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EMPIRICS OF STRATEGIC INTERDEPENDENCE: THE CASE OF THE RACIAL TIPPING POINT

William Easterly

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Empirics of Strategic Interdependence: The Case of the Racial Tipping Point William Easterly NBER Working Paper No. 15069 June 2009 JEL No. D85,O10,R0,Z13

ABSTRACT

The Schelling model of a "tipping point" in racial segregation, in which whites flee a neighborhood once a threshold of nonwhites is reached, is a canonical model of strategic interdependence. The idea of "tipping" explaining segregation is widely accepted in the academic literature and popular media. I use census tract data for metropolitan areas of the US from 1970 to 2000 to test the predictions of the Schelling model and find that this particular model of strategic interaction largely fails the tests. There is more "white flight" out of neighborhoods with a high initial share of whites than out of more racially mixed neighborhoods

William Easterly New York University Department of Economics 19 W. 4th Street, 6th floor New York NY 10012 and NBER william.easterly@nyu.edu

1. Introduction

Models of strategic interaction are common in the economic growth literature, as well as in many other fields. For example, in human capital spillover models of economic growth, your incentive to acquire human capital depends on the human capital of others. If spillovers take place within neighborhoods, then strategic interactions affect neighborhood formation, human capital of different ethnic groups, and overall inequality (Borjas 1993, 1996, Benabou 1993, 1996, Durlauf 2002, 1999, 1996). These models often feature multiple equilibria and sensitivity to initial conditions. Although the theory is well developed, there has been only limited empirical testing of strategic interactions and sensitivity to initial conditions.²

One of the most famous models of strategic interaction in economics is Thomas Schelling's (1971) elegant model of racial segregation (see its coverage in Dixit and Nalebuff 1991, for example). He shows how only a modest preference of whites to live next to other whites could result in nearly complete residential segregation, because of the instability of intermediate points where one agent's residential location depends on the actions of other agents in the neighborhood. In this model, even a relatively small fraction of nonwhites could cause the neighborhood to "tip" from completely white to completely nonwhite. The fraction at which this happens is called the "tipping point."

Segregation outcomes might seem to reflect segregationist preferences by whites, but in the Schelling model the degree of segregation exceeds what all but a small minority of the white population desires. If there are differences in average human capital

² Borjas 1993, 1996 shows empirically that outcomes for individuals are affected by "neighborhood capital" and "ethnic capital", but does not test for sensitivity to initial conditions in neighborhood formation.

between whites and blacks, and there are spillovers within neighborhoods, then residential segregation has important implications for black-white income differences. Card and Rothstein (2007) find that the black-white test score gap is higher in more segregated cities. Hence, Schelling's model is potentially one of the important building blocks in understanding inequality (Durlauf 2002 cites it in this context).

The tipping view of neighborhood change had been around long before Schelling's piece. Schelling (1971) says he was inspired by articles from the 1950s, where the tipping process was described as universal, as was the instability of mixed neighborhoods. Once a neighborhood had begun to change from white to black, there was rarely a reversal. The process was very nonlinear. An article in 1960 defined it thus:

Although the movement of whites out of the area may proceed at varying rates of speed, a "tipping point" is soon reached which sets off a wholesale flight of whites. It is not too long before the community becomes predominantly Negro.³

The idea of the "tipping point" is very much alive today both in academic literature and in popular folklore. Sociologists Douglas Massey and Nancy Denton (1993) describe how a white "majority still feel uncomfortable in any neighborhood that contains more than a few black residents; and as the percentage of blacks rises, the number of whites who say they would refuse to enter or would try to move out increases sharply." Ellen (2000) sums up the current conventional wisdom similarly: "racially integrated neighborhoods cannot stay that way for long…because as soon as the black population in a neighborhood has reached some "tipping" point, whites move away in droves."⁴ A recent paper by Card, Mas, and Rothstein (2008) (discussed further below) reaffirms the

³ Oscar Cohen quoted in Wolf (1963).

⁴ Ellen (2000) does not share this conventional wisdom, arguing for a more nuanced view of "white avoidance" of integrated neighborhoods for reasons unrelated to race. She argues that racially mixed neighborhoods in 1990 were more common and more stable than the conventional wisdom would have it.

story that, in their words, "once the minority share exceeds the tipping point, the neighborhood transitions rapidly to a very high minority share."

A large social science literature studies racial segregation. However, the Schelling model has undergone surprisingly little large-scale empirical testing on residential neighborhoods. There has been extensive empirical testing of the determinants of segregation using survey methods to ascertain people's preferences for segregation, or testing small samples of neighborhoods or school districts, or testing cross-city determinants of segregation, some of which address the Schelling hypothesis (with mixed results).⁵ In a Galllup survey in 1997, 25 percent of whites said they would move if blacks came in "great numbers" into their neighborhood (which was a large decrease from 73 percent in a similar survey in 1966). This seems to indicate an increased tolerance for racial integration among whites over time. However, the researchers who report this survey result (Schuman et al. 1997) still believe in the tipping point model: "the upward trend does not seem to match reality if compared with the exodus of white families that often occurs when large numbers of black families move into a previously white neighborhood" (pp. 152-153).

The Multi-City Study of Urban Inequality (O'Connor, Tilly and Bobo 2001) conducted a more nuanced survey in Atlanta, Boston, Detroit, and Los Angeles. They showed whites five different cards indicating neighborhood composition ranging from

⁵ See Clark (1991), Galster (1988), Clark (1988), Massey and Gross (1991), Wilson (1985), Farley et al. (1994), Giles (1978), Farley and Frey (1994), Hwang and Murdock (1998), Giles et al. (1975), Wolf (1963), and Schwab and Marsh (1980). Denton and Massey (1991) analyze transition matrices for a large sample of metropolitan census tracts from 1970 to 1980, but do not test the nonlinear dynamic equation implied by the Schelling model. Massey and Denton (1993) extensively discuss neighborhood transitions with census tract data, but do not test the tipping point hypothesis. Crowder (2000) does do a regression of individual level mobility on neighborhood nonwhite share that shows a highly nonlinear relationship as predicted by the tipping point model, but his setup does not lend itself to explicitly testing for a tipping point. Clotfelder (2001) finds the growth of white enrollment in schools declines almost linearly over most of the range of exposure to nonwhites in school districts.

all-white to majority black and asked them if they would "feel comfortable" in such neighborhoods or would be "willing to move in" to such a neighborhood. The affirmative response by whites falls fairly sharply as the black share rises, which would be more consistent with the tipping point hypothesis (for example, only 30 percent of whites would be willing to move into a neighborhood with an 53 percent black majority, with slightly more "feeling comfortable.")⁶

Despite the popularity of the tipping model, there has been little in the way of full-scale test of the tipping point hypothesis with nationwide data on American metropolitan neighborhoods.⁷ The major exception is Card, Mas, and Rothstein 2008, who use the same data as this paper (the data will be described next) and a regression discontinuity design to test for racial tipping points based on the local stability of intermediate points of minority share. They did find evidence of tipping at relatively low levels of minority share. Their methodology has the important advantage that they can derive from the data a different tipping point for each metropolitan area, allowing for different sensitivity to minority share across metropolitan areas. This paper (whose first version preceded Card et al.) differs from Card et al by estimating the global dynamics of tipping based on initial racial composition. To accommodate the highly non-linear prediction of the Schelling model, I estimate the change in white share as a function of a fourth-order polynomial of initial white share. I first assume that the tipping point is the same in all neighborhoods, and then will allow tipping points to vary parametrically with other neighborhood characteristics. These two different methodologies allow for testing

⁶ Charles 2001, pp. 233-237

⁷ Aaronson (2001) and Ellen (2000) also run regressions for neighborhood dynamics using census tract data from 1970 to 1990, but they do not explicitly test the tipping point hypothesis. Both have indirect results that tend to indicate stability of neighborhood racial composition, which would be in line with this paper's conclusions.

of different predictions of the tipping model – the model has predictions both for local instability and for global dynamics. The Card et al. approach economizes on assumptions about parametric forms, however it does so at the cost of being only a test of local instability. Local instability is necessary but not sufficient for confirmation of the tipping model. The advantage of this paper's approach is that it tests also the global dynamics predictions that are also required to confirm the tipping story.

These tests have become feasible thanks to the availability of a new database from the Urban Institute and a firm called Geolytics.com, which matches census tract information from the U.S. censuses for 1970, 1980, 1990, and 2000. This is called the Neighborhood Change Data Base (NCDB). The database covers metropolitan areas; it does not include rural areas.

This database confirms that American neighborhoods continue to be highly segregated in the year 2000, despite some decrease in segregation and despite years of rhetoric and legal action in favor of integration. Nonwhites made up 28 percent of the sample population in the NCDB in 2000. Blacks make up 14 percent of the sample population. If each neighborhood were a random draw of whites and nonwhites, with the probability of drawing a nonwhite = .28, the odds against a neighborhood nonwhite share of less than 10 percent would be astronomical. Yet 35 percent of all census tracts had nonwhite shares less than 10 percent. Similarly, the probability that a nonwhite would live in a neighborhood where the nonwhite share exceeds 50 percent would be extremely low if the population were distributed randomly. Yet the median black lived in a neighborhood that was 52 percent black. The Tauber dissimilarity index, a widely used indicator of segregation, was .53 in the year 2000 for America as a whole (the index

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ranges from 0 if nonwhites are evenly distributed across neighborhoods to 1 if whites and nonwhites are completely segregated). The index can be interpreted as the fraction of either whites or nonwhites that would have to move to achieve even distributions of racial groups across neighborhoods.⁸ I do not attempt in this paper to cover the rich literature on the historical and modern mechanisms determining racial segregation; I am just doing a test of one particular model of segregation.⁹

Of great relevance for the tipping point hypothesis, changes in neighborhoods from majority white to majority nonwhite are common in the dataset. Of the 41,321 urban census tracts in the NCDB that have data for both 1970 and 2000, 3965 neighborhoods had a drop in white share of .5 or greater from 1970 to 2000. Thus nearly 10 percent of the neighborhoods in the sample changed drastically from majority white to majority nonwhite over these 30 years. A weaker definition of tipping, the change from any white majority in 1970 to a nonwhite majority in 2000, reveals 14 percent of the neighborhoods tipped during this 30 year period.

This paper uses this database to conduct tests of some of the predictions of the Schelling "tipping point" model. It will ask the fundamental question of whether the high degree of segregation observed in American neighborhoods is a consequence of the dynamic instability of intermediate points due to strategic interdependence, with only weak preferences for living next to your own racial group.

⁸ See the discussion by Cutler, Glaeser, and Vigdor 1999 of different measures of racial segregation. They also present evidence that segregation declined from 1970 to 1990.

⁹ See Massey and Denton (1993), Ellen (2000), and Meyer (2000) for a richer treatment of the complexities of residential segregation.

2. Schelling's model

Schelling's model is simple and elegant. Suppose that whites' preferences for neighborhood segregation between whites and blacks can be summarized as follows: each white individual j has an individual-specific preference to live in a neighborhood that has at least w_j percent of whites. If white individual j finds himself in a neighborhood containing less than w_j percent of whites, then he will exit the neighborhood. As long as the neighborhood contains more than w_j percent of whites, then individual j will stay in the neighborhood. Whites have diverse preferences for racial segregation ranging from integrationist to segregationist, which can be summarized by a cumulative density function increasing from zero to one over w_j from zero to one. Thus, the cumulative density function gives us the percent of whites that have an w_j less than or equal to w. The CDF therefore shows the percent of whites that will live in a neighborhood that is w percent white – it is all those who have an w_j less than or equal to w.

To relate the CDF to the whites who desire to live in the neighborhood as a fraction of the neighborhood population, one set of assumptions consistent with his model is that whites have a right of first refusal on the homes in any neighborhood – so all the homes are offered to a representative sample of whites, F(w) of whom accept. The remainder of homes are then occupied by non-whites. Hence F(w) also gives the ratio of whites desiring to live in the neighborhood to the total neighborhood population.

Note that Schelling's basic model assumes the outcome is entirely driven by whites' preferences. This assumption is debatable (and perhaps even offensive), but it reflects the traditional view of neighborhood segregation as mainly driven by whites' behavior. It could be justified by saying that whites have stronger preferences about getting their preferred racial composition than nonwhites, and hence will pay more to live in their preferred neighborhood (note that the basic Schelling model does not have any role for housing prices). Schelling actually did a version of the model that also incorporated nonwhites' preferences for neighborhood composition, but it has not caught on like the original model and it did not dramatically change the predictions of the model.

The point where the cumulative density function crosses the 45 degree line is where the fraction of whites willing to live in a neighborhood that is w percent white is in fact equal to w:

(1)
$$w = F(w)$$

This is an equilibrium outcome for racial composition of the neighborhood; there will be more than one such point since CDFs satisfy F(0)=0 and F(1)=1. The tipping point story only makes sense if (1) also holds for an intermediate point between 0 and 1.

The dynamics of the white share can be specified by giving the change in white share as the distance between the CDF and the 45 degree line.

(2)
$$\Delta w = F(w) - w$$

This is the equation that will actually be estimated in the empirical section, using a very flexible fourth order polynomial.

Now suppose also that (3) holds.

(3) F'(w) >1 evaluated at a point strictly less than 1 and strictly greater than 0 where(1) holds.

If (3) holds, then one of the points where (1) holds is an unstable equilibrium. In other words, (1) and (3) define a tipping point. Any w above this point will spiral upwards

towards greater segregation of mostly white neighborhoods, and any w below it will show white flight and more segregated black neighborhoods.¹⁰

Suppose for illustration that the CDF is of the normal distribution with mean μ and variance σ^2 , F(w; μ , σ^2), For example, assuming just for illustration that μ =.75 and σ =.1, Figure 1 shows the corresponding cumulative density function.

 $^{^{10}}$ (3) could hold at w=1, in which case w=1 is a tipping point. However, this is not what Schelling had in mind, since he discusses a shift from a stable neighborhood above the tipping point.



Figure 1: The cumulative normal distribution for racial preferences

The CDF is highly nonlinear, with flat segments at either end but climbing steeply in the middle. This reflects the characteristics of the normal distribution, with flat tails but steep increases in the number of individuals contained in the middle. The actual fraction of whites who live in the neighborhood is given by the 45 degree line.

Note that the tipping point is a higher fraction of whites w than the mean of the normal distribution of white preferences. For example, in the figure the equilibrium point is .86, while the mean of the normal distribution was .75. Any mean of the normal distribution greater than .5 cannot be an equilibrium or a tipping point, because only .5 of whites are willing to live in the neighborhood with the mean of the normal distribution for fraction of whites. The tipping point always lies above the mean in this case.

If there is a disturbance to a neighborhood in the vicinity of the tipping point such that a few whites leave the neighborhood or a few nonwhites enter and the white share drops below equilibrium, then the fraction of whites willing to live in the neighborhood falls below the actual share. There is a further decrease in white share, and yet a further white exodus. This process does not stop until the neighborhood becomes completely nonwhite – a white share of 0 is a stable equilibrium. The neighborhood has "tipped" from being majority white to completely nonwhite.

Conversely, any deviation of the white share above the equilibrium will lead to a fraction of whites willing to live in the neighborhood that is greater than the actual share. This will cause the white share to increase. A new equilibrium is not reached until the cumulative density function intersects the 45 degree line from above. In the diagram, this happens at a white share of about .992. Hence, the remarkable outcome of Schelling's

model is that the long run equilibrium is for neighborhoods to be either entirely nonwhite or 99.2 percent white, despite the preferences of the median white for a mixed neighborhood that is 75 percent white and 25 percent nonwhite.

Although the tipping point idea is linked historically to racial scare-mongering about the "threat" of nonwhites moving into the neighborhood, Schelling's contribution actually gives quite a different perspective on racial segregation. The point of Schelling's model was that the strategic interdependence of weak preferences for living next to people of the same race could lead to an outcome of almost total segregation. Suppose a small increase in nonwhites around the tipping point of a neighborhood with high white share directly causes only the most racist white to exit the neighborhood. However, the departure of the most racist white causes a further decrease in white share, which now leaves the second-most racist white uncomfortable with fewer white neighbors, and he also leaves. (I do not mean to imply that people have to move sequentially and gradually, this is just for illustration.) This in turn leaves the third most racist white discomfited, and he leaves. So things keep unraveling until even relatively integrationist whites wind up exiting, until the whole neighborhood tips over, all because of an initial small increase in nonwhites that only directly bothered the most racist white. This contrasts with the view that segregation reflects whites having a very strong preference for having white neighbors. Hence a test of Schelling's model is a test of whether residential segregation simply reflects the interaction of what could be weak average preferences for same-race neighbors. The alternative is that segregation is something more fundamental driven by other factors.

In figure 1, the fall in white share is dramatic below the tipping point, reflecting the rapid movement through the fat part of the normal distribution of the w_j . This accords well with the classic story of tipping as a rapid exodus of whites out of the neighborhood once tipping begins. However, we have no evidence that the distribution is normal or any other distribution, and the prediction of very rapid exodus comes out of some distributions and not others. A CDF could be much closer to the 45 degree line below the tipping point and still satisfy conditions (1) through (3).

In general, some CDFs do not fit the classic "tipping point" narrative, even though they have tipping points. For example, think of a simple distribution where there are only three discrete groups of whites, each containing one-third of the white population. The first has a w_i of 0.3, the second of 0.6, and the last of 0.9. This would generate the CDF shown in Figure 2. This CDF features no less than 4 stable equilibria (zero white, minority white, majority white, and all white) and 3 tipping points. Tipping is a relatively more modest affair between these stable equilibria, and each group has a neighborhood close to their preferences, in contrast to the massive reversal and difference between preferences and outcomes in the normal distribution tipping story. The classic tipping story relies on a distribution of whites who are fairly similar to one another and thus vulnerable to chain reactions; more heterogeneity of preferences leads to more stable outcomes closer to preferences. The advantage of this paper's methodology in estimating the entire distribution (2) is that it allows for the "classic" tipping point story to be compared to two alternatives: (a) there are no tipping points, and (b) there are tipping points but the CDF does not fit the "classic" global tipping story. The Card et al. 2008 approach, in contrast, can only rule out (a), not (b).



Figure 2: Tipping Points with 3 heterogeneous groups of whites

One last set of considerations when taking the model to the data is considering the overall outcome. When whites exit a neighborhood that is tipping nonwhite, where do they go? Obviously, they would go to a neighborhood with a higher white share, and so they become part of those neighborhoods that are tipping towards greater white share.

However, note that the Schelling model is not a general equilibrium model. There is no adding up constraint to enforce that the population-weighted average of neighborhoods' white share be equal to the system-wide share of whites in the population. Because all neighborhoods are subject to random shocks of varying intensity, the Schelling model does not place any restrictions on how many of the neighborhoods will be in the segregated nonwhite equilibrium or in the higher segregated white equilibrium. Hence, one cannot reject a particular estimated tipping point on the grounds that it is inconsistent with overall white share. However, other possible estimated outcomes of equation (2) could be inconsistent with overall population structure. If (2) shows a globally stable intermediate equilibrium which is different than the overall white share, then that does violate adding up constraints. Similarly, if the estimated equation (2) implies a dynamic structure in which all neighborhoods converge to all-white (or all-nonwhite), then that would also obviously violate the adding up constraint. Such violations would call into question that what has been estimated is in fact a global dynamic structure, as opposed to a relationship between initial white share and predicted changes in white share (which could be one time changes) that are explained by other stories besides the Schelling model.

3. The data

The database used in this analysis was originally called the Underclass Database (UDB). It was put together for 1970, 1980, and 1990 by the Urban Institute, a nonpartisan think tank in Washington DC. Given the interests of the Institute, the data covered metropolitan neighborhoods (where "metropolitan" is defined as in the census to include central city, inner suburbs, and outer suburbs). The database has been updated to include the 2000 census by a commercial firm called Geolytics Inc.¹¹ The unit of analysis in the database is the census tract, a division meant to approximate a "neighborhood", usually containing between 2500 and 8000 people. The tract boundaries are chosen to capture neighbors with similar social characteristics (which means that measures of segregation based on tract data will tend to exaggerate segregation). Tract boundaries do not cross county, metropolitan area, or state boundaries.¹²

¹¹ The new database is available on CD-ROM from geolytics.com. The description of the data contained here is based on the NCDB Data User's Guide, including Appendix J on tract matching.

¹² Except in New England, some tracts cross metropolitan area boundaries.

There are several difficult issues surrounding the data construction. Of those tracts that have data for both 1970 and 2000, two-thirds changed boundaries. Some tracts were merged into a single tract, and some single tracts were divided into multiple tracts. Unfortunately, in the majority of tract changes, there are boundary changes that are not simple mergers or divisions of existing tracts. The constructors of the database addressed this problem in several different ways, depending on what data was available for different census years. They used geographic information software (GIS) to overlay 2000 tract boundaries on earlier tract boundaries. They then used 1990 block data to estimate the proportion of the old tracts in various racial categories that went into the new tract, and then recalculated the 1990 tract data using the 2000 tract boundaries.

Block data located spatially were not available for 1970 and 1980. The 1980 tracts were matched to the 1990 tracts and 1970 tracts matched to 1980 tracts using Census Bureau information on tract correspondence based only on spatial changes in tract boundaries. Hence, the 1970 and 1980 tract matching to 1990 and 2000 is less accurate than the tract matching between 1990 and 2000.

The database includes an indicator of which tracts changed boundaries. The use of the full sample could be justified if we think any errors introduced by boundary changes are random, i.e. uncorrelated with the right hand side variables in my regressions below. However, I will run a robustness check of my results by running them on the sub-sample which did not change boundaries between 1970 and 2000.

Some 2000 tract boundaries include areas that were not covered at all by 1970 data. As long as the covered area is a random sample of the whole tract, with the error term uncorrelated with the 1970 white share, the use of the full sample could still be

justified. Nevertheless, I will run another robustness check by omitting these observations from the sample.

Census data has the commonly known problem that it undercounts the population because some people are harder to reach for enumeration. Of concern for our exercise, the undercount is thought to be proportionally greater for nonwhite populations. The undercount percentage has been falling over time. I do not have any solution to this problem, but hope that it is of small enough magnitude not to distort the results. In 1990, the Census estimated the overall undercount as 2 percent, down from 5 percent in 1950. The undercount for blacks was estimated at 5.7 percent in 1990, an increase from 4.5 percent in 1980.¹³

Table 1 shows the variable definitions and summary statistics. The sample is all available data in the NCDB, which as I noted is mainly for metropolitan census tracts (Map 1 shows the coverage of NCDB for 1970). Census tracts have a mean population in 1970 of 3,208 people. I eliminated any census tracts with a population of less than 100 in either 1970 or 2000 from the sample so as to avoid extreme outcomes in very small census tracts. The maximum population of census tracts in the sample is 31, 903 in 1970 and 36,146 in 2000.

¹³ Another problem was that the 2000 census introduced a change in its racial classification methodology. Racial classification is done by self-identification. In 2000, individuals were allowed to select more than one race to describe themselves, in contrast to earlier years when they could only pick one. 2.4 percent of respondents chose multiple races in 2000. To match 2000 data to earlier years, the NCDB creators used the principle that anyone who selected a nonwhite category, even if it was in addition to white, would be classified as nonwhite. Since this conforms to the social convention for defining nonwhites, which probably influenced individuals' self-classification in prior years, and since the number choosing multiple races is small, I do not think this will overly distort the results. For some reason, the database authors violated this rule only with Native Americans, who were counted as Native Americans only if they did not also choose "white." However, the proportion of Native Americans in the sample is small in any case. Other racial issues arise with classifying Hispanics. "Hispanic" is a national origin classification, which is different than racial classification. There is a category "other" in the racial classification, which in earlier work co-authors and I have found to be strongly correlated with "Hispanic" (Alesina, Baqir, and Easterly 1999).



The restriction of the NCDB to metropolitan census tracts is fine for my purposes,

since the tipping model is mainly about urban neighborhoods.

Table 1: Variable Definitions and Summary Statistics								
Variable	DSHRWHT70	SHRWHT7	LPOPDENS7	LFAVINC7				
Deficition	Change in white share from 1970 to	White share of population	Log of population density in	Log of median family income in				
Definition	2000	IN 1970	1970	1970				
Mean	-0.185	0.894	7.451	9.323				
Median	-0.117	0.983	7.851	9.320				
Maximum	0.813	1.000	12.394	12.178				
Minimum	-1.000	0.001	-2.197	6.957				
Std. Dev.	0.207	0.217	1.987	0.318				
Skewness	-1.119	-2.739	-0.633	0.340				
Kurtosis	4.435	9.740	3.250	4.882				
Observations	41321	41321	41321	41284				

4. Empirical testing 1970 to 2000

Note from Table 1 that the mean white share declined considerably from 1970 to 2000, reflecting the faster growth of nonwhite population than white population in metropolitan areas. While the white population only increased by 15% from 1970 to 2000, the nonwhite population nearly tripled over the same period. Both blacks and other nonwhites shared in this rapid population increase. We could think of this influx of nonwhite population as a natural experiment of the Schelling model – predicting that neighborhoods in the vicinity of the tipping point would flip over to nonwhite majorities, while neighborhoods well above the tipping point would have retained stable white majorities.

4.1 Basic estimation

Using the NCDB, I estimate dynamic equations for the change in white share as a function of initial white share. I first assume that the tipping point is the same in all neighborhoods, and then will relax that assumption. To accomodate the highly non-linear prediction of the Schelling model, I estimate the change in white share as a function of a fourth-order polynomial of initial white share.¹⁴ I test first the change in white share from 1970 to 2000, and then I will test the change over each decade. I will first assume that all neighborhoods in the sample have the same tipping point, but shortly I will relax this assumption. Table 2 shows the basic regression for this fourth-order polynomial. All of

¹⁴ I experimented with a fifth-order polynomial also, but it did not make a difference to the shape of the curve described below.

the polynomial terms are significant, which does confirm the highly nonlinear dynamics

of the white share. ¹⁵

Table 2: Regressions of change in white share on nonlinear function of initial white share

Dependent variables: change in white share from 1970 to 2000 Same constant for all Different constants neighborhoods for metro areas 0.103*** 0.206*** Constant (0.00573)(0.00881)White share, 1970 -2.018*** -1.515*** (0.11)(0.115)7.578*** 5.027*** White share^2, 1970, (0.464)(0.467)-12.00*** -7.980*** White share³, 1970 (0.664)(0.663)White share^4, 1970 6.176*** 4.192*** (0.307)(0.305)R squared 0.071 0.194 Number of observations 41912 41912

The predicted change in white share for initial white share is shown in Figure 3. Figure 3 does not have a tipping point (except for zero and unity as discussed below). We would need some predicted increases in white share at a high enough initial share to get an intermediate tipping point of the kind that Schelling had in mind. There is a large predicted decrease in white share for all mostly white neighborhoods no matter how high the initial white share. Only at very low values of white share is there a predicted increase in white share. Hence, we fail to identify any such tipping point using the simple structure of the Schelling model.

¹⁵ This is somewhat different from the results of Ellen (2000) for change from 1980 to 1990, who did not find the linear term for black share in 1980 to be significant in a regression for percent change in whites. She specifies the relationship as quadratic, but did not find different results in a spline regression for black share.





We can reconstruct the implied CDF by adding w to the predicted dw for each w, following the rigid assumptions of the Schelling model as specified above (i.e. we assume that only the distribution of preferences for racial shares causes racial shifts, and not any other factors). Figure 4 shows the implied CDF as a function of initial white share. The implied estimated CDF has F(0) > 0 and F(1) < 1. F(0) > 0 suggests that about 8 percent of whites have NO tipping point, they will not leave a neighborhood no matter how low the share of whites. There is therefore a singularity at zero, shown here as a vertical segment of the CDF. Zero is an unstable equilibrium. There will be a stable equilibrium at a white share that includes the zero-tipping-point group and any other

whites with w_j between zero and that stable equilibrium. This would cause any neighborhood between zero and the low stable equilibrium to tip upward.¹⁶

Neighborhoods) 0.9 х df unconstrained 0.8 0.7 0.6 0.5 0.4 0.3 0.2 0.1 ٥ 0.1 0.15 0.2 0.25 0.3 0.35 0.4 0.45 0.5 0.55 0.6 0.65 0.7 0.75 0.8 0.85 0.9 0.95 0 0.05

Figure 4: CDF implied by predicted change in white share from Figure 3

Estimated Cumulative Density Function of Tipping Points (Assumed to Be the Same for All

At the other extreme, there is also a singularity at unity, shown again as a vertical segment of the CDF. This means that for 18 percent of whites, anything arbitrarily close to 1 from below is a tipping point; they will exit if even a single black moves into the neighborhood. When we consider the historical anecdotes of extreme aversion to integrated neighborhoods, this is not completely implausible, although it would be more surprising that such extreme segregationist preferences still exist today among a

¹⁶ Taken literally, we have the paradoxical implication that 8 percent of whites want to live in a neighborhood with no whites! Being less literal so that we consider neighborhoods with an epsilon share of whites as being arbitrarily close to zero, this just suggests that some group of whites residing in a neighborhood will never exit even if they are the only remaining white in the neighborhood.

nontrivial share of whites. Note that this extreme literalist implication would hold for any negative predicted change at an initial white share of 1. More likely than this extreme implication, some other factors probably cause all white neighborhoods to have an average decrease in white share.

Thus the implied density curve from the initial estimates is very different from the kind of density curve suggested by Schelling for the tipping point story. He thought in terms of a bell curve around some mean high tipping point that was significantly less than one, which meant that virtually no one had extreme tipping points (0 and 1). The estimates above imply preferences that are very polarized with density spikes at 0 and 1.

The problem with the estimated curve above is that there are no stable equilibria with high white share; the only stable equilibrium has a very low white share far below the average share of whites in the population and thus violates an adding up constraint in the long run. We will explore next whether this problem can be fixed by allowing the tipping point to vary continuously with other neighborhood characteristics.

So far the initial results do not support Schelling's "tipping point" story as an explanation for neighborhood dynamics from 1970 to 2000 (although Schelling's story could have explained neighborhood dynamics for pre-1970 periods). The pattern does suggest that segregation was diminishing from 1970 to 2000, as neighborhoods with high white shares had the biggest drop in those shares. This is confirmed by the aggregate statistics: while the Tauber dissimilarity index for nonwhites was 0.53 in 2000, it had been 0.75 in 1970.¹⁷ Decreasing segregation by itself does not prove or disprove the tipping model. If there had been a tipping point, there still could have been some

¹⁷ Cutler, Glaeser, and Vigdor 1999 also present evidence that segregation declined from 1970 to 1990, as does Ellen (2000).

decreased segregation as many mostly white neighborhoods had a fall in white share as they were in the process of tipping over to a nonwhite neighborhood, in response to the influx of nonwhites. However, neighborhood change did not follow the dynamics of tipping, in which some segregated white neighborhoods remained stable or showed an increase in white share. Instead a high degree of "white flight" happened in all neighborhoods with high initial white shares.

4.2 Allowing tipping points to vary across cities and neighborhoods

The models estimated so far were restricted in that the dynamics (and the potential tipping point) was assumed to be equal in all neighborhoods in the sample. Another logical step is to allow for differences across the 202 metropolitan areas in the sample. I put metro dummies, allowing the intercepts to vary. At the average intercept for the metro areas, the shape of the curve is little different from figure 2. The intercept does vary considerably across metro areas, from a low of -.045 (Albany, Georgia) to a high of 0.297 (Johnson City-Kingsport-Bristol, Tennessee). Johnson City, Tennessee is the ONLY metro area with a tipping point, as shown in Figure 4.

Figure 4: Allowing intercepts and tipping points to vary by metropolitan

area



Predicted change in white share controlling for metropolitan dummies

I next consider other control variables that allow the tipping point to be different in different neighborhoods as a continuous function of these variables. The two most important candidate variables are income and population density, as rich neighborhoods may be more stable and have lower tipping points than working class neighborhoods, and dense inner city neighborhoods may have a higher tipping point than less dense suburban neighborhoods. Hence, I also introduce the log of initial population density as a control for change in white share.

Table 3 also shows the regression of change in white share on these right hand side variables. All of the polynomial terms for initial white share are still highly significant. The population density variable is significant with an extremely high tstatistic, which (with the polynomial estimated) suggests a higher tipping point the lower is population density. (I also tried estimating separate polynomials for the central city sample and for the sample in the suburbs, but it made little qualitative difference in the results described here.) Family income is also very significant, suggesting the tipping point goes up with income (again for the estimated polynomial). All variables together have decent explanatory power for such a noisy variable in a large sample, with an Rsquared of .226. Of course, there could be alternative explanations for the effect of income and density rather than thinking of them only as changing tipping points. As an alternative to the tipping story in general, the higher white flight out of denser, lower income neighborhoods could reflect a large explanatory power for the "white suburbanization" hypothesis for changing white share.

Figure 5 shows the shape of the relationship between initial white share and change in white share at mean values of population density and family income, and then considers shifts in income and density. At the mean values, the curve in Figure 5 looks similar to Figure 3 except with a higher stable equilibrium at a white share of around 0.2. This curve has all the same problems as the curve in Figure 3, as the stable equilibrium white share is inconsistent with the share of whites with mean income and population density.

Figure 5



Predicted change in white share controlling for income and population density

Figure 5 shows how the curve would look if the initial log population density were two standard deviations below its mean, and in addition, the log family income were two standard deviations above its mean. While we do get a predicted increase in white share at high initial values of white share, the large shifts in income and density mean that we are describing only a very tiny part of the sample: 6 out of 41,865 neighborhoods to be precise. For this miniscule set, we get an unstable "tipping point" equilibrium at a white share of around .97, but even this is far from the large tipping over to majority nonwhite envisaged by the tipping point hypothesis. The drop in white share is fairly modest below .97 and there is a stable equilibrium at a white share of around .7. Below .7, there is a predicted increase of white share which becomes quite large at low initial white share. Note that the singularity at zero has an even higher share of whites (above 40 percent) who have no tipping points. For the (mostly out of sample) "rich and spacious" neighborhoods there are two stable equilibria: 100 percent white share and 70 percent white share.

The adding up constraint is not automatically violated as long as all such neighborhoods have a total share of whites above 70 percent. Anything above this could be consistent with some indeterminate number of neighborhoods at 100 percent white share. This seems much more plausible than the other estimated curves, and there is something of a tipping point story, but these neighborhoods are not the ones that will tip over to majority nonwhite and so do not really fit the original tipping point story.

To sum up, even controlling for density and income, we do not see anything like the kind of dynamic behavior of neighborhoods predicted by the tipping point model. *4.3 Further robustness checks*

One issue that obviously follows from the white suburbanization hypothesis is that there is a high degree of spatial correlation in the data. A central city neighborhood with a declining white share is not independent of its neighbors, who also often turn out to be neighborhoods with declining white share. If the assumption of independence was violated, as seems certain, that would imply that the standard errors and hence t-statistics were incorrectly estimated in the regressions above.

Hence, I run another set of regressions with clustered standard errors. I use three different definitions of clustering. First, each zip code typically contains a handful of

census tracts, and so correcting for clustering by zip code will take into account very local spatial dependence.¹⁸ This yields 8227 clusters. Second, it may be as suggested by the suburbanization hypothesis that tracts in high density and low density areas of each metropolitan area behaved similarly to other tracts in those same areas. Hence, I define a new set of groups that are first broken down by metropolitan area, and then broken down into tracts above median density and those below median population density for the whole sample. This second method yields 404 clusters (i.e. 202 metropolitan areas, with low and high density areas in each one). A third method aims at capturing the same idea with jurisdictional boundaries – whether the tract lies in the central city or the suburbs for each metropolitan area.

Another robustness check I perform is to enter the dummies for metropolitan areas at the same time as the income and density terms. Finally, I omit observations that may be questionable for reasons described in the data section. There are two types of problematic observations: 1) those in which the 1970 data apply to only part of the area contained in the 2000 tract boundaries, and 2) those in which the tract definition changed from 1970 to 2000. Note that 1) is a subset of 2). 1) is the most problematic kind of tract change, because there is simply missing information on a part of the tract for the year 1970. For other tract changes, there was an attempt by the database builders to map from the old tract data to the new tract boundaries, as described in the data section above.

¹⁸ Zip code boundaries are independent of census tract boundaries, so they will split some tracts into 2 different zip codes. The tract is assigned to the zipcode that accounts for the majority of the tract.

-	[1]	[2]	[3]	[4]	[5]	[6]	[7]
Constant	-0.113	-0.432	-0.106	-0.112	-0.141	-0.177	-0.303
	-4.34	-10.42	-2.12	-1.11	-1.36	-1.58	-3.63
White share, 1970	-2.089	-1.546	-2.114	-2.082	-2.132	-1.923	-1.426
	-20.87	-12.95	-15.08	-8.96	-8.02	-8.54	-7.77
White share ^{^2}	7.163	4.395	7.270	7.154	7.395	6.569	4.344
	17.30	9.82	12.41	6.76	6.29	6.08	5.97
White share ^{^3}	-11.260	-6.846	-11.370	-11.252	-11.677	-10.542	-7.747
	-19.10	-11.26	-13.34	-6.87	-6.5	-6.17	-6.86
White share^4	5.802	3.57	5.830	5.795	6.012	5.505	4.458
	21.37	13.09	14.58	7.3	6.97	6.53	7.92
Log (Population/							
Land Area), 1970	-0.041	-0.04	-0.040	-0.040	-0.041	-0.047	-0.046
	-86.41	-82.83	-35.95	-10.31	-11.11	-20.12	-13.51
Log Family							
Income, 1970	0.068	0.11	0.066	0.066	0.072	0.079	0.093
	23.29	33.64	11.94	4.72	4.94	6.12	9.72
Number of							
observations	41284	41351	41862	41304	31985	32407	11773
R squared	0.226	0.1924	0.2169	0.2155	0.2246	0.2431	0.278
# Clusters	none	none	9099	404	403	431	368
# metropolitan							
dummies	none	198	none	none	none	none	none
Cluster definition	none	none	zipcodes	metro areas (Low and High Density)	metro areas (Low and High Density)	metro areas (central city and suburbs)	metro areas (Low and High Density)
Excluded observations	none	none	none	none	1970 coverage of 2000 tract<98 percent	1970 coverage of 2000 tract<98 percent	Any changes in tract definitions

Table 3: Robustness checks for metropolitan dummies, clustered standard errors, and restricted sample

The results of clustered standard errors are shown in Table 3. The t-statistics do fall drastically, especially on the population density variable, but also on the initial white

share. All variables remain significant at the 1% level, however. The results (Table 3) are also qualitatively similar with metropolitan dummies, with the plot for predicted change in white share much the same as in figure 3.

Table 3 also shows what happens when observations falling under either 1) or 2) are eliminated. All of the variables are still statistically significant in the smaller, more reliable samples. The coefficients are relatively unchanged for the sample that omits observations in which 1970 data did not cover the whole 2000 tract. The coefficients do change quite a bit in the restricted sample with no tract redefinitions at all. However, the picture of the predicted changes in white share looks qualitatively similar with these coefficients to that shown in figure 3.

I next consider a more extensive set of ancillary variables: percent of population under 18, percent over 65, percent of population who are homeowners, and percent of tract located in the central city. These additional variables are chosen based on what the previous literature considered; I did not do any searching among alternative sets of variables. All of the additional variables are significant, except for percent of population under 18, with intuitive signs. However, they do not alter the coefficients on white share very much and the shape of the curve with these variables is similar to that shown in figure 3. As with the earlier exercise with income and density, a positive or negative sign on these variables can be interpreted as a positive or negative shift of the tipping point (for the estimated polynomial shape). So a higher share of population over 65 and share of homeownership increases the tipping point, while an increase in the percent of the neighborhood lying in the central city decreases the tipping point. The share of children under 18 is not statistically significant.

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Constant-0.104-0.104-0.104 -3.09 -0.94 -1.78 White share, 1970 -1.868 -1809 -19.86 -8.33 -14.8 White share^2, 1970, 5.976 5.765 14.72 5.5 10.74 White share^3, 1970 -9.368 -9.141 -9.368 -9.141 -9.368 -15.88 -5.48 -11.31 White share^4, 1970 4.842 4.766 4.842 4.766 4.842 17.62 5.76 12.3 Log Family Income, 1970 0.057 0.058 0.057 0.058 0.057 16.72 4.56 9.35 Log (Population/Land Area), 1970 -0.036 -0.036 -0.24 -0.08 -0.14 Percent over age 65, 1970 0.102 0.100 15.67 4.52 9.27 Percent of homeowners, 1970 15.81 2.75 8.35 Percent of tract in central city, 1999 -0.001 -0.001 -27.43 -2.8 -11.91 Number of observations 41139 40533 41139
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Number of observations 41139 40533 41139
R squared 0.270 0.2679 0.2696
Clusters None 434 9480
metro areas (central
Cluster definition None suburbs) zip codes

 Table 4: Robustness checks for additional variables with clustered standard errors

Using these additional variables, we could check how much of a shift is necessary to give something like a tipping point story. If we shift all variables by x standard deviations in the direction that would increase the tipping point, I choose the x that produces a tipping story. X turns out to be about 1.4 (standard deviations). So neighborhoods that have higher income, lower density, higher share over 65, higher share of homeowners, and lower share of neighborhood in central city, each by 1.4 standard deviations compared to the mean, would have a tipping point (Figure 6). The tipping point story is once again not fully matching the qualitative features of the Schelling model, as the stable equilibrium is at about.75 white share, and the tipping point is between .96 and .97. Unfortunately, this is an out of sample prediction, as there are no neighborhoods out of 41,139 observations that satisfy all these criteria.

Figure 6

Allowing tipping points to vary with income, density, share over 65, homeownership, and central city



A final robustness check is to regress the change in *black* share from 1970 to 2000 on a polynomial for initial *black* share in 1970. This tests whether the dynamics of tipping are any different when we focus on blacks rather than all non-whites. The tipping point hypothesis would imply a positive relationship between initial black share and change in black share, peaking at an intermediate level of black share, before declining mechanically because the maximum increase in black share possible is 1-initial black share. The story would predict that neighborhoods with a black share below the tipping point would have a fall in black share, with the tipping point where the curve crosses the x-axis from below.

The polynomial terms for initial black share are all significant (not shown). The implied curve is shown in figure 7. The curve is the mirror image of the white share dynamics in figure 3 above. The relationship between change in black share and initial black share has only a small upward sloping portion. Over most of the range of black share, the line is downward sloping – suggesting that neighborhoods are moving away from the extremes of all black or all non-black, which is just the opposite of the tipping prediction. The change in black share is much larger at low initial levels of black share rather than at intermediate levels, contradicting the tipping point hypothesis.



Figure 7: Change in black share as function of initial black share 1970 to

2000

5. Decade to decade changes in white share

The results are similar when I look at the individual decade changes from 1970 to 1980, 1980 to 1990, and 1990 to 2000. Table 5 shows these three regressions.

$\begin{array}{c c c c c c c c c c c c c c c c c c c $	White Heteroskedasticit	y-Consistent S	tandard Eri	ors & Covaria	ince		
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S.E. of regression0.1180.0810.082Mean dependent var-0.073-0.050-0.063S.D. dependent var0.1300.0890.091Observations412844121841205	Adjusted R-squared	0.175		0.183		0.192	
Mean dependent var-0.073-0.050-0.063S.D. dependent var0.1300.0890.091Observations412844121841205	S.E. of regression	0.118		0.081		0.082	
S.D. dependent var0.1300.0890.091Observations412844121841205	Mean dependent var	-0.073		-0.050		-0.063	
Observations 41284 41218 41205	S.D. dependent var	0.130		0.089		0.091	
	Observations	41284		41218		41205	

Table 5: Estimates of dynamic equations for white share 1970-1980,1980-1990, and 1990-2000 Method: Least Squares

Again, population density is by far the strongest predictor of change in white share. The effect of initial family income is weak in the 1980 to 1990 regression and insignificant in the 1990 to 2000 regression. The nonlinear terms for initial white share are significant, but much less so than density. The following figures show the dynamic curves for each regression, comparing the curve at mean log population density with that with density 1.96 standard deviations below the mean, and then both density and income 1.96 standard deviations away from the mean. The curves are quite different from one decade to the next, but none of them fit comfortably with the picture predicted by the tipping point model. At mean density and income, all of them show the highest predicted drop in white share to be at high initial levels of white share rather than the intermediate levels (Figure 8 a,b,c).





Change in white share from 1970 to 1980





Change in white share from 1980 to 1990



Change in white share from 1990 to 2000



There are multiple equilibria for some low values of density in these graphs, but the lower stable equilibrium is one with a mixed neighborhood. At mean density, all of the neighborhoods with high white share show a decline in white share, with only a modest trough at intermediate values of the white share. (The curvature in this zone is consistent with the predictions of the normal CDF for preferences, but we do not find a tipping point except at low density.)

The curve for changes from 1990 to 2000 comes the closest to fitting the tipping model. At low density, the stable equilibria are a white share equal to one, and a white share equal to about .4. This captures the idea that neighborhoods could tip from homogeneous white neighborhoods to minority white neighborhoods. However, the lower equilibrium of .4 is much higher than in the typical view of the tipping point. And the tipping point itself is implausibly high – any white share less than .99 will tip over to

the minority white neighborhood. While providing some support for the tipping point view, these parameters do not portray a very plausible tipping story.

It would be nice to have data from earlier decades to assess tipping. It may be, as the survey evidence suggests, that whites' behavior and attitudes have changed in the course of the 20th century. It is possible that "tipping" is a good description of neighborhood change before 1970. Unfortunately, we do not have the data to assess this conjecture.

6. Conclusions

Although a significant fraction (about 10 percent) of the sample of urban American neighborhoods did change from majority white to majority nonwhite over 1970 to 2000, they did not do so as the "tipping point" hypothesis suggests. The main factor in neighborhood change was arguably a movement of whites from central cities and inner suburbs to outer suburbs in metropolitan areas. The relationship between change in white share and the initial white share does not fit the "tipping point" model. In this dataset, the dynamics of neighborhood composition do not suggest the instability of strategic interdependence as modeled by Schelling. This result differs from the results of Card et al. (2008), who found locally unstable tipping points in an approach that attractively economized on any assumptions about functional forms. However, these locally unstable equilibria are only necessary conditions, not sufficient ones to confirm the tipping story (recall Figure 2's demonstration of multiple unstable equilibria in a form that did not fit the normal tipping narrative). The tipping story also makes predictions about the global dynamics of racial share, which this paper finds to be contradicted by the data, using as flexible a parametric form for such dynamics as possible.

Models of strategic interdependence are very slippery creatures to test. It could be that there were other long-run factors not captured by the empirical analysis that determined changes in white share, and that conditional on these other factors there was still strategic interdependence that would lead to tipping. I have sought in the robustness checks to explore some of the obvious candidates (which also allow tipping points to be heterogeneous based on these other factors), but the list of possible third factors is endless. I can only say that the simplest tests of the tipping model, conditional on the seemingly most obvious third factors, fail to confirm the model.

Schelling's model remains a classic theoretical milestone for understanding instability of interdependent behavior. Perhaps we need to distinguish two intellectual tasks: (1) the theoretical demonstration that massive tipping could occur through strategic interaction despite only weak preferences for same-race neighbors, and (2) the empirical explanation of segregated neighborhoods. Schelling's tipping point model remains a masterpiece as far as task (1). It is not so surprising that it is, however, too simple to actually do task (2), i.e. explain the patterns of neighborhood change in the real world. These empirical results should induce some caution as to the widespread use of the phrase "tipping point" as a sufficient explanation for real world segregation outcomes. Aaronson, Daniel. 2001. Neighborhood Dynamics. Journal of Urban Economics 49, 1-31

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