lines are due to the change in the upper bound as a result of the change in the  $ACORR_d$  variable. Similarly, Figure 4.5 shows the results for all coins. The same conclusion can be drawn from the positive curves for all denominations. In sum, an increase in  $ACORR_d$  will lead to an increase in the expected value of  $Y_d$ , for all  $ACORR_d$  and all notes.

The graphic representation of the estimation results in Figure 4.4 shows another striking result. The curve for NLG 50 lies beneath the curves of the other denominations. This difference is significant as can be shown by taking into account the 95 percent confidence intervals of the expected value, as depicted in Figure 4.6 and Figure 4.7. To make this difference even clearer, we zoom into these two denominations in Figure 4.8. The conclusion can be drawn that the NLG 50 has a lower probability for being chosen as payment than, say, NLG 100 for all values of  $ACORR$ .

At this point we return to the central question posed in the beginning of this section. To test the premise of indifference of individuals towards the use of denominations in payments, we use the estimation results presented above. The idea was that the use of denominations should be studied under equal payment circumstances for proper comparison. This is exactly what the graphic representation, as depicted in Figures 4.4 and 4.5, does. The following example may provide additional insight. We compare denominations NLG 100 and NLG 50 by choosing a reference point at the relevant curves in Figure 4.4. The following holds.

	<b>NLG 100</b>	<b>NLG 50</b>
to pay	NLG 200.00	NLG 100.00
$ACORR_d$	0.69	0.69
$E[Y_d]$	1.02	0.20

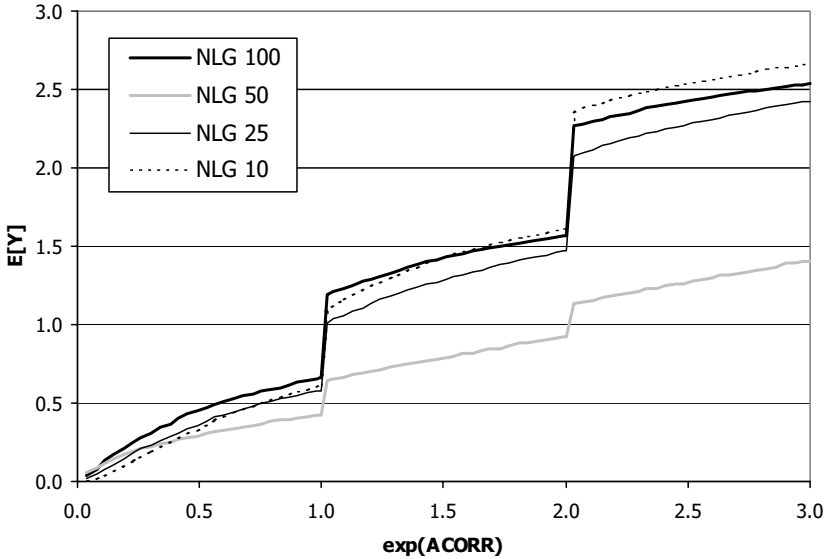


Figure 4.4: Expected value of  $Y_d$  for the banknotes (lower bound equals 0)

Suppose that two individuals are faced with a payment choice problem. The first individual has to pay a remaining amount of NLG 200 and disposes of 3 bank notes of NLG 100. As this individual's wallet contains enough bank notes and coins of lower denominations to pay for the NLG 200, the lower bound for choosing to pay with NLG 100 is equal to 0. The second individual has a remaining amount of NLG 100 to pay and needs to decide how many of his bank notes of NLG 50 to use. This individual has enough bank notes of NLG 50 to pay for the amount, but is not obliged to do so as he or she has smaller denominations in his wallet that also sum up to NLG 100. The situations of the two individuals are equal, as was required for comparison. The estimated cash payment model now provides a tool to investigate the difference in use of NLG 50 and NLG 100 under equal circumstances. The results indicate that the expected value of NLG 50 will be lower than the expected value of NLG 100.

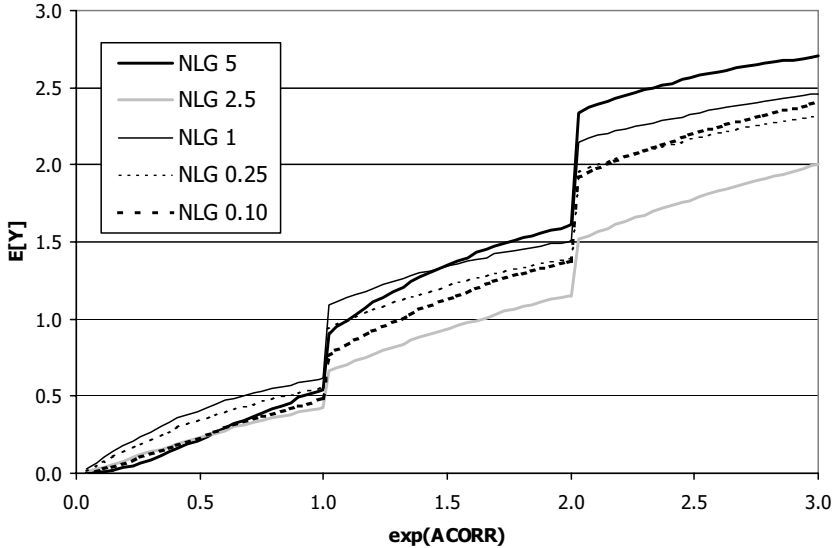


Figure 4.5: Expected value of  $Y_d$  for the coins (lower bound equals 0)

The top panel of Figure 4.4 shows that the expected value is higher for NLG 100 than for NLG 50 for all values of  $ACORR$ . The bottom panel shows a similar difference. From this analysis using the estimation results of our cash payment model it can be concluded that under equal circumstances NLG 50 is less preferred (or popular) than NLG 100 in cash payments. This contradicts the premise of indifference, commonly used in currency research. A similar kind of analysis can be done for coins. The results indicate that the  $2\frac{1}{2}$  guilder coins is less preferred than both the 5-guilder and 1-guilder coin in similar payment situations.

## 4.6 Conclusions

In this chapter we studied cash payment behavior in guilders. We specifically examined whether Dutch individuals showed preferences for one or more denominations in the cur-

rency range. For this purpose we estimated the cash payment model with a large data set of cash payments that were observed in retail outlets in 1998. To account for the absence of matching wallets in the data set, we collected an additional data set of wallet contents.

The estimation results indicate that the 50-guilder bank note was less preferred than other bank notes in similar payment situations. The same holds for the  $2\frac{1}{2}$ -guilder coin. The presented analysis does not explain these differences, but readers that are familiar with these denominations might not be too surprised with this result. The 50-guilder bank note was introduced far later than other bank note denominations and had a different design than the others. And as we learnt from 20th century history, see Section 1.1, this denomination had been introduced before but failed to be successful then. The dislike of the  $2\frac{1}{2}$ -guilder coin brings back the discussion on the optimal range of denominations, see Section 2.1. Maybe the value of  $2\frac{1}{2}$  guilders is not easy to calculate with. If so, then this result would suggest the general idea that the 1-2-5 range is best for currency.

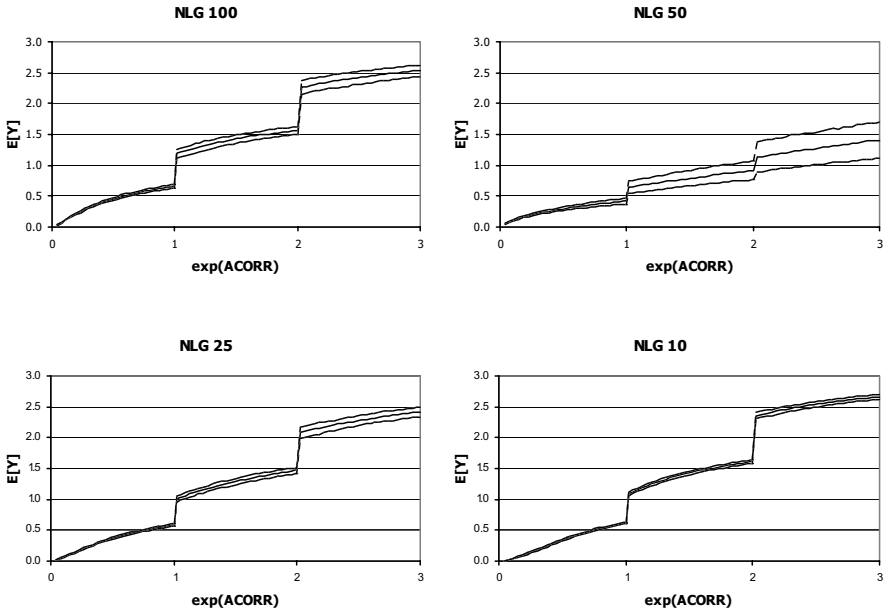


Figure 4.6: 95%-confidence intervals around  $E[Y_d]$  for each of the notes (lower bound equals 0)



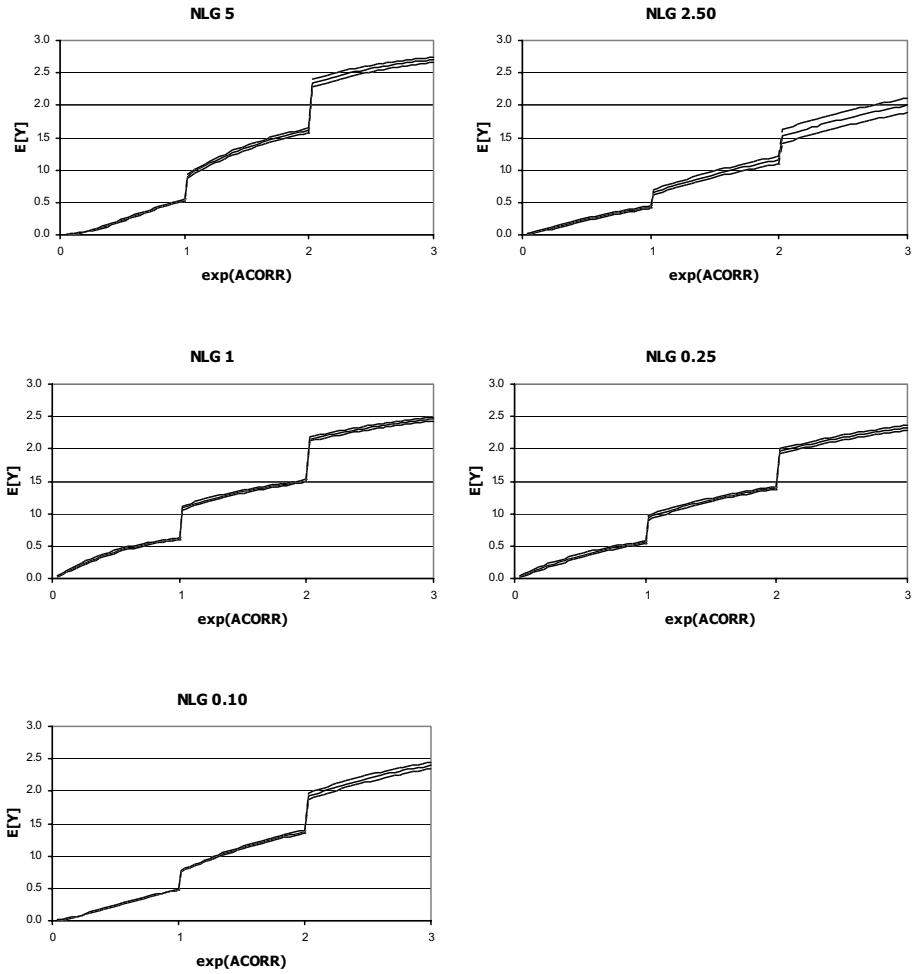


Figure 4.7: 95%-confidence intervals around  $E[Y_d]$  for each of the coins (lower bound equals 0)

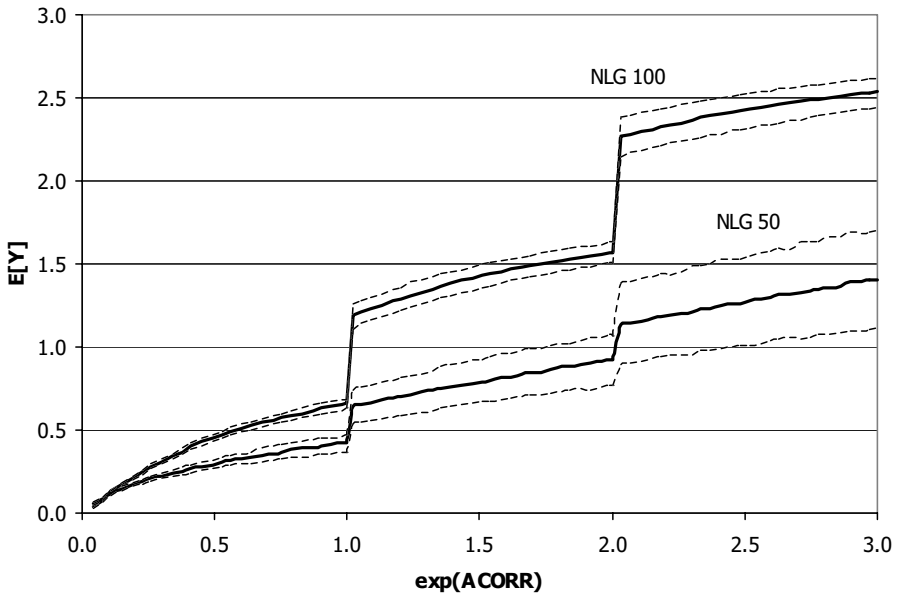


Figure 4.8:  $E[Y_{50}]$  and  $E[Y_{100}]$  and their 95%-confidence intervals (lower bound equals 0)



# Chapter 5

## Empirical results II: the Euro

### 5.1 Introduction

A second empirical analysis of cash payments is exercised, but now related to the new euro notes and coins. Euro cash consists of seven bank notes with values 500, 200, 100, 50, 20, 10 and 5, and eight coins with values 2, 1, 0.5, 0.2, 0.1, 0.05, 0.02, 0.01. This currency range contains one bank note and two coins more than the former guilder range. Furthermore, it is based on the 1-2-5 range, as opposed to the  $1-2\frac{1}{2}$ -5 range applied to guilders. In Chapter 2.2 we examined the theoretical differences between the two. It was concluded that the advent of euro cash hardly improved the payment system in the Netherlands, that is, when it is evaluated on the basis of “the principle of least effort”. In Chapter 4 we studied cash payment behavior in practice. This was the first empirical analysis of individual cash payments. A large data set of cash payments in guilders was used. Unfortunately, the data set did not include information on the wallet contents. We used a second data set of wallet contents to generate multiple feasible wallets for each payment. We examined the extent to which the Dutch public paid efficiently, and we estimated the parameters of the cash payment model. The main conclusion was that the 50-guilder note was less preferred than other denominations.

An empirical study of cash payments in euros is a natural follow-up, and this is what we pursue in this chapter. Given the changes to the denominational structure, it is of interest to study cash payment behavior again. The analysis we present here is based on a unique data set of euro cash payments that was collected by means of observation at two different retail locations in the Netherlands. As we collected this data especially, we made sure we obtained matching wallets to the payments. We also have a second, survey-based, data set, for validation of the results. The characteristics of both unique data sets are

described in Section 5.2. We study the data of euro cash payment in a two-step empirical analysis. In Section 5.3 we zoom in on the cash payments and categorize them according to a number of distinguishable payment principles that individuals seem to have applied in their choice of denominations for making cash payment. One of them is the “principle of least effort”, as was first presented by Cramer (1983) and discussed in Section 2.2. The question is whether in practice individuals also strive for efficiency in the Cramer sense with euro cash, is answered in this section. Next we move on from this preliminary exercise to a more sophisticated evaluation of the use of euro cash in payments. We estimate the parameters of the cash payment model, described in Chapter 3. The aim is to determine whether a preference ranking for denominations, as was present in guilder payments, is also recognized for certain denominations in euro payments. The relevant discussion appears in Section 5.4.

## 5.2 Data

In this section we discuss the properties of the two data sets we use for empirical analysis.

### 5.2.1 Main data set

The first data set, collected by three students through a time-consuming method, comprises a sample of cash payments that includes (i) the set of coins and bank notes in possession of the paying individual prior to the cash payment, that is, the wallet, (ii) the amount to be paid, (iii) the notes and coins selected by the payer to make the final payment, and (iv) the notes and coins returned as change. As the use of money is a delicate and private subject, an adequate method of data collection had to be chosen to minimize non-response. This concerns especially the collection of information regarding the wallet content, which, as mentioned, was not present in the data set used in Chapter 4. Furthermore, attention had to be paid to the timing of the survey in order to avoid any influence on the choice for notes and coins by the respondent.

In October 2002, the students posted at two different retail locations, a supermarket in an urban area and an appliance store (like ‘Home Depot’) in a rural area. During the cash payments, they registered the payment amount and the notes and coins used by the paying individual. Subsequently, the customer of the store that just had made the payment was asked for cooperation in the research. If so, the customer was assisted in completing a survey. In the survey the respondent was also asked for the content of his wallet. This means that wallet content was registered after the cash transaction took place.

Furthermore, the respondent was asked for other information regarding demographics (e.g. age and gender), possible personal experience as being a cashier and the weekly shopping frequency. The latter two questions were included to check for any influence on cash payment behavior. Finally, the respondents were given a cognitive test, meant to measure computational skills, again with the purpose to test the influence of these skills on a cash payment behavior. The test consisted of four series of numbers, the respondents were asked to complete. The tests were weighted according to their level of difficulty, such that a score between 0 and 10 could be assigned to each individual. Table 5.1 summarizes the main characteristics.

During four full days of posting, a total of 272 surveys were taken. On 40 occasions the customer was asked by the cashier to add a small amount to make change easier. As this does not reflect the original intended cash payment behavior by the paying individual, these observations were corrected in order to represent the original intention of the paying individual. The surveys finally resulted in a data set of 272 observations on cash payments and their matching wallets. Figure 5.1 presents the distribution of amounts, Figures 5.2 and 5.3 present the average number of bank notes and coins used for payment, and the average number of bank notes and coins in the wallet of individuals.

Table 5.1: Characteristics of respondents in main data set, where ‘Gamma’ is the appliance store and ‘Plusmarkt’ is a supermarket for FMCGs

	‘Gamma’	‘Plusmarkt’	All observations
Age (average)	47.4	49.9	48.1
Gender (1=male)	64%	16%	51%
# weekly shopping trips	3.4	3.6	3.5
Experience as a cashier? (1=yes)	22%	24%	23%
Computing skills <sup>a</sup>			
series a, % correct answers	90%	93%	91%
series b, % correct answers	62%	47%	58%
series c, % correct answers	81%	65%	77%
series d, % correct answers	46%	32%	42%
Score, computing skills	6.2	4.9	5.9

<sup>a</sup> Respondents were asked to complete the following series.

a: 2 - 4 - 6 - 8 - 10 - ...

b: 5 - 7 - 4 - 6 - 3 - ...

c: 1 - 2 - 4 - 8 - 16 - ...

d: 1 - 2 - 4 - 7 - 11 - ...

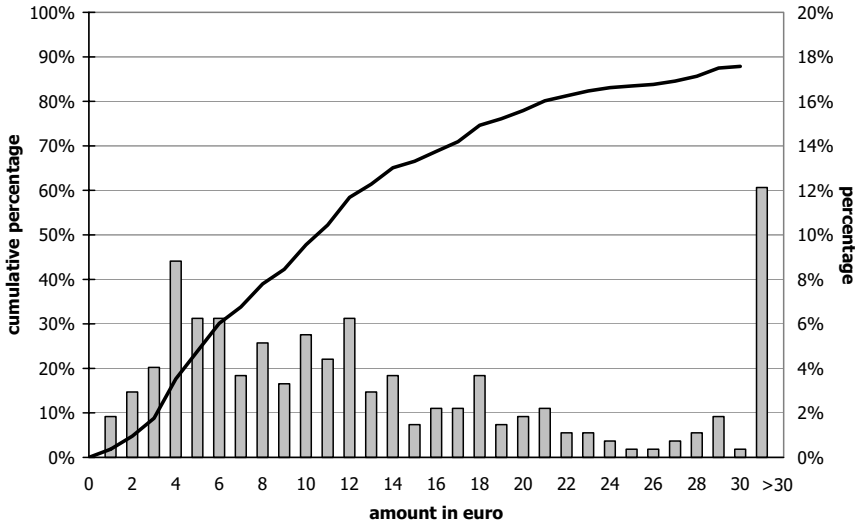


Figure 5.1: Distribution of amounts in sample of 272 cash payments in euro

### 5.2.2 Validation data set

On the authority of De Nederlandsche Bank, a survey on payment behavior was held among Dutch respondents of 12 years and older in the week of April 3 through April 9, 2003. The respondents were selected from the TNS NIPO Capi@Home panel, a large database of households who regularly participate in surveys for TNS NIPO using their personal computer at home. The survey on payment behavior was distributed among a sample of households on seven different weekdays. The response was about equally spread across the days of the week. The respondents were asked about their payment behavior the day before. Almost 1300 surveys were returned.

Additional to more general questions on payment behavior, the survey contained detailed questions on the specifics of the most recent cash payment the respondents made. About 26% of the respondents indicated not to have made any cash payment the day before. Furthermore, 1% did not remember the specifics of their last cash payments. The remaining 952 respondents were able to indicate the payment amount and the notes and coins they had used for payment, as well as the notes and coins they received as change. In order to relate the cash payments to the notes and coins they disposed of at the time

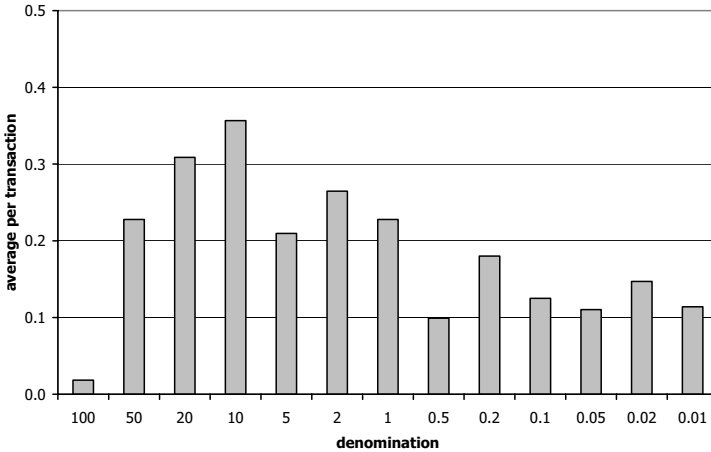


Figure 5.2: Average number of tokens used in sample of 272 cash payments in euro

of the payment, the respondents were asked to count the notes and coins in their wallet at the time of the survey. Subsequently, they were asked if they had made their recent cash payment from their wallet (and not with some loose coins or bank notes from their pocket, for example) and if their wallet content had changed between the recent cash payment and the time of the survey. From these answers we were able to construct the wallet content after payment and subsequently re-construct the wallet content before payment, in a similar way as we did for the main data set. As it turned out, 63 respondents did not pay from their wallet and 42 respondents were not willing or able to give their wallet content.

The result was a data set with all the specifics on the recent cash payment and the matching wallet contents for 847 individuals. Incorrect entries with extremes, such as unlikely high payment amounts and payments with 20 or more bank notes were removed from the data set. Ultimately, the data set contained 840 observations with information on the payment amount, the notes and coins used for payment by the paying individual and the notes and coins in their wallet prior to the payment.

As the first data set concerns actual - and not stated - payment behavior, we will use this second data set merely for validation of the results obtained with the first data set.



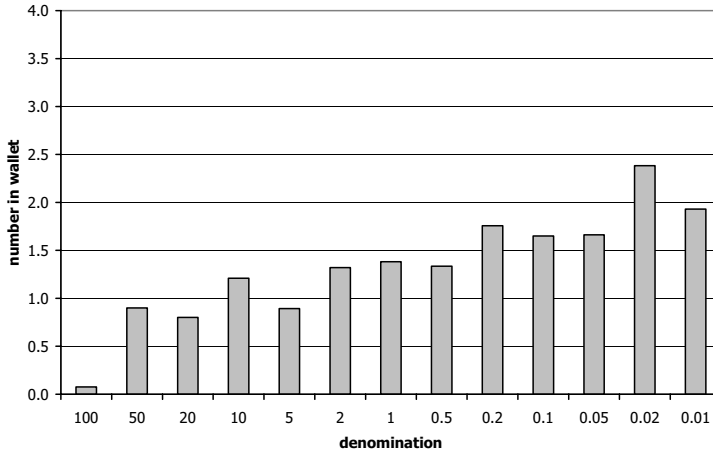


Figure 5.3: Average number of euro banknotes and coins in sample of 272 wallets

### 5.3 Efficiency

Also in a euro cash environment, we want to answer the following question. According to which principles do individuals make their cash payment choice? As we now have a data set of cash payments with matching wallets, we have the opportunity to analyze observed payment behavior in more detail. For our main data set, we search for such principles in euro payments. A starting point is again the “principle of least effort” based on the model of Cramer (1983), which we described in Section 2.2.1. This model assumes that individuals are triggered to make a payment in such a way that the smallest number of notes and coins is exchanged between payer and receiver, including change in case of overpayment.

We examine whether the “principle of least effort” has been applied for the observations in the main data set. We use the efficient payment schemes for the amounts ranging 0.01 to 100 euro, generated using Cramer’s algorithm (see Appendix A.1 and Section 2.2 for further details). We compare the payment schemes in our sample of actual payments with the efficient payments. We concentrate on the payment scheme used by the paying individual, as we are interested in the intention of this individual. Therefore, if this scheme coincides with the efficient payment scheme for the same amount, we consider the

payment to be efficient, irrelevant of change given back by the cashier. The rationale is that the paying individual did intend to make the payment as efficient as possible.

In our data set 1, the wallet content prior to each of the payments is known, and hence we are interested in the payment that was efficient, given the wallet contents. For this purpose, we again use the optimizing problem with a wallet restriction included. See Section 4.3 for further details. If this individual has used the notes and coins available to him or her as efficient as possible, the cash payment still constitutes efficient behavior, but the wallet simply did not allow for a pure efficient payment scheme in the Cramer sense. We additionally generate efficient payment schemes for each payment in our data set, given the wallet restriction and the payment amount. The algorithm used is described in full detail in Appendix A.2. The results of the comparison are presented in Table 5.2.

Table 5.2: Efficient versus non-efficient payment schemes in our cash payment dataset 1

	Efficient payments	Non-efficient payments	All payments
Wallet facilitates pure efficient payment <sup>a</sup>	80	53	133
Wallet does not facilitate pure efficient payment	85	54	139
Total	165	107	272
Percentage	61%	39%	100%

<sup>a</sup> Pure efficient payment according to Cramers definition.

The table shows some interesting results. First, 61% of the payments were done in an efficient manner. This constitutes a statistically significant share of the individuals. It can therefore be concluded that individuals are more inclined to make an efficient payment than a non-efficient payment. Second, efficient and non-efficient payments show similar results for the group of individuals with or without the appropriate wallet content to make a pure efficient payment. Statistically speaking, there is no significant difference. This means that individuals are equally inclined to make an efficient payment, irrelevant whether the wallet facilitates the pure efficient payment or not. These results indicate that individuals do seem to strive for efficiency.

Still, 39% of the individuals in our sample did not pay efficiently. Apparently, these individuals had their reasons not to employ the efficient payment. Apart from some obvious reasons, such as computational effort or time pressure, a number of alternative principles could have been applied, conscious or unconscious from the viewpoint of the

individuals. A first check we perform is to see, by using a logit model, if the characteristics in Table 5.1 have explanatory value for paying ‘yes’ or ‘no’ efficiently. The estimation results are summarized in Table 5.3.

Table 5.3: Parameter estimates with standard errors in parentheses for the logit model. Dependent variable is ‘yes’ or ‘no’ efficient payment. Regressors are characteristics of respondents.

variable	parameter	Se
constant	1.634	(0.630)
gender	-0.558	(0.276) <sup>a</sup>
# weekly shopping trips	-0.087	(0.056)
experience as cashier	-0.017	(0.331)
computing skills score	-0.025	(0.040)
LR statistic (5 df)	8.182	

<sup>a</sup> Significantly different from zero at the 5 percent level.

Except for gender, none of the covariates has a significant impact on paying efficiently, yes or no. Hence, more cognitive skills or experience as a cashier do not lead to more payment efficiency. Overall, the LR-test does not reject the joint null hypothesis that all slope coefficients except the constant are zero.

Next, one can think that individuals feel a desire to empty the coins from their wallet, or feel a necessity to receive change for a subsequent payment. If these motives were applied randomly, no pattern could be deducted. However, if there were preferences for certain denominations, there would be a structural deviation from efficient behavior. Then, one could conclude that the available range of denominations employed is sub-optimal. Before we formally test the possibility of preferences by means of our cash payment model, in Section 5.4, we first carry out a preliminary search for patterns and payment principles by examining the cash payment behavior of the 53 individuals in our sample who did not pay efficiently while their wallet did facilitate the pure efficient payment. This simple counting exercise is executed manually and mainly meant to give some detailed insight into the payment behavior of a small group of individuals. There were 22 individuals who paid with only one token, usually being a bank note. These individuals applied a principle of least effort from their point of view, or simply wanted to ‘break’ the bank note to receive change. Another 10 individuals paid with many coins and, considering their wallets, probably applied the principle of rebalancing their wallet

content. For the other 21 individuals no logical payment principle could be deducted, although 8 of them seemed to have tried to make an efficient payment. Their choice seemed efficient at first sight, but when looking at the payment schemes generated with Cramer's algorithm, it turned out that the true efficient payment needed less bank notes and coins. Given the complexity of these efficient payments, it could have been too much effort to think of the proper combinations under time pressure.

The counting exercise becomes a bit more involved when we want to assess the use of denominations as compared to others. About 35% of the non-efficient payments were paid with one denomination. One might wonder if this is the obvious denomination given the wallet and payment amount. This question cannot be answered with this small sample, and not by means of such a simple counting exercise. In the next section we therefore proceed by following our modeling approach to compare the use of denominations in a given payment situation. Our intention is to draw statistical conclusions on the use of euro denominations in the Netherlands and to assess any preferences for some notes or coins.

## 5.4 Estimation results

The analysis in the previous section involves a tedious counting exercise, while no conclusive statement can be made on the use of different denominations in relation to each other. In this section we formalize our exercise with the purpose of identifying systematic preferences or non-preferences for denominations, by estimating the parameters of our cash payment model with the data collected on euro cash payments. The model, as defined by equation (3.2) and (3.9), is described in full detail in Chapter 3. It can generally be used for any sample of cash payments with any given denominational range. In Chapter 4 it was applied to a sample of 2,000 guilder coins and notes. It was found that the Dutch public showed a preference for paying with a 100-guilder bank note over payment with a 50-guilder note in similar payment situations. In fact, the 50-guilder note was less preferred than any other bank note. The same was concluded for the  $2\frac{1}{2}$ -guilder coin in relation to other guilder coins.

In contrast to the data set of cash payments in guilder, we now have matching wallets with the observed cash payments in euro. This enables us to apply the cash payment model directly. We estimate the parameters of the cash payment model with the sample data of 272 euro cash payment using Maximum Likelihood estimation. In addition to an intercept we include the explanatory variable  $ACORR$ , defined by (3.12), in our cash

payment model. This variable is a function of the amount to be paid in relation to the face value of the relevant denomination. The scaling allows us to compare the effect of the values of *ACORR* across denominations. Added payments are again excluded from the data set, as the cash payment model cannot use these for estimation. The reduced data set contains 240 observations. The highest denominations, 200 and 500 euro, are excluded from analysis as with these bank notes hardly any payments were registered. Furthermore, given the specification of our model, parameter estimates are only based on those observations for which the upper bound in (3.7) exceeds the lower bound in (3.4) or (3.6). If the lower bound equals the upper bound, the paying individual has no freedom of choice and has to pay with the number of tokens determined by the coinciding bounds. These observations do not contribute to the likelihood. Given the hierarchical structure of our model, the number of observations in which free choice can happen decreases as the denomination lowers. Ultimately, no parameters for the lowest denomination, the 1-euro cent, can be estimated, because the lower bound then equals the upper bound for all observations. As the number of effective observations becomes quite low for the denominations smaller than 1 euro, we focus on the denominations 1,2,5,10, 20, 50 and 100 euro. The resulting number of effective observations for the relevant denominations is high enough to statistically draw conclusions on possible preferences by means of a likelihood ratio testing. We refer to Section 3.4 for further details. We will discuss below how we utilize the LR-test for inference.

The estimation results are presented in Table 5.4. The parameters for *ACORR* are all significantly different from zero, and obtain the expected positive sign.

Table 5.4: Parameter estimates with standard errors in parentheses for the cash payment model. The samples size of this data set is 240

denomination	constant		<i>ACORR</i>		effective observations
	parameter	Se	parameter	Se	
100	0.855	(0.829)	2.174 <sup>a</sup>	(0.465)	10
50	0.519	(0.435)	1.413 <sup>a</sup>	(0.363)	76
20	0.122	(0.162)	1.019 <sup>a</sup>	(0.187)	76
10	0.332	(0.161) <sup>a</sup>	1.188 <sup>a</sup>	(0.257)	66
5	0.139	(0.263)	1.754 <sup>a</sup>	(0.576)	48
2	0.142	(0.262)	1.633 <sup>a</sup>	(0.390)	46
1	-0.032	(0.218)	1.780 <sup>a</sup>	(0.376)	39

<sup>a</sup> Significantly different from zero at the 5 percent level.

We visualize the estimation results for the different denominations by plotting expected values for each denomination on the basis of the estimated parameters, and for a range of values for ACORR. We generate a large sample of expected values for a range of values of  $ACORR$  by means of simulation. We refer to Section 4.5 for a description of this simulation scheme. Various resulting curves are presented in Figure 5.4.

If there were no preferences, the resulting curves would coincide for all denominations. Indeed, Figure 5.4 shows that most curves are relatively close to each other. However, the curves for the 50-euro note and 20-euro note seem to deviate. To illustrate how parameter uncertainty propagates in the expected values, we also plot a point-wise 95% confidence interval of the  $N$  simulated expected values given the range of ACORR in Figure 5.5. Obviously, the confidence interval would become smaller as the parameters of the model are estimated for a larger sample of cash payments. From Figure 5.5, it can be observed that the difference between the curves for the 50-euro and 20-euro note is not significant.

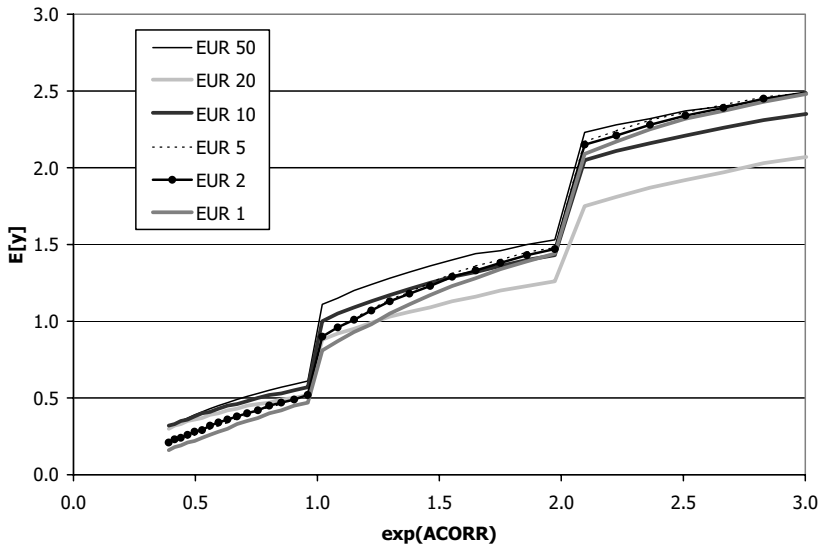


Figure 5.4: Expected value of  $Y_d$  for denominations 1 euro through 50 euro.

To formally test if there are differences in preference, we use the likelihood ratio (LR) test, that was discussed in detail in Section 3.4. Indifference implies that intercept

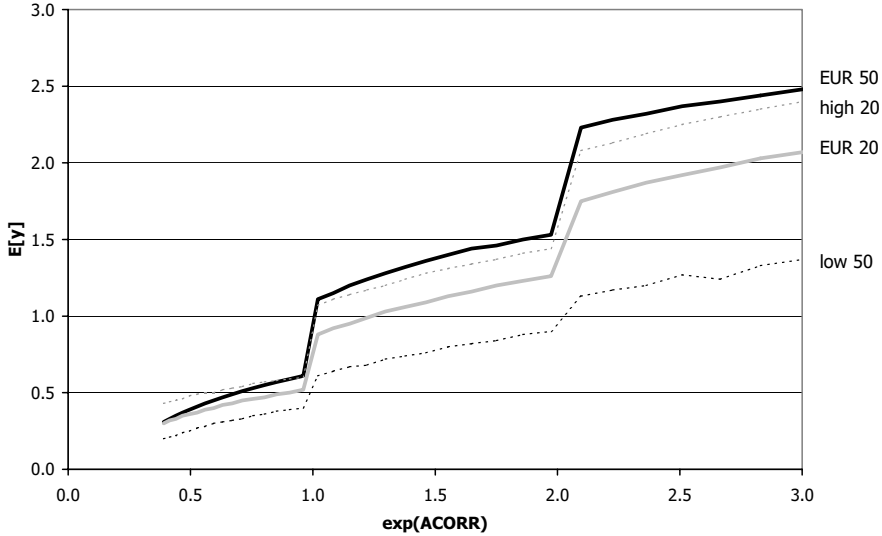


Figure 5.5: Relevant 95%-confidence bounds around  $E[Y_d]$  for denominations 20 and 50 euro.

parameters are the same across notes, and also that the ACORR parameters are the same across the notes. This is the null hypothesis of the LR-test. Rejection of the null hypothesis suggests that the parameters significantly differ across denominations. In that case, there is no indifference towards denominations by individuals, and certain denominations are (more or less) preferred. The LR-statistic obtains a value of 5.03, which does not exceed the 5% critical value of 18.31 of the chi-square distribution with 10 degrees of freedom. We can therefore conclude that there is no strong preference for one or more denominations in euro cash payments.

To validate this finding for the main data set, we now turn to the validation data set. We report the estimation results in Table 5.5. We observe that the parameter estimates bear strong similarities with those in Table 5.4. The LR statistic obtains a value of 10.85, which again is far from the relevant 5% critical value.

Table 5.5: Parameter estimates with standard errors in parentheses for the cash payment model. The samples size of this data set is 840

denomination	constant		<i>ACORR</i>		effective observations
	parameter	Se	parameter	Se	
100	-1.789	(2.755)	4.744	(3.083)	9
50	-0.245	(0.330)	1.379 <sup>a</sup>	(0.326)	211
20	-0.509 <sup>a</sup>	(0.141)	1.949 <sup>a</sup>	(0.207)	330
10	-0.156 <sup>a</sup>	(0.091)	1.991 <sup>a</sup>	(0.173)	387
5	-0.293 <sup>a</sup>	(0.099)	1.937 <sup>a</sup>	(0.199)	302
2	0.240 <sup>a</sup>	(0.095)	1.725 <sup>a</sup>	(0.146)	288
1	-0.230 <sup>a</sup>	(0.095)	1.575 <sup>a</sup>	(0.171)	227

<sup>a</sup> Significantly different from zero at the 5 per cent level.

## 5.5 Conclusions

In this chapter we examined whether the Dutch public has preferences for using certain euro coins and notes. An important motivation concerned the results in Chapter 4. There it was documented that such a preference ranking existed for the Dutch guilder, which had the 1-2 $\frac{1}{2}$ -5 based denominational range. Based on two new and unique data sets, our cash payment model and a statistical test with good properties, we conclude that the Dutch public does not have particular preferences regarding the new euro currency with its 1-2-5 denomination range. Hence, for the Netherlands, the introduction of the euro seems to have facilitated actual payment behavior.





# Chapter 6

## Empirical results III: the experimental data

### 6.1 Introduction

In this chapter we present the third empirical study of cash payments, in continuation of the former two studies. In Chapter 4 a large sample of cash payments in guilders was analyzed. An important result is the observation that Dutch individuals were not at all indifferent to various coins and notes. In fact, the  $2\frac{1}{2}$ -guilder coin and the 50-guilder note were used less often than would have been implied by theory. Chapter 5 describes a study that aimed to redo this analysis for euro payments. The main conclusion from this second study is that Dutch individuals did not have a strong preference for any euro denomination, in contrast to the findings for the guilder range.

The key features of the data in this study are that (i) individuals who were involved in the data collection stage could have had very different wallet contents, that (ii) some individuals might have faced more “difficult” amounts to pay, and that (iii) individuals were buying very different products, which could in turn involve different preferences. Additionally, the data were collected in a supermarket and in a hardware and home improvement store, and these stores could have used different pricing and rounding strategies.

It would now be of interest to collect data in a controlled setting. Such a setting would be very difficult and expensive to realize in an actual store or supermarket. Also, to make individuals to buy the same type of products would not be easy. Therefore, we decided to collect data while observing individuals playing a game. The European edition of Monopoly<sup>®</sup> turned out to be the best kind of game, as during such a game, people face the same items to purchase, use the same transaction amounts, and their

utility functions are similar as they all want to win the game. The European edition of Monopoly<sup>®</sup> uses (toy-) notes that all look very similar to the euro bank notes currently in use, but are much smaller. This method of data collection through experiments was rather labor-intensive, but delivered some unique data sets on cash payments.

In the present empirical study we will examine if the findings in Chapter 5 carry over to the case where individuals pay with euro notes in an experimental setting. A second aspect of collecting data through an experiment is that we are able to manipulate the settings. In order to have a better understanding of payment behavior, we decided to play various games (or better: have the games played, as we ourselves were not involved), while we each time removed one of the euro notes. In this way, we are able to study what happens when certain notes are not available. In practice this can happen when banks or stores run out of certain denominations. The theoretical results in Section 2.2 suggest that having no 100-euro and 10-euro notes has the smallest deteriorating effects on making payments, while the lack of 50-euro notes gives substantial problems in terms of efficiency. It seems unlikely that we can ever obtain real-life data for these situations, and therefore we resort to experiments. With these we can see how payment behavior changes in case of the unavailability of a single denomination. This can provide useful information for banks and cash counters.

## 6.2 Data

In this section we discuss how we collected the data. We also discuss the minor modifications we made to the rules of the game, to ensure that we would collect adequate data.

First, we deal with the sample size, as data collection is rather time-consuming in our case. From the empirical application in Chapter 5, and the simulation results described in 3.4, we learned that we need about 250 observations in order to have a reasonable power of the statistical test we aim to carry out in our model to draw conclusions. It turned out that playing the game twice with 3 or 4 players for a period of about 2 hours could ensure this amount of observations. The person, who played as bank, kept records of all transactions. After one hour there was a short break so that we could write down the wallet contents, in order to allow for an intermediate check. We played two games with all denominations, that is, with 500, 200, 100, 50, 20, 10, 5, and 1. Additionally, we conducted five experiments of two games where each time we removed one of the denominations. The 500-euro note and the 5-euro note turned out to be mandatory

for their use in all relevant cases. Also, our cash payment model needs a bottom-valued benchmark denomination, which here amounts to the 1-euro coin. We therefore conducted five experiments, each consisting of two games, one without the 200-euro note, one without the 100-euro, without 50, without 20 and finally without the 10-euro note. Hence, in total we had 12 games played during the period of September 2002 through January 2003.

The individuals who participated in the games ranged in the age of 18 to 56, and there were about as many men and women. The players had different educational backgrounds, as they consisted of undergraduate students, academic colleagues and administrative assistants. In total there were 51 participants. None of them were informed about the purpose of the experiments. Actually, we informed them that the data were collected to understand strategic behavior in games, which in fact was not really true. When asked for it afterwards, the players indeed indicated not to be aware of the purpose of the experiment.

In the European edition of the Monopoly<sup>®</sup> game transaction between 2 and 2000 euro can occur. The game uses notes of 500, 100, 50, 20, 10, and 5, and a coin of 1. As the genuine denominational range of the euro also contains 200, we had to make a new note. It turned out that nobody of the participants was familiar with this edition, so neither one of them was aware that our 200-euro note was new. Specifically for this purpose we had the note made, and it is depicted in Figure 6.1 Like the other toy notes, its size is much smaller than a genuine note, and it is only printed on one side, but the design resembles genuine euro notes. In order to make sure that we would never run out of notes of any value, we also had additional copies made of the available notes. Hence, in all situations the bank (within the game) would have an abundant availability of notes.

There were two more modifications to the game as it is outlined in the booklet of the European edition of Monopoly<sup>®</sup> and these are the following. Sometimes people play with an additional fund, where fines can be deposited. This possibility was excluded in our experiment. Also, we issued genuine wallets to all players and they were told not to inform the other players what was in their wallet. This prevented that they could help with meeting payment amounts by matching. Naturally, all players started with the same amount in the wallets, consisting of one or more notes and coins of each denomination in the game.

In the end, we collected amounts of observations within the range of 210 to 280 per experiment. All data on (i) wallets contents prior to each transaction, (ii) the way of payment at the transaction occasion, and (iii) the money received in return were checked for correctness. The return cash was used to keep track of the wallet contents.



Figure 6.1: The newly constructed 200-euro toy notes (its true size is equal to the size of the other toy notes and its color is yellow)

### 6.3 Estimation results

We estimated the parameters in the cash payment model, given by (3.2) and (3.9), with the experimental data and explanatory variables  $ACORR$ , as defined by (3.12) and an intercept. We use the LR test, described in Section 3.4, in two ways. First, we test whether all parameters are equal. Rejection of the test then means that there is no indifference towards denominations. Second, we reduce the model by setting parameters equal until the LR test rejects further hypotheses of equal parameters.

Furthermore, we again visualize the outcomes of all final models by plotting expected values on the basis of the estimated parameters, and for a range of values of the  $ACORR$  variable. That is, for a given value of  $ACORR$ , we compute the choice probabilities between the upper bounds and the lower bound 0 and we construct the expected value of  $Y$ . Various resulting curves are present in Figure 6.2 and the following graphs. Note that we do not make use of simulation techniques, like in Chapters 4 and 5. The reason is that we make use of the LR-test to draw statistical inference on the existence of preferences, as opposed to 95% confidence intervals we used in Chapter 4. Therefore, we do not need to resort to simulation techniques to draw inference. For interpretation of the empirical results we switch between LR test and graphs. The LR-test will tell us if the hypothesis of indifference is rejected or not. The graphs will give us insights into the denominations causing differences in preferences in case of rejection.

We first discuss the case with all notes, and next we turn to the five cases where each time a note has been removed.

### 6.3.1 All denominations

To give an impression of typical estimation results to be obtained from our model, we give the estimated parameters for the unrestricted model for the “all notes” case in Table 6.1. Almost all intercept parameters are not significantly different from zero, while all ACORR parameters are significant and within the range of 1.5 to 3.

Table 6.1: Parameter estimates with standard errors (Se) for the full model. The denominations are 500, 200, 100, 50, 20, 10, 5 and 1. The sample size of this data set is 217.

denomination	constant		<i>ACORR</i>	
	parameter	Se	parameter	Se
500	-0.049	(0.845)	2.969 <sup>a</sup>	(1.474)
200	-0.200	(0.261)	1.846 <sup>a</sup>	(0.438)
100	0.038	(0.341)	2.237 <sup>a</sup>	(0.399)
50	-0.044	(0.389)	2.770 <sup>a</sup>	(0.640)
20	-0.663 <sup>a</sup>	(0.299)	1.640 <sup>a</sup>	(0.705)
10	0.105	(0.342)	1.574 <sup>a</sup>	(0.538)
5	0.311	(0.509)	1.817 <sup>a</sup>	(0.905)

<sup>a</sup> Significantly different from zero at the 5 per cent level.

The first row of Table 6.2 gives the LR statistic for the restriction that all intercepts are equal and that all ACORR parameters are equal. We give the LR test result for the case where the parameters for the 500-euro note are unrestricted. This is relevant as the parameter values for the 500-euro note tend to be higher than the others due to the limited range of ACORR values for which the parameter is estimated, see also Table 6.1. Hence, the LR test could likely suggest rejection due to the deviating parameters for the 500-euro note. We therefore focus on indifference towards all denominations while excluding the 500-euro note, and focus on the LR test of equal parameters across these denominations as presented in Table 6.2.

The LR statistic for the experiment with all notes included equals 19.567, which is significant at the 5 per cent level, compared with the chi-square (10) distribution. This means that our finding for the experimental data is not in accordance with the finding in Chapter 5. There, it was concluded on the basis of observed euro cash payments in retail stores, that there appeared to be no preferences for using certain euro notes. While there were no significant preferences for these payments, now significant preferences seem to be present in the payments observed in this experimental setting. Indeed, the curves in Figure 6.2 clearly do not coincide.

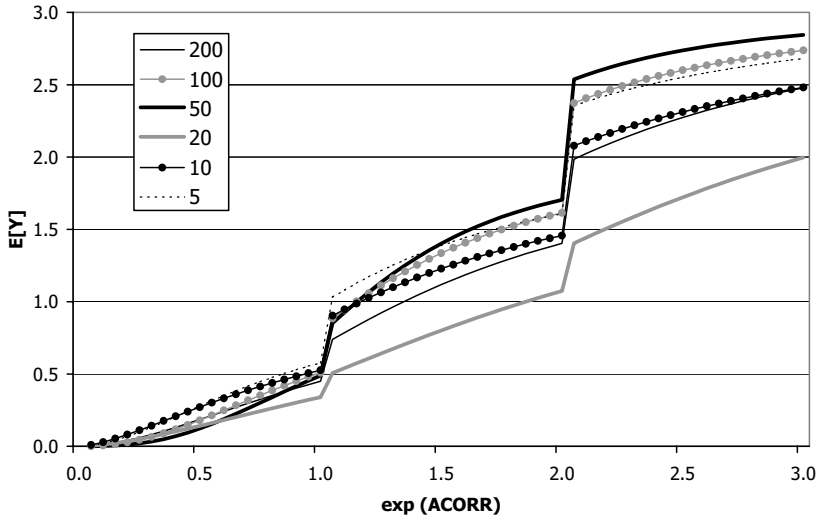


Figure 6.2: Expected values of  $Y_d$  for full model (lower bound equals 0)

Table 6.2: Likelihood Ratio tests for equality of the intercept parameters and for the ACORR parameters, where the parameters for the 500-euro note are left unrestricted.

Denominations	Sample size	LR test
All	217	19.567 <sup>a</sup>
No 200	235	18.314 <sup>a</sup>
No 100	226	9.576
No 50	280	12.186
No 20	210	18.024 <sup>a</sup>
No 10	210	14.451

<sup>a</sup> Significantly different from zero at the 5 per cent level. The LR test should be evaluated using the critical values of the chi-square (10) distribution in case of all notes and of the chi-square (8) distribution in case one of the notes is excluded.

We reduce the model until a set of restrictions is found for which the LR test no longer rejects. The first panel in Table 6.3 presents the parameters of the reduced model for the experiment with all notes. From this, we can conclude that the use of the 50-euro note is significantly different from the use of the other notes. From Figure 6.2 we can tell that the 50-euro note is used far more in the upper range of ACORR values, while it is used less than the others in the lower range of ACORR values. Apparently, higher amounts in the game are paid more with 50-euro notes than one would expect.

The difference in payment behavior between the experimental setting and real-life situations should be sought in the difference of payment amounts. The median payment amount in the experiment with the full bank note range is 100 euro, while in real-life cash payments usually concern lower amounts with a median around 10 euro. Although the players of the game encountered rather unfamiliar high payment amounts, they were able to pay with notes that resemble the euro notes they use in real life. Apparently, the players resort to the familiar 50-euro note to make high amount payments.

### 6.3.2 Leaving out one note

Table 6.4 to 6.8 give the parameter estimates for the cases where we each time leave out a single note. Table 6.2 gives the relevant LR tests and Figures 6.3 to 6.7 present the corresponding graphs. We first observe from the graphs that payment behavior becomes less erratic, in the sense that less lines intersect, if the 100-, 50- or 10-euro note is left out. This is confirmed by the outcomes of the LR tests displayed in Table 6.2, as the null hypothesis of equal intercepts and *ACORR* parameters across the denominations cannot



be rejected for these cases.

Consider the case where the range of notes in the game does not include a 200-euro note, as in the original Monopoly<sup>®</sup> game. The LR test suggests rejection, and from Figure 6.3 it is clear that this is caused by a preference for the 50-euro note for payment amounts higher than its nominal value. The final model for which the parameter estimates are given in the second panel of Table 6.3 confirms this. Apparently, the preference for a 50-euro note is not cancelled out by removing the 200-euro note from the range.

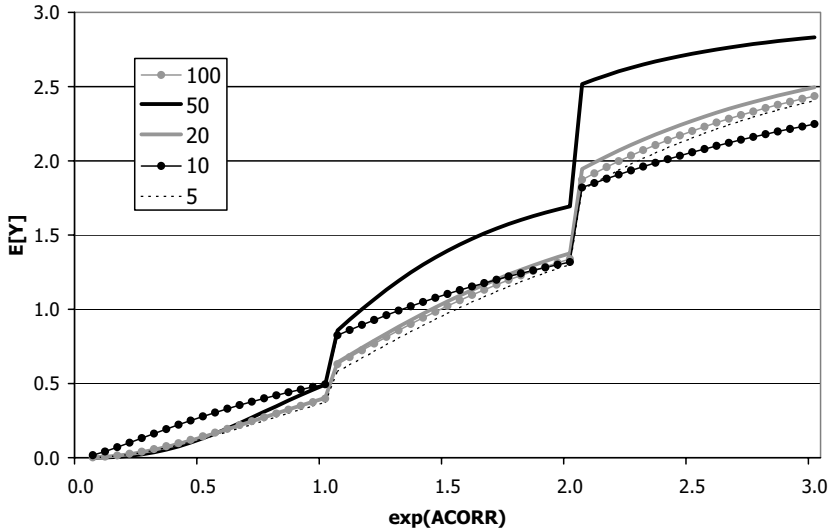


Figure 6.3: Expected value of  $Y_d$  for model without 200 (lower bound equals 0)

In contrast, the removal of the 100-note clearly does make the 50-euro less preferred, as we can tell from Figure 6.4, and from the outcome of the LR test in Table 6.2. In case we leave out the 50-euro note, the curves of the expected values of  $Y$  in Figure 6.5 are remarkably close together. Indeed the LR test confirms indifference towards denominations in this experiment (see again Table 6.2). However, in case the 20-euro note is removed from the range, aberrant payment behavior with the 50-euro note shows up again, as it also did in the full range case. Furthermore, as we can see from the final models presented in Table 6.3 and the curves in Figure 6.6, only for a few denominations equal payment

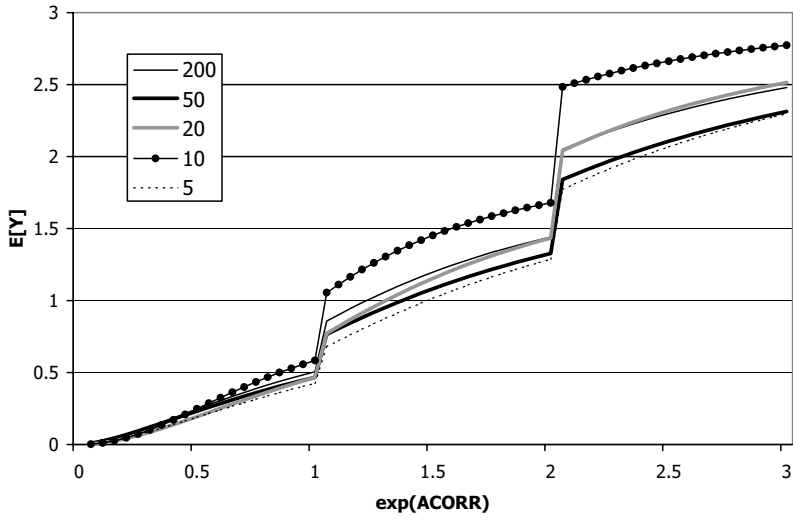


Figure 6.4: Expected values of  $Y_d$  for model without 100 (lower bound equals 0)

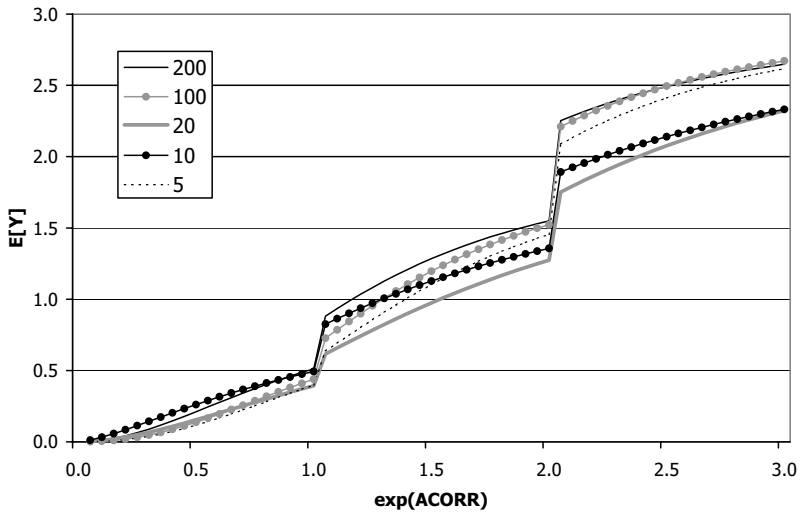


Figure 6.5: Expected value of  $Y_d$  for model without 50 (lower bound equals 0)

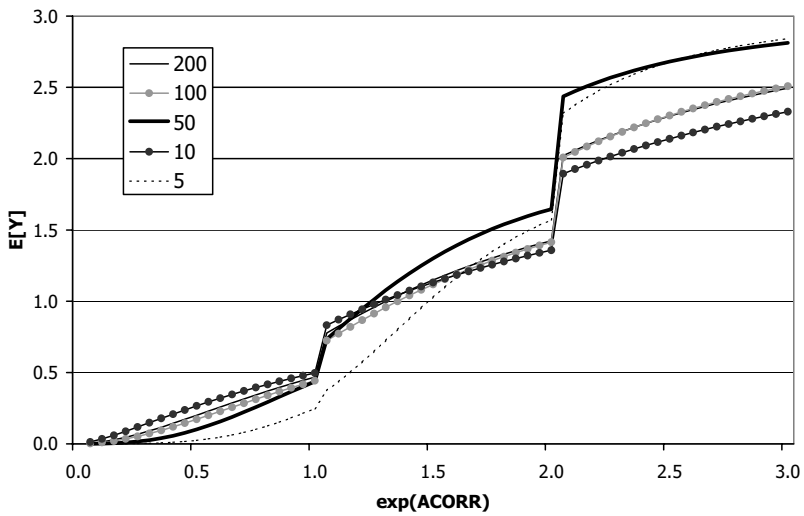


Figure 6.6: Expected value of  $Y_d$  for model without 20 (lower bound equals 0)

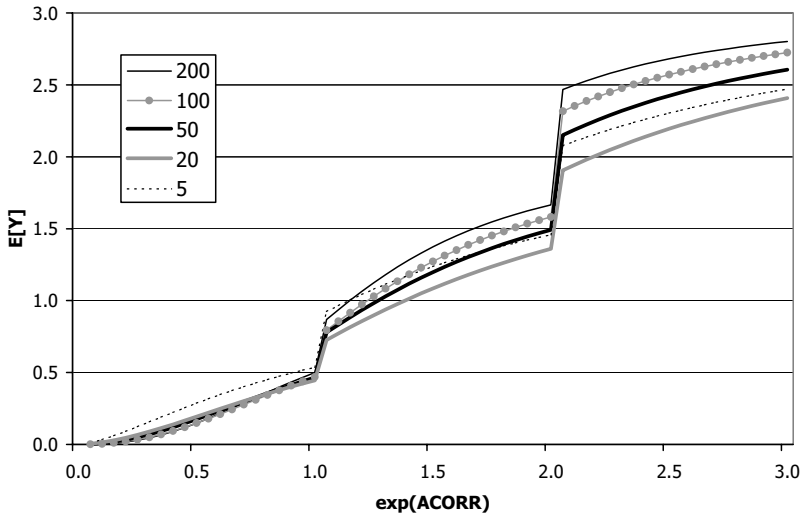


Figure 6.7: Expected value of  $Y_d$  for model without 10 (lower bound equals 0)

behavior emerges. Hence, removing the 20-euro note has created additional preferences for other notes.

Finally, the experiment without the 10-euro note shows a balanced use of all remaining denominations. The curves of expected use in Figure 6.7 lie close together, and the LR statistic in Table 6.2 is 14.451, which is not significant at the 5 per cent level.

## 6.4 Conclusions

The empirical results in this chapter, based on some unique data sets collected through observing various games of the European edition of Monopoly<sup>®</sup>, suggest the following conclusions. The first is that if all notes are available, the participants of the game use the 50-euro note in another way than other denominations in similar payment situations, which is in contrast to the findings of Chapter 5. Hence, the settings of the Monopoly<sup>®</sup> experiment apparently induce preferences that do not exist in real life payment situations. The atypical use of the 50-euro note is however not surprising. This is the highest valued note Dutch individuals are familiar with, as it is the highest valued note that is distributed through ATMs and therefore more regularly used in cash payments than higher valued bank notes.

A second conclusion is that the removal of the 100-euro or 10-euro note has a positive influence on payment behavior. This is in accordance with the theoretical results obtained in Section 2.2. In these cases the aberrant payment behavior related to the 50-euro note is cancelled out. This suggests that banks or cashiers may, for a short while, not issue 100-euro or 10-euro notes, perhaps without substantial problems. As a consequence, payment behavior improves and the remaining range of bank notes is used optimally.

A third conclusion is that the 200-euro and 20-euro notes are the most crucial notes. They cannot be left out as payment behavior does not improve in these cases. Payment behavior even deteriorates when the 20-euro note is excluded.

Finally, removing the 50-euro note itself has a rather positive effect on payment behavior, that is, the other notes are used in the way they should be in case the paying public would be indifferent to the notes. Apparently, taking out the most familiar note is caught up perfectly by the remaining notes.

Table 6.3: Estimation results for final models. The results for the 500-euro note are not reported

denomination	constant		<i>ACORR</i>	
	parameter	Se	parameter	Se
	<i>All denominations</i>			
All but 50	-0.202	(0.138)	1.812 <sup>a</sup>	(0.169)
50	-0.044	(0.392)	2.770 <sup>a</sup>	(0.637)
	<i>All denominations except 200</i>			
All but 50	-0.316	(0.140)	1.817 <sup>a</sup>	(0.233)
50	-0.032	(0.380)	2.680 <sup>a</sup>	(0.275)
	<i>All denominations except 100</i>			
All	-0.045	(0.136)	1.583 <sup>a</sup>	(0.073)
	<i>All denominations except 50</i>			
All	-0.263 <sup>a</sup>	(0.120)	1.632 <sup>a</sup>	(0.064)
	<i>All denominations except 20</i>			
200-100	-0.170	(0.211)	1.861 <sup>a</sup>	(0.148)
50-5	-0.464	(0.307)	2.971 <sup>a</sup>	(0.639)
10	-0.012	(0.372)	1.444 <sup>a</sup>	(0.332)
	<i>All denominations except 10</i>			
All	-0.138	(0.093)	1.646 <sup>a</sup>	(0.024)

<sup>a</sup> Significantly different from zero at the 5 per cent level.

Table 6.4: Parameter estimates with standard errors (Se) for the model without 200. The denominations are 500, 100, 50, 20, 10, 5 and 1. The sample size of this data set is 235.

denomination	constant		<i>ACORR</i>	
	parameter	Se	parameter	Se
500	1.497	(0.794)	4.219 <sup>a</sup>	(1.330)
100	-0.415	(0.284)	1.972 <sup>a</sup>	(0.347)
50	-0.028	(0.386)	2.693 <sup>a</sup>	(0.279)
20	-0.394	(0.259)	2.053 <sup>a</sup>	(0.675)
10	-0.021	(0.306)	1.344 <sup>a</sup>	(0.475)
5	-0.521	(0.373)	2.020 <sup>a</sup>	(0.908)

<sup>a</sup> Significantly different from zero at the 5 per cent level.

Table 6.5: Parameter estimates with standard errors (Se) for the model without 100. The denominations are 500, 200, 50, 20, 10, 5 and 1. The sample size of this data set is 226.

denomination	constant		<i>ACORR</i>	
	parameter	Se	parameter	Se
500	1.649	(0.817)	3.514 <sup>a</sup>	(1.166)
200	0.020	(0.255)	1.674 <sup>a</sup>	(0.364)
50	-0.136	(0.280)	1.534 <sup>a</sup>	(0.140)
20	-0.140	(0.302)	1.854 <sup>a</sup>	(0.504)
10	0.338	(0.341)	2.091 <sup>a</sup>	(0.666)
5	-0.303	(0.398)	1.664 <sup>a</sup>	(0.782)

<sup>a</sup> Significantly different from zero at the 5 per cent level.

Table 6.6: Parameter estimates with standard errors (Se) for the model without 50. The denominations are 500, 200, 100, 20, 10, 5 and 1. The sample size of this data set is 280.

denomination	constant		<i>ACORR</i>	
	parameter	Se	parameter	Se
500	0.468	(1.253)	4.280 <sup>a</sup>	(2.271)
200	0.048	(0.242)	1.970 <sup>a</sup>	(0.336)
100	-0.244	(0.296)	2.295 <sup>a</sup>	(0.581)
20	-0.436	(0.260)	1.821 <sup>a</sup>	(0.201)
10	-0.027	(0.248)	1.461 <sup>a</sup>	(0.268)
5	-0.418	(0.379)	2.319 <sup>a</sup>	(0.896)

<sup>a</sup> Significantly different from zero at the 5 per cent level.



Table 6.7: Parameter estimates with standard errors (Se) for the model without 20. The denominations are 500, 200, 100, 50, 10, 5 and 1. The sample size of this data set is 210.

denomination	constant		<i>ACORR</i>	
	parameter	Se	parameter	Se
500	0.079	(0.819)	1.816 <sup>a</sup>	(0.784)
200	-0.130	(0.278)	1.809 <sup>a</sup>	(0.237)
100	-0.233	(0.303)	1.929 <sup>a</sup>	(0.424)
50	-0.255	(0.348)	2.795 <sup>a</sup>	(0.695)
10	-0.013	(0.374)	1.445 <sup>a</sup>	(0.335)
5	-1.114	(0.829)	3.723 <sup>a</sup>	(1.658)

<sup>a</sup> Significantly different from zero at the 5 per cent level.

Table 6.8: Parameter estimates with standard errors (Se) for the model without 10. The denominations are 500, 200, 100, 50, 20, 5 and 1. The sample size of this data set is 210.

denomination	constant		<i>ACORR</i>	
	parameter	Se	parameter	Se
500	4.551	(4.064)	9.650	(7.281)
200	-0.002	(0.071)	2.520 <sup>a</sup>	(0.904)
100	-0.125	(0.334)	2.341 <sup>a</sup>	(0.824)
50	-0.137	(0.031)	2.037 <sup>a</sup>	(0.171)
20	-0.218	(0.306)	1.747 <sup>a</sup>	(0.422)
5	0.142	(0.435)	1.521 <sup>a</sup>	(0.319)

<sup>a</sup> Significantly different from zero at the 5 per cent level.

# Chapter 7

## Summary and conclusions

### 7.1 Summary

In this thesis we presented the first empirical analysis of the use of currency denominations in individual cash payments. In Chapter 1 we showed that such an approach contributes to existing currency research that mainly concentrates on currency use at the aggregate level. In fact, no more than theoretical assumptions have been made on individual cash payment behavior in economic literature. In Chapter 2 we summarized the discussion on what values should be assigned to denominations for a currency range to be optimal. This optimizing problem can be approached from two different angles. The first argues that the optimal range should provide for cash payments in which, on average, a minimum number of notes and coins is exchanged. This is called the “principle of least effort” that suggests that currency should provide for the most efficient cash payments. The second approach has the view that the number of different denominations should be minimized, while allowing any amount to be paid. The reasoning being, that the lower the number of denominations is, the lower the costs of issuing, developing and producing denominations are, and the lower the risk of confusion is with the public. No single optimum was presented, but it was generally agreed that the predominantly used 1-2-5 range worldwide is an acceptable compromise that more or less meets both criteria, and coincides with the decimal system to facilitate easy mental calculations. This conclusion does not clearly make the euro currency range, that was introduced in 12 European countries in 2002 and follows the 1-2-5 range perfectly, superior to the former Dutch guilder range that was based on a  $1-2\frac{1}{2}$ -5 range. In the second part of Chapter 2, we theoretically examined the difference between both ranges in terms of efficiency. We made use of an algorithm developed by Cramer (1983) to compute all payments in which the least number of notes

and coins are exchanged for a range of payment amounts. As the algorithm can be applied to any currency range, we did so for the current euro range as well as the former guilder range. The results indicated that the euro range is not more efficient than the guilder range, theoretically at least. However, if the 1- and 2-cent coins are excluded from the euro range, efficiency does improve substantially.

The concept of efficiency in cash payments is appreciated for its elegance and simplicity. That is, it provides a model by which cash payments can be simulated and concurs with rational behavior, a common assumption in economics. However, all theoretical results described above are valid only if individuals actually behave accordingly. In Section 2.3 we argued that this is a rather restrictive assumption. Other reasons than efficiency may lie at the basis of a payment choice, and individuals might have a preference for one denomination over others, consciously or unconsciously. Furthermore, individuals are restricted to the content of their wallet, and hence do not have unlimited access to all relevant denominations. Another practical matter is that not all amounts occur equally frequently. For these reasons, we developed in Section 2.3 an alternative, more general, framework for individual cash payment behavior taking all these observations into account. We thereby considered a cash payment as a choice process from the perspective of the paying party, leaning heavily on concepts developed in behavioral science and brand choice research. We concluded that an individual probably forms a consideration set of notes and coins before making a payment. In general, individuals form such consideration sets to simplify the choice process, which is a well known phenomenon in brand choice. The existence of consideration sets in cash payments could mean that there is a possibility that denominations are not considered for payment, even though they might be the most useful ones to include in a payment. This implies that one of the basic underlying assumptions that individuals are indifferent towards denominations, might not hold.

After analyzing these theoretical issues in Chapter 2, we then argued that one should study actual cash payments to draw any conclusions on payment behavior. In first instance, it is of interest to test whether individuals pay efficiently in practice. But more generally, one should study actual cash payments to draw conclusions on the use of denominations of a currency range. Such analysis is of importance as the issuing authorities of currency strive for an efficient currency mix that accommodates the demand of the public. A currency that is not used to its full potential affects all cash users, and drives up the costs associated with cash distribution. Such a situation occurs if some denominations are more preferred than others, or otherwise stated, if individuals are not indifferent towards denominations. As the behavioral framework we developed in Section 2.3 furthermore

suggests, one should take the influence of the payment amount and wallet content on the payment choice into account.

In Chapter 3, we built a statistical model accordingly. The model describes the probability of choosing a combination of denominations for a cash payment from the viewpoint of the paying individual. The choice set is then determined by the wallet content. The model differs from familiar choice models, such as those developed for branch choice, to account for special features of a cash payment process. First, a hierarchical structure is imposed as the paying individual chooses for a combination of bank notes and coins rather than a single item. That is, the probability of choosing a certain denomination for payment is modeled given the choice made for higher denominations. The conditional probabilities then account for the interdependency of denominations in cash payments. Second, the choice set varies with the wallet content of the paying individual and the payment amount. The conditional probability of the number of tokens of a certain denomination chosen for payment is modeled as a truncated Poisson distribution in which the choice set is restricted between an upper bound and a lower bound that are dependent on the wallet content and the payment amount. The main explanatory variable is given by the payment amount, scaled to the face value of the relevant denominations. This scaling allows a direct comparison of parameters across denominations.

We next presented three empirical studies in Chapters 4 through 6. The first application, described in Chapter 4, made use of a large data set of cash payments in guilders. The data was collected on the authority of De Nederlandsche Bank in 1998. In a representative sample of Dutch retail stores the specifics of cash payments were registered, such as the payment amount, and the bank notes and coins used for payment and received as change, if applicable. Unfortunately, no wallet contents were registered prior to the cash payment. To account for the unobserved wallets, additional data was used, containing wallet contents collected through an e-mail survey. Initially, the observed payments were compared to their relevant efficient payment schemes, given the wallet content. Cramer's original algorithm was modified to compute restricted efficient payment schemes for all amounts in the data set, and 10 feasible wallets for each payment. These were selected from the data set of wallet contents. The results, averaged over 10 wallets, showed that about 34% of the observed payments were done in an efficient manner. For the remaining 66% more tokens were used than would have been needed out of 1 or more of 10 possible wallets. Next, the cash payment model was estimated with the cash payment data for guilders, again with 10 feasible wallets selected for each payment. The estimation results are interpreted by means of graphical representation. The results showed a strong pref-

erence for payment with 100-guilder bank notes as compared to the 50-guilder note in similar payment situations. In fact, the 50-guilder note seemed to be less preferred than any other bank note. Hence, it can be concluded that the guilder range of denominations was not optimally used.

Chapter 5 reported on the second empirical application in euro as a natural follow up to the first. A data set of cash payments was especially collected for this purpose, whereby the importance of information on the wallet content was taken into account. A survey in two different retail stores in a rural and urban area in the Netherlands in 2003 resulted in just over 270 cash payments in euro. After payment, paying individuals were invited to participate in a survey, in which they were asked, among other things, to report their wallet contents. Hence, it was possible to construct matching wallets to the observed cash payments. It was calculated that over 60% of the payments were done in an efficient manner, given the restriction of the wallet content. This constituted a significant part of the group, indicating that paying individuals indeed strive for efficiency. For a more general conclusion on the use of euro cash, the cash payment model was estimated with the data. The results were interpreted by means of likelihood ratio testing. That is, the graphical representation of the expected values did not show coinciding curves, but nevertheless seemed to lie close together. The likelihood ratio test of equal parameters across denominations, confirmed indifference towards denominations in euro cash payments. Given the sample size, it was concluded that there was no strong preference present for euro denominations. The parameters of the cash payment model were estimated for a second data set, collected through an internet survey. As these payments concern stated rather than observed behavior, it was merely used for validation. Its estimation results supported the earlier finding.

Finally, in the third empirical study that is described in Chapter 6, experiments were conducted to simulate real life payment situations. Such an experimental setting allowed us to change the denominational structure and study its effect on cash payment behavior. Obviously, such an experiment cannot be conducted in real life. Each experiment consisted of two games of Euro Monopoly<sup>®</sup> played with 3 or 4 participants each to obtain a sufficiently large data set. The payments, as well as the wallets of the players, were registered. The participants were not aware of the purpose of the experiment. In the first experiment, all euro toy notes, that resemble the design of the real euro bank notes, were included. In five subsequent experiments each time one denomination was removed from the game. The resulting data sets of cash payments gave us the opportunity to study the effects of cancelling one euro bank note denomination on cash payment behavior. The

estimation results for the first experiment showed that the 50-euro was used differently than other denominations in similar payment situations. This result did not concur with that of Chapter 5. Apparently, the experimental setting does not induce similar payment behavior, which is probably due to the relative high payment amounts in the Monopoly<sup>®</sup> game. We observed that removing the 50-euro note resulted in a well-balanced use of the remaining denominations. This was also the case for experiments without the 10- and 100-euro notes. Removal of the 20-euro note, however, caused more aberrant cash payment behavior, indicating that this denomination is crucial in cash payments.

## 7.2 Conclusions

From the research presented in this thesis we can draw the following main conclusions.

First, if one wants to investigate payment behavior, it should be studied empirically at the individual payment level. This calls for costly, and labor-intensive data collection methods, that should take into account the delicate nature of money. Data analysis should be done with the use of a statistical model, that allows for various specific circumstances at the time of payment. The model presented in this thesis enables such an analysis. The estimation results can answer the question if some denominations in a denominational range are preferred over others in similar payment situations, indicating suboptimal use of the denominational range. Any denominational range can be used, so it is a method that can be applied to any country, or even in an experimental setting as we demonstrated in this thesis.

Second, our empirical results suggest that the euro range outperforms the former guilder range in the Netherlands, in the sense that no clear preference can be spotted in the use of euro denominations as opposed to the preference ranking found in guilder payments. As the euro range was found to be theoretically equally efficient, one can conclude that the cash system in the Netherlands seems to have improved with the introduction of euro cash. Unfortunately, we were not able to carry over these findings to an experimental setting. But the experiments, in which bank note denominations were subsequently removed from the range, did indicate that the absence of a denomination may cause changes in payment behavior.

Third, it is worthwhile mentioning that the removal of a 1- and 2 euro cent from the Dutch payment system would improve efficiency of cash payments. At least, this is suggested by theory, but not examined empirically. This might be a suggestion for future research. One could expect that current payment behavior with these low valued coins

is far from efficient, as they might get lost in circulation, or not used as efficiently as calculated by theory. Therefore, cancelling these denominations might even prove more efficient than theory suggests.

Our research has made a start with studying cash payment behavior empirically at the individual payment level. In light of the conclusions drawn above, we see many avenues for further research in this area. It would of course be interesting to see if our current findings for the Netherlands also hold for other countries. Of particular interest are (i) those countries where they do not (yet) have the euro, like Denmark, Sweden and England, (ii) those countries which use a mixture of the 1-2-5 and  $1-2\frac{1}{2}$ -5 ranges like the US (where there are quarter dollars and 2-dollar notes) and (iii) those countries where the coins and notes substantially differ in size and shape, like in Australia where the 50-cent coin is rather large.

Furthermore, it might be worthwhile to investigate how the experimental setting can be changed, for example by changing the Monopoly payment amounts, to lead to results that are more consistent with day-to-day payments in retail stores. The emphasis would then be on the notes regularly used for payment, that is, the 50, 20-, 10- and 5-euro notes. That would give the opportunity to start experimenting with abnormal denominational ranges, a luxury a central bank does not have in real life. For example, it would be exciting to see how people would react if we were to introduce 25-euro notes, 40-euro notes or 300-euro notes, to mention just a few, or if we were to introduce a whole new unfamiliar denominational range, such as 1-3-9-.... Also, it would be quite interesting to find out what, from a behavioral point of view, would be the optimal density of a range. This may shed light on the discussion that too many different denominations would cause inefficient payment behavior. By experimenting one could start with studying a small number of denominations, and expand with one denomination each time, to conclude at what point denominations start to fall out of use. Furthermore, one could experiment with the characteristics of bank notes and coins, such as design, size and weight, to study their effect on payment behavior. Such experiments might shed some light on the phenomenon that some denominations are preferred over others, which would be essential information to issuing authorities.

In general, we believe that the type of research presented here, has many opportunities. The cash payment process is an interesting choice process in and of itself. We discovered that it is quite challenging to capture this process in a probability model. The search for an appropriate model could be extended, especially in the direction of forecasting power of a cash payment model. A forecasting model would enable simulation of cash

payments. Such an application could be of interest to retailers, for example, to optimize change needed in cash registers. Another application might be to investigate the effect of changing the wallet content. For example, what would be the effect on cash payments if banks were to change the denominational mix in their ATMs, which would be reflected in a changing composition of the wallet content. If a large enough data set were available, including matching wallet contents to observed cash payments, it might be of interest to compare payment behavior for different categories of payment amounts. Finally, we viewed cash payments as the outcome of a choice process, whereby we focused on the choice between denominations. Such a choice process might be extended to the choice between electronic means of payment and cash payments, considering the content of the wallet and the potential complexity of a cash payment.

With this final remark we end up where we started at the beginning of this thesis, the future might lie in the increasing use of modern means of payment, but many questions on how the most traditional means of payment is used, namely cash, are still unanswered. We think to have contributed to a better understanding of currency use in this thesis.





# Chapter 8

## Nederlandse samenvatting (Summary in Dutch)

### 8.1 Inleiding

Contant betalen is het meest populaire betaalmiddel, ook al nemen elektronische alternatieven zoals pinnen en chippen een steeds groter aandeel van de omzet in de detailhandel voor hun rekening. Nog altijd betaalt het publiek het meest met bankbiljetten en munten bij kleine bedragen, gemiddeld zo'n  $1 \frac{1}{2}$  keer per dag. Hiervoor is het nodig om een uitgebreid en kostbaar distributiesysteem van bankbiljetten en munten in stand te houden, waarvan zowel de centrale bank, het bankwezen en de detailhandel deel uitmaken. In een recent onderzoek van het Maatschappelijk Overleg Betalingsverkeer, een platform waarin alle betrokken partijen vertegenwoordigd zijn, worden de jaarlijkse kosten voor contant betalen in Nederland geschat op 2 miljard euro jaarlijks voor zo'n 7 miljard betalingen in totaal. Deze cijfers geven aan dat het in ieders belang is om de kosten zoveel mogelijk te drukken door het betaalsysteem zo efficiënt mogelijk in te richten.

De monetaire autoriteit speelt daarin een belangrijke rol. Niet alleen is zij als laatste schakel in de chartale keten verantwoordelijk om te allen tijde aan de vraag naar contant geld te voldoen, ook bepaalt zij de coupurereeks waarin bankbiljetten en munten worden uitgegeven. Contant geld wordt in verschillende coupures uitgegeven om contante betalingen te vergemakkelijken. De monetaire autoriteit is erop gericht de coupurereeks naar de wens van het publiek samen te stellen. Daarvoor kan het nodig zijn de coupurereeks van tijd tot tijd te wijzigen, bijvoorbeeld als een nieuwe hoogste coupure gewenst is als gevolg van langdurige inflatie, of als een coupure steeds minder gebruikt lijkt te worden. Het onderzoek in dit proefschrift is erop gericht individueel betaalgedrag empirisch te

onderzoeken. Doel is om te ontdekken of een coupurereeks optimaal wordt gebruikt. Daarvoor is Nederland bij uitstek een interessant onderzoeksonderwerp. Het Nederlandse geld had namelijk de unieke  $1-2\frac{1}{2}-5$  structuur, terwijl de recente introductie van euro bankbiljetten en munten heeft geleid tot een overgang naar de meer gebruikelijke  $1-2-5$  structuur waarop nagenoeg alle coupurereeksen in de wereld zijn gebaseerd.

Bestaande literatuur naar het gebruik van contant geld is omvangrijk, maar heeft zich voornamelijk gericht op de geaggregeerde vraag naar bankbiljetten en munten. Zo heeft een reeks aan publicaties zich beziggehouden met het ontwikkelen van een voorspelmodel voor de vraag naar chartaal geld. Een dergelijk model is van belang voor de taak van een centrale bank om haar geldvoorraad te plannen. Een tweede reeks aan publicaties is gericht op het houderschap van voornamelijk hoge coupures. Met behulp van economische macrogegevens zijn schattingen gemaakt van dat deel van de uitstaande geldcirculatie dat wordt opgepot, en dat deel dat wordt gebruikt voor transactiedoeleinden. Een derde reeks van publicaties presenteert methodes om het gebruik van een coupurereeks in de praktijk te beoordelen op effectiviteit. Veelal gaat het hier om methodes die theoretische concepten over coupuregebruik in individuele transacties koppelen aan de geobserveerde vraag naar deze coupures op geaggregeerd niveau. Uit dit literatuuronderzoek trekken we de algemene conclusie, dat als men het gebruik van contant geld werkelijk wil onderzoeken, het nodig is om bij de bron te beginnen, namelijk door betaalgedrag te observeren op het individuele transactieniveau. Immers, op geaggregeerd niveau worden de cijfers vervuld door veelvuldig oppotten van geld en geld dat verloren gaat. Theorieën over individueel betaalgedrag zijn in de bestaande literatuur veelvuldig gebruikt, maar niet in de praktijk getoetst. Omdat individueel betaalgedrag bepalend is voor het functioneren van het betaalsysteem, kan dit als een omissie in de literatuur worden aangemerkt. Het onderzoek dat we in dit proefschrift presenteren, is daarom vooral gericht op een empirische analyse van contant betaalgedrag op individueel niveau. Doel is om inzicht te krijgen in coupuregebruik en vervolgens conclusies te trekken over de performance van een coupurereeks in de praktijk. Ons onderzoeksonderwerp is Nederland. Het contante betaalgedrag in dit land is met name interessant vanwege de recente overgang van de gulden naar de euro in 2002. Guldenbiljetten en -munten waren immers gebaseerd op een  $1-2\frac{1}{2}-5$  reeks, terwijl de euro de meer gangbare  $1-2-5$  reeks volgt. Het onderzoek dat we hier presenteren, kan echter op elke willekeurige coupurereeks, en dus in elke willekeurig land worden toegepast.

## 8.2 Theorie

In hoofdstuk 2 beschrijven we verschillende theoretische concepten die ons verder helpen in het formuleren van een onderzoeksvraag. Eerst gaan we in op de vraag welke coupurereeks theoretisch optimaal is. Dit is het onderwerp van een recente reeks artikelen, die er niet in slagen een eenduidig antwoord te geven. Omdat ons empirische onderzoek zich met name richt op de gulden- en euroreeks van bankbiljetten en munten, vragen wij ons af of de theoretische analyse naar de optimale coupurereeks in genoemde artikelen licht kan werpen op de vraag of de euroreeks theoretisch beter is dan de voormalige guldenreeks. Het blijkt dat de optimale reeks de oplossing is van een multicriteria optimalisatieprobleem. Enerzijds moet de optimale reeks zorgen voor betalingen waarvoor gemiddeld zo min mogelijk bankbiljetten en munten nodig zijn. Anderzijds moet een coupurereeks zo min mogelijk verschillende coupures hebben, aangezien iedere extra coupure kosten met zich meebrengt, zoals ontwikkelkosten en drukkosten. Afhankelijk van welk criterium het meeste gewicht wordt gegeven, en afhankelijk van welke rekenmethode gebruikt wordt, blijkt de optimale oplossing een reeks met 1-2-4... of 1-3-9... te zijn. Wil men het criterium eraan toevoegen dat het gebaseerd moet zijn op het decimale stelsel, ofwel dat 10 de basis vormt van het stelsel, dan lijkt 1-2-5 het ideale compromis. Toch kunnen we in deze artikelen geen aanleiding vinden om te denken dat de wereldwijd meest gebruikte 1-2-5 reeks voor contant geld theoretisch efficiënter is dan de  $1-2\frac{1}{2}-5$  reeks. Enkele artikelen bevestigen dat ze in gelijke mate aan de gestelde criteria voldoen.

Om definitief uitsluitsel te geven in deze kwestie, voeren we een theoretische analyse uit op beide reeksen. Hiervoor maken we gebruik van een theoretisch concept van efficiënte betalingen, dat is geïntroduceerd door Cramer (1983). Efficiënte betalingen zijn gedefinieerd als betalingen waarbij zo min mogelijk bankbiljetten en munten worden uitgewisseld. Ieder betaalbedrag heeft één of meer efficiënte betaalschema's. Met behulp van een algoritme, kan voor iedere coupurereeks en ieder betaalbedrag uitgerekend worden wat de efficiënte betaalschema's zijn. Alhoewel het onwaarschijnlijk is dat in de praktijk alle betalingen efficiënt worden gedaan, biedt het concept een eenvoudige manier om de theoretische verschillen tussen coupurereeksen te vergelijken. Eerst bepalen we de efficiënte betaalschema's voor alle guldenbedragen tussen NLG 0,05 en NLG 220,35, en voor dezelfde omgerekende bedragen in euro. De statistieken laten zien dat de guldenreeks en de euroreeks vrijwel even efficiënt zijn. De euroreeks biedt meer mogelijkheden om efficiënt te betalen, dat wil zeggen er zijn gemiddeld meer efficiënte betaalschema's beschikbaar per bedrag, maar gemiddeld genomen zijn er minder guldenbiljetten- en munten nodig om een bedrag te betalen. Deze verschillen zijn echter niet groot, en als men deze beide

aspecten van efficiënt betalen gecombineerd beschouwd, dan is er nagenoeg geen verschil. Kortom, theoretisch gezien, is het Nederlandse publiek er niet op vooruit, maar ook niet op achteruit gegaan. Echter, als we deze exercitie herhalen voor een euroreeks zonder de kleinste munten die ook in de guldenreeks niet bestonden, namelijk de 1- en 2-cent, dan blijkt dat de euroreeks wel degelijk efficiënter wordt. Gemiddeld zouden er per bedrag dan 0,8 munten minder nodig zijn. De vergelijking is daarmee nog niet geheel compleet, want de guldenreeks had geen munt van 50 cent. Een definitieve vergelijking tussen een pure 1-2-5 reeks en een pure  $1-2\frac{1}{2}$ -5 reeks voor bedragen 0 tot 100 geldeenheden, laat uiteindelijk zien dat de verschillen van efficiëntie tussen beide reeksen nihil zijn. Dit zijn echter alleen theoretische resultaten. In de volgende hoofdstukken gaan we betaalgedrag met verschillende reeksen empirisch onderzoeken. Tot slot doen we nog enkele rekenexercities waarbij we steeds één biljetcoupure uit de euroreeks halen. De theoretische resultaten duiden erop dat wanneer het 10- of 100-euro biljet niet beschikbaar is voor betalingen, theoretisch de minst negatieve effecten te verwachten zijn. Het 50-eurobiljet daarentegen, is het minst misbaar. Deze resultaten gebruiken we opnieuw in hoofdstuk 6 van dit proefschrift.

De theorie van efficiënte betalingen is ons inziens te restrictief in de praktijk. De onderliggende aannames zijn namelijk dat (i) individuen bankbiljetten en munten op een efficiënte manier gebruiken, (ii) individuen geen voorkeuren hebben voor betalen met bepaalde coupures, (iii) individuen bij iedere betaling de beschikking hebben over alle benodigde bankbiljetten en munten, en (iv) betaalbedragen even vaak voorkomen. Premisse (i) en (ii) betreffen veronderstellingen over betaalgedrag die nog niet empirisch getoetst zijn. Of individuen daadwerkelijk streven naar betalingen waarin zo min mogelijk bankbiljetten en munten bij betrokken zijn, is onbekend. Verder kunnen individuen redenen hebben om bepaalde voorkeuren voor bankbiljetten of munten te hebben, of juist niet. Zo zouden kleine munten minder geschikt kunnen worden gevonden, of is men sneller geneigd bankbiljetten met makkelijke nominale waarden te gebruiken. In dat geval wordt de potentiële efficiëntie van een coupurereeks niet ten volle benut. Premisse (iii) en (iv) zijn onmiskenbaar onjuist in de praktijk. Iedere individu is bij een contante betaling beperkt tot de inhoud van zijn of haar portemonnee. Verder is het bekend dat de frequentieverdeling van betaalbedragen geconcentreerd is rond kleine bedragen. Om de twee laatstgenoemde redenen is het van belang empirisch onderzoek te doen. Voor dit onderzoek verkennen we een alternatief theoretisch raamwerk voor individueel betaalgedrag, dat minder restrictief is dan efficiëntie. Daarvoor leunen we sterk op concepten uit de gedragswetenschap en merkkeuze onderzoek. We presenteren een raamwerk dat ervanuit

gaat dat contante betalingen de uitkomst zijn van een individueel keuzeprocess. In het algemeen houdt zo'n keuzeprocess in dat individuen informatie verzamelen over verschillende beschikbare alternatieven en deze informatie gebruiken om de alternatieven te vergelijken en een definitieve keuze maken. De theorieën over keuzeprocessen in de gedragswetenschap geven aan dat het waarschijnlijk is dat individuen het keuzeprocess vereenvoudigen, en dat de mate waarin dat gebeurt afhankelijk is van de complexiteit van de keuze en van omgevingsfactoren. Wij beargumenteren dat de karakteristieken van een contante betaling aanleiding geven om te denken dat in het betaalproces wel degelijk sprake is van vereenvoudiging. Het betalend individu moet immers berekeningen doen om na te gaan welke combinatie van bankbiljetten en munten een gezamenlijke waarde heeft die hoger is dan het te betalen bedrag. Daarbij is het nodig om in te schatten welke combinaties handig zijn voor de kassière, of welk wisselgeld het individu eventueel terugkrijgt. Hoe meer bankbiljetten en vooral munten de portemonnee bevat, hoe meer combinaties het individu zou moeten aflopen. Daarbij kan er tijdsdruk zijn om de betaling zo snel mogelijk te doen, bijvoorbeeld als er een lange rij staat voor de kassa. Gegeven de mogelijkheid dat een betalend individu zijn betaalproces vereenvoudigt, leggen we verband met onderzoek naar merkkeuze. Shocker et al. (1991) bijvoorbeeld, beschrijft een raamwerk voor een merkkeuzeprocess waarin het individu de beschikbare alternatieve in verschillende stappen verkleint om tot een uiteindelijke keuze te komen. De keuzeset van alternatieven die het individu serieus overweegt, wordt de overwegingsset genoemd. In onderzoek naar merkkeuze is deze overwegingsset een bekend fenomeen. Wij gebruiken hetzelfde concept voor een raamwerk voor betaalkeuze. De achterliggende gedachte is dat individuen niet alle beschikbare bankbiljetten en -munten serieus overwegen voor betaling, maar slechts een deelverzameling. In het uiterste geval kiest het individu voor vereenvoudiging door het eerste bankbiljet te gebruiken dat hem of haar opvalt in de portemonnee.

Met deze theoretische verkenning van betaalgedrag komen we tot het formuleren van een gespecificeerde onderzoeksvraag: hoe kunnen we empirisch vaststellen of alle coupures gelijkelijk worden overwogen voor een betaling? Ofwel, hoe kunnen we voorkeuren voor coupures empirisch vaststellen? In hoofdstuk 3 ontwikkelen we hiervoor een econometrisch model, dat we op verschillende data sets van geobserveerde betalingen toepassen in hoofdstukken 4 tot en met 6.

### 8.3 Een econometrisch model

Het model beoogt de keuze van bankbiljetten en munten voor een specifieke betaling te beschrijven. Vanwege de karakteristieken van contante betalingen kunnen standaard keuzemodellen niet zonder meer toegepast worden. Als de uitkomst van een keuzeproces een combinatie van bankbiljetten en munten is, en omdat een coupurereeks al snel 12 verschillende coupures heeft, wordt het onmogelijk om aan iedere mogelijke combinatie afzonderlijk een kans toe te kennen. De kans op een zekere betaalcombinatieuitkomst wordt daarom ontbonden in een opeenvolgende reeks van geconditioneerde kansen. Daarbij is de kans op betaling met een zeker aantal coupures  $d$  geconditioneerd op de betaling met coupures  $d+1$  en hoger. Deze modelleerwijze impliceert een sequentieel betaalproces, waarmee het individu keuzes voor coupures doorloopt van hoog naar laag. Alhoewel er wellicht andere opties zijn, blijkt deze modelkeuze het betaalproces goed te beschrijven. Iedere conditionele kans wordt verondersteld een Poisson verdeling te volgen, waarbij de keuzeset afgeknot wordt door een ondergrens en bovengrens. De bovengrens wordt bepaald door de beperking van de portemonnee-inhoud, een individu kan immers niet met meer biljetten betalen dan hij of zij in de portemonnee heeft. Daarnaast kan een bovengrens van toepassing zijn, omdat een betaling met meerdere coupures haar nut verliest boven een zeker aantal. Zo zal een individu niet met 2 biljetten van 100 gulden betalen als het te betalen bedrag lager is dan 100 gulden. In andere gevallen is het noodzakelijk om met een zeker aantal coupures te betalen, dit is dan de ondergrens. Bijvoorbeeld, als men moet kiezen met hoeveel biljetten van 100 te betalen, 2 biljetten van 100 beschikbaar heeft, het betaalbedrag 140 bedraagt, en de rest van de portemonnee maar een waarde van 50 heeft. In dit geval moet met minimaal 1 biljet van 100 betaald worden, en is 1 dus de ondergrens.

Voor het resulterende geknottede Poisson regressiemodel is de verwachting dat het te betalen bedrag de grootste verklarende waarde zal hebben. Hiertoe wordt naast een constante een variabele gedefinieerd die de waarde heeft van het bedrag dat nog betaald moet worden, gegeven de betaling met hogere coupures, geschaald naar de nominale waarde van de betreffende coupure. De verwachting is dat de parameterwaarde voor deze variabele positief is. Dat wil zeggen, een toename in het betaalbedrag leidt tot een toename van het verwachte aantal coupures in de betaling. Dus, hoe verder het bedrag ligt van de waarde van de coupure die wordt overwogen, hoe meer biljetten of munten van die coupure in de betaling zullen worden opgenomen. Door de schaling is het mogelijk de effecten van deze variabele op de betaalkeuze van coupures onderling te vergelijken. De parameters van het model kunnen geschat worden met maximum likelihood. Om antwoord te geven op

onze onderzoeksvraag, kunnen we een likelihood ratio toets uitvoeren op schattingsresultaten van het hierboven beschreven betaalmiddel. Daartoe worden de uitkomsten van een geresliceerd model vergeleken met de uitkomsten van een ongeresliceerd model. De restrictie is in dit geval dat de parameters van de variabelen gelijk zijn voor verschillende coupures. De getoetste hypothese is dan dat alle coupures gelijke kans hebben om te worden gekozen voor betaling in gelijke betaalomstandigheden. In ons betaalmiddel worden deze betaalomstandigheden bepaald door de verklarende variabelen en de afknotting, ofwel de relatieve afstand van het te betalen bedrag tot de waarde van de coupure die in overweging is en de begrenzing van de keuzeset (gelijke portemonnees bijvoorbeeld). Als de parameterwaarden voor twee coupures gelijk zijn, is de kans even groot dat met een gelijk aantal biljetten of munten van deze coupures in gelijke betaalomstandigheden wordt betaald. Is dat niet het geval, dan is er sprake van een voorkeur van de ene coupure boven de andere. Als dat voor één of meerdere coupures in een coupurereeks geldt, dan wordt de betreffende coupurereeks suboptimaal gebruikt.

## 8.4 Empirische resultaten voor de Nederlandse gulden

In de eerste empirische toepassing van ons betaalmiddel, maken we gebruik van een grote data set van guldenbetalingen die in 1998 zijn waargenomen bij zo'n 70 winkels in de detailhandel. Dit onderzoek, dat in opdracht van De Nederlandsche Bank is uitgevoerd, leverde gedetailleerde gegevens op van bijna 40.000 contante betalingen. Daarvan is genoteerd welke coupures als betaling werden gebruikt, welke bankbiljetten en munten eventueel als wisselgeld werden teruggegeven, en wat het te betalen bedrag betrof. Destijds is niet de portemonnee-inhoud behorend bij al deze betalingen genoteerd. Omdat kennis van de portemonnee-inhoud noodzakelijk is voor ons betaalmiddel, is een separaat onderzoek gedaan, waarin de respondenten per email werd gevraagd hun portemonnee-inhoud op te geven. Dit onderzoek leverde een tweede data set van 840 waarnemingen op. Uit de eerste data set van betalingen zijn bijna 2000 betalingen willekeurig geselecteerd. Voor iedere betaling zijn 10 mogelijke portemonnees geselecteerd uit de tweede data set. Voor schatting van de parameters in het betaalmiddel is gemiddeld over deze portemonnees. Daarnaast is ook gekeken naar de mate waarin individuen efficiënt hadden betaald. Hiervoor is het algoritme van Cramer aangepast om efficiënte betaalschema's te kunnen berekenen gegeven een portemonnee-inhoud. Het blijkt dat de betalingen van 34% van de individuen voor alle 10 portemonnees efficiënt was. In de overige 66% betalingen waren meer bankbiljetten en munten gebruikt dan nodig voor minstens 1 van de 10 gegenereerde



portemonnees.

Naar verwachting waren de geschatte parameters voor de bedragvariabele in het betaalmodel significant en positief. Om de relatie tussen het relatieve betaalbedrag en de verwachtingswaarde voor een zekere coupure inzichtelijk te maken, is deze grafisch weergegeven met behulp van simulaties. Daarin is een groot aantal parameterwaarden gegenereerd met behulp van de schattingsresultaten, en vervolgens de verwachtingswaarde bepaald. Hierbij is aangenomen dat de ondergrens en bovengrens van de keuzeset constant is over coupures. Met behulp van deze simulatieresultaten zijn verschillende curves gepresenteerd, namelijk de verwachtingswaarde voor een reeks van relatieve betaalbedragen voor iedere coupure en het bijbehorende 95% betrouwbaarheidsinterval. De resulterende curves zijn oplopend, hetgeen de positieve relatie tussen verwachtingswaarde en het relatieve betaalbedrag weerspiegelt. De grafische presentatie geeft verder weer dat de curve van het 50-guldenbiljet duidelijk onder de overige curves valt. Indien coupures in gelijke mate worden overwogen voor betaling onder gelijke betaalomstandigheden, ofwel indien er sprake is van indifferentie, dan zouden de curves voor verschillende coupures samenvallen. De betrouwbaarheidsintervallen voor het 50-guldenbiljet in vergelijking tot die van het 100-guldenbiljet, laten verder zien dat dit verschil significant is. De conclusie is dan ook dat het 50-guldenbiljet minder populair was dan de andere guldenbiljetten in contante betalingen. Dezelfde conclusie kan getrokken worden voor de rijksdaalder in vergelijking met andere guldenmunten. Kortom, er bestond een voorkeursrangorde voor guldencoupures in Nederland.

## 8.5 Empirische resultaten voor de euro

In een tweede toepassing van het betaalmodel gebruiken we een speciaal voor dit doel verzamelde data set van eurobetalingen. Hiertoe hebben studenten econometrie gedurende meerdere dagen in twee winkels 272 betalingen geobserveerd. Voor elke transactie is het betreffende individu achteraf gevraagd om mee te werken aan het onderzoek. Degenen die daartoe bereid waren hebben een enquête ingevuld waarin onder andere gevraagd werd om de portemonne-inhoud op te geven. Zodoende was het mogelijk te herleiden welke portemonnee-inhoud bij de waargenomen betaling hoorde. In de enquête waren eveneens vragen opgenomen over demografische kenmerken, ervaring als kassière, en aankoopfrequentie. Tot slot hebben de respondenten vier wiskundige reeksen ingevuld. Deze aanvullende informatie is gebruikt om te toetsen of dergelijke karakteristieken invloed hebben op wel of niet efficiënt betalen met behulp van een logit model. Geen van de karakteristieken

bleek efficiënt betaalgedrag significant te beïnvloeden. Als de 272 waargenomen betalingen werden vergeleken met de efficiënte betaalschema's behorend bij het betaalbedrag en gegeven de portemonnee, dan bleek ruim 60% efficiënt betaald te hebben. Dit is een significante meerderheid.

Met deze 272 eurobetalingen hebben we eveneens de parameters van ons betaalmodel geschat. De schattingsresultaten geven geen indicatie van duidelijke voorkeuren voor bepaalde eurocoupures. Daarmee lijkt het erop dat de euroreeks in Nederland beter gebruikt wordt dan de voormalige guldenreeks gebruikt werd in 1998. Dit resultaat wordt bevestigd door de schattingsresultaten van een tweede data set. Dit betreft een grotere data set van 840 betalingen, die via een internetsurvey zijn verzameld. Omdat dit verklaard betaalgedrag achteraf betreft, in plaats van waargenomen betaalgedrag ten tijde van de transactie, gebruiken we deze data set alleen voor validatiedoeleinden. Ook voor deze data set blijkt dat indifferentie niet wordt verworpen door de LR-toets.

## 8.6 Empirische resultaten in een experimentele setting

Een derde toepassing van het betaalmodel gebruikt waargenomen betalingen in een experimentele setting. Daarvoor laten we verschillende keren het Euro Monopoly<sup>®</sup> spel spelen. In dit spel worden transacties verricht met speelgoedbiljetten die lijken op eurobiljetten. Het originele spel heeft alle biljetcoupures, behalve de 200-euro, en een 1-euro munt. Om te zorgen dat er voldoende 'geld' in het spel is, hebben we extra speelgoedbiljetten laten maken, en bovendien de reeks aangevuld met een soortgelijk speelgoedbiljet van 200-euro. Vervolgens hebben we het spel twee keer laten spelen met 3 of 4 spelers om voldoende waarnemingen te verzamelen. Om zoveel mogelijk normale betaalsituaties na te bootsen, hebben we de spelers voorzien van portemonnees. Gedurende het spel zijn alle transacties genoteerd, en de bijbehorende portemonnee geregistreerd. Dit experiment hebben we zes keer uitgevoerd, één keer met alle eurocoupures, en daarna vijf keer met steeds één coupure verwijderd. Dit betroffen achtereenvolgens de coupures 200, 100, 50, 20 en 10 euro. Daarmee hadden we de mogelijkheid om betaalgedrag te bestuderen als één van de eurocoupures niet beschikbaar is. Met de resulterende transactie- en portemonneedata hebben we voor ieder experiment de parameters van ons betaalmodel geschat. Met behulp van LR-toetsen konden we vervolgens vaststellen óf er zekere voorkeuren voor biljetten te signaleren waren, en zo ja, welk(e) biljet(ten) het dan betrof. Het bleek dat het experiment met alle eurocoupures afwijkend betaalgedrag vertoonde voor het 50-euro biljet. Dit

resultaat vormt een contrast met de resultaten van waargenomen betaalgedrag in de praktijk uit hoofdstuk 5. Deze afwijking kan mogelijk worden verklaard uit het verschil tussen de transactie-omvang van contante betalingen in de dagelijkse praktijk en die van de betalingen in Monopoly<sup>®</sup>. Deze zijn veelal hoger en vaak een veelvoud van 50. Bij de experimenten waarin de 100- en 10 euro was weggelaten bleek dit afwijkende betaalgedrag verdwenen. De LR-toets verwierp de hypothese van indifferentie voor deze coupurereeksen niet. Ook als het 50-euro biljet zelf uit de coupureeks werd weggelaten, verbeterde het betaalgedrag, omdat geen afwijkend gebruik van coupures in vergelijkbare betaalsituaties meer vast te stellen was. Tot slot, door het weglaten van een 200- of 20-euro biljet verslechterde het betaalgedrag nog verder, dat wil zeggen de verschillen in betaalgedrag tussen coupures werden groter. Deze resultaten geven aan dat de 200- en 20-euro cruciaal zijn voor betaaltransacties, en de overige coupures gemist kunnen worden. Dit empirische resultaat komt overeen met de theoretische conclusie in hoofdstuk 2.

## 8.7 Conclusies

Uit het in dit proefschrift gepresenteerde onderzoek kunnen we het volgende concluderen. Ten eerste, om betaalgedrag te onderzoeken is het nodig om empirisch analyse te doen op het individuele betaalniveau. Dit vraagt om kostbare, arbeidsintensieve, dataverzameling, waarbij het delicate karakter van geld bij het publiek in ogenschouw moet worden genomen. Het is echter niet genoeg om alleen data beschikbaar te hebben. Omdat betaalgedrag afhankelijk is van de specifieke betaalomstandigheden, zoals bedrag en portemonnee, is het nodig om een modelleerbenadering te volgen. Het model dat we in dit proefschrift hebben gepresenteerd, biedt de mogelijkheid om de vraag te beantwoorden of zekere coupures in de praktijk meer of juist minder populair zijn. Daarbij kan het model feitelijk in ieder land en op iedere coupurereeks worden toegepast, mits de data beschikbaar is. Ten tweede suggereren de onderzoeksresultaten voor Nederland dat met de introductie van eurobankbiljetten en -munten in 2002 het contante betaalsysteem in Nederland is verbeterd. In tegenstelling tot de vastgestelde voorkeursrangorde voor gulden coupures in 1998, lijken de euro coupures even populair te zijn. Helaas konden we dit resultaat niet overhevelen naar een experimentele setting. Wel kon worden vastgesteld dat het weglaten van een biljet coupure wel degelijk effect heeft op betaalgedrag, waarbij het wegnemen van de ene coupure negatief kan zijn, terwijl het wegnemen van een andere juist evenwichtiger betaalgedrag tot gevolg lijkt te hebben. Tot slot is het de moeite waard te benadrukken dat de theoretische analyse laat zien dat het de efficiëntie van betalen ten goede komt als

de 1- en 2-eurocent uit het betalingsverkeer verdwijnen. Dit is overigens niet empirisch getoetst, maar te verwachten valt dat de kleine munten in de praktijk minder efficiënt worden gebruikt dan in de theoretische analyse is verondersteld. Dan zou het verwijderen van de laagste eurocenten uit het betalingsverkeer een nog groter positief effect hebben op de efficiëntie van contant betalen.

Ons onderzoek heeft een start gemaakt met de empirische analyse van individueel contant betaalgedrag. Zulk onderzoek kan nog in vele richtingen worden voortgezet. Het zou bijvoorbeeld interessant zijn of de conclusies die getrokken zijn voor Nederland ook gelden voor andere landen. In het bijzonder gaat het dan om landen die de euro (nog) niet hebben ingevoerd, zoals Denemarken, Zweden en Engeland, of die landen waar ze een mix van de 1-2-5 en  $1-2\frac{1}{2}$ -5 reeks hebben in hun coupurereeks, zoals de VS. Verder zou het de moeite waard zijn om te onderzoeken hoe de experimentele setting met Monopoly kan worden veranderd zodat zij beter aansluit op de dagelijkse betalingen in winkels. Dan zou het immers mogelijk zijn om te experimenteren met bijzondere coupurereeksen, een mogelijkheid die een centrale bank nooit zal hebben. Stel bijvoorbeeld, dat we kunnen observeren hoe het publiek zou reageren op de introductie van bankbiljetten met waardes als 25, 40 of 300 euro. Ook zou het interessant zijn het gebruik van geheel onbekende coupurereeksen te onderzoeken, zoals 1-3-9-... Daarnaast speelt de vraag hoeveel coupures een reeks idealiter zou hebben. In een experimentele setting zou het mogelijk zijn te starten met een minimaal aantal, vervolgens steeds coupures toe te voegen om te concluderen bij welk aantal coupures buiten gebruik beginnen te vallen. Tot slot kan worden geëxperimenteerd met afwijkende karakteristieken van bankbiljetten of munten, zoals opvallende kleuren of juist onopvallende kleuren, aantrekkelijke of onaantrekkelijke ontwerpen, en verschillende formaten.

Contante betalingen zijn op zichzelf al een interessant genoeg keuzeproces om verder te onderzoeken. We hebben ontdekt dat het uitdagend is dit keuzeproces in een geschikt model te vatten. De zoektocht naar andere geschikte modellen kan worden uitgebreid, vooral in de richting van voorspelmodellen. Met een goede voorspelkracht, zou een dergelijk model gebruikt kunnen worden om betalingen te simuleren. Dit zou een interessante toepassing kunnen zijn voor bijvoorbeeld de detailhandel, om hun kassa-inhoud te optimaliseren, of de samenstelling van het wisselgeld dat ze geregeld bij banken moeten bestellen. Daarnaast kan de mogelijkheid onderzocht worden in hoeverre het effect van veranderingen in portemonnee-inhouden op betaalgedrag gemeten kan worden. Dit is bijvoorbeeld het geval als banken besluiten de coupuresamenstelling van hun geldautomaten te wijzigen. Als een dataset van betalingen groot genoeg is, en de bijbehorende

portemonnee-inhouden bekend zijn, dan is het wellicht mogelijk om betaalgedrag voor verschillende bedragcategorieën te vergelijken. Tot slot kan het concept van een contante betaling als keuzeproces worden uitgebreid naar de keuze van elektronisch betalen en contant betalen, waarbij de complexiteit van de contante betaling, en de inhoud van de portemonnee van invloed kan zijn op de keuze om wel of niet contant te betalen.

Hiermee zijn we weer terug bij het begin van dit proefschrift. Gezien de huidige ontwikkelingen lijken moderne betaalmiddelen de toekomst te hebben, maar over het gebruik van het meest traditionele betaalmiddel, namelijk bankbiljetten en munten, blijven nog steeds veel vragen onbeantwoord. We menen echter met dit onderzoek te hebben bijgedragen aan een beter begrip van contant betalen.

# Appendix A

## Efficient payment algorithms

This appendix describes the algorithm that can be used to compute efficient payment schemes, as defined by Cramer (1983), for any range of amounts and currency range. It is used in Sections 2.2, 4.3 and 5.3. We first describe Cramer's original algorithm for the guilder range, see Cramer (1986). Note that it applies similarly to the euro range. We next describe a modified algorithm for computing efficient payments, that takes the restriction of the wallet into account.

### A.1 Cramer's algorithm

We take a range of amounts that are of interest, say NLG 0.05 until NLG 100 with intervals of NLG 0.05. This yields a list of 2,000 amounts in total. The goal of the algorithm is to cover each amount in this list with an efficient combination of tokens. The steps of the algorithm are as follows:

1. The algorithm starts by covering all amounts in the list that can be paid by only one token. For example, in the Dutch system and with the range given above, the amounts NLG 0.05, 0.10, 0.25, 1, 2.50, 5, 10, 25 and 100 in our list would be covered.
2. Next, all amounts that can be paid with 2 tokens, either given by the consumer as payment or by the retailer as change, are computed. If in this step we find an amount that was already covered with only one token in the previous step, such as NLG 50 + NLG 50, we do not add this pair of two tokens to the list, since two tokens is not efficient for this amount. An example amount that would be covered in this step is NLG 0.20. The combination needed here is NLG 0.25 and NLG 0.05

as change. Another scheme with the same amount of tokens is NLG 0.10 + NLG 0.10. Both these example schemes are stored in this step.

3. To the pairs that were found efficient in the previous step, we add each token once, both with positive and negative sign. For a given pair, this results in  $2D$  extra potential combinations with an additional token each. Adding a token with a positive sign to a combination, which has this same token with a negative sign, would yield a combination with less tokens and is therefore ignored. Also, we have the restriction that the highest token has a positive sign.

With these combinations, we cover all resulting amounts, provided they were not already covered by less tokens.

4. We repeat step 3 until all amounts on the list are covered.

This algorithm results in a number of efficient combinations for each possible amount in the range specified. Many amounts can be paid efficiently with more than one combination.

## A.2 Cramer's algorithm, modified for wallets

If a certain individual  $i$ , with his or her amount  $A_i$  and wallet  $w_i$  can make an efficient payment (as computed with the standard Cramer algorithm, and the wallet  $w_i$  suffices to pay according to the efficient scheme), we can simply take this efficient scheme. However, if the wallet contents of the individual is not enough to pay according to the efficient scheme, we need to find the restricted efficient scheme, that is, the combination of tokens that minimizes the numbers of tokens used, given the wallet contents. In this case, the following algorithm is executed for this particular amount  $A_i$  and wallet  $w_i$ :

1. The algorithm starts by computing all amounts in this range that can be covered by only one token. If the wallet contents  $w_i$  of the individual suffices, the combination is stored.
2. Next, all amounts that can be paid with 2 tokens, either given by the consumer or by the retailer as change, are computed. Again, only if the wallet contents of the individual suffices, the combination is stored.
3. To the pairs that were found efficient in the previous step, we add each token once, both with positive and negative sign. With these combinations, we fill the list with

the resulting amounts, provided they are not already covered by less tokens, and provided the combination fits in the individual's wallet  $w_i$ .

4. We repeat step 3 until the amount  $A_i$  is covered. Note that it is not enough to only store the combination which pays the given amount  $A_i$  exactly. We need to store each combination that fits in the individual's wallet, since we use these as the starting point for the schemes in the next step.

This algorithm is executed for each transaction for which the wallet does not suffice to pay the standard efficient scheme.





# Appendix B

## Computer code in Eviews

This appendix gives the relevant Eviews code, that can be used to estimate the parameters of our cash payment model, as it is described in Chapter 3. The code presented below is an example of a model that includes denominations 100, 50, 20, 10, 5, 2 and 1. The relevant restrictions used to conduct the likelihood ratio test, as it is described in Section 3.4, are included as well.

```
@logl logl1
```

```
@temp c1 c2 c3 c4 c5 c6 c7 l1 l2 l3 l4 l5 l6 l7
```

```
lambda1=@exp(c(11)+c(12)*acorr500)*(ub500>lb500)+3*(ub500=lb500)
```

```
lambda2=@exp(c(21)+c(22)*acorr200)*(ub200>lb200)+3*(ub200=lb200)
```

```
lambda3=@exp(c(31)+c(32)*acorr100)*(ub100>lb100)+3*(ub100=lb100)
```

```
lambda4=@exp(c(41)+c(42)*acorr50)*(ub50>lb50)+3*(ub50=lb50)
```

```
lambda5=@exp(c(51)+c(52)*acorr20)*(ub20>lb20)+3*(ub20=lb20)
```

```
lambda6=@exp(c(61)+c(62)*acorr10)*(ub10>lb10)+3*(ub2=lb10)
```

```
lambda7=@exp(c(71)+c(72)*acorr5)*(ub5>lb5)+3*(ub5=lb5)
```

```
c1=@cpoisson(ub500,lambda1)-@cpoisson(lb500-1, lambda1)*(lb500>0)
```

```
c2=@cpoisson(ub200,lambda2)-@cpoisson(lb200-1, lambda2)*(lb200>0)
```

```
c3=@cpoisson(ub100,lambda3)-@cpoisson(lb100-1, lambda3)*(lb100>0)
```

```
c4=@cpoisson(ub50,lambda4)-@cpoisson(lb50-1, lambda4)*(lb50>0)
```

```
c5=@cpoisson(ub20,lambda5)-@cpoisson(lb20-1, lambda5)*(lb20>0)
```

```
c6=@cpoisson(ub10,lambda6)-@cpoisson(lb10-1, lambda6)*(lb10>0)
```

```
c7=@cpoisson(ub5,lambda7)-@cpoisson(lb5-1, lambda7)*(lb5>0)
```

```

l1=(ub500-lb500>0)*(-lambda1+b500*@log(lambda1)-@log(c1)-@log(@fact(b500)))
l2=(ub200-lb200>0)*(-lambda2+b200*@log(lambda2)-@log(c2)-@log(@fact(b200)))
l3=(ub100-lb100>0)*(-lambda3+b100*@log(lambda3)-@log(c3)-@log(@fact(b100)))
l4=(ub50-lb50>0)*(-lambda4+b50*@log(lambda4)-@log(c4)-@log(@fact(b50)))
l5=(ub20-lb20>0)*(-lambda5+b20*@log(lambda5)-@log(c5)-@log(@fact(b20)))
l6=(ub10-lb10>0)*(-lambda6+b10*@log(lambda6)-@log(c6)-@log(@fact(b10)))
l7=(ub5-lb5>0)*(-lambda7+b5*@log(lambda7)-@log(c7)-@log(@fact(b5)))

```

```
logl1 = l1+l2+l3+l4+l5+l6+l7
```

Restriction corresponding with indifference, all notes

```
c(11)=c(21)=c(31)=c(41)=c(51)=c(61)=c(71)
```

```
c(12)=c(22)=c(32)=c(42)=c(52)=c(62)=c(72)
```

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Jeanine Kippers (1973) obtained her master's degree in econometrics from the Erasmus University Rotterdam in 1997. In the same year she was employed by De Nederlandsche Bank. She participated in the preparations of the cash changeover from guilder to euro in 2002, and contributed to the development of a European cash distribution system and the restructuring of the national distribution system. During her employment at the central bank she started her research on currency use, which resulted in several papers that have been published or are currently under review. In January 2004 she joined the Pension- and Insurance Supervisory authority, as a policy advisor in the (international) field of insurance supervision.



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## Empirical Studies on Cash Payments

Cash is still the most common means of daily payments. The large number of cash payments is supported by a costly distribution system in which retailers, banks and central banks participate. Currency is issued in a range of bank note and coin denominations to facilitate efficiency in cash payments.

The purpose of this thesis is to study the performance of a currency range in practice. It presents a number of empirical studies on cash payments at the individual payment level, whereby cash payments are viewed as the outcome of a choice process. This type of analysis calls for labor-intensive data collection methods and the development of a sophisticated econometric model. This thesis introduces such a model for cash payments, and it reviews its application to three unique data sets of cash payments. The data concern payments in the Netherlands before and after the transition from the guilder to the euro in 2002, and payments in an experimental setting. The estimation results allow for an assessment of the use of different bank note and coin denominations in cash payments, and of possible preferences for one of more denominations.

Currency research generally approaches currency use from a macro-economic point of view. The novelty of this thesis is that it contributes to currency research by focusing on individual cash payments.

## ERIM

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