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Endogenous Growth and Technological Progress with Innovation Driven by Social Interactions

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Abstract. We analyze the implications of innovation and social interactions on economic growth in a stylized endogenous growth model with heterogenous research firms. A large number of research firms decide whether to innovate or not, by taking into account what competitors (i.e., other firms) do. This is due to the fact that their profits partly depend on an externality related to the share of firms which actively engage in research activities. Such a share of innovative firms also determines the evolution of technology in the macroeconomy, which ultimately drives economic growth. We show that when the externality effect is strong enough multiple BGP equilibria may exist. In such a framework, the economy may face a low growth trap suggesting that it may end up in a situation of slow long run growth; however, such an outcome may be fully solved by government intervention. We also show that whenever multiple BGP exist, the economy may cyclically fluctuate between the low and high BGP as a result of shocks affecting the individual behavior of research firms.

Keywords: Economic Growth, Innovation; Firms Interaction; Low Growth Trap; Fluctuations

JEL Classification Numbers: C60, D70, O40

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1 Introduction

Technological progress is by far the most important determinant of economic growth in industrialized economies. During the last two decades, after the seminal works of Romer (1986), Aghion & Howitt (1992), and Grossman & Helpman (1994), many efforts have been put forward to try understanding and explaining the sources of technological advances. All the resulting works take the nature of the research sector as given, and the interaction among firms in the research industry has never been analyzed in depth thus far[1]. This is however one of the main fields of interest of computational economics; heterogeneous agent models, for instance, may help in explaining how innovation occurs, which are the dynamics of innovation and how innovation determines technological progress (see Dawid (2006) for an extensive survey). The goal of this paper is to bridge these two different branches of literature by developing a stylized but analytically tractable and micro-founded agent based model of innovation to shed some light on the role that interactions among research firms might play in the process of economic growth. Once an almost traditional economic growth model is extended to allow some form of interaction among research firms along the lines outlined in Brock & Durlauf (2001) and Blume & Durlauf (2003), some traditional results, like the uniqueness of equilibrium, found in growth theory vanish. Indeed, such an interaction among research firms, by determining the rate of technological progress, plays a critical role in shaping the whole macroeconomic dynamics. We show that, under certain parameter conditions, the economy may be characterized by a multiplicity of balanced growth path (BGP) equilibria, and a situation of low growth trap. We also show that the economy may eventually (endogenously) fluctuate between the low and high BGP generating thus a growth cycle in which periods of low and high economic growth rates follow one another. In this context economic policy, aiming to modify the incentives associated with research activities, may be very effective in order to completely solve the low growth trap problem, avoiding thus further fluctuations in economic activity.

Our paper is thus related to different branches of the economic literature, namely computational economics, economic growth and business cycles theory. From the computational economics literature we simply borrow the interest in analyzing the interaction between research firms and its eventual implications for technological progress and the long run economic growth (Dawid (2006), Dosi et al. (2010)). However, from a methodological point of view our approach is substantially different since we develop a very simple and tractable model, in which most of the results are analytically derived; simulations in our paper play only a marginal role and are instrumental to exemplify some interesting and potential outcomes. Economic growth theory is the main benchmark for our analysis since the model is an almost standard continuous time model of optimal growth with endogenous technological progress (Acemoglu (2009)). With respect to what traditionally assumed in this literature (Romer (1986), Grossman & Helpman (1994)), we allow for a certain degree of diffusion in the pattern

[1] Schumpeterian growth models to some extent model the interaction in the research sector by allowing for a business-stealing effect, determining the likelihood that an incumbent innovator looses its monopoly power because of a success in the innovation process by a new entrant (Acemoglu (2009)). Apart from this type of characterization, the endogenous growth literature has not emphasized how the choice of research firms are related and interdependent.

of innovation, meaning that in our framework technical progress is driven by the interaction among research firms.[2] To the best of our knowledge, no other study has thus far focused on the firms interaction in the research industry in a way comparable to ours, and moreover all the works identify a unique BGP equilibrium thus cyclical behavior cannot occur. The understanding and characterization of cyclical patterns is the main interest of the business cycle theory[3] (Kydland and Prescott (1982), King et al. (1988a), King et al. (1988b)), which besides adopting a discrete time framework[4] (Evans et al. (1998); Furukawa (2007)), it also relies upon stochastic growth models in which the source of the shock is completely exogenous (Walde (2005)). Probably, the work most close to ours is Bambi et al. (2014), which analyzes an endogenous growth model with expanding product variety showing that cyclical fluctuations may arise as a result of implementation delays in the innovation process. Despite their setting is quite similar to ours (an almost traditional endogenous growth model) the mechanism underlying output fluctuations is substantially different since we do not allow for time delays but simply for some sort of interaction among firms operating in the research industry. Moreover, different from them, our model shows the existence of a growth trap threshold, allowing to clearly distinguish economies which will experience low and high growth rates, which is again simply due to the interaction among research firms.

Therefore, our paper is also closely related to the literature on poverty traps. The eventual existence and characteristics of poverty traps have been extensively analyzed in literature since the seminal work by Skiba (1978). Different explanations of why multiplicity of equilibria and thus poverty traps may exist have been put forward, and they include increasing returns and imperfect competition, coordination failure, matching problems and increasing returns (see Azariadis and Stachurski (2005) for an exhaustive survey). However, all these theories proposed thus far outline sources of multiplicity in levels, suggesting thus that under certain conditions an economy may eventually end up in poverty, that is a situation of stagnation with no long run growth. Our model instead suggests the potential existence

[2]This is in line to what suggested by the seminar work by Bass (1969) in the context of diffusion of durables. The Bass model is a particular case of a larger class of *epidemiological models*. We refer the reader to Hethcote (2000) for a recent survey on the topic.

[3]Cyclical outcomes are also analyzed in growth theory by characterizing the eventual existence of equilibrium indeterminacy (Benhabib and Farmer (1994); Benhabib and Farmer (1998)). Also this approach is substantially different from ours, since our BGP equilibria are all determinate.

[4]Because of the similarity with our paper and their qualitative results, the seminal work by Evans et al. (1998) deserves some specific comments. Indeed, also Evans et al. (1998) show that under specific conditions a stylized economic growth model may give rise to a low growth trap and a growth cycle in which the economy stochastically switches between periods of low and high growth. However, the underlying argument and the type of dynamics at the basis of their analysis is substantially different from ours, since, apart from relying on a discrete time setup, the driver of the entire economic dynamics in their model is represented by shocks on agents' expectations which affect the learning dynamics associated with multiple perfect-foresight equilibria. Our results, instead, are derived in a micro-founded model where firm-specific shocks within the research industry, by determining the evolution of technology, propagate in the whole economy eventually generating growth cycles; the concept of endogenous fluctuations we describe is thus not related to either expectational indeterminacy or self-fulfilling growth cycles, which represent the traditional mechanisms discussed in hte business cycle literature (Evans et al. (1998); Furukawa (2007)). The fact that such very different setups allow to generate qualitatively similar dynamics suggest that endogenous growth cycles and low growth traps are not only rare theoretical possibility but rather outcomes quite common whenever we depart from the traditional economic growth framework.

of equilibrium multiplicity in growth rates, meaning that an economy may eventually end up in a situation of long run growth characterized by low growth rates. In order to distinguish this result from what traditionally discussed in the poverty traps literature we refer to such an outcome as a *"low growth trap"*. To the best of our knowledge, apart the very recent paper by Agénor and Canuto (2015) in an overlapping generation setting, there is no other study characterizing the eventual existence of low growth traps. The implications of the existence of a low growth trap threshold are however very intuitive and in line with empirical evidence: some countries will experience fast economic growth while others slow economic growth, meaning that income gaps will tend to widen over time characterizing thus a situation of long run divergence, as traditionally found in the empirics on economic growth, especially between developed and developing countries[5] (Dowrick (1992), Pritchett (1997)). Finally, our model predicts a very important role for economic policy, since in the case of a low growth trap the government, by simply rising the level of taxation on households in order to increase the revenues granted to research firms, may be able to completely solve the trap problem. This does not simply mean that the low growth trap threshold may be exceeded, as the traditional policy implication of poverty trap models (see for example Sachs et al (2004), or more recently La Torre et al. (2015)), but that the threshold itself will cease to exist ensuring thus that the economy is able to experience fast economic growth.

The paper proceeds as follows. Section 2 focuses on the research industry and describes its peculiarities without considering its implications for the whole economy. Specifically, the research industry is populated by a large number of profit-seeking firms facing a dichotomous choice. On the one hand, these firms are heterogeneous in their propensity to innovate, and on the other hand, their decision whether to innovate or not is partly affected by the behavior of other firms in the industry through an externality component. We characterize the research industry dynamics deriving an explicit expression which allows us to describe the (aggregate) behavior of research firm in terms of the share of firms actively engaged in research activities. Section 3 integrates the research industry in a traditional macroeconomic model of endogenous growth, where the government finances research by taxing households, and the overall level of technology in the economy depends on the share of firms engaged in innovation. Section 4 shows that the balanced growth path (BGP) equilibrium, which strictly depends upon the behavior of research firms, may or may not be unique according to the magnitude of the externality-induced profit component; we also characterize the dynamic properties of different BGP equilibria, identifying the eventual existence of a low growth trap along with its policy implications, and the possibility of cyclical behavior. In Section 5 we discuss how our model relates to the middle-income trap hypothesis, suggesting that after a first stage of take off characterized by rapid growth developing countries may face a significant growth slowdown; differently from previous research which identify mainly intersectoral dynamics as a potential source of growth slowdowns, we argue that this may also be the result of intra-sectoral dynamics (driven by social interactions and technology diffusion) within the research industry. Section 6 presents concluding remarks and proposes directions

[5]Despite the existence of some (absolute) convergence within a small number of industrialized countries (see, for example, Barro and Sala–i–Martin (1995)), convergence clubs represent more the exception rather than the rule in the empirics of economic growth.

for future research.

2 Research Activities and Intra-Industry Interactions

We consider a research industry populated by a large number of research firms which try to maximize the profits associated with their research activities; specifically, there exist N firms indexed by $i = 1, \ldots, N$. For the sake of simplicity we assume that the research choice is just binary, thus we do not try to properly quantify research efforts. Thus, any research firm needs to decide whether to engage in research activities or not, thus it needs to compare the profit it will obtain by performing research with the zero-profit associated with no research activities.

If a firm actively engages in research activities it will give rise with no uncertainty to an innovation, which generates a given (fixed) amount of revenues $h \geq 0$ associated with the sale of the (unitary) innovation[6]. In order to produce one unit of innovation, the firm faces a (stochastic) production cost $z + \zeta_i$, where $z \geq 0$ denotes the cost common to all the firms and ζ_i is a random firm-specific shock. Apart from these private components of the profit structure, research profits are also affected by a social component associated with the number of firms actively engaged in research activities. Specifically, the size of the research industry through an externality[7] channel determines whether profits, ceteris paribus, tend to rise or fall. There are two different cases that need to be considered: an increase in the number of firms actively engaged in research may increase the profit for the whole research industry and thus rise the profit of the individual research firm; alternatively, an increase in the number of firms actively engaged in research may decrease the profit for the whole research industry and thus lower the profit of the individual research firm. The former case represents the so-called "standing-on-the-shoulder effect", that is innovation by some firms increases the possibility of further innovation by others, while the latter case the "fishing-out effect", that is innovation by some firms decreases the possibility of further innovation by others (Jones (2005)). Formally, we model the individual firm research profits as in random utility models (see Brock & Durlauf (2001) and Barucci & Tolotti (2012)). Each firm is thus characterized by its specific innovative attitude $\omega_{i,t} \in \{0; 1\}$, where $\omega_{i,t} = 1$ ($\omega_{i,t} = 0$) denotes that firm i is (is not) innovating at time t. The decision to engage in research activities to produce innovation is based on the following profit structure:

$$\pi_i(\omega_i) = \omega_i \left[h - (z + \zeta_i) + J \left(\tilde{x}_i^e - \frac{1}{2} \right) \right]. \tag{1}$$

If the firm does not innovate ($\omega_{i,t} = 0$) the profit above is simply null, $\pi_i(0) = 0$. If the firm does innovate ($\omega_{i,t} = 1$) the profit is equal to $\pi_i(1) = [h - (z + \zeta_i) + J(\tilde{x}_i^e - 1/2)]$,

[6] For the time being we do not look at the demand side of the innovation market, but this will be introduced in a very stylized way in section 3, where we assume that the government buys such an innovation. The amount of revenue h can thus be interpreted as the incentive provided by the government to induce firms to perform research activities, or alternatively as the price at which it purchases the innovation from research firms.

[7] This externality in research profits may be interpreted in terms of the availability of potential trading partners for the innovation, which reflects into a larger or smaller willingness to produce according to the sign of J in (1). With this respect, the market for innovation is similar to the trading market proposed in Diamond (1982).

where the first two terms represent the private component of profit while the third term is the social component related to the effect of externalities. The impact of the research externality is equal to $J(\tilde{x}_i^e - 1/2)$, where $J \in \mathbb{R}$ determines the sign and the magnitude of the externality effect and $\tilde{x}_i^e$ is the expectation of firm i about the average of the choices of others firms: $\tilde{x}_i^e = \frac{1}{N-1}\mathbb{E}[\sum_{j \neq i} \omega_j]$. Note that the sign of J determines the type of externality affecting research firms: whenever $J > 0$ individual profits tend to increase as a result of the research performed by others (standing-on-the-shoulder effect), while whenever $J < 0$ individual profits tend to fall (fishing-out effect). The term $(\tilde{x}_i^e - 1/2)$ states that in quantifying the impact of the (positive or negative) externality-induced profit component firms look at what the majority of other firms does. Indeed, the term $1/2$ refers exactly to one half of the total population of research firms, thus if $\tilde{x}_i^e > 1/2$ then firm i will expect more than half of the firms to do research. Finally, the random components of cost, ζ_i, $i = 1, \dots, N$ are i.i.d. random shocks drawn from a common distribution η, which affect with different intensity the perceived profit of individual firms. Two remarks on the profit structure are needed. First of all, note that the profit π depends on the subjective expectation of the firm about others' actions. With this respect, it can be seen as the realized profit once conditioned on agent's expectation about others' actions. Secondly, the random component of the profit is entirely related to the cost structure. We could in principle build a profit structure where randomness may inpact jointly or separately both revenues and costs. Besides amounting in a more complicated probabilistic structure, this woud not have any significant qualitatve implications. For a more comprehensive discussion about the rationale behind the profit structure as in (1), we refer the reader to Appendix A.

It can be easily verified that profits as in (1) turn into a probabilistic choice model where:

$$\mathbb{P}(\omega_i = 1 | \, \tilde{x}_i^e) = \eta \left[h - z + J \left(\tilde{x}_i^e - \frac{1}{2} \right) \right]. \tag{2}$$

As shown in the literature on social interactions (see Blume & Durlauf (2003)), a dynamic counterpart of such a model can be derived. Define $x_t^N = \frac{1}{N}\sum_{i=1}^N \omega_{i,t}$ as the fraction of innovative firms at time t and assume this quantity is observable; we refer to x_t^N as the "innovation share". Similarly as in the static model, we assume that firms can decide whether to invest or not at any time t by considering its potential revenue h, cost z and the current value of the innovation share $\widetilde{x}_i^e$. Indeed,

$$\mathbb{P}(\omega_{i,t+\Delta t} = 1 | \, \omega_{i,t}, x_t^N) = \eta \left[h - z + J \left(x_t^N - \frac{1}{2} \right) \right]. \tag{3}$$

It turns out that the Markovian dynamics induced by (3) are difficult to study in the finite dimensional population model; nevertheless, it is possible to describe in closed-form the (deterministic) dynamics emerging from the asymptotic system when letting the number of research firms go to infinity. In particular, the following result describes the time evolution of x_t which is the fraction of innovative firms at time t when we let $N \to \infty$. To this aim, we assume that the shocks ζ_i follow a centered logistic distribution[8] with parameter $\beta > 0$

$$\eta(x) = \mathbb{P}(\zeta_i \le x) = \frac{1}{1 + e^{-\beta x}}.$$

In this context, β is a measure of the dispersion of opinion in the population of firms: $\beta = 0$ would represent a situation in which the firms decide to invest or not by tossing a coin; on the contrary, $\beta \to \infty$ would mean that the firms do not receive any stochastic signal (i.e., the random cost component) and decide just by looking at the sign of $h - z + J\left(x_t^N - \frac{1}{2}\right)$. In the next proposition we state a convergence result in the number of firms and we describe the dynamics of x_t, the equilibrium fraction of research firms.

Proposition 1. *Let $x_t^N = \frac{1}{N}\sum_{i=1}^N \omega_{i,t}$ be the share of innovative firms at time t. Suppose $\lim_{N\to\infty} x_0^N = x_0$. Then, when $N \to \infty$, the family of stochastic processes $(x^N)_{N\ge 0}$, where $x^N := (x_t^N)_{t\ge 0}$, converges almost surely to $x := (x_t)_{t\ge 0}$, where x_t solves*

$$\dot{x}_t = \frac{1}{2}\tanh\left\{\beta\left[h - z + J\left(x_t - \frac{1}{2}\right)\right]\right\} - x_t + \frac{1}{2}, \tag{4}$$

for a given initial condition x_0.

Proof. We can recover the standard Blume & Durlauf (2003) framework by rearranging the state variables to take values on $\{-1; +1\}$. Define $\zeta_i = 1$ when $\omega_i = 1$ and $\zeta_i = -1$ when $\omega_i = 0$. In this case, we have that

$$\mathbb{P}(\zeta_{i,t} = 1 |\, \zeta_{i,t}, m_t^N) = \eta(h - z + J/2 \cdot m^N(t)), \quad \mathbb{P}(\zeta_{i,t} = -1 |\, \zeta_{i,t}, m_t^N) = 1 - \eta(h - z + J/2 \cdot m^N(t)),$$

where now $m^N(t) = \frac{1}{N}\sum_i \zeta_{i,t}$ takes values on $[-1, 1]$. Arguing similarly as in Barucci & Tolotti (2012), it can be shown that, under the assumptions of Proposition 1,

$$\lim_{N\to\infty} m_t^N = m_t,$$

where m_t is the unique solution to

$$\dot{m}_t = \tanh\left\{\beta\left(h - e + J \cdot \frac{m_t}{2}\right)\right\} - m_t; \quad m_0 = 2x_0 - 1.$$

Since $x_t = \frac{m_t + 1}{2}$, equation (4) immediately follows. ■

The quantity x_t characterizes the fraction of innovative firms in a large economy of research firms subject to externalities and private signals. Since (4) provides us with an explicit expression for describing the behavior of research firms, as we shall see in the next section, it is now straightforward to incorporate the research industry in a canonical endogenous growth model. This allows us to understand to what extent the presence of firm interactions in the research industry is going to affect the macroeconomic outcome, further distinguishing between the standing-on-the-shoulders and the fishing-out cases. Since the role of the fixed cost z is negligible in our setting, in the remainder we will set it equal to zero.

[8] We could in principle use any continuous probability distribution. The logistic is vastly used in the context of random utility models. One reason being that the dynamics obtained under this assumption have a logistic shape which seems to represent patterns underlying many social phenomena (see Anderson et al. (1992)).

3 The Macroeconomic Model

Apart from the characterization of the research market which to some extent resembles what discussed in Marchese at al. (2014), the model is an almost standard endogenous growth model characterized by households, productive and research firms, and a government. Households try to maximize their lifetime welfare, by determining how much to consume given the dynamic evolution of capital. Productive firms produce competitively the unique final consumption good, by determining how many workers and how much capital to employ given the available technology. Research firms determine whether to invest or not in innovation, and overall technological progress depends on the share of research firms which actively engage in research activities. The government aiming at maintaining a balanced budget at any point in time levies taxes on households to finance such research activities. Households and productive firms are homogeneous, thus we analyze their behavior as traditional representative agents. Research firms are instead heterogeneous in their propensity to innovate, and their behavior is consistent with what discussed in the previous section.

The representative household's problem consists of maximizing its welfare given its initial capital endowment k_0 and the law of motion of capital, k_t, by choosing how much to consume, c_t, and supplying inelastically labor. The household size, L, is constant and it is assumed to be infinitely large. Welfare is defined according to the average utilitarian criterion[9], thus it is equal to the infinite discounted sum (ρ is the pure rate of time preference) of instantaneous utilities, which depend solely upon consumption. The instantaneous utility function is assumed to take the following isoelastic form: $u(c_t) = \frac{c_t^{1-\sigma}-1}{1-\sigma}$, where $\sigma > 1$ is the inverse of the intertemporal elasticity of substitution. As usual lowercase letters denote per capita variables while uppercase letters aggregate variables. The household's problem in per capita terms can be written as:

$$\max_{c_t} \quad W = \int_0^\infty \frac{c_t^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt \tag{5}$$

$$s.t. \quad \dot{k}_t = (1-\tau_t)(r_t k_t + w_t) - c_t, \tag{6}$$

where r_t is the capital rental rate, w_t the wage rate and τ_t a (time varying) income tax rate. The first terms in the RHS of (6) represent the disposable income which needs to be allocated among consumption (c_t), and capital investments ($\dot{k}_t$).

Output is produced by competitive productive firms according to a Cobb-Douglas production function, combining labor, L (inelastically supplied by households), and capital, K_t. The production function in per capita terms takes the following form:

$$y_t = A_t k_t^\alpha \tag{7}$$

where $\alpha \in (0, 1)$ is the capital share while A_t a technological factor, representing total factor

[9]Note that since household size is constant, in our model the difference between welfare as defined according to either the average or total utilitarian criterion is simply a constant, equal to household size (see Marsiglio (2014) for a recent discussion of the implications of average and total utilitarianism on economic growth). However, since the size of household is assumed infinitely large (why this is needed will become clear later) we cannot rely on total utilitarianism since this would imply that household's objective function is infinite.

productivity. Productive firms take the level of technology as given and maximize their instantaneous profits, determining thus the rental rate of capital, r_t (and the wage rate, w_t).

Research firms indexed by $i = 1, \ldots, N$ are heterogeneous in their propensity to innovate $\omega_{i,t}$ and try to maximize the profits associated with their research activities. Their behavior is identical to what discussed in the previous section, thus is determined by the comparison between their profit when innovating ($\omega_{i,t} = 1$) and when not ($\omega_{i,t} = 0$). Whenever innovating they will sell their innovation at a price $\tilde{h}_t = hy_t$ to the government, which does somehow finance the research activities in the overall economy. We assume the number of research firms is infinitely large such that Proposition 1 holds.

The government by taxing households collects the tax revenue $\tau_t y_t$, which is used to buy innovations at a price (in units of output) $\tilde{h}_t$ from each research firm actively engaged in research activities, $\sum_i \omega_i$. Thus, the government's budget constraint reads as follows:

$$\tau_t \ell = hx_t, \tag{8}$$

where x_t is the share of innovative firms whose dynamics is given in (4) and $\ell = \lim_{(L,N)\to(\infty,\infty)} \frac{L}{N} > 0$ is the household to research firm ratio.[10] Once an innovation is bought by the government, it is immediately released in the public domain to allow productive firms to use such an innovation for free to produce the final consumption good (Marchese at al. (2014)).

By financing research activities the government determines the time evolution of the total factors productivity. Indeed, the overall level of technology is determined by the interaction among research firms. Specifically, we assume that it evolves according to the following law of motion:

$$\dot{A}_t = \phi x_t A_t, \tag{9}$$

where $\phi > 0$ is a scale parameter and x_t represents the share of research firms which actively engage in innovative activities. According to (9) for technological progress to occur it does not matter the size of the research industry (i.e., how many research firms exist) but the relative size of innovative firms with respect to the industry. If none does research ($x_t = 0$) then technological progress does not occur, while if all firms do research ($x_t = 1$) then technological progress occurs at a strictly positive rate ϕ. For any situation different from these two extreme cases, the rate of technological progress will lie between 0 and ϕ, and which specific rate will arise depends on the behavior of research firms and their interaction within the research industry.

In general equilibrium, all agents maximize their objective function and all markets clear. The economy is completely characterized by the following system of differential equations

[10] Note that the budget constraint is written assuming an infinite number of research firms as in the asymptotic model described by Proposition 1. In case of N and L finite, it reads $\tau_t y_t L = hy_t \sum_i \omega_{i,t}$ implying that $\tau_t y_t \frac{L}{N} = hy_t \frac{\sum_i \omega_{i,t}}{N}$. Letting N (and L) go to infinity, we are back to (8).

and the given initial conditions k_0, x_0 and A_0:

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\sigma}\left[\left(1-\frac{h}{\ell}x_t\right)\alpha A_t k_t^{\alpha-1} - \rho\right] \tag{10}$$

$$\dot{k}_t = \left(1-\frac{h}{\ell}x_t\right) A_t k_t^{\alpha} - c_t \tag{11}$$

$$\dot{x}_t = \frac{1}{2}\tanh\left\{\beta\left[h + J\left(x_t - \frac{1}{2}\right)\right]\right\} - x_t + \frac{1}{2} \tag{12}$$

$$\dot{A}_t = \phi x_t A_t \tag{13}$$

Apart from the case in which x_t converges to zero (which however will never be an equilibrium), the above system (10), (11), (12) and (13) is not stationary (i.e., it does not show any equilibrium at all), thus in order to study its dynamic behavior it may be convenient to recast the system in a stationary system as traditionally done in the endogenous growth literature. From the equilibrium properties of this latter system, we will then be able to infer the properties of the BGP equilibrium associated with (10), (11), (12) and (13). A BGP equilibrium denotes a situation in which all variables grow at a constant (possibly non-negative) rate, and deriving and discussing the characteristics of the BGP equilibrium is our main goal in next section.

4 BGP Equilibrium

By introducing the variables $\chi_t = \frac{c_t}{k_t}$ and $\varphi_t = A_t k_t^{\alpha-1}$, denoting the consumption to capital ratio and the average product of capital respectively, it is possible to recast the above system in the following stationary system:

$$\frac{\dot{\chi}_t}{\chi_t} = \chi_t - \frac{\rho}{\sigma} - \frac{\sigma-\alpha}{\sigma}\left(1-\frac{h}{\ell}x_t\right)\varphi_t \tag{14}$$

$$\frac{\dot{\varphi}_t}{\varphi_t} = \phi x_t - (1-\alpha)\left(1-\frac{h}{\ell}x_t\right)\varphi_t + (1-\alpha)\chi_t \tag{15}$$

$$\dot{x}_t = \frac{1}{2}\tanh\left\{\beta\left[h + J\left(x_t - \frac{1}{2}\right)\right]\right\} - x_t + \frac{1}{2} \tag{16}$$

At equilibrium the above system is characterized by the following steady state values:

$$\overline{\chi} = \frac{(1-\alpha)\rho + (\sigma-\alpha)\phi\overline{x}}{\alpha(1-\alpha)} \tag{17}$$

$$\overline{\varphi} = \frac{(1-\alpha)\rho + \sigma\phi\overline{x}}{\alpha(1-\alpha)(1-\frac{h}{\ell}\overline{x})}, \tag{18}$$

$$\overline{x} = \frac{1}{2}\tanh\left\{\beta\left[h + J\left(\overline{x} - \frac{1}{2}\right)\right]\right\} + \frac{1}{2} \tag{19}$$

where $\overline{x}$ cannot be determined explicitly. However since $x_t \in [0,1]$ it follows that $\overline{x}$ will always be non-negative. This means that provided that $\ell > h$, the steady state values $\overline{\chi}$ and

$\overline{\varphi}$ will be strictly positive. We summarize results about the BGP equilibria and their stability in the following proposition.

Proposition 2. *Assume $\ell > h$; then along a BGP equilibrium, the economic growth rate, γ, is strictly positive and given by the following expression:*

$$\gamma \equiv \gamma_c = \gamma_k = \frac{\gamma_A}{1-\alpha} = \gamma_y = \frac{\phi\overline{x}}{1-\alpha} > 0, \tag{20}$$

where $\overline{x}$ denotes the steady state value of x_t. Moreover, there exists a threshold $J^t = \frac{2}{\beta}$ such that:

i) if $J < 2/\beta$, there exists a unique γ^* and the unique BGP equilibrium is saddle-point stable with a two-dimensional stable manifold;

ii) if $J > 2/\beta$, there exists a threshold level $h^t(J,\beta) > 0$ such that:

a) if $h > h^t(J,\beta)$, there exists a unique γ^* and the unique BGP equilibrium is saddle-point stable with a two-dimensional stable manifold;

b) if $h < h^t(J,\beta)$, there exist three BGP equilibria corresponding to three values $\gamma_L < \gamma_M < \gamma_H$. The intermediate one is saddle-point stable with a one-dimensional stable manifold, whereas the two extreme ones are (locally) saddle-point stable, each with a two-dimensional stable manifold.

Proof. By plugging the steady state values of χ_t and φ_t back in the original equations (10)–(13), it is straightforward to derive the BGP growth rate γ, as in (20). The characteristics of γ strictly mimic those of $\overline{x}$. Indeed, multiplicity is due to the possible multiplicity of the steady states of equation (4). As already shown in the literature (see Brock & Durlauf (2001)), it turns out that, depending on the values of the parameters, we can have a unique stable equilibrium ($\bar{x}$) for (4) or three equilibria ($\overline{x}_L < \overline{x}_M < \overline{x}_H$), two of which are locally stable ($\overline{x}_L$ and $\overline{x}_H$). A similar threshold value for J is also derived by Brock & Durlauf (2001).[11] From (20), if there are multiple equilibria for x, then the system admits multiple equilibria as well.

Concerning stability, by linearization around a steady state it is possible to analyze the (local) stability properties of the above system by deriving the following Jacobian matrix:

$$J(\overline{\chi},\overline{\varphi},\overline{x}) = \begin{bmatrix} \overline{\chi} & -\frac{\sigma-\alpha}{\sigma}(1-\frac{h}{\ell}\overline{x})\overline{\chi} & \frac{\sigma-\alpha}{\sigma}\frac{h}{\ell}\overline{\varphi\chi} \\ (1-\alpha)\overline{\varphi} & -(1-\alpha)(1-\frac{h}{\ell}\overline{x})\overline{\varphi} & \phi\overline{\varphi} + (1-\alpha)\frac{h}{\ell}\overline{\varphi}^2 \\ 0 & 0 & \Lambda \end{bmatrix}, \tag{21}$$

where $\Lambda = \frac{\partial \dot{x}_t}{\partial x_t}|_{x_t=\overline{x}}$. It is straightforward to show that the eigenvalues are given by the following expressions $\lambda_1 = \Lambda$, and $\lambda_{2,3} = \frac{\Delta\pm\sqrt{\Delta^2+\Theta}}{2}$, where $\Delta = \overline{\chi} - (1-\alpha)(1-\frac{h}{\ell}\overline{x})\overline{\varphi} > 0$ and $\Theta = 4\frac{\alpha}{\sigma}(1-\alpha)(1-\frac{h}{\ell}\overline{x})\overline{\chi}\ \overline{\varphi} > 0$, from which it directly follows that $\lambda_2 > 0$ and $\lambda_3 < 0$. Independently of what the sign of Λ is, there exists at least one positive and one negative eigenvalue, thus any possible equilibrium is saddle-point stable. Moreover, it is possible to

[11] In Brock & Durlauf (2001), the threshold is $1/\beta$. The factor 2, appearing in our statement, depends on the transformation from the variable m_t to the rescaled variable x_t as shown in the proof of Proposition 1.

show that $\Lambda < 0$ for $\bar{x}_L$ and $\bar{x}_H$ and $\Lambda > 0$ for $\bar{x}_M$. Therefore, the stable manifold associated with the three equilibria has dimension 2 for γ_L and γ_H and dimension one for γ_M. ■

The parameter condition required by Proposition 2 is needed in order to ensure that the BGP equilibrium is well defined. Intuitively, it requires that the household to research firm ratio (ℓ) is large enough to provide the government with the resources needed to promote research activities (h). Along a BGP the economic growth rate γ depends negatively on α and positively on ϕ and, more importantly on the equilibrium share of innovative firms $\overline{x}$. This means that our model economy does not show any scale effect, since the growth rate is independent of any aggregate variable[12]. However, since the equilibrium share of research firms may not be unique, also the BGP equilibrium turns out to be not unique, and this is strictly related to the size of the externality parameter, J. Indeed, Proposition 2 suggests that in the fishing-out case ($J < 0$) there always exists a unique saddle-point stable BGP equilibrium; however, in the standing-on-the-shoulder case ($J > 0$) there is a richer variety of possible outcomes. Whenever the standing-on-the-shoulder effect is weak (i.e., the magnitude of the positive externality is small) a unique stable equilibrium will emerge. In the case of a sufficiently large externality, then the number of equilibria depends on the value of the incentive mechanism provided by the amount of revenues obtained, h. A large h makes the equilibrium unique, whereas, a small h gives rise to the presence of two locally stable equilibria.[13] As a matter of expositional simplicity, in the following we will refer to the case (i) in Proposition 2 as the "small externality case" and to the case (ii) as the "large externality case". Note that the macroeconomic behavior closely resembles the behavior on the innovation share, and when the equilibrium innovation share is unique (multiple) then the BGP equilibrium is unique (multiple) as well. Specifically, in the case of multiple equilibria, if $x_0 < \overline{x}_M$ then x_t will converge to $\overline{x}_L$ (and the BGP growth rate will be low, γ_L), while if $x_0 > \overline{x}_M$ then $\overline{x}_H$ will be reached instead (and the BGP growth rate will be high, γ_H). Thus, the initial fraction of innovative firms plays a crucial role in determining which BGP equilibrium will be effectively achieved.[14]

In order to understand more in depth what are the characteristics of the BGP equilibrium, we now analyze the behavior of the economy under a realistic model's parametrization. Specifically, we set the inverse of the intertemporal elasticity of substitution, σ, equal to 2, the rate of time preference, ρ, to 0.04, the capital share, α to 0.33 (Mullingan and Sala–i–Martin (1993)); the scale parameter determining the rate of growth of technology, ϕ is calibrated to 0.4, in order to obtain an economic growth rate equal to 0.03 (in the case in which the equilibrium share of innovative firms is exactly equal to one half); the other parameter values, are set arbitrarily in order to make sure that the assumption required in Proposition 2 is met and

[12]An increase in the number of firms in the research industry does not rise the overall economic growth rate. This rate can increase only if the equilibrium share of innovative firms rises.

[13]Note that the intermediate equilibrium γ_M, although saddle point stable, is derived from an innovation share $\overline{x}_M$ which is linearly unstable on its own. Therefore, unless we assume that the economy is exactly tuned on $x_0 = \bar{x}_M$, this equilibrium will never emerge. For this reason, we will not consider it as a possible realist economic outcome.

[14]The importance of the initial share of innovative firms for the model's outcome is further discussed in Section 4.1 where we focus on the finite-number of research firms case. We will show that in such a (stochastic) framework the presence of multiple equilibria might give rise to growth cycles.

that our qualitative results are as clear as possible. We thus set the households to research firm ratio, ℓ, equal to 1000, the measure of the dispersion of opinion in the population of research firms, β equal to 1, while we let the revenue provided to research firms, h, and the size of the externality parameter, J, vary in order to see how they affect the BGP economic growth rate γ. Table 1 summarizes the parameter values employed in our analysis.

σ	ρ	α	ℓ	ϕ	β
2	0.04	0.33	1000	0.4	1

Table 1: Parameter values employed in our simulation.

In Figure 1 we show how the BGP growth rate γ varies for different values of the externality parameter,[15] J, whenever the revenue parameter, h is set equal to 0. As expected from Proposition 2, for negative and positive but small enough values of the externality parameter a unique BGP and thus a unique economic growth rate, $\gamma^* = \frac{\phi \overline{x}^*}{1-\alpha}$ (equal to 0.03), exist. For larger values, three equilibria, namely $\gamma_L = \frac{\phi \overline{x}_L}{1-\alpha}$, $\gamma_M = \frac{\phi \overline{x}_M}{1-\alpha}$ and $\gamma_H = \frac{\phi \overline{x}_H}{1-\alpha}$ with $\gamma_L < \gamma_M < \gamma_H$, exist and the gap between the high and low economic growth rate, $\gamma_H - \gamma_L$ rises with J.

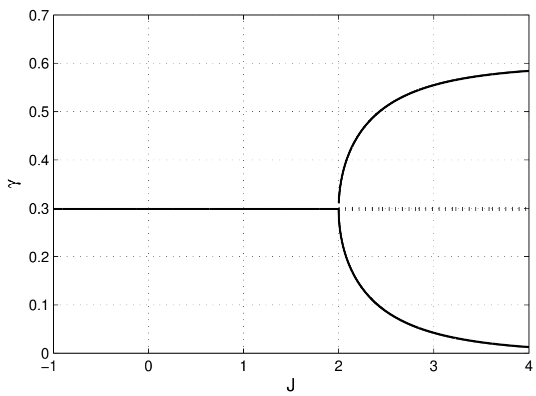

Figure 1: Changes in the economic growth rate, γ, for different values of the externality parameter, J (revenue parameter, h set equal to 0).

Since the existence of either a unique or multiple BGP equilibrium is related to the size of the externality parameter, it may be convenient to separately analyze the cases in which the externality parameter is either small or large. In Figure 2 we thus consider two alternative values $J = 1.9$ and $J = 2.5$, lying below and above the threshold value $J^t = 2$ (see Figure 1) respectively, and show how the BGP growth rate γ varies with the revenue parameter, h. As discussed above, the small externality case represents a situation in which the research sector is characterized by either fishing-out ($J < 0$) or weak standing-on-the-shoulder ($0 < J \leq J^t$)

[15] J varies between 0 and 4. This range has been proved to be large enough to show the desired transition effects.

effects. In both the cases, equation (16) shows a unique stable equilibrium and consequently the BGP equilibrium is unique as well: $\gamma^* = \frac{\phi \overline{x}^*}{1-\alpha}$. The convergence to the steady state of the system (14)–(16) will occur along a two-dimensional stable manifold. We can see that the unique economic growth rate increases with h, thus the higher the incentive for research firms to engage in research activities the faster the economic growth (Figure 2, left panel). The large externality case represents instead a situation in which the research sector is characterized by a strong standing-on-the-shoulder ($J > J^t$) effect. In this case, equation (16) shows three equilibria ($\overline{x}_L < \overline{x}_M < \overline{x}_H$), two of which are locally stable ($\overline{x}_L$ and $\overline{x}_H$). As a consequence, the BGP equilibrium is not unique as well: we need to distinguish three BGP equilibria, characterized by an economic growth rate equal to $\gamma_L = \frac{\phi \overline{x}_L}{1-\alpha}$, $\gamma_M = \frac{\phi \overline{x}_M}{1-\alpha}$ and $\gamma_H = \frac{\phi \overline{x}_H}{1-\alpha}$ with $\gamma_L < \gamma_M < \gamma_H$, respectively. As seen from Proposition 2, the convergence to such three steady states of the system (14)–(16) will occur either along a two-dimensional stable manifold (for γ_L and γ_H) or along a one-dimensional stable manifold (for γ_M). We can see that the high and low economic growth rate, γ_H and γ_L increase with h, while the medium one γ_M falls with h; thus the higher the incentive for research firms to engage in research activities the faster the economic growth in each of the two stable equilibria (Figure 2, right panel). Note that the threshold value for h is $h^t \approx 0.078$, meaning that only whenever $0 \leq h < h^t$, three equilibria exist.

Figure 2 suggests some interesting policy implications, since it clearly shows how the revenue parameter impacts on the equilibrium economic growth rate. Indeed, in the large externality case whenever the revenue provided to research firms is small ($h < h^t$), three different BGP equilibria exist, and this is strictly related to the existence of three different equilibrium values for the innovation share. Therefore, the same economy may experience different growth rates according to how many research firms actively engage in research activities: if this share is small the economic growth rate will be low while if it is large the economic growth rate will be high. This means that the economy is potentially faced with a low growth trap, which may condemn it to grow slower than what it could potentially do. In such a framework it is natural to wonder what policymakers can do in order to deal with this problem. As traditionally discussed mainly in the context of poverty traps (Sachs et al (2004)), an economy may escape its low growth trap by increasing the innovation share, allowing thus the initial share of research firms (x_0) to exceed its unstable middle equilibrium ($\overline{x}_M$). Such an outcome might be implemented by simply opening the economy to international trade and providing some incentive for foreign firms actively engaged in research activities to operate also on the domestic market; research activities at international level may thus provide the economy with the push it needs to achieve fast economic growth. However, policymakers may do much more than this, since they can effectively allow the economy not only to escape its low growth trap, but to even solve completely the trap problem. Indeed, by rising enough the revenue provided to each research firm such that $h > h^t$, the innovation share will naturally converge towards its unique (higher) equilibrium value and thus the economic growth rate at equilibrium will be high and thus the economy will not be trapped into a low growth equilibrium. Such an outcome can be easily implemented by increasing the tax rate applied to households' income in order to finance the increase in the revenue parameter. Indeed, in our model's parametrization the tax parameter $\hat{\tau}$ needed

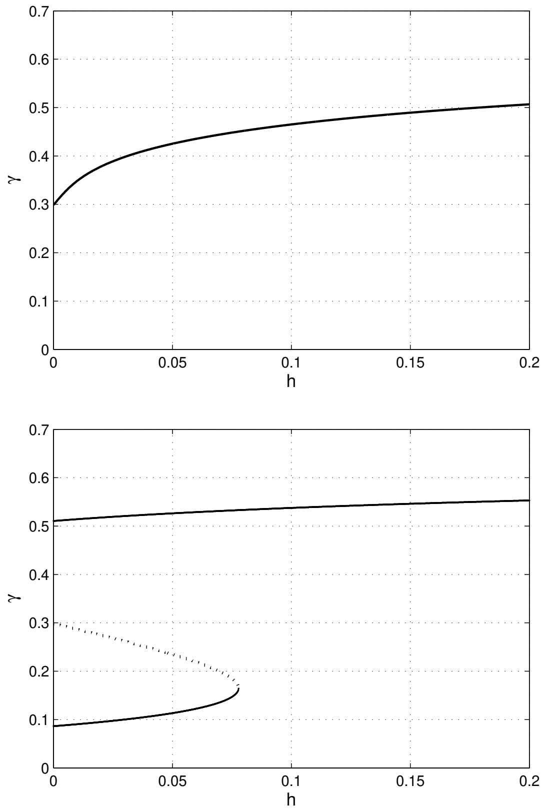

Figure 2: Changes in the growth rate, γ, for different values of the revenue parameter h ($0 \leq h \leq 0.2$); the externality parameter, J set equal to either 1.9 (left panel) or 2.5 (right panel).

to escape the low growth trap is $\hat{\tau} = \frac{h}{\ell}\overline{x} \approx 0.0413\%$. The result should be clear from Figure 2, and it can also be seen from Figure 3 where we plot the equilibrium values of $\overline{x}$ for two different values of the revenue parameter h. This clearly show that with a higher h a unique equilibrium $\overline{x}^*$ (and thus also a unique BGP) may exist.

4.1 Endogenous Cycles

We conclude the discussion about the potential implications of our model by analyzing through a simple example the finite version of the research industry. Indeed, recall that x_t describes exactly the deterministic evolution of the research industry under the modeling assumption that N is infinite, thus what we have analyzed thus far represents simply an approximation of the true behavior of the system. We thus now focus on the case in which N is finite in order to understand what the presence of random shocks to the cost components of firms might imply for the economy. Let us focus on the large externality case, and specifically on a situation in which multiple BGP equilibria exist (case ii.b in Proposition 2). In such a

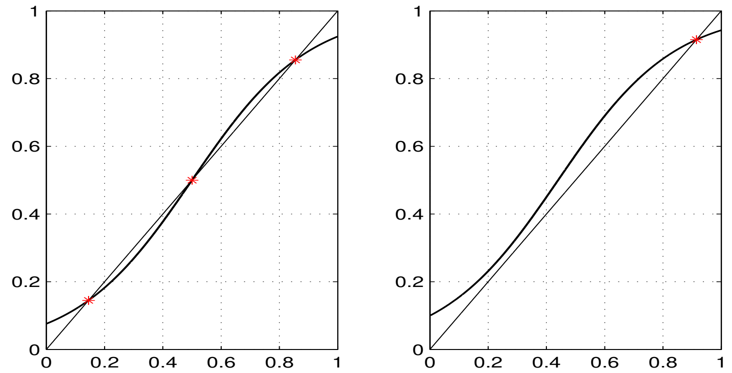

Figure 3: Equilibrium values $\overline{x} \in [0, 1]$ (marked with a star) found as the intersection between the bisector line and $f(x) = \frac{1}{2} \tanh \left\{ \beta \left[h + J \left(x - \frac{1}{2} \right) \right] \right\} + \frac{1}{2}$ (see equation (19)). Parameters values: $h = 0$ (left panel) and $h = 0.15$ (right panel), with $J = 2.5$.

case two stable BGP equilibria exist, and as discussed above which equilibrium our economy will achieve depends upon the initial conditions (χ_0, φ_0, x_0), and in particular a critical role is played by the initial condition on x_t. Indeed, the initial share of innovative firms determines whether the equilibrium share will be high or low, determining thus whether the economic growth rate, γ, will be high or low. While this outcome is clear in the infinitely large number of research firms version of the problem, whether this holds true also for the finite version is not so obvious. In fact, in the finite version of the model, research firms are subject to random shocks which determine whether they will decide to innovate or not; in the infinite version the effects of such shocks cannot be analyzed since equation (16) turns out to be completely deterministic.

Let us denote by x_t^N the proportion of innovative firms at time t among a total population of N firms. As theoretically proved for a wider class of probabilistic models in Mathieu and Picco (1998), under the assumptions of case ii.b in Proposition 2, the trajectory x_t^N has a *metastable behavior*[16]: it fluctuates close to one of the two equilibria, say $\bar{x}_H$ and, after a random time, it suddenly jumps to values close to $\bar{x}_L$. This gives rise to a *cycling* behavior of the finite dimensional system. Note that this random jumps happen with probability one for each trajectory, although the jump times could be possibly large. A formal statistical analysis of the finite dimensional model and the calibration of such a random time is out of the scope of this paper, but the theory of metastability suggests that this random time is exponential in N and depends on the parameters of the model and on the shape of the basin of attractions of the two locally stable equilibria. Indeed, it turns out that, for values of J close to J^t, the random time needed to exit the basin of attraction of the two locally stable

[16]A probabilistic system exhibits a metastable behavior when it remains for long times close to an apparent equilibrium, (called metastable), then it suddenly relaxes to the true equilibrium state. It can be proved that, on a suitable time-scale, the process therefore behaves like a pure jump process with two states. For more details, see for instance Mathieu and Picco (1998).

equilibria is relatively small and trajectories showing growth cycles arise. In Figure 4 (right panel), we provide an example showing that, for $J = 2.05$ (recall that $J^t = 2$), this random time is reached early enough to be seen in the trajectory. More precisely, we show that the (stochastic) time series of x_t^N may deviate from its expected behavior predicted by equation (16). Recall that x_t describes exactly the deterministic evolution of the system under the modeling assumption that N is infinite. In the left panel we show that x_t^N may converge to the equilibrium it is not supposed to achieve. Indeed, since the initial condition $x_0 = 0.1$ is greatly lower than $\overline{x}_M = 0.4178$, we would expect the time series of x_t^N to fluctuate around the red-dashed trajectory $x_t^{(L)}$ leading to the low equilibrium. However, in this particular simulation, this is not the case: the trajectory deviates and start fluctuating around the high equilibrium $\overline{x}_H$. In the right panel, as said, we show that the finite dimensional trajectory x_t^N may spend quite a long time close to one of the two equilibria and then depart from it to reach the other one. What discussed for x_t^N has clear implications also in terms of the macroeconomic outcome: differently from what suggested by the (deterministic) theory, the system, even when the initial conditions are very close to the high BGP equilibrium, may converge towards the low BGP equilibrium or oscillate between the two BGP equilibria without converging to a steady state. Note that in the small externality case, in which the equilibrium is unique, such an effect naturally disappears. This suggests that government intervention may be essential not only to allow the economy to solve its eventual low growth trap problem but also to reduce the fluctuations in economic activity.

5 Middle-Income Trap

An interesting line of interpretation of our stylized model is related to the middle-income trap hypothesis[17]. This refers to the experience common to many developing countries (especially in Latin America and in the Middle East) in the second half of the XIX century, in which growth has significantly slowed down after a first stage of take off characterized by rapid growth (see Gill and Kharas (2007), Commission on Growth and Development (2008)). This development process has allowed these economies to quickly move from a low-income to a middle-income status, but not to make the further leap needed to become high-income economies. This has advanced the hypothesis that there may exist a middle-income trap, preventing thus some economies to fill the gap with more advanced countries. What might be the specific hindrances affecting this second stage of economic development is still an open question, but these are likely to be substantially different from those involving the first stage in which traditional poverty traps are in place.

Understanding thus what may be the reason why some fast growing economies have failed to achieved a high-income status is an active and recent research question with clear policy implications. While empirical evidence supporting the existence of a middle-income trap seems robust and convincing, much less clear is from a theoretical point of view why fast growth might come to an end. On the empirical side, Eichengreen et al. (2012) show that growth tends to slowdown at levels of per capita income of about \$15,000 (at 2005 constant

[17] The term "middle-income trap" has been originally introduced by Gill and Kharas (2007), and the notion has also often been referred to as "growth slowdown" (Eichengreen et al. (2012)).

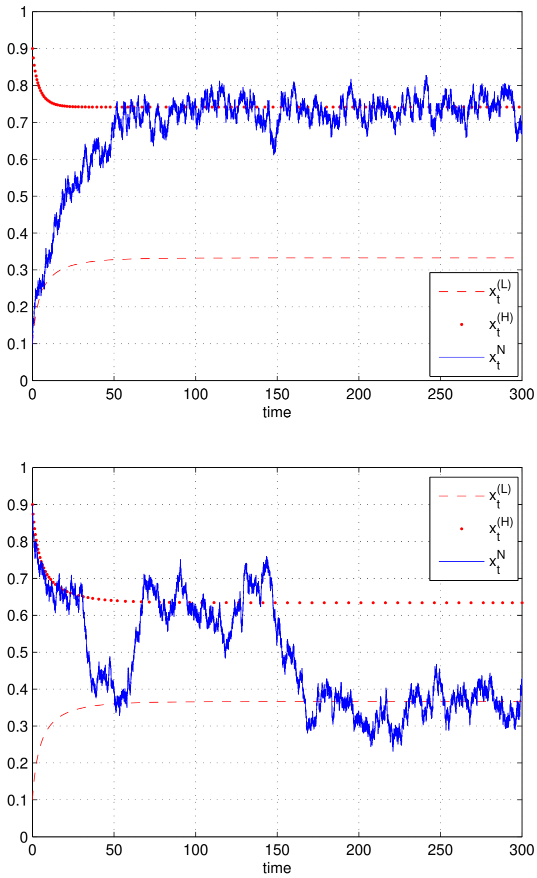

Figure 4: The evolution of x_t^N (in blue) for a finite dimensional system of $N = 1,000$ firms. In the left panel, we see the deviation from the expected trajectory $\overline{x}_t^{(L)}$ suggested by the model in favor of $\overline{x}_t^{(H)}$. In the right panel we have a trajectory fluctuating around the two attractors. Parameters are as in Table 1 with $h = 0.01$ and $J = 2.14$ (left panel) and $h = 0$ and $J = 2.05$ (right panel).

international PPP prices), suggesting that a critical role is played by a reduction in the growth rate of the total factor productivity (TFP); specifically, a drop in TFP growth represents about 85% of the fall in per capita income growth. Eichengreen et al. (2013) provide some additional evidence, showing that the distribution of growth slowdowns is not necessarily unimodal, and in particular two modes, one around \$15,000 and another around \$11,000, exist. On the theoretical side, very few works have tried to provide some explanation of growth slowdowns in middle-income countries, and they focus on reallocation or misallocation of workers between different economic sectors. A traditional argument suggests that while in earlier stages of development it may be possible to raise productivity by shifting workers from agriculture to industry, this process may come to an end whenever the share of workers employed in agriculture falls enough (Lewis (1954)). A more recent explanation emphasizes that a low allocation of high skilled individuals in the research sector may give rise to low productivity growth; however, this situation of potential slow growth can be fixed by policy interventions (Agénor and Canuto (2015)). Differently from these works in which inter-sector dynamics is the driver of eventual growth slowdowns, our paper provides an alternative explanation based entirely on social interactions and technology diffusion.

Along the lines of Agénor and Canuto (2015), whenever the economy experiences multiple BGP equilibria (Proposition 2, case *ii.b*), the intermediate BGP equilibrium γ_M (i.e., the low growth trap threshold) can be clearly interpreted as a middle-income trap, separating fast and slow growing economies. Note that the eventual existence of such a trap is determined by the outcome in the research industry, which is completely driven by social interactions among research firms. Thus, the research intra-sector dynamics only might explain why technological progress and thus economic growth tend to be high or low in specific economies. In order to relate this to the pattern advanced by the middle-income trap hypothesis we need to understand why an economy initially (during a first stage of economic development) in a BGP with high economic growth rate, γ_H, may end up (in the second stage of development) in a BGP equilibrium with low growth rate, γ_L, later. In our setting this is equivalent to a either a fall in the number of innovative firms x_t or a rise in the intermediate equilibrium of the innovation share, $\overline{x}_M$; both the cases imply that an economy with an original high innovation share may end up with a low (compared with the critical threshold) innovation share and thus experiencing a slowdown in its economic growth. The former case may be triggered by a change in international policy, and specifically it may occur as a result of the introduction of tariffs or other restrictive policies, which by providing negative incentives for foreign firms to operate on the domestic market, leads some foreign firms engaged in research activity to exit the domestic research market. The latter case may instead be triggered by a change in domestic economic policy, and it may occur as a result of a reduction in the support provided to research firms which, by determining the amount of revenue received by innovative firms, tends to increase the intermediate equilibrium value of the share of firms engaged in research activities. While empirical evidence seems to supports our conclusions related to the negative relation between growth slowdowns and openness (Eichengreen et al. (2012)), the available evidence does not allow to either support or refute those related to the positive (up to a certain point) nexus between growth slowdowns and research-enhancing policies.

Apart from the eventual existence of such a middle-income trap, our model differently from Agénor and Canuto (2015) suggests that also growth cycles may occur. This implies that also fast growing economies cannot claim to have definitely escaped their middle-income trap, since they may be cyclically pulled into situations of growth slowdowns. This reinforces our previous conclusions that policymakers can play a critical role in the development process. By actively intervening with specific policies they can completely solve the trap problem dampening the size of the growth fluctuations, promoting a smooth process of fast growth allowing the economy to eventually catch up with more advanced economies and become a high-income country.

6 Conclusion

Technological progress is by far the most important determinant of economic growth over the long run. However, whether and how the interaction among research firms in the research industry might determine the technological progress has never been analyzed thus far in the growth literature. Thus, in this paper we try to fill this gap by allowing a certain degree of firms interaction. Specifically, we assume that firms decide whether to innovate or not by taking into account also what other research firms do. Such an interaction among research firms, by determining the rate of technological progress, plays a critical role in shaping the whole macroeconomic outcome. Indeed, we show that under certain parameter conditions, by mimicking the behavior of the share of innovative firms, the economy may be characterized by a multiplicity of balanced growth path (BGP) equilibria, and eventually may face a situation of low growth trap. We have also shown that the economy may eventually (endogenously) fluctuate between the low and high BGP generating thus a growth cycle in which periods of low and high economic growth rates follow one another. The potential existence of low growth traps and endogenous growth cycles suggest that the government might play an essential role in order to contrast such negative effects. In particular, by rising enough the tax rate applied to households' income it could completely solve the low growth trap problem, avoiding thus further fluctuations in economic activity.

This paper represents a first attempt to enrich the macroeconomic dynamics in traditional models of endogenous growth by allowing a certain extent of externality in the level of research innovation. The approach followed is thus quite simplistic on purpose in order to show in the simplest possible way which might be the potential implications of allowing for social interactions in traditional macroeconomic models. Of course, our framework has several limitations which need to be accounted for in future research. Specifically, the law of motion of the share of innovative firms turns out to be independent of the other macroeconomic variables; this assumption needs to be relaxed in order to establish a mutual nexus between the macroeconomic setup and the degree of social interactions. Also the specification of the research market is overly simple, and adopting a more traditional setup with either horizontal or vertical product differentiation (Acemoglu (2009)) may shed some further light on the impacts of social interactions on macroeconomic outcomes. Extending the analysis along these lines is left for future research.

A The Rationale behind Random Utility Models

In this appendix we briefly summarize the main ideas recovered by Brock & Durlauf (2001) and leading to the profit structure defined in (1). Suppose that a research firm faces the binary decision *to innovate* or *not to innovate*. We define the binary random variable $\omega \in \{0, 1\}$ accordingly. The main assumption behind random utility models is that the profit π related to the innovation has the following general structure:

$$\pi(\omega_i) = R(\omega_i, \mu_i^e(\omega_{-i}), h) - \zeta(\omega_i),$$

where revenues R depend on the choice made by the firm, on the price h received by the buyer of the innovation and by an externality term. Indeed, each firm i estimates the conditional probability measure μ_i^e on the choices of others, where ω_{-i} denotes the vector of actions deprived of the i-th component. As seen in Section 2, costs are random and denoted by ζ. For the moment, we set $z = 0$ for simplicity.

We now make some further (minimal) assumptions to came up with a tractable profit structure.

i) $\pi(0) = 0$. This is an obvious normalization. Both R and ζ are zero if no research activity is in place. Therefore, we concentrate on $\pi(1)$ (we call it simpy π). Rearranging variables and notations we have:

$$\pi = R(\mu_i^e(\omega_{-i}), h) - \zeta_i.$$

ii) Externalities due to the behavior of competitors, only depend on the average action of others' choice. This implies that $\mu_i^e(\omega_{-i})$ is substituted by the (simpler) statistics $x_i^e = \frac{1}{N-1}\sum_{j\neq i} x_{ij}^e$, where $x_{ij}^e = \mathbb{E}^{(i)}[\omega_j]$ denotes the expectation of firm i about the choice of competitor j. Therefore,

$$\pi = R(x_i^e, h) - \zeta_i.$$

Concerning the information structure of the model, we also assume that $\mathbb{E}^{(i)}[\cdot] = \mathbb{E}^{(j)}[\cdot]$ for all $i, j = 1, \ldots, N$. This amounts in saying that all firms share the same expectations about others' choices.

iii) We assume that $\frac{\partial\pi}{\partial x_i^e} = J$. This simplifying assumption introduces a unique parameter J measuring the degree of dependence (or the force of externality) due to the others' actions. Note that $J > 0$ resembles a staying-on-the-shoulder situation, whereas $J < 0$ a fishing-out case. Secondly, as obvious, $\frac{\partial\pi}{\partial h} > 0$.

iv) We assume that the monetary effects due to the sale of the technology and the externalities are additive. Moreover, for sake of simplicity, we assume a linear dependence. This fact, together with assumption iii), produces the following payoff

$$\pi = h + Jx_i^e - \zeta_i.$$

v) Finally, we slightly correct x_i^e by substituting it with $x_i^e - \frac{1}{2}$. The reason is that we want the decision to be driven by what *the majority of the population of firms is doing.* The quantity $x_i^e - \frac{1}{2}$ reflects exactly this goal: it is positive if and only if the majority of the research firms produces an innovation. Therefore, in case of a positive J, the single firm is more prone to be aligned with the majority. On the contrary, if $J < 0$, the firm will tend to behave in the opposite direction. We obtain

$$\pi = h - \zeta_i + J\left(x_i^e - \frac{1}{2}\right).$$

Therefore, by reintroducing a private cost z and recalling that $\pi(0) = 0$, we obtain the general expression for π as it appears in (1):

$$\pi(\omega_i) = \omega_i\left[h - (z + \zeta_i) + J\left(x_i^e - \frac{1}{2}\right)\right].$$

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