Endogeneity Corrected Stochastic Production Frontier and Technical Efficiency

Apurba Shee and Spiro E. Stefanou

Abstract

A major econometric issue in estimating production parameters and technical efficiency is the possibility that some forces influencing production are only observed by the firm and not by the econometrician. Not only can this misspecification lead to a biased inference on the output elasticity of inputs, but it also provides a faulty measure of technical efficiency. We extend the Levinsohn and Petrin (2003) approach and provide an estimation algorithm to overcome the problem of endogenous input choice in stochastic production frontier estimation by generating consistent estimates of production parameters and technical efficiency. We apply the proposed method to a plant-level panel dataset from the Colombian food manufacturing sector for the period 1982-1998. This dataset provides the value of output and prices charged for each product, expenditures and prices paid for each material used, energy consumption in kilowatt per hour and energy prices, number of workers and payroll, and book values of capital stock. Empirical results find that the traditional stochastic production frontier tends to underestimate the output elasticity of capital and firm-level technical efficiency. The evidence in this research suggests that addressing the endogeneity issue matters in stochastic production frontier analysis.

Keywords: Colombian food industry, Endogeneity of input choice, Maximum likelihood, Semi-parametric estimation, Stochastic production frontier, Technical efficiency

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Introduction

Estimating the production technology is fundamental to assessing the production potential of firms or sectors. Increased availability of large firm-level micro datasets of inputs and outputs and the interest in analyzing production efficiency in relation to any change in policy in production processes has led to renewed interest in productivity and efficiency analysis. From an econometric perspective, the stochastic production frontier approach has been a standard starting point for modeling technical efficiency (Kumbhakar and Lovell 2000; Greene 2008).

A major econometric issue in estimating production parameters and technical efficiency is the possibility that some determinants of production are only observed (or predictable) by the firm and not by the econometrician. The firm’s input allocation is chosen by its optimizing behavior where input choices may be correlated with these observed (or predictable by the firm) components. Traditionally, stochastic production frontier models assume that input choices are independent of the efficiency and productivity term. If a firm observes some part of its efficiency and productivity, its input choices may be influenced, resulting in an endogeneity problem in the stochastic production frontier estimation. This misspecification leads to a biased inference on measurement of input elasticities and the economies of scale, and provides a faulty measure of firm technical efficiency.

The concerns about endogeneity in production function estimation are well documented in the literature (Marschak and Andrews 1944; Griliches and Mairesse 1995; Olley and Pakes 1996; Levinsohn and Petrin 2003; Ackerberg, Caves, and Frazer 2006). Quantities of inputs are likely to be correlated with productivity shocks, which lead to biased estimates of production function parameters. The traditional approaches to addressing endogeneity in production function estimation employing instrumental variables and fixed effects are problematic on both theoretical and empirical grounds. Olley and Pakes (1996) address endogeniety by focusing on investment to control for the unobserved productivity shock, while Levinsohn and Petrin (2003) and Ackerberg, Caves, and Frazer (2006) use intermediate inputs as a means to control for the unobserved shocks. These approaches assume that firms operate efficiently to obtain maximum potential output given the firm's resources and information at a given time. However, the firms may not necessarily make optimal decisions in every period. The discrepancy between optimal and observed quantities is derived as a measure of technical efficiency in the stochastic frontier literature.

Kutlu (2010) and Tran and Tsionas (2013) modify the widely used Battese and Coelli (1992) approach to deal with the endogeneity problem in the case of stochastic production frontier estimation. Mutter et al. (2013) also address the endogeneity issue but in a stochastic cost frontier setting. However, these latter studies do not model shocks to the production that are predictable by the firms but unknown to the econometricians.

Overall, the stochastic frontier literature has largely ignored the advances made in firm production function estimation using inputs to control for unobservables. Our approach extends the semi-parametric estimation approach of Levinsohn and Petrin (2003) and provides an estimation algorithm to address the endogeneity of the input bias problem within the stochastic production frontier framework to generate consistent estimates of the production parameters and technical efficiency. We apply the proposed method to plant-level panel data for the Colombian food manufacturing sector and find that addressing the endogeneity issue significantly impacts stochastic production frontier estimation.

The next section addresses the issue of the endogeneity of input choice and presents a semi-parametric approach to the stochastic production frontier estimation that corrects for the input choice endogeneity. The following two sections present the data and estimation results, with the final section providing concluding comments.

Endogeneity and the Stochastic Production Frontier

Firm output is bounded from above by a frontier that is stochastic in the sense that it varies randomly across firms. The starting point is the stochastic production frontier for a sample of N firms for T time periods, and can be written as

(1) $$Y_{it} = A_{it} f(X_{it};\beta)e^{v_{it}-u_{it}} \qquad i=1,...,N; t=1,...,T;$$

Y_{it} denotes production of i[th] firm at time period t, X_{it} is a vector of input quantities of i[th] firm at t time period, β is a vector of unknown parameters to be estimated, and A_{it} is the (unobserved) production shock component. The model combines two random error components; $v_{it} \sim N(0,\sigma_v^2)$, a standard noise component, and $u_{it} \sim N^+(\mu,\sigma_u^2)$, a non-negative term reflecting technical inefficiency.

We focus on the log-linear form of the Cobb-Douglas production frontier with technical efficiency presented as

(2) $$y_{it} = \beta_0 + \beta_l l_{it} + \beta_m m_{it} + \beta_e e_{it} + \beta_k k_{it} + \delta t + a_{it} + v_{it} - u_{it}$$

where y, l, m, e, and k refer to the natural logarithm of output, labor, material, energy, and capital inputs, respectively, while β_l, β_m, β_e, and β_k are the coefficients associated with inputs

l, m , e , and k; t is the proxy for exogenous technical change; u_{it} represents technical inefficiency; and v_{it} is random statistical noise. We can create a composed error term $(\varepsilon_{it} = a_{it} + v_{it} - u_{it})$ with the following rationale. The shocks to production that are predictable by firms when making input decision are denoted a_{it} and can be influenced by factors like expected rainfall at the firm's location, managerial ability of the firm, expected breakdowns, strikes, etc. The pure random deviation or measurement error, v_{it}, is not observable by the firm when making its input choices. The deviations from the 'best-practice' firm are captured by u_{it}. All the predictable components of the productivity and efficiency are embodied in the a_{it} term to address endogeneity.

If a firm observes some part of its efficiency and productivity, its input choices may be influenced, resulting in a simultaneity problem in the stochastic production frontier estimation. These production input decisions can be influenced by common causes impacting efficiency and, hence, the simultaneity problem emerges. Inputs are likely to be correlated with the components of productivity and efficiency that are observed by the firm but unobserved by the econometrician. This problem is more pronounced for inputs that adjust quickly, such as labor and materials. The omission of some explanatory variables leads to biased likelihood estimation of the stochastic production frontier models.

Semi-parametric approach to stochastic production frontier estimation

Olley and Pakes (1996) overcome the simultaneity problem by using investment as a proxy for the unobserved productivity shock. When investment is discontinuous, Levinsohn and Petrin (2003) suggest that investment may not respond fully to the productivity shocks and propose using intermediate inputs to control for the simultaneity problem. Two important conditions must be met for intermediate inputs to be a valid proxy for controlling for simultaneity. First, there should be a strict monotonicity assumption on the intermediate input demand functions, which follows the basic economic primitives of a profit maximizing firm. If more productive firms find it profitable to produce more than the less productive firms for a given capital stock, more productive firms will demand more of that intermediate input. Second, the market environment is assumed to be competitive and firms face common input and output prices. This assumption relates to the monotonicity condition. If the market structure is not competitive, it is not obvious that the firms with a greater productivity shock will produce more output, and hence will use more intermediate input. In an oligopolistic market structure, for example, the more productive firms do not necessarily produce more due to price differences.

To correct for the simultaneity issue in stochastic production frontier estimation, we modify the structural estimation methodology proposed by Levinsohn and Petrin (2003) for obtaining consistent estimates of production parameters and technical efficiency. The estimation stages proceed as follows:

Stage 1

The first stage employs energy as the proxy for the unobserved productivity shock. Using the assumptions mentioned above, specifying the input demand function for energy as

(3) $$e_{it} = e_{it}(a_{it}, k_{it}),$$

we employ the monotonicity condition to invert (3) and generate the energy demand equation

(4) $$a_{it} = a_{it}(e_{it}, k_{it}).$$

By expressing the intermediate input demand as only a function of a_{it} and k_{it}, we implicitly invoke the perfect competition assumption, which further assumes input and output prices are identical across firms. However, indexing the input demand function by t allow these prices to change over time, with prices being common across firms, allowing us to express the intermediate input demand function with just two state variables.[1]

In estimating (2), we follow Battese and Coelli (1992), $v_{it} \sim N(0,\sigma_v^2)$, $u_{it} \sim N^+(\mu,\sigma_u^2)$, and time-varying technical efficiency is defined by $u_{it} = u_i \exp(-\zeta[t-T])$, with u_i reflecting the firm-specific, base-period efficiency component, where the sign of the estimated ζ governs the change in technical inefficiency over time. While the production shock a_{it} is a state variable that influences the firm's decision, the remaining error $v_{it} - u_{it}$ has no impact on the firm's decision. Substituting (4) into (2) yields

(5) $$y_{it} = \beta_l l_{it} + \beta_m m_{it} + \delta t + \phi_t(e_{it}, k_{it}) + v_{it} - u_{it}$$

where

(6) $$\phi_t(e_{it}, k_{it}) = \beta_0 + \beta_k k_{it} + \beta_e e_{it} + a_{it}(e_{it}, k_{it}).$$

Following Levinsohn and Petrin (2003), we specify a third-order polynomial approximation in k_t and e_t in place of $\phi_t(e_t, k_t)$ or $\phi_t(e_t, k_t) \approx \sum_{i=0}^{3}\sum_{j=0}^{3-i} c_{ij} k_t^i e_t^j$. Maximum likelihood estimation with no intercept leads to consistent estimates of the coefficients of freely variable inputs except the proxy from (5). The time-varying technical efficiency parameter is also estimated in this stage using the Battese and Coelli (1992) error component model.

Stage 2

The coefficients of the proxy input and capital are identified in this stage. Coefficients of capital and energy enter twice in (6) and cannot be identified without further restrictions. Building on Levinsohn and Petrin (2003), identification is facilitated by assuming that capital is a state variable and does not instantaneously adjust to the unexpected part of productivity shock, although it might adjust to the predicted part. This notion is formalized by assuming that productivity is governed by an exogenous first-order Markov process

(7) $$p\left(a_{it} \mid \{a_{i\tau}\}_{\tau=0}^{t-1}, I_{it-1}\right) = p\left(a_{it} \mid a_{it-1}\right)$$

where I_{it-1} is the firm's information set at $t-1$. The evolution of a firm's productivity over time is such that a firm having just observed a_{it-1} at $t-1$ infers that the distribution of a_{it} will be $p\left(a_{it} \mid a_{it-1}\right)$. We can decompose a_{it} into its conditional expectation given the information available to the firm at $t-1$ (denoted by I_{it-1}) and a residual in a_{it}

(8) $$a_{it} = E\left(a_{it} \mid I_{it-1}\right) + \xi_{it} \ .$$

Using the assumption that productivity follows a first-order Markov process as given in (7) we know that firms, realizing the value of a_{it-1} at $t-1$, form expectations of productivity at t and hence we obtain

(9) $$a_{it} = E\left(a_{it} \mid a_{it-1}\right) + \xi_{it} .$$

Further, we assume that the non-forecastable part of productivity is uncorrelated with capital, leading to the two moment conditions

(10) $$E\left[(\xi_{it} + v_{it})k_{it}\right] = E\left[\xi_{it} k_{it}\right] + E\left[v_{it} k_{it}\right] = 0$$

(11) $E\left[(\xi_{it}+v_{it})e_{it-1}\right]=E\left[\xi_{it}e_{it-1}\right]+E\left[v_{it}e_{it-1}\right]=0$

The first moment condition (10) states the assumption that capital does not respond to the innovation in productivity. Capital stock in period *t* is determined by investment decisions from previous periods and does not respond to this period's productivity innovation ξ_t. The second moment condition (11) reflects last period's electricity choice and is uncorrelated with innovation in productivity. We employ the Generalized Method of Moments (GMM) to estimate the parameters of capital and energy, which involves choosing a starting value β_e^* and β_k^* for the estimation algorithm. For any candidate values, we re-write (2) to yield

(12) $y_{it}-\hat{\beta}_l l_{it}-\hat{\beta}_m m_{it}-\beta_e^* e_{it}-\beta_k^* k_{it}-\hat{\delta}t+\hat{u}_{it}=a_{it}+v_{it}$

Substituting (9) into (12) yields

(13) $y_{it}-\hat{\beta}_l l_{it}-\hat{\beta}_m m_{it}-\beta_e^* e_{it}-\beta_k^* k_{it}-\hat{\delta}t+\hat{u}_{it}-E(a_{it}\mid a_{i,t-1})=\xi_{it}+v_{it}$

Conditional on our candidate values (β_e^*,β_k^*), (12) implies estimate of $\widehat{a_{it}+v_{it}}$

(14) $\widehat{a_{it}+v_{it}}=y_{it}-\hat{\beta}_l l_{it}-\hat{\beta}_m m_{it}-\beta_e^* e_{it}-\beta_k^* k_{it}-\hat{\delta}t+\hat{u}_{it}$

With $E(a_{it}\mid a_{i,t-1})$ unknown, we estimate $E(a_{it}\mid a_{i,t-1})=E\left(a_{it}+v_{it}\mid a_{i,t-1}\right)$. From (6) in Stage 1 we obtain

(15) $\hat{a}_{i,t-1}=\hat{\phi}_{i,t-1}-\beta_e^* e_{i,t-1}-\beta_k^* k_{i,t-1}$

By performing local least squares regression[2] on $\widehat{a_{it}+v_{it}}$ by $\hat{a}_{i,t-1}$ we estimate $E(a_{it}\mid a_{i,t-1})$, which now allows us to compute an estimate of the residual $\widehat{\xi_{it}+v_{it}}\left(\beta^*\right)$ using (13) where $\beta^*=(\beta_k^*,\beta_e^*)$. We perform local least squares with dependent variable $\widehat{a_{it}+v_{it}}$ and independent variable $\hat{a}_{it-1}$, specifying a local quadratic kernel-based estimation that weights the observations closest to the point of evaluation more heavily. We then employ the GMM criterion to estimate the unknown parameters $\beta^* = (\beta_k^*, \beta_e^*)$

(16) $$\min_{\beta^*}\left[\left(\sum_i\sum_t\left(\widehat{\xi_{it}+v_{it}}\right)k_{it}\right)^2+\left(\sum_i\sum_t\left(\widehat{\xi_{it}+v_{it}}\right)e_{i,t-1}\right)^2\right]$$

and use a two-dimensional grid search to obtain the global minimum of this objective function by allowing the candidate values for β_k^* and β_e^* to vary from 0.01 to 0.99, in increments of 0.01. The moment condition represents the distance between the observed moments and zero. The two moment conditions (10) and (11) state that the residual term $\widehat{\xi_{it}+v_{it}}\left(\beta^*\right)$ is mean independent of k_{it} and e_{it-1}.

In *stage 2,* the estimated coefficients from *stage 1* are fed into the regression equations to compute $\widehat{a_{it}+v_{it}}$ and $\hat{a}_{i,t-1}$. Local quadratic least square estimation is executed using these estimators. Both the estimated *stage 1* coefficients and the predicted values from the local least square regression are then combined in the GMM estimation routine to estimate the coefficients of the capital and the proxy. All the preliminary estimators are used more than once and they introduce noise into the estimation routine. We use the bootstrap approach to estimate the standard errors where the observed data are used to approximate the true population distribution of the data and are sampled repeatedly to compare the variability of the estimates across these samples.

Data Description

Our dataset is sourced from the Colombian Annual Manufacturers Survey (AMS) covering 1982 to 1998. The AMS is an unbalanced panel of plant-specific quantities and prices for both output and inputs, and is suitable for estimating the gross output physical production frontier. The data are provided by *Departamento Administrativo Nacional de Estadistica* (DANE) and were created originally to study the impact of structural reforms on productivity and profitability enhancing reallocation in the Colombian manufacturing industry (Eslava et al. 2004). The same database is used by Eslava et al. (2010) to investigate the plant-level adjustment dynamics of capital and labor and their joint interactions in the context of deregulated Colombian manufacturers.

The dataset is comprised of Colombian manufacturing plants with more than 10 employees or sales over US$35,000 in 1998[3], and contains annual plant-level information on the following: i) the value of output and prices charged for each product; ii) cost and prices paid for each material used; iii) energy consumption in kilowatt per hour and energy prices; iv) number of workers and payroll; and v) book values of capital stock (buildings, structures, machinery, and equipment)[4]. In contrast to the literature measuring productivity by deflating sales by an industry-level price index, these data eliminate a common source of measurement error in production function estimation.

The plant-level price indices of output and materials are constructed using Tornqvist indices. While the quantities of materials and output are constructed by dividing the cost of materials and value of output by the corresponding price indices, the quantities of energy consumption are directly reported in the data. The capital stock variable is constructed by the perpetual inventory method using the book values and capital expenditure together with gross capital deflators and the depreciation rate of capital. Capital in period t is calculated by combining deflated investment in new capital with depreciated capital from $t-1$.[5] Labor is measured as total hours of employment, which is an improvement over the number of employees as a labor variable. Since these data do not present worker hours, a sector-level measure of average hours per laborer is constructed as the ratio of earnings per worker and the sectoral wage, which is obtained from the Monthly Manufacturing Survey of various years.

This study focuses on the Colombian meat, dairy products, bakery products, and confectionary industry indicated by 4-digit ISIC codes 3111, 3112, 3117, and 3119, respectively. We estimate the production frontier model at the 4-digit ISIC level to address as homogenous a sample of producers as possible. These data are annual time-series observations for 93 meat manufacturing firms with 1032 observations, 99 dairy firms with a total of 1219 observations, 363 bakery firms with 4049 observations, and 46 chocolate and confectionary firms with 551 observations. Summary statistics for the key variables are presented in Table 1 where the means and standard deviations of the logarithm of plant-level physical quantity and price of output and input variables are presented. The units for energy consumption and labor use are kilowatt hours and hours of employment, respectively. Output, capital, and materials are expressed in thousands of pesos based on the constant price index for 1982 being 100. The level of inputs and output differs across the food sub-sectors. Meat and dairy product firms are comparatively large in terms of average annual output, capital and employment. There are significant differences in material and energy consumption among the sectors. Meat and dairy product firms are more material- and energy-intensive than bakery and confectionary product firms.

The prices for output, materials, and energy are expressed as real prices relative to the yearly producer price index (PPI) to discount inflation. The mean of this relative price should be close to zero if appropriately weighted by output since the PPI value is dominated by manufacturing industries. A positive price variable can be interpreted as an increase in price relative to yearly PPI, whereas, a negative price variable shows a decrease in price relative to yearly PPI.

These constructed price indices are used to obtain plant-specific physical quantities by deflating the value of output and inputs and represent an important advantage over deflating sales by industry-level aggregate price deflators. In the next section we use these variables to estimate the production parameters and the technical efficiency by using a capital-labor-energy-material (KLEM) physical production frontier.

Estimation Results

Table 2 presents the stochastic production frontier parameter estimates using the traditional production frontier and the endogeneity corrected production frontier method. The standard errors are reported in parentheses and all dependent and independent variables are in log form. As a baseline, the traditional production frontier parameters are estimated using the Battese and Coelli (1992) error component model and the maximum-likelihood method with time-varying technical efficiency. The endogeneity corrected stochastic production frontier is estimated by the two-stage semi-parametric method outlined earlier where energy acts as proxy for the productivity shock[6]. The coefficients of labor and materials are estimated in the first stage whereas the coefficients of capital and energy are estimated in the second stage of the estimation procedure. The parameters of the production frontiers are significantly different across the four industries, but none have radically different point estimates. The estimates from both methods differ and provide insights into the endogeneity issue in stochastic production frontier estimation. As the semi-parametric approach points out, these elasticities are likely to be biased if productivity shocks are correlated with input choices. The coefficient of materials is the largest and lies in the range 0.7-0.8 for all individual food industries and the aggregate food sector. The output elasticities of labor and material are similar within the four industries, but that of energy is different in both methods. The coefficient of capital is consistently higher in the endogeneity corrected method than in the traditional stochastic frontier method in all four industries and the food manufacturing sector in aggregate. Compared to the endogeneity-corrected model, the estimates of the traditional stochastic frontier models find labor and materials coefficients to be equivalent or slightly overestimated for all industries. The energy coefficients, on the other hand, are underestimated for 4 out of the 5 regressions in the traditional stochastic frontier model. Consistently better log-likelihood values are generated with the endogeneity-corrected method than in the traditional method across industries.

For a two-input production function, with one variable input and one quasi-fixed (say capital) input, Marschak and Andrews (1944) suggest that the coefficient of the variable input is likely to be biased upward while the capital estimate is likely to be biased downward, provided the capital is not correlated or weakly correlated with this period's productivity shock. But in the presence of endogeneity, it is generally impossible to sign the biases of the production function coefficients when there are many inputs (Levinsohn and Petrin 2003). All of the inputs may be correlated with the error to varying degrees. The estimation bias of the production function coefficients depends not only on the correlation of the input variables with the unobserved productivity shock but also on the correlation between the input variables. With the energy proxy controlling for the unobserved productivity shock that is correlated with variable inputs, the evidence suggests that addressing the endogeneity issue matters in stochastic production frontier estimation to generate consistent estimates of production parameters for this sample of

Colombian food industry firms. The average rate of technical progress in all food manufacturing sectors is positive. For the industry-level estimation, the average rates of technical progress for meat and dairy product sectors are higher than that for bakery and confectionary product sectors. The annual rate of technical progress is highest in the meat industry with an estimate of 2%, and lowest in the bakery product industry with an estimate of 1.2%. The returns to scale estimates for the four food industries are 1.035, 1.118, 1.234, and 1.173 for meat, dairy, bakery, and confectionary product sectors, respectively, although estimates are not significantly different from constant returns to scale as judged by the Wald test at the 5% significance level.

Technical efficiency is estimated for each observation based on maximum likelihood estimation in the first stage. The point estimator for technical efficiency is calculated as the mean of the conditional distribution of u_{it} given $v_{it} - u_{it}$. The coefficient γ denotes the variance of the inefficiency component divided by total variance of the composed error term. The estimates of γ in Table 2 for all industries and the sector in aggregate are statistically significant at least at the 5% significance level, implying that technical inefficiency exists in all food sectors regardless of whether or not endogeneity is corrected for. The estimates of the time-varying efficiency component, ζ, are negative and statistically significant for all industries, implying that technical efficiency in Colombian food manufacturing sectors trending toward being less efficient. The significant coefficient γ, along with negative and significant ζ, implies that technical efficiency is present and decreases over time. Estimates of technical efficiency vary considerably both across firms and across time periods. The average technical efficiency for all food industry firms is 62.1%. Meat and bakery product industries have the highest average technical efficiency estimates, both being 66%, and the dairy sector has the lowest average technical efficiency estimate, 56%. Firm-level point estimates of technical efficiency are higher for most firms in the endogeneity corrected method than in the traditional stochastic frontier model for all food industries we examined. Figure 1 provides kernel density plots of technical efficiency for both the traditional stochastic frontier and the endogeneity corrected stochastic frontier methods. The plots show that the distributions of TE for the endogeneity corrected method are shifted rightward consistently for all the sectors. The firm-level estimates of technical efficiency increase because of the correction of endogeneity by conditioning out correlated unobserved shocks in production in stage 1 of the estimation procedure. Overall, low technical efficiency estimates indicate that the rate of technology diffusion in Colombian food firms was slow.

The average technical efficiency is found to be deteriorating through the sample period for all selected food manufacturing sectors. The rate of technical efficiency change consistently hovers around -1%, resulting in a steady negative impact on technological progress. Firm-level net effect of technological progress and technical efficiency change for the selected Colombian food manufacturing sectors are summarized by quintiles in Table 3. The results indicate that the gains in technological progress were reduced by the decrease in technical efficiency over time, but the net effects were positive for most food firms. Due to slow technological progress in bakery and confectionary product industries, the firms in the lowest quintile face a net loss effect. Overall, annual technological progress of 1.6% is offset by the negative estimate of average technical efficiency change of -1%, resulting in a net annual shift of 0.6% for all Colombian food manufacturing firms.

The competitive environment suggests that a time-varying specification of technical efficiency is desirable, particularly if a long panel dataset is available. Differences in managerial ability and education can impact the firm’s technical efficiency (Mundlak, 1961; Stefanou and Saxena, 1988; Battese and Coelli, 1995; Kalaitzandonakes and Dunn, 1995). Evidence of deteriorating technical efficiency is not new in the literature. Other studies finding decreasing technical efficiency over time include Spanish dairy farming (Cuesta, 2000), Korean textile manufacturing (Kim and Han, 2001), and Malaysian manufacturing (Kim and Shafii, 2009).

Concluding Comments

In order to correct for the endogeneity of input choice problem in the stochastic production frontier estimation, this study controls for the unobserved productivity shock using an intermediate input as a proxy and compares the results concerning the information about endogeneity in the stochastic frontier framework. We find that the output elasticity of capital is consistently higher when correcting for endogeneity compared to the traditional stochastic frontier method in all four food sectors. The traditional stochastic frontier analysis approach tends to underestimate the output elasticity of capital and firm-level technical efficiency for the Colombian food manufacturing industry. Although the coefficients of variable inputs are not widely different in the two methods, labor and materials are slightly overestimated in the traditional stochastic frontier method. The distributions of firm-level technical efficiency are found to be shifted rightward in endogeneity corrected method because of the correction of endogeneity in stage 1 of the estimation procedure. The average technical efficiency for all food industries is approximately 62% and is found to be deteriorating through time. The results also suggest that the gains in technological progress are reduced by the decrease in technical efficiency over time, resulting in a modest net annual shift for all Colombian food manufacturing firms. Low technical efficiency estimates indicate that the rate of technology diffusion in the Colombian food firms is slow. Hence, it is important to encourage policies that promote the efficient use of the existing technology to catch up to the technology frontier in the Colombian food manufacturing industry.

The level of efficiency speaks to the competitiveness of plants and their ability to compete, survive and grow. More efficient sectors can exploit greater gains from the resources expended, with greater efficiency translating into productivity gains. Correcting for the endogeneity of input choice leads to an increase in estimated technical efficiency for plants in each industry. By providing a methodology and estimation algorithm for correcting endogeneity of input choice problem, this study overcomes a major limitation in existing stochastic frontier research and provides more accurate estimates of production parameters and technical efficiency that are critical for policy analysis. The evidence suggests that addressing the endogeneity issue matters in the stochastic production frontier estimation to generate consistent estimates of production parameters and technical efficiency.

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Table 1. Industry-wise Summary Statistics of Key Variables

Variables	Butchering and Meat Canning	Dairy Products	Bakery Products	Cocoa, Chocolate and Confectionary	All Food
Output	11.582	12.035	9.779	10.637	10.976
	(1.580)	(1.673)	(1.287)	(1.937)	(1.809)
Capital	9.259	9.912	7.717	8.633	8.828
	(1.655)	(1.648)	(1.558)	(2.104)	(1.949)
Labor	11.244	11.541	10.508	10.956	10.881
	(1.239)	(1.086)	(1.015)	(1.298)	(1.198)
Energy	12.404	13.195	11.183	11.362	12.211
	(1.580)	(1.454)	(1.186)	(1.961)	(1.719)
Materials	11.276	11.687	9.341	10.140	10.637
	(1.695)	(1.690)	(1.252)	(1.962)	(1.857)
Output prices	-0.109	-0.024	0.110	0.050	0.053
	(0.299)	(0.287)	(0.338)	(0.432)	(0.328)
Energy prices	0.394	0.365	0.381	0.425	0.349
	(0.489)	(0.430)	(0.425)	(0.396)	(0.455)
Material prices	-0.143	-0.083	0.014	-0.012	-0.018
	(0.331)	(0.223)	(0.221)	(0.284)	(0.268)
No. of plants	93	99	363	46	1029
No. of obs.	1032	1219	4049	551	10772

Note: This table reports mean and standard deviations (in the brackets) of the log of quantity variables and log of prices deviated from yearly producer price indices to discount inflation. The units of the labor and energy variables are hours of employment and kilowatt hours respectively. The other variables are expressed in thousands of pesos based on constant price index for 1982 being 100.

Table 2. Stochastic Production Frontier Estimates

Variables	Butchering and meat canning		Dairy Products		Bakery Products		Chocolate and confectionary		All Food	
	Stochastic frontier	Endogeneity corrected stochastic frontier	Stochastic frontier	Endogeneity corrected stochastic frontier	Stochastic frontier	Endogeneity corrected stochastic frontier	Stochastic frontier	Endogeneity corrected stochastic frontier	Stochastic frontier	Endogeneity corrected stochastic frontier
Const.	-0.817		-1.214		-0.459		-0.522		-0.693	
	(0.293)		(0.308)		(0.172)		(0.491)		(0.139)	
lnL	0.106	0.107	0.187	0.168	0.116	0.111	0.224	0.236	0.138	0.131
	(0.017)	(0.032)	(0.019)	(0.037)	(0.010)	(0.020)	(0.027)	(0.035)	(0.007)	(0.013)
lnM	0.760	0.758	0.787	0.780	0.862	0.852	0.772	0.777	0.821	0.816
	(0.014)	(0.023)	(0.015)	(0.032)	(0.009)	(0.020)	(0.021)	(0.042)	(0.006)	(0.012)
lnE	0.056	0.030	0.015	0.080	-0.002	0.200	-0.008	0.020	0.013	0.030
	(0.014)	(0.015)	(0.015)	(0.016)	(0.007)	(0.225)	(0.020)	(0.324)	(0.005)	(0.015)
lnK	0.057	0.140	0.042	0.090	0.024	0.070	0.052	0.140	0.033	0.080
	(0.012)	(0.046)	(0.013)	(0.049)	(0.005)	(0.029)	(0.016)	(0.067)	(0.004)	(0.029)
t	0.020	0.020	0.021	0.018	0.015	0.012	0.012	0.014	0.018	0.016
	(0.003)	(0.004)	(0.003)	(0.006)	(0.002)	(0.003)	(0.005)	(0.010)	(0.002)	(0.002)
σ_s^2	0.139	0.140	0.135	0.126	0.124	0.126	0.223	0.240	0.135	0.137
	(0.024)	(0.025)	(0.017)	(0.016)	(0.009)	(0.010)	(0.061)	(0.076)	(0.007)	(0.008)
γ	0.746	0.747	0.699	0.691	0.720	0.727	0.820	0.835	0.728	0.732
	(0.045)	(0.047)	(0.040)	(0.040)	(0.022)	(0.023)	(0.050)	(0.053)	(0.015)	(0.016)
μ	0.497	0.480	0.759	0.729	0.651	0.569	0.675	0.607	0.764	0.687
	(0.093)	(0.094)	(0.074)	(0.073)	(0.056)	(0.043)	(0.159)	(0.188)	(0.042)	(0.040)
ζ	-0.037	-0.037	-0.037	-0.036	-0.038	-0.038	-0.036	-0.043	-0.034	-0.036
	(0.008)	(0.008)	(0.005)	(0.005)	(0.003)	(0.003)	(0.009)	(0.009)	(0.002)	(0.003)
mean TE	0.653	0.660	0.562	0.581	0.633	0.663	0.624	0.645	0.585	0.621
	(0.126)	(0.126)	(0.137)	(0.131)	(0.134)	(0.138)	(0.154)	(0.161)	(0.131)	(0.135)
LLR	112.460	113.930	75.904	105.932	535.462	548.676	27.679	32.578	654.374	683.705

Table 3. Firm-level Net Effect of Technical Change and Technical Efficiency Change Corrected for Endogeneity

	Quintile	Butchering and Meat canning	Dairy Products	Bakery Products	Chocolate and Confectionary
Net change	1 (lowest)	0.007	0.006	-0.001	-0.001
	2	0.009	0.006	0.000	0.000
	3	0.010	0.007	0.001	0.002
	4	0.011	0.008	0.003	0.004
	5 (highest)	0.014	0.011	0.007	0.008
TEC	1 (lowest)	-0.013	-0.013	-0.013	-0.015
	2	-0.011	-0.012	-0.012	-0.014
	3	-0.010	-0.012	-0.011	-0.012
	4	-0.009	-0.011	-0.009	-0.010
	5 (highest)	-0.006	-0.007	-0.006	-0.006
TP	1 (lowest)	0.020	0.018	0.012	0.014
	2	0.020	0.018	0.012	0.014
	3	0.020	0.018	0.012	0.014
	4	0.020	0.018	0.012	0.014
	5 (highest)	0.020	0.018	0.012	0.014

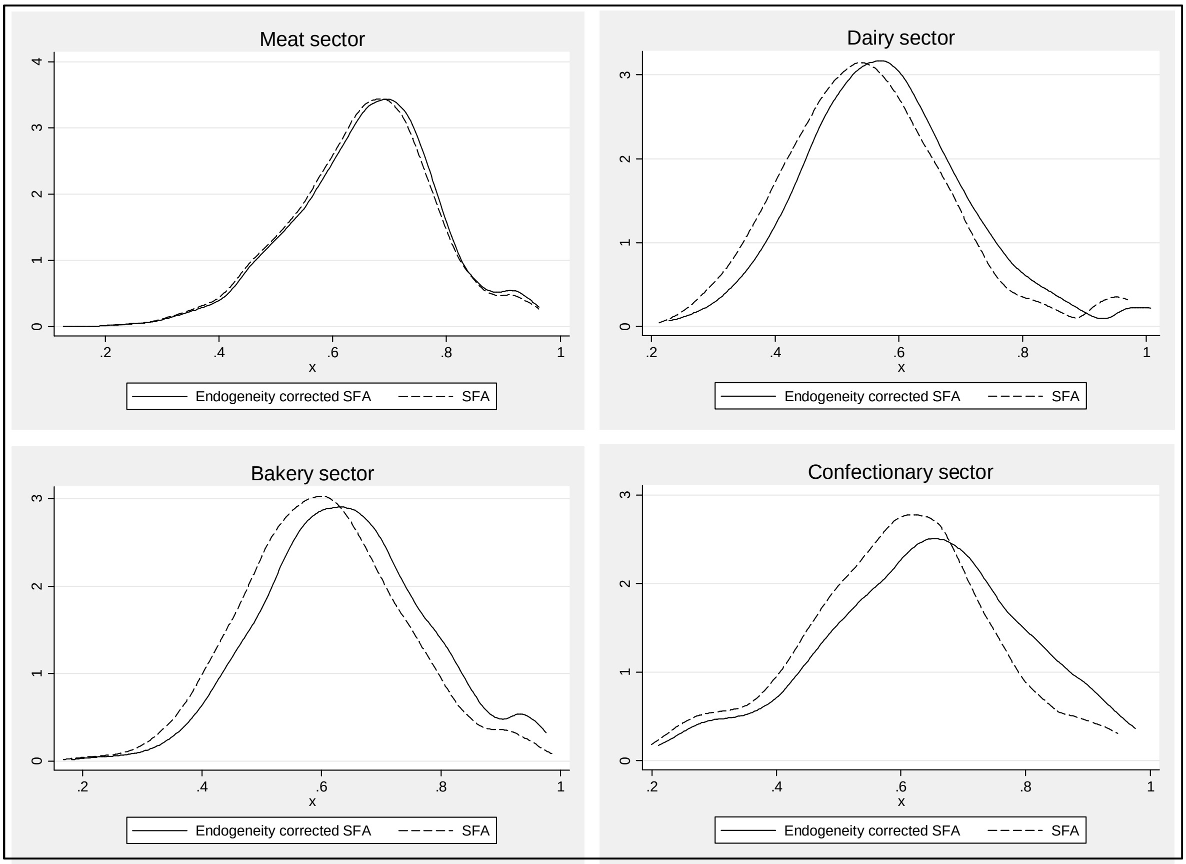

Figure 1. Kernel density plots of TE for endogeneity corrected and standard stochastic frontier for 4 sectors

Appendix A

A stochastic frontier production model representing panel data can be written as

$$Y_{it} = X_{it}^{\beta} \exp(v_{it} - u_{it})$$

Taking logarithmic transformation and writing log terms in small letters

(A1) $y_{it} = x'_{it}\beta + \varepsilon_{it}$

where $\varepsilon_{it} = v_{it} - u_{it}$ is composed error. Following Battese and Coelli (1992), time varying technical inefficiency can be written as

(A2) $u_{it} = \zeta_{it} u_i = \exp[-\zeta(t-T)]u_i$; $t \in \phi(i)(i = 1, 2, .., N)$,

where ζ is an unknown scalar parameter to be estimated and $\phi(i)$ represents the set of T_i time periods among the total T periods for which observation for the i^th^ firm are obtained. Assuming $v_{it} \sim iidN(0, \sigma_v^2)$ and $u_i \sim iidN^+(\mu, \sigma_u^2)$ and u_i's and v_{it}'s are independent

(A3) $\varepsilon_{it} = v_{it} - \zeta_{it} u_i$

(A4) $$f(u) = \frac{1}{[1 - F(-\frac{\mu}{\sigma_u})](2\pi)^{1/2}\sigma_u} \exp\left[-\frac{1}{2}\left(\frac{u-\mu}{\sigma_u}\right)^2\right]$$

(A5) $$f(v) = \frac{1}{(2\pi)^{1/2}\sigma_v} \exp\left[-\frac{1}{2}\left(\frac{v}{\sigma_v}\right)^2\right]$$

Since u and v are independent, the joint density is the product of their individual densities

$$\text{(A6)} \qquad f(u,v) = \frac{1}{[1-F(-\frac{\mu}{\sigma_u})]2\pi\sigma_u\sigma_v} \exp\left[-\frac{1}{2}\left(\frac{u-\mu}{\sigma_u}\right)^2 - \frac{1}{2}\left(\frac{v}{\sigma_v}\right)^2 \right]$$

In (A3) the density function of u is independent of time whereas the density of v is time dependent. In vector notation, let v_i be the $(T_i \times 1)$ vector of v_{it}'s for T_i observations for the ith firm $v_i = (v_{i1},...,v_{iT_i})'$. Using the results from multivariate normal distribution when there are T_i independent observations, we obtain

$$\text{(A7)} \qquad f(u_i,v_i) = \frac{1}{[1-F(-\frac{\mu}{\sigma_u})](2\pi)^{(T_i+1)/2}\sigma_u\sigma_v^{T_i}} \exp\left[-\frac{1}{2}\left(\frac{u_i-\mu}{\sigma_u}\right)^2 - \frac{1}{2}\left(\frac{v_i' v_i}{\sigma_v^2}\right) \right]$$

Using $\varepsilon_{it} = v_{it} - \zeta_{it}u_i$ and ε_i being $T_i \times 1$ vector of ε_{it}'s for T_i number of observations or

$\varepsilon_i = (v_{i1} - \zeta_{i1}u_i,..., v_{iT_i} - \zeta_{iT_i}u_i)'$ and in vector form $v_i = \varepsilon_i + \zeta_i u_i$ where ζ_i is $T_i \times 1$ vector of ζ_{it}

or $\zeta_i = (\zeta_{i1},...,\zeta_{iT_i})$, the joint distribution of u_i and ε_i is given by

$$\text{(A8)} \qquad f(u_i,\varepsilon_i) = \frac{1}{[1-F(-\frac{\mu}{\sigma_u})](2\pi)^{(T_i+1)/2}\sigma_u\sigma_v^{T_i}} \exp\left[-\frac{1}{2}\left(\frac{u_i-\mu}{\sigma_u}\right)^2 - \frac{1}{2}\left(\frac{(\varepsilon_i+\zeta_i u_i)'(\varepsilon_i+\zeta_i u_i)}{\sigma_v^2}\right) \right]$$

The marginal density function of ε_i is obtained by integrating out u_i or $f(\varepsilon_i) = \int_0^\infty f(u_i,\varepsilon_i)du_i$

$$\text{(A9)} \qquad f(\varepsilon_i) = \frac{\left[1-F(-\frac{\mu_i^*}{\sigma_i^*})\right]}{\left[1-F(-\frac{\mu}{\sigma_u})\right](2\pi)^{T_i/2}\sigma_v^{T_i-1}\left[\sigma_v^2 + \zeta_i'\zeta_i\sigma_u^2\right]^{1/2}} \exp\left[-\frac{1}{2}\{\left(\frac{\varepsilon_i'\varepsilon_i}{\sigma_v^2}\right) + \left(\frac{\mu}{\sigma_u}\right)^2 - \left(\frac{\mu_i^*}{\sigma_i^*}\right)^2\} \right]$$

where

$$\text{(A10)} \qquad \mu_i^* = \frac{\mu\sigma_v^2 - \zeta_i'\varepsilon_i\sigma_u^2}{\sigma_v^2 + \zeta_i'\zeta_i\sigma_u^2}$$

(A11) $\sigma_i^{*2} = \dfrac{\sigma_u^2\sigma_v^2}{\sigma_v^2 + \zeta_i'\zeta_i\sigma_u^2}$

The conditional density of u_i given ε_i is

(A12) $f(u_i \mid \varepsilon_i) = \dfrac{f(u_i, \varepsilon_i)}{f(\varepsilon_i)} = \dfrac{1}{[1 - F(\frac{\mu_i^*}{\sigma_i^*})](2\pi)^{1/2}\sigma_i^*} \exp\left[-\dfrac{1}{2}\left(\dfrac{u_i - \mu_i^*}{\sigma_i^*}\right)^2\right]$

This is the density function of the positive truncation of the $N(\mu_i^*, \sigma_i^{*2})$. Hence the estimation of technical efficiency of the ith firm at time period t is given by

(A13)
$$E[\exp(-u_{it}) \mid \varepsilon_i] = \int_0^\infty \exp(-\zeta_{it}u_i) f(u_i \mid \varepsilon_i) du_i$$
$$= \left[\frac{1 - F[\zeta_{it}\sigma_i^* - (\mu_i^* / \sigma_i^*)]}{1 - F(-\mu_i^* / \sigma_i^*)}\right] \exp\{-\zeta_{it}\mu_i^* + \tfrac{1}{2}\zeta_{it}^2\sigma_i^{*2}\}$$

The density function of y_i, a $T_i \times 1$vector of y_{it}'s for the ith firm, can be obtained from (A9) by substituting $(y_i - x_i'\beta)$ for ε_i, where x_i is a $T_i \times k$ matrix of x_{it}'s for the ith firm. The log likelihood function for the sample observations $y = (y_1', y_2', ..., y_N')'$ is given by

(A14)
$$LL(\theta^*; y) = \tfrac{1}{2}(\sum_{i=1}^N T_i)\ln(2\pi) - \tfrac{1}{2}\sum_{i=1}^N (T_i - 1)\ln(\sigma_v^2) - \tfrac{1}{2}\sum_{i=1}^N \ln(\sigma_v^2 + \zeta_i'\zeta_i\sigma_u^2)$$
$$- N\ln[1 - F(-\mu / \sigma_u)] + \sum_{i=1}^N \ln[1 - F(-\mu_i^* / \sigma_i^*)]$$
$$- \tfrac{1}{2}\sum_{i=1}^N [(y_i - x_i\beta)'(y_i - x_i\beta) / \sigma_v^2] - \tfrac{1}{2}N(\mu / \sigma_u)^2 + \tfrac{1}{2}\sum_{i=1}^N (\mu_i^* / \sigma_i^*)^2$$

where $\theta^* = (\beta', \sigma_u^2, \sigma_v^2, \mu, \zeta)'$

Appendix B

Estimation algorithm

Stage one:

1. Create a third-order polynomials in k and the proxy e or $\phi_t(e_t, k_t) \approx \sum_{i=0}^{3}\sum_{j=0}^{3-i} c_{ij} k_t^i e_t^j$.

2. Run Battese and Coelli maximum likelihood estimation with no intercept using the freely variable inputs (except the proxy) and the constructed polynomial terms as independent variables.

The key estimated parameters from this stage are all the freely variable inputs except the proxy and the technical (in)efficiency, or $\hat{\beta}_l$, $\hat{\beta}_m$, $\hat{\delta}$, and $\hat{u}_{it}$.

Stage two:

1. Choose starting candidate values for (β_k, β_e) say (β_k^*, β_e^*) for estimation algorithm. Although starting value is not critical, a good guess for beginning would be OLS estimates.

2. Compute $\widehat{a_{it} + v_{it}} = y_{it} - \hat{\beta}_l l_{it} - \hat{\beta}_m m_{it} - \beta_e^* e_{it} - \beta_k^* k_{it} - \hat{\delta} t + \hat{u}_{it}$, We call this variable 'A'.

3. Compute $\hat{a}_{i,t-1} = \hat{\phi}_{i,t-1} - \beta_e^* e_{i,t-1} - \beta_k^* k_{i,t-1}$ and call the variable 'B'.

4. Regress 'A' on 'B' using locally weighted least squares. Take the predicted value and call it 'C' which is equal to $E(a_{it} \mid a_{i,t-1})$.

5. Compute $\xi_{it} + \widehat{v_{it}(\beta^*)} = y_{it} - \hat{\beta}_l l_{it} - \hat{\beta}_m m_{it} - \beta_e^* e_{it} - \beta_k^* k_{it} - \hat{\delta} t + \hat{u}_{it} - E(a_{it} \mid a_{i,t-1})$ which is basically 'A'- 'C'. This enters into moment equation in GMM estimator.

6. Perform a grid search to obtain the global minimum of the GMM objective function and iterate the previous steps by allowing the candidate values for β_k^* and β_e^* from 0.01 to 0.99, in increments of 0.01.

The key estimated parameters from this stage are proxies input (energy) and capital.

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[1] The within-year variation of the price indices for output, material and energy are fairly tight, with a coefficient of variation falling in the 15-20% range on average. Consequently, we can reasonably expect that these markets are perfectly competitive.

[2] Local least squares regression is a nonparametric kernel-based estimation method is discussed in Pagan and Ullah (1999).

[3] For a more detailed description of the data, see Eslava, et al. (2004).

[4] We treat plants as firms although there are multi-plant firms in the sample because of data restriction. The AMS does not provide any information on which plants are firms and which plants belong to a firm (or group).

[5] Industry-level depreciation rates are obtained from Pombo (1999).

[6] We also estimate the model using materials as the proxy and find the parameters of the production frontier to be very similar. We present the estimation results using energy as the intermediate proxy for transmitted productivity shock.