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Endogenous growth mechanism  
as a source of medium term  
fluctuations in the labor market.  
Application to the US economy

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# Introduction

A considerable part of the economic literature focuses on the sources and mechanisms of economic cycles. The bulk of this literature, including Real Business Cycle theory (see e.g. papers of Kydland and Prescott 1982, Prescott 1986, King, Plosser, and Rebelo 1988) or New-Keynesian theory (see e.g. Woodford 2003, Smets and Wouters 2003, Christiano, Eichenbaum, and Evans 2005), aim at explaining business cycles. Business cycles are usually defined, following a seminal contribution of Burns and Mitchell (1946), as fluctuations with periodicity between 1 and 8 years. But in the recent years, there has been a growing recognition of the importance of economic cycles that last more than 8 years – the so called medium term cycles. Blanchard (1997) and Solow (2000) were among the first, who stressed the importance of research on this issue and the need to develop models accounting for medium term fluctuations.

The most apparent empirical evidence on the importance of medium term cycle is the behavior of unemployment rate in the US economy. Unemployment was relatively low in the 1950s and 1960s of the last century, then increased on average for roughly next 20 years and later, in the 1990s, went back to a lower level. These fluctuations occur with periodicity far greater than a decade. Also the divergence of unemployment experience in US and large continental European countries in the 1970s and 1980s (for a discussion, see e.g. Blanchard 2006) is an indirect evidence of the importance of medium term fluctuations. The literature on changes in productivity growth trend in US (see e.g. Basu, Fernald, and Shapiro 2001) documents another important aspect of this issue.

An important paper of Comin and Gertler (2006) documents, in a rigorous way (using band-pass filters of Christiano and Fitzgerald 1999), various facts on medium term fluctuations in goods and capital markets. The paper also defines medium term cycles as fluctuations of periodicity up to 50 years. Comin and Gertler proposed a theoretical framework well suited for analyzing medium term cycles – they introduced concepts from the endogenous growth theory into the RBC model. Their approach follows a seminal paper by Romer (1990), with modifications accounting for the Jones' critique of Romer's model (see Jones 1995). Within their framework, short term shocks affect the profitability of production activity and influence the incentives to innovate and develop new products. Ultimately, it induces fluctuations in

the number of available products, resulting in medium term fluctuations of the whole economy. One of the main findings of Comin and Gertler is that medium term fluctuations can be explained by the same factors as business cycle fluctuations<sup>1</sup>. What is important from our perspective, Comin & Gertler focused on capital and goods markets, leaving the analysis of labor market for further research. This study aims at filling this gap.

The empirical evidence on medium term fluctuations in the labor market is presented e.g. in the papers of Hall (2005d) and Hall (2005c). He stressed the importance of fluctuations in medium term frequencies in many macro variables. Additionally, he hypothesized that medium term cycles can result from adjustments that take place in an asymmetric information environment. Alternative explanations of the lower frequency variation in the labor market variables focus more closely on factors specific to the labor market itself. One of them is the hysteresis effect (see e.g. Blanchard and Summers 1986, Blanchard and Summers 1987), as predicted by the insider-outsider theory. Another branch of the literature highlights the role of demography in generating low-frequency labor market volatility, e.g. the prolonged effects of baby-boom generations (see e.g. Flaim 1990) or the changes in participation rates (see e.g. Juhn and Potter 2006) .

In this study we focus on the question if medium term fluctuations in the labor market may be explained by the prolonged effects of short lived shocks coming from the goods market. We focus simultaneously on the short term component and the medium term component of medium term cycle in a unified way. As the data suggest that the medium term volatility is present in various markets of the economy, we do not explain lower frequency variation in the labor market with factors specific only to labor market, but instead we look for a common source of volatility in various markets of the economy. So, in other words, this study aims at answering the question, whether the shocks, believed to be the source of traditional business cycles, are able to generate substantial medium term fluctuations in the labor market.

The main theses of this study can be stated as follows:

1. Variation of economic activity in medium term frequencies is substantial and comparable to the variation in business cycle frequencies.
2. A large part of medium term fluctuations in both labor and goods markets may be explained by the same sources.
3. Endogenous growth mechanism is able to explain a large part of variation in medium term fluctuations.

We construct a theoretical model (with explicitly specified micro foundations), that belongs to a class of Dynamic Stochastic General Equilibrium

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<sup>1</sup>See also Growiec (2005) for a discussion on the endogenous growth models and a brief description of their results.



models, and then we calibrate (and partially estimate) it and verify its predictions against the data. As our analysis requires longer time series, we decided to focus on the US economy. Additionally, there have been many empirical papers analyzing US economy, which simplifies the calibration of the model. Following Romer (1990), we use the endogenous growth framework augmented with the search-matching description of the labor market. It follows closely the Diamond-Mortensen-Pissarides<sup>2</sup> framework (the notion of this framework originates from a seminal contributions by Diamond 1982, Mortensen 1982, Pissarides 1985). The search theory introduces an inherent friction into the functioning of the decentralized labor market and allows to model unemployment as an equilibrium phenomenon.

As a source of volatility we use the technology shock, as it is commonly used in the Real Business Cycle literature and, as Hall (2005c) noticed, could also be the main driving force of the medium term labor market fluctuations. The literature acknowledges the fact that the standard search-matching models underestimate the volatility of unemployment, as observed in the data (see e.g. Costain and Reiter 2003, Shimer 2005, Hall 2005b). Thus, we will analyze two extensions of the model that address this issue: shocks to matching technology and real wage rigidity.

Our framework focuses on the consequences of the changes in the developments of the goods market for the labor market. Thus, we do not model explicitly the labor supply decisions and treat them as exogenously given. We admit, that labor supply shifts could be an important source of economic fluctuations, also in the medium term, but to simplify the analysis we are leaving it outside the model. It allows us to see how important the main mechanisms of our model are in explaining the patterns in the data. Introduction of the endogenous labor supply could only improve the model performance.

This theory has at least two implications. First, in order to account for economic variation in medium term frequencies, it seems that there is no need for a new generation of models, but it is enough to augment the current generation of DSGE models with elements of the endogenous growth theory. Second, if our theory is true, the effects of economic policies are more persistent than it is usually implied by the standard DSGE framework. The last issue may be especially important for the monetary policy, but we will leave this for further research.

The study is organized as follows. First, we briefly discuss the existing literature in the context of the issues that are important from the perspective of the study. Then, we describe the US data features, concentrating on the medium term characteristics and derive a set of "stylized facts" that will be useful from the modeling perspective. Next, we present the details and derivations of the theoretical model, which includes both the endogenous

<sup>2</sup>This labor market theory is commonly called the DMP or search-matching theory.

growth component and the search-matching mechanism on the labor market. This section also discusses the steady state properties and restrictions imposed on the model structure by the balanced growth path assumption. Next, we discuss our calibration strategy, along with the data and information sources used for this purpose. As the stochastic parameters of the model, together with stochastic shocks, are estimated from the US data, this section also addresses the estimation issues. The last section presents the predictions of the estimated model and verifies them against the US data. As the basic version of the model understates the extent of labor market volatility, we also extend our analysis in two distinct dimensions. Firstly, we introduce shocks to matching technology and secondly - real wage rigidities, both extensions aimed at resolving the volatility issue. In each case, we check the model predictions and verify its properties. The last part of this section compares the predictions of the model extended for wage rigidities with the predictions of the benchmark model - basic RBC model with search-matching and wage rigidities - and presents some evidence on the importance of the endogenous growth component. The last section concludes and discusses some implications for our results for policy and the economic modeling.

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# Chapter 1

## Overview of the literature

This section reviews some concepts from the literature that are useful in the context of the model designed to reflect the medium term fluctuations of economic activity, that we will present in next sections. The first part of the section focuses on the literature on business cycles. Next subsection focuses on the existing literature of medium term fluctuations, with the special focus on the endogenous growth mechanisms of Romer (1990) used in the model of Comin and Gertler (2006). The last part deals with the aspects of the search and matching theory of Diamond (1982), Mortensen (1982) and Pissarides (1985), that are of interest in our study. These two main building blocks, integrated within a standard one sector dynamic stochastic general equilibrium model with rational expectations, will allow us to study medium term fluctuations of both goods and labor markets.

### 1.1 Business cycles

The first serious attempt to analyze economic fluctuations was the research program launched in the 1930s by Burns and Mitchell. This program was summarized in their study *Measuring Business Cycle* (see Burns and Mitchell 1946). Their methodology was criticized by some economists at that time (see Koopmans 1947) but the main reason for their methods not being adopted by the profession was the revolution, triggered by the contribution of Keynes (1936), which attracted a lot of attention of economists for the next few decades. The methods and results of Burns and Mitchell were undust again by the early proponents of the real business cycle theory. The analysis of Hodrick and Prescott (1980), reexamined the empirical regularities of the business cycles using modern analytical tools (nowadays widely known HP detrending procedure). They found that these regularities are strikingly robust across different cycles. Additionally, their results supported the evidence presented by Burns and Mitchell, despite being discovered using completely different tools. That was the time, when Lucas (1977) said that “business cycles are all alike”, suggesting that the nature of business cycles is country

and time independent, giving a hope to construct a unified theory of the business cycles.

The unified theory of business cycle fluctuations, so called real business cycle theory (RBC), was introduced into the economic profession by Kydland and Prescott (1982) and Long and Plosser (1983), giving rise to the revolution of the way the economic research is conducted till now. The real business cycle theory builds on a core neoclassical growth models of Solow (1956) and of Ramsey (1928)<sup>3</sup>. This core growth model was made stochastic by Brock and Mirman (1972), being an important early contribution to the real business cycle theory.

The way of analyzing economic fluctuations, introduced by the proponents of the RBC paradigm, has become standard in economic profession. The approach begins with a general equilibrium model of rational agents (with the usual assumption of homogeneity across agents), who decide on allocations in the economy, given the prices which equilibrate demand with supply in each market. The preferences of the households, technologies of production processes and, if necessary, parameters of market structures are specified and calibrated on the basis of both microeconomic and macroeconomic evidence. Then, given the realization of the stochastic shocks governing the model dynamics, the variables described by the model are simulated and moments of variables are computed and compared with their data counterparts. The evaluation of the performance of the basic RBC model shows that it is able to reflect a lot of the properties and stylized facts of the US economy (see e.g. the analysis in King, Plosser, and Rebelo 1988).

Most of the RBC models use exogenous productivity shocks as a driving force of economic fluctuations, although there are also models that emphasize to role of government spending as a source of stochastic disturbances - see e.g. Baxter and King (1993) in this context. But government spending cannot be the only source of economic fluctuations, as the standard models predict a decline of private consumption after positive government spending shock, due to the negative wealth effect triggered by an increase of taxes needed to finance growing debt. The resulting countercyclicity of private consumption is contrary to the data (see the discussion in Barro and King 1984), although Correia, Neves, and Rebelo (1992) documents that there were some periods with large shocks to the government expenditures when consumption was indeed countercyclical. For a more elaborate discussion on both theoretical and empirical findings with regard to the government spendings, see Bukowski, Kowal, Lewandowski, and Zawistowski (2005).

Subsequent research on extending the basic RBC model is quite vast. The extension important in the context of our research, also emphasized by King and Rebelo (1999), is the introduction of indivisible labor and lotter-

<sup>3</sup>The latter being re-invented and introduced into the economic profession by Cass (1965) and Koopmans (1965).

ies<sup>4</sup>, developed by Rogerson (1988) and applied to business cycles by Hansen (1985). This extension addresses one of the difficulties of basic RBC model, i.e. this model needs substantial labor supply elasticity (relative to the evidence from micro studies) to generate enough variation in labor input, as observed in macroeconomic data. The contribution of Rogerson and Hansen introduces labor adjustments only on extensive margin<sup>5</sup> (changes in the number of workers employed, rather than changes in the number of hours worked), breaking the link between individual labor supply elasticity (which is in this context irrelevant) and the labor supply elasticity of an representative agent, which matters for aggregate fluctuations and can be calibrated to match business cycle facts. The approach used in our study also focuses on the labor adjustments on extensive margins, but does not use the concepts of Rogerson (1988) and instead applies the search-matching theory of Diamond-Mortensen-Pissarides to describe the behavior of labor market. So, our approach emphasizes the role of demand rather than supply in determining the behavior of labor market. We leave the discussion on DMP framework to next sections.

The burst of the RBC theory in the 1980s and its success to build a unified, theoretically elegant, coherent and empirically plausible theory of economic fluctuations have left behind the second main branch of macroeconomic thinking, namely Keynesians. The contribution of Mankiw (1985) was one of the first attempt to assess the welfare and business cycles consequences of price stickiness (that arise due to the menu costs), launching the literature that gave microfoundations to Keynesian ideas. The early contributions to so called New Keynesian theory were compiled in two volumes of Mankiw and Romer (1991) and were focusing mostly on microeconomic ingredients that could produce Keynesian behavior of the economy. Later contributions focused on introducing different kinds of real and nominal rigidities into a general equilibrium model with microfoundations (often called Dynamic Stochastic General Equilibrium models - DSGE)<sup>6</sup> that give rise to sluggish response of output and prices to exogenous shocks and to the short run non-neutrality of monetary policy. The New Keynesian literature focused on seeking various sources of exogenous shocks that could induce business cycles. One of the most important shock that, according to

<sup>4</sup>Lotteries are added to the consumption set, making it possible to study a competitive equilibrium by solving a representative agent problem. They also imply that the firm is providing full employment insurance to the workers.

<sup>5</sup>In the US data most of the variation in total hours work comes from adjustments on extensive margin, rather than intensive margin (per capita hours worked). For further discussion, see e.g. Cho and Cooley (1994), King and Rebelo (1999) or Fang and Rogerson (2007).

<sup>6</sup>Important contributions in the class of modern New Keynesian DSGE models include e.g. Erceg, Henderson, and Levin (2000), Christiano, Eichenbaum, and Evans (2005) or Smets and Wouters (2003). There are also studies, attempting to specify and estimate a DSGE type of models to mimic the behavior of the Polish economy. Wróbel-Rotter (2007) and Grabek, Kłos, and Utzig-Lenarczyk (2007) are examples of a closed-economy model and a small open-economy model, respectively.

the proponents of New Keynesian paradigm, induces economic fluctuations is the shock to the monetary policy<sup>7</sup>, but the literature also highlights other sources of fluctuations, like shocks to price or wage markups.

The New Keynesian literature is very vast and extends basic DSGE models in many directions, the discussion of which is not the subject of this study. One issue that is relevant from our perspective is an ongoing discussion between the proponents of RBC approach to business cycle fluctuations (which is deeply rooted in neo-classical way of thinking of macroeconomy) and the New Keynesians. The debate concerns in principle the sources of economic fluctuations and the mechanisms of propagation of economic shocks into the economy. One of the latest voice in this discussion, that is also important in our context, is the paper of Chari, Kehoe, and McGrattan (2007). They propose a method of business cycle accounting that assigns the sources of economic fluctuations to different kind of wedges: efficiency wedge, labor wedge, investment and government consumption wedge. The domination of a given type of wedge in accounting for business cycle fluctuations should give rise to research on microfoundations of model ingredients that result in endogenous fluctuations of this wedge. Chari, Kehoe, and McGrattan (2007) conclude that in the case of the US economy, the most important wedge that drives a substantial part of economic fluctuations is the efficiency wedge (which is associated with the fluctuations of efficiency in the use of factor inputs in production process). Basing on the results presented in Chari, Kehoe, and McGrattan (2007), our approach uses an exogenous technology process as the source of economic fluctuations. Additionally, the endogenous growth component of the model adds an endogenously determined component to overall productivity, also enhancing the role played by efficiency wedge in explaining economic fluctuations. Moreover, the search-matching theory used to model labor market in our approach constitutes a mechanism of labor market behavior that endogenously generates fluctuations in the labor wedge - the second important source of economic fluctuations identified by Chari, Kehoe, and McGrattan (2007)<sup>8</sup>.

## 1.2 Medium term cycles

The low frequency cyclical fluctuations of the economic activity are familiar to the economists. The research on longer-term fluctuations was initiated in the 1930s, but was attenuated by the Keynesian revolution (both by the way of thinking and methodological tools), which attracted most of the attention

<sup>7</sup>For a discussion on the structure and application of DSGE models (based on New Keynesian paradigm) to a conduct of the monetary policy, see e.g. Walsh (2003) or Kokoszcyński (2004).

<sup>8</sup>The authors conclude that *...the efficiency and labor wedges together account for essentially all of the fluctuations; the investment wedge plays a decidedly tertiary role, and the government consumption wedge plays none.*

of the economists at that time. Joseph Schumpeter in his famous book (see Schumpeter 1939) synthesized the research on economic fluctuations in the following classification of cycles, based on their duration:

- Seasonal cycles - within a year,
- Kitchin inventory cycles - 3 years,
- Juglar fixed investments cycles - 9-10 years (also called “the” business cycle),
- Kuznets infrastructural investments cycles - 15-20 years,
- Kondratiev innovation cycles - 48-60 years.

The medium term cycles<sup>9</sup> include, together with business cycles, Kuznets cycles and, to a certain extent, also Kondratiev cycles. Till the beginning of the 90-ties<sup>10</sup>, the literature on longer term fluctuations was rather limited and focused mainly on their empirical properties. Moreover, the definition of cycle (business cycle - fluctuations of periodicity up to approximately 8 years) adopted by the RBC and growing popularity of Hodrick-Prescott filtering with standard smoothing parameter values have in effect assigned longer term fluctuations as movements in trend.

Most of the work on the long-term characteristics of the economic growth is based on the deterministic models of growth. The literature on this topic is well developed and includes e.g. the books of Barro and Sala-i Martin (2003), Aghion and Howitt (1998) or Gomulka (1998). Deterministic models of economic growth are also presented in Tokarski (2001) and Tokarski (2005). Among the theories of endogenous growth<sup>11</sup> that attracted the most attention of economists are based on the concepts of human capital accumulation and investments in research and development. The literature on the former includes e.g. Lucas (1988), Romer (1989) or Zajączkowska-Jakimiak (2006). The latter concept is inspired by the influential work of Romer (1990), who created a model of endogenous growth, based on the plausible assumption that the intentional creation of new specialized intermediate goods stemming from R&D activity is the source of technological change and drives the longer term growth of the economy<sup>12</sup>.

<sup>9</sup>Following Comin and Gertler (2006), we define medium term cycles as fluctuations of duration up to 50 years. See section 2.1 for deeper discussion on the definition of the medium term.

<sup>10</sup>This time lag was partly motivated by the short data spans describing the evolution of post-war economies.

<sup>11</sup>As was mentioned, the endogenous growth literature covers various models. An interesting and modern model of endogenous growth, utilizing the concepts of incremental and radical innovations, described by Olsson (2005), is developed and analyzed by Growiec and Schumacher (2007).

<sup>12</sup>Additionally, an interesting contribution to the literature on endogenous growth with R&D investments is the study of Panek (1994).

Several papers, including Evans, Honkapohja, and Romer (1998) or Fatas (2000), utilized the Romer's framework in a stochastic environment and tried to build a model of longer term economic fluctuations. Fatas (2000) investigated, using a variation of the Romer's framework, the strong positive correlation between long-term growth rates and the persistence of output fluctuations, being a feature of cross section of countries he analyzed. Evans, Honkapohja, and Romer (1998) also developed an endogenous growth model to study fluctuations over medium term horizons. In their framework aggregate growth alternates between a low growth and a high growth state. They emphasize the sunspot fluctuations in the growth rate implied by the framework. This expectational indeterminacy is induced by complementarity between different types of capital goods.

The most interesting stochastic framework, from our perspective, is developed by Comin and Gertler (2006)<sup>13</sup>. They consider a two-sector version of a reasonably conventional RBC model, enhanced with endogenous productivity of final goods, endogenous capital-specific productivity that allows them to distinguish between embodied and disembodied technological progress. They also model the diffusion lags of technological innovations, following the evidence presented e.g. in Rotemberg (2003). They additionally use capacity utilization and variation in entry and exit of firms, induced by variation in the degree of competition (the precise formulation of endogenous competition mechanism follows Gali and Zilibotti 1995). They use the stochastic exogenous process of market power of labor supply (wage markup) as the main driving force of economic fluctuations. Diego Comin and Mark Gertler defined the medium term fluctuations (in our study we apply the same definition of the medium term, see section 2.1 for further details) and show how their framework induces longer term swings in economic activity. Their model implies large amplification and propagation mechanisms and allows to generate medium term fluctuations in economic activity induced by short term changes in economic environment. As the Comin and Gertler (2006) framework proves to be successful in explaining medium term fluctuation in goods and capital markets, we use (somewhat simplified, in order to focus only on the most important aspects of their model) their framework and enhance it with search-matching mechanism of labor market functioning to focus on the determination of unemployment.

Why do we want to focus on the medium term variation in labor market and in particular - of unemployment? Our attempt is motivated by several papers that emphasize the empirical and, to a certain extent, theoretical aspects of medium term fluctuations in the labor market. One of them is Blanchard (1997), who focused on medium-run evolution of OECD countries, emphasizing the role of labor supply and demand shifts in explaining

<sup>13</sup>Their model is also described and discussed in the context of economic fluctuations by Growiec (2005).



the persistence of unemployment fluctuations. More recently, Hall (2005d) documented the medium term evolution of labor market variables, emphasizing their comovement with variables describing other aspects of economic activity<sup>14</sup>. He has not proposed any modeling framework to describe this phenomenon but hypothesized that medium run variation in the data can be induced by slow-moving changes in parameters of the information distribution across different agents operating in the economy. Additionally, the literature of longer term differences in unemployment evolution across continental Europe and US indirectly indicates the existence of medium term fluctuations in the labor market. The literature on this topic includes e.g. Blanchard (2006), Prescott (2004), Rogerson (2004), Rogerson (2007) or Ljungqvist and Sargent (1998). These papers focus on different explanations of the divergence of European and US unemployment rates, including taxes, different structure of the economies or growing economic turbulence. These papers emphasize the role of both labor demand and supply as a source of medium term fluctuations, so it is hard to ultimately assign the source of medium term fluctuations to either of the sides of the labor market.

The papers dealing with the medium term characteristics of the labor market variables are focusing mainly on data evidence and do not seek to propose a unified model of medium-run labor market fluctuations. Our study tries to extend the research area in this direction and answers the Solow's postulate (see Solow 2000) to build a unified model of medium term fluctuations, that describes both goods and labor markets<sup>15</sup>.

### 1.3 Search-matching theory of unemployment

There are several theories of unemployment, emphasizing various sources of the existence of this socio-economic phenomenon<sup>16</sup>. It is not our goal to describe all of them, but let us only mention some books, dealing with the sources of unemployment. These include e.g. Layard, Nickell, and Jackman (1991), Pissarides (2000) or Kwiatkowski (2002).

The roots of the search-matching theory lies in the pioneering work of Stigler (see Stigler 1962), solved mathematically by McCall (1970). Both these authors proposed a framework to think of the process of the search for

<sup>14</sup>Also Shimer (2005) indirectly includes medium term fluctuations in his definition of the cycle by filtering his data with HP filter with very high smoothing parameter, which implies very smooth trends and much more volatile cycle.

<sup>15</sup>Additionally, Solow (2000), stated that: *... among the services that such a hybrid model [of medium term fluctuations] should be able to provide are interpretations of divergent trends in unemployment in Europe in the 1980s and 1990s...* Although this line of research is a very interesting and promising venue, the scope of this study is limited only to the US economy. We leave the issue of explaining different unemployment experiences of US and continental Europe countries with the theoretical model developed in this study (see section 3) for further research.

<sup>16</sup>See Góra (2005) for a brief discussion of various sources of unemployment and their implications in case of the Polish economy.

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jobs or other opportunities in an economically valid fashion. McCall (1970) characterized solution to the search problem - i.e. job decision in terms of the worker's reservation wage - the lowest wage that the worker is willing to accept in exchange for the job contract. The optimal strategy of the worker (for given job characteristics) is to accept offers with wages above reservation wage and decline job offers that does not compensate for the reservation wage.

The job search framework of Stigler was integrated with a matching theory into a more comprehensive model of labor market by Diamond (1982), whose framework was extended by Mortensen (1982) and Pissarides (1985)<sup>17</sup>. The search-matching theory distinguishes between jobs and workers and describes the process of both searching and matching unfilled jobs (vacancies) with workers searching for a job (unemployed) in a given instant.

Models in the spirit of the DMP framework describe labor market in the continuous equilibrium. There is no economic agent who waits to change a price or allocation, once the change is merited. Unemployment arises as an equilibrium phenomena, as job seekers and prospective employers face a friction that limits their flow of meetings. The friction in the labor market arises due to the fact that labor is not a homogeneous commodity - services provided by individual workers in various occupations may differ. Also vacancies are heterogeneous - there are various skills of a job candidate that the prospective employer is looking for. So, it is nontrivial to match a given worker with a vacancy to achieve a contract with appropriate level of productivity.

Employers decide on the level of recruiting effort - they post vacancies whenever the gain (marginal product of labor net of labor cost - approximately the employer's reservation wage) from employing additional worker(s) is higher than the cost associated with the effort required to get in touch with the worker. A matching technology (or a matching function) relates open vacancies with workers seeking for a job (unemployed) and determines the number of new matches in a given instant. There are several microeconomic models that result in the aggregate matching function (see e.g. Hosios (1990) for a brief description of some micro-founded models that share the same reduced form of a matching function)<sup>18</sup>. When an employer with open vacancy meets a job seeker, they determine if their prospective relationship has a surplus<sup>19</sup>. If there is a positive surplus from this relationship the parties

<sup>17</sup>The search-matching framework is often called DMP theory, following the names of its authors. The DMP framework is also discussed in Pissarides (2000) or in Ljungquist and Sargent (2000).

<sup>18</sup>The concept of aggregate matching function is quite similar to the concept of aggregate production function. It also share similar problems as aggregate production function, description of which dates back to the classical contribution of Houthakker (1955-1956). For an interesting and a more recent treatment, also focusing on consequences of labor market frictions for TFP, see Lagos (2006). See Pissarides (2000) for a discussion on the problems with the concept of aggregate matching function.

<sup>19</sup>The surplus from a match is a difference between the reservation wage of an employer and the reservation wage of a job seeker.

engage in wage bargaining in order to split the surplus. The commonly used assumption in search-matching models is the Nash bargain, which splits the surplus proportionally between the two parties.

When engaging in employment relationship and wage bargaining, both parties internalize the fact that the relationship will hold for some time and take into account the present discounted value of all future benefits from the contract when making their decisions. So, the relationship between worker and employer has a longer-term character. It dissolves when the gain from continuing this relationships is no longer profitable to either of the party. Some models in the search-matching framework simplify the the problem of braking down the job contract and assume that the relationship dissolves exogenously. This simplification is based on the evidence described e.g. by Hall (2005a), Hall (2005c) or Shimer (2005), who argue that the separations, although slightly countercyclical, exhibit little variation and could be treated as relatively constant within the cycle. Therefore, most of the modern models abstract from decisions on job separations and assume constant separation rate. We also follow this line and choose to simplify (in this dimension) the description of the labor market functioning in the model developed in the study presented here.

The search-matching approach to understanding unemployment flourished during the 1980s and 1990s. Incorporating the simple observation that searching is costly into a theory of labor markets has resulted in a rich set of models which have helped economists not only to understand how unemployment responds to various policies and regulations, but also to gain a better understanding of other labor market issues including job creation and destruction, business cycle characteristics, and the effects of labor market policies on the aggregate economy more generally.

From the very broad spectrum of the literature in the search-matching framework, we are going to focus on two aspects, that are important from the perspective of our study. The first is the incorporation of the search-matching principles into a fully specified general equilibrium model of economic activity and the second is related to the issues of wage determination and its consequences for the volatility of labor market variables.

The first attempt to introduce search-matching mechanism into the core RBC model was due to Merz (1995) and Andolfatto (1996). They utilized the fact that labor adjustments (measured by total hours worked) during the business cycle, takes place mainly on extensive (employment), rather than intensive (average hours) margin (see e.g. Cho and Cooley (1994) and King and Rebelo (1999) or a discussion in section 1.1). But they followed the route different than the one taken by Rogerson (1988) and Hansen (1985). Merz (1995) and Andolfatto (1996) abandoned the standard Walrasian approach to model labor market<sup>20</sup> and focused on determination of flows of

<sup>20</sup>This standard approach uses the classical theory of labor market, in which the supply

workers between different states of economic activity. These two influential studies show that (apart from fitting the labor market data better) including a matching function improves the behavior of the RBC model by increasing the persistence of fluctuations. Additionally, Cole and Rogerson (1999) argued that the search-matching model can reasonably account for the business cycle facts on employment, job creation and job destruction, provided that the spell of unemployment is relatively high.

The success of early generations of general equilibrium models with labor market modeled in the DMP tradition, have met with a growing interest of New Keynesians and researchers in central banks. They studied the consequences of non-Walrasian labor market for the inflation determination (see e.g. Christoffel and Linzert (2005) or Krause, Lopez-Salido, and Lubik (2007)), monetary policy and monetary transmission mechanism (see e.g. Trigari (2004), Blanchard and Gali (2006), Trigari (2006) or Gertler and Trigari (2006)) or for the optimality of the monetary policy - see Arseneau and Chugh (2007)<sup>21</sup>. The search-matching framework adds complexity and enhances the plausibility of the monetary transmission mechanism, enriching both the description of marginal costs and inflation determination and the way the monetary policy shocks are propagated into prices and real variables.

Another aspect of the ongoing research on general equilibrium models with the DMP labor markets, that is important from our perspective, is the issue of volatility of unemployment and wage rigidity. The contributions of Costain and Reiter (2003), Hall (2005b) or Shimer (2004) show that the standard search-matching model (with reasonable calibration, especially with respect to the replacement ratio) have difficulty in matching the volatility of one of its central elements - the unemployment<sup>22</sup>. One of the possible sources of this shortcoming of the standard DMP model is the issue of the wage determination. The literature stresses that the search-matching theory determines only the bargaining set - the range of feasible wages that are acceptable to both worker and employer. In other words, the theory focuses on determination of reservation wages of both parties of the contract, resulting in a match surplus that can be divided by the negotiated wage between both parties. Any wage within the bargaining set is efficient, in the sense that it does not distort the individual decisions of agents and leads to a successful

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of labor is derived by households from the utility maximizing principles and the demand for labor is decided by firms in their profit optimization program. Wages are determined by the equilibrium condition that relates the marginal rate of substitution, as perceived by households, to the marginal product of labor, as perceived by producers. The Walrasian model of labor market implies that the labor market is always in equilibrium with full employment, so there is no possibility of unemployment to arise (in other words, unemployment is treated equally with economic inactivity). In the search-matching framework, due to frictions that limits the flows of meeting between workers and employers, unemployment arises naturally as an equilibrium phenomenon.

<sup>21</sup>The contribution of Arseneau and Chugh (2007) shows that the way the labor market is modeled matters a lot for the properties of the optimal monetary policy.

<sup>22</sup>The first authors who stressed the role of wage determination in explanation of observed fluctuations in unemployment was Veracierto (2002) and Shimer (2003).

job contract. But simultaneously, the way that the negotiated wage splits the joint surplus from a match between the two parties matters for the vacancy posting activity of employers and thus for the aggregate conditions on the labor market.

The standard search-matching models (see e.g. Mortensen and Pissarides 1994) use the Nash bargaining solution (see Nash 1953) to pin down the wage and to select the particular equilibrium form the range of possible equilibrium. But the Nash solution implies that the match surplus is divided proportionally between both parties in each instant, implying in turn that the negotiated wage follows closely the evolution of productivity - the approximate gain from a successful match for an employer. As the DMP model assumes that the employer decides over the possible contract, such considerations imply that with changing environment (e.g. with productivity shock) the employer has limited incentives to post new vacancies and the volatility of labor market variables during the cycle (e.g. unemployment or vacancies) is lower, than observed in the data.

Robert E. Hall (see Hall 2003, Hall 2005b) proposed a different equilibrium selection rule to pin down the wage within the bargaining set (a selection rule that introduces wage rigidity). He followed the idea of Akerlof, Dickens, Perry, Gordon, and Mankiw (1996) and used previous period's wage as a norm for this period's wage - the adaptive wage equilibrium selection rule. What is more important, although the wage selection rule proposed by Robert E. Hall introduces rigidity in the wage formation process, it does not distort the formation of efficient matches, as it assures that the realized wage lies in the bargaining set. So, inefficiencies associated with perspective matches that cannot be realized due to wage being outside the bargaining set, cannot occur and this kind of wage stickiness is immune to the Barro's critique<sup>23</sup>. Additionally, there is a vast literature on the existence and nature of wage rigidity (inertia) in price and wage determination, that starts from seminal papers of Friedman (1968) and Phelps (1967).

In one of the recent publications, Christopher A. Pissarides argues with Robert E. Hall's and Robert Shimer's proposal of wage stickiness as the answer to "the unemployment volatility puzzle" (see Pissarides 2007). Pissarides stresses that bulk of the literature focuses on models with job creation being the main source of labor market volatility, ignoring the role of job separations or treating them as exogenous and subject to cyclical shocks<sup>24</sup>. He notes that the introduction of cyclical job separations contributes substantially to the cyclical volatility of unemployment, pushing the volatility of the

<sup>23</sup>Barro (1977) criticized sticky wage models, like the one of Calvo (1983) or other stressed by the New Keynesian literature, for introducing arbitrary restrictions that intelligent agents could easily avoid. In the case of time-dependent wage stickiness, the equilibrium inefficiency introduced by wage rigidity could be easily overcome by agents negotiating over wages in each period.

<sup>24</sup>This assumption follows the evidence presented in Hall (2005c) and Shimer (2005), who show that job separations exhibit relatively low volatility over the business cycle.

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model generated unemployment to the levels observed in the data<sup>25</sup>. Unfortunately, Pissarides does not address the negative consequences of cyclical job destruction for the Beveridge curve, as is apparent from the analysis presented by Shimer (2005) (and, to some extent, in the case of our model, see section 5.2). Introduction of cyclical volatility in job destruction drives the Beveridge curve (the negative relation between unemployment and vacancies) towards zero, which is contrary to the data. Thus, in our analysis we decided to specify the model without job destruction cyclicality and introduce wage rigidity to bring the model predictions closer to the data in as many dimensions as possible (although we also analyze the consequences of exogenous shifts to the Beveridge curve for our results).

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<sup>25</sup>Pissarides also stresses that there is important difference whether the cyclicality in job separations is a result of endogenous job destruction decisions or exogenous shocks. In the case of optimal job destruction decisions only jobs with net productivity close to zero are destroyed whereas in the case of exogenous shocks all jobs, regardless of their net productivity, could be destroyed. So it follows that in the former case job destruction has no impact on job creation, while in the latter case job destruction have a negative effect on job creation, enhancing the overall volatility of labor market variables.

## Chapter 2

# Evidence on medium term cycle from the US economy

In this section we will present the evidence based on US data on the importance of the medium term cycle in both goods and labor markets, with a special emphasis on the latter. In order to filter out the medium term cycle, we will apply the Christiano-Fitzgerald band-pass filter (for the reference, see Christiano and Fitzgerald 1999). This filtering method allows to define the range of frequencies of fluctuations that one wants to extract from the raw data, so it is well suited to the exercise we are intending to perform<sup>26</sup>. Additionally, we present some evidence regarding the medium term cycle that is based on spectral decomposition of the time series. It allows us to assess quantitatively the role played by medium frequency component of the cycle in the overall variation of economic variables.

The first part of this chapter defines the concept of medium term cycle. Further sections describe the data sources and present the evidence on medium term cycles following closely the route marked by Comin and Gertler (2006), Hall (2005d) and Hall (2005c).

### 2.1 Definition of medium term cycle

As the research on the medium term business cycle is relatively new in the economic literature, there is no widely accepted definition of the medium term cycle. Literature on economic fluctuations concentrates on the so called business cycle fluctuations. These fluctuations are conventionally defined as fluctuations of economic variables within the frequencies between 2 and 32 quarters. This definition follows from a seminal contribution of Burns and Mitchell (1946) and is formalized e.g. in Baxter and King (1999) and Christiano and Fitzgerald (1999). Standard parametrization of the widely used in the business cycle literature Hodrick-Prescott filter (for further reference, see Hodrick and Prescott 1980) implies that the definition of the business

<sup>26</sup>This way of expressing the data was also applied in the paper of Comin and Gertler (2006) as a good illustration of data properties.

cycle roughly corresponds to fluctuations with frequencies between 2 and 32 quarters.

The standard approaches assign all fluctuations with periodicity over 8 years to the trend. However, as we will see shortly, this procedure implies that the trend is relatively volatile. What we could do is to redefine the notion of a trend, in order to allow it to be very smooth. But, the question is which fluctuations should be treated as medium term component of the cycle and which we should treat as a trend volatility. Comin and Gertler (2006), having analyzed the US data properties, decided to treat all fluctuations with periodicity above 50 years as trend. Additionally, they stressed the importance of analyzing the business cycle fluctuations and medium term fluctuations together, so they defined the **medium term cycle** as fluctuations in frequencies between 2 and 200 quarters. So, within the medium term cycle we may distinguish:

- the business cycle component of the medium term cycle (**high frequency component**, with periodicity between 2 and 32 quarters)
- the medium term component of the cycle (**medium frequency component**, with periodicity between 32 and 200 quarters).

We will follow the definition and naming convention introduced by Comin and Gertler (2006), as some results emerging from the variance decomposition using spectral methods seems to justify this definition (see section 2.4). Additionally, the definition applied here allows for very smooth nonlinear trends of the data. Simultaneously, it is much better than simple log-linear data filtering, as there is a number of factors, such as demographics, that are likely to introduce low frequency variation in the data. Linear filtering is not able to account properly for such a long-term fluctuations in the data<sup>27</sup>.

## 2.2 Measurement and data sources

Our analysis concerning data properties concentrate on macroeconomic variables describing both goods and labor market of the US economy. We are using several publicly available data sources, most of them published by US federal agencies, such as the Bureau of Labor Statistics or the Bureau of Economic Analysis. We present most of the variables normalized using the size of the labor force, which is consistent with the model that we develop in the next sections<sup>28</sup>.

<sup>27</sup>Another thing worth noting is that with a linear trend, the estimates of some moments of the data becomes imprecise. So, our definition of the medium term cycle brings together reasonably smooth trends and reasonably precise estimates of volatility of the filtered data. For further reference on this point, see Comin and Gertler (2006).

<sup>28</sup>Many studies use the size of population as a normalizing variable. The choice of normalization does not affect the results discussed here. For the discussion on the medium term properties of the US data normalized by the size of population, see Comin and



The size of the labor force, as well as employment and unemployment are taken from the Current Population Survey, published by the Bureau of Labor Statistics, US Department of Labor. The data are measured quarterly (calculated as a mean of respective monthly data) and cover the period from 1Q1948 to 4Q2006. Wages are calculated as real compensation of employees (in chained 2000 dollars, taken from National Income and Product Accounts, published by the Bureau of Economic Analysis, US Department of Commerce) divided by the size of employment from the CPS and cover the period from 1Q1948 to 4Q2006. Labor share is measured as real compensation of employees per Gross Domestic Product (in chained 2000 dollars, taken from NIPA).

Employment and unemployment rates are measured by the number of employed and unemployed respectively per the size of labor force (by construction, these two rates sum to one). Vacancies are measured with the Help Wanted Advertising Index (in real terms, 1987 = 100, converted to quarterly frequency by averaging of monthly observations), published by The Conference Board and are expressed per labor force. The data on vacancies cover the period from 1Q1951 to 3Q2006. The job finding probability, covering the period 1Q1948 – 4Q2006, is constructed by Robert Shimer (for more details, see Shimer 2007) and taken from his website<sup>29</sup>. Shimer calculated these probabilities using the publicly available CPS data. As the Shimer's data are expressed in monthly terms, we transformed them to quarterly frequency using the formula  $p_{quarterly} = 1 - (1 - p_{monthly})^3$ .

The measure of output used for the US economy is real Gross Domestic Product (in chained 2000 dollars, taken from NIPA) per labor force. Consumption is measured by real personal consumption expenditures on non-durables and services (all data from NIPA) per labor force (from CPS). Investment outlays are proxied by real gross private domestic investments plus real consumption expenditures on durables, per labor force. All data from NIPA cover the period from 1Q1948 to 4Q2006.

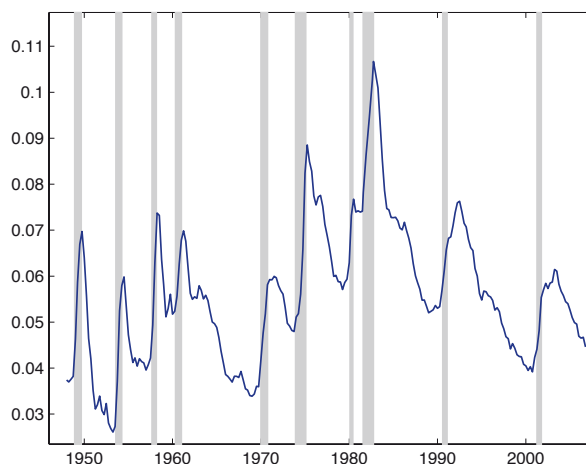
Real interest rates are measured by nominal market yield on US Treasury securities at 1-year constant maturity (published on monthly basis by the Federal Reserve Board, we transformed the raw data to quarterly frequency by averaging the monthly observations), deflated by expected inflation. The latter variable is proxied by next 4 quarters change of personal consumption expenditures deflator (taken from NIPA).

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Gertler (2006). We also performed similar exercise (not shown here) using population as a normalizing variable. This exercise confirmed that the basic results of this section are not affected by the choice of the normalization variable.

<sup>29</sup>See [robert.shimer/googlepages.com/flows](http://robert.shimer/googlepages.com/flows)

Figure 1: Unemployment rate in US (1948-2006)



Shaded areas coincide with NBER recession periods  
 source: Current Population Survey, Bureau of labor Statistics

### 2.3 Evidence on the medium term cycle

The behavior of unemployment rate in the US economy is a very good illustration of the medium business cycle. Figure 1 depicts the evolution of the unemployment rate since 1948 in the US economy. The unemployment was relatively low in the 50-ties and 60-ties of the last century, then increased for roughly next 20 years and then, since the 90-ties, went back to the lower levels. These fluctuations occur with periodicity far greater than a decade and are rather attributed to the medium frequency component of the medium term cycle. Of course, the behavior of unemployment rate is also subject to fluctuations in higher frequencies, and these are usually associated with booms and recessions (shaded areas on the Figure 1 represent recession periods, announced by National Bureau of Economic Research, NBER). In the context of this study, we will focus both on the medium term and short term evolution of unemployment and other macroeconomic variables. We will try to examine whether both these phenomena have the same origins and could be explained simultaneously.

In order to extract information on the medium term cycle and its higher and medium frequency components we apply the band-pass filters<sup>30</sup> devel-

<sup>30</sup>The introduction of approximated optimal filters, specified in frequency domain, into economics is due to Baxter and King (1999). The algorithm used in their paper have some limitations as it does not allow to compute filtered components at the beginning and at the end of sample period. This is due to the fact that approximation to the optimal band-pass filter is a symmetric two-sided filter, with coefficients computed from the correlogram of the time series. In other words, in order to compute the filtered series in a given period one need the information from both the preceding and succeeding periods, so one cannot compute filtered series at the beginning and end of the whole sample. This shortcoming of

oped by Christiano and Fitzgerald (1999). In line with the discussion in section 2.1, we define the medium term cycle as fluctuations with periodicity between 2 and 200 quarters and the higher and medium frequency components of the cycle as fluctuations with periodicity in the range  $[2, 32]$  and  $[32, 200]$  respectively. It follows that we define the trend in the data as fluctuations with periodicity above 200 quarters.

In order to apply the band-pass filter<sup>31</sup> to the data in the case of non-stationary series (like wages or GDP and its components), we first convert raw data into growth rates by taking log differences. Then, after applying band-pass filters to the growth rates, we cumulate the resulting components of the analyzed series into log levels<sup>32</sup>. The stationary series (like unemployment rate, vacancy rate, job finding probability, labor share or interest rate) were directly filtered in log levels. Figure 2 depicts the results of the filtering procedure for the measures of unemployment and employment rates, real wages, labor share, vacancies (per labor force) and job finding probabilities (for the discussion on the measurement issues and data sources used, see section 2.2). Graphs on Figure 2 show the medium term cycle (fluctuations in the range between 2 and 200 quarters) of the economic variables, as well as the medium frequency component of the cycle (variation in the frequencies between 32 and 200 quarters). The difference between the two series on the graph shows the higher frequency component of the cycle (usually associated with the notion of the business cycle).

What stems from Figure 2? First, the intuition gained from the visual inspection of the raw unemployment rate data is confirmed when using more elaborate econometric tools. There is substantial variation of unemployment in medium term frequencies. Second, the medium term component of the cycle is very pronounced, also in the case of other aspects of the labor market, especially in the case of real wages. Third, there is a lot of business cycle variation in the data (in frequencies between 2 and 32 quarters) in case of unemployment, labor share, vacancies and job finding probability. The magnitudes of fluctuations of the two components of the whole medium term cycle is at least comparable (in case of wages, the variation in medium term frequencies seems to prevail over variation in business cycle frequen-

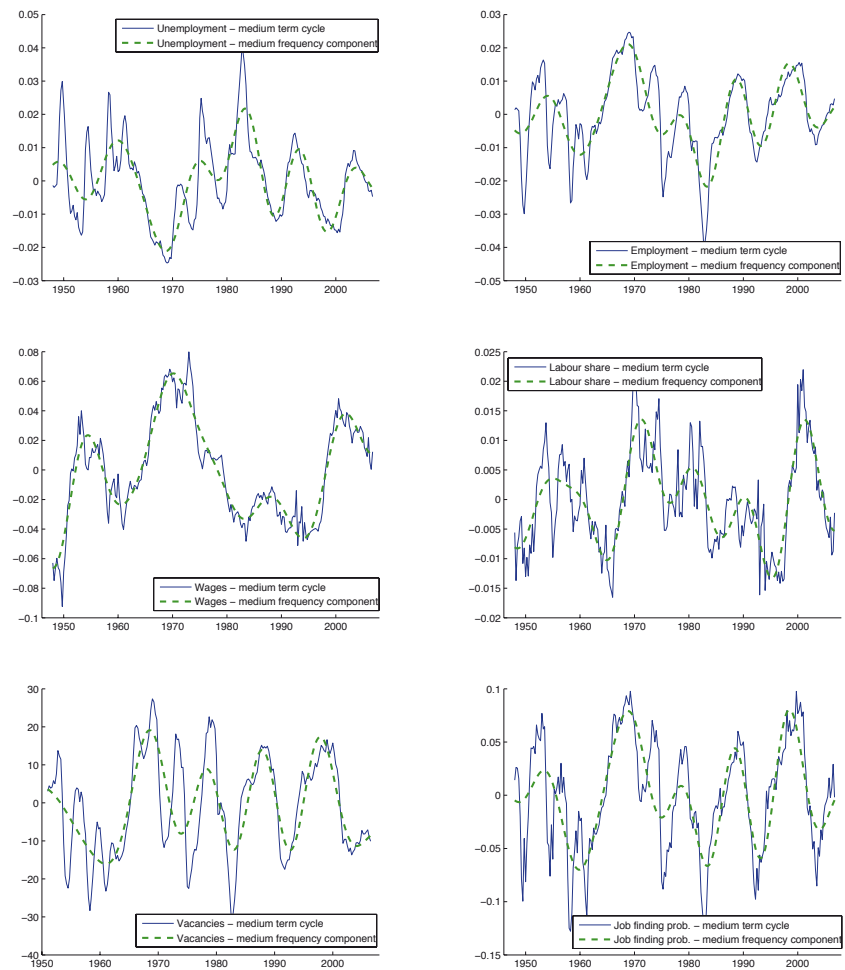
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the Baxter-King filter is addressed by Christiano and Fitzgerald (1999), who proposed a modification of the filter at the sample ends that uses more information from the available data to dampen the negative effect of non symmetry of the filter in the neighborhood of the sample ends. So, when using the band-pass filter of Christiano and Fitzgerald, one can compute the filtered series for all time periods covered by the data, but at the cost of phase shift between the raw and filtered series at the beginning and at the end of the sample.

<sup>31</sup>We use the algorithms developed by Christiano and Fitzgerald. The codes for Matlab we used are publicly available on the Federal Reserve Bank of Cleveland website (see <http://www.clevelandfed.org/research/models/bandpass/Index.cfm>).

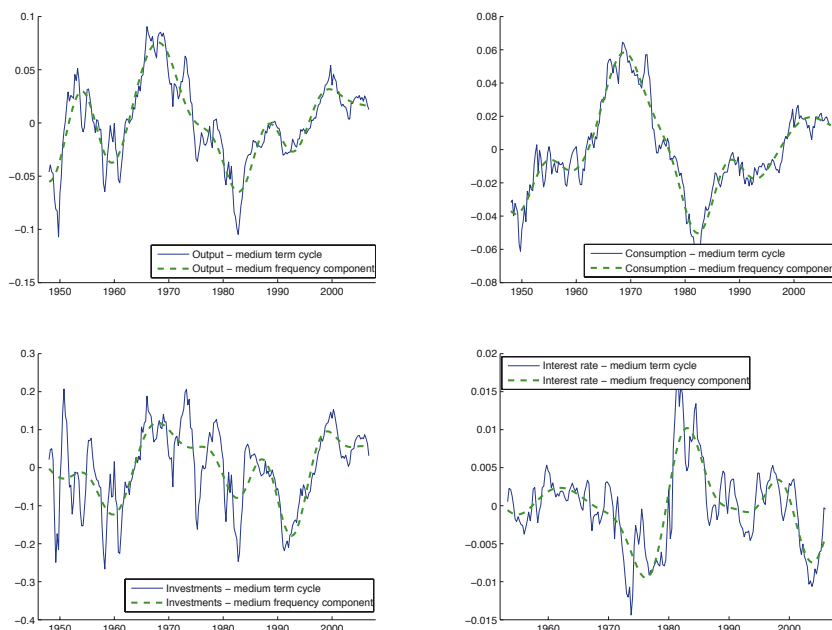
<sup>32</sup>Spectral methods are designed to operate on stationary data, so we apply the procedure on growth rates rather than levels. We obtain virtually the same results, in the case of nonstationary series, by filtering the data in log levels, after first removing a linear trend.

Figure 2: Medium term cycle in US labor market



source: own calculations

Figure 3: Medium term cycle in US goods market



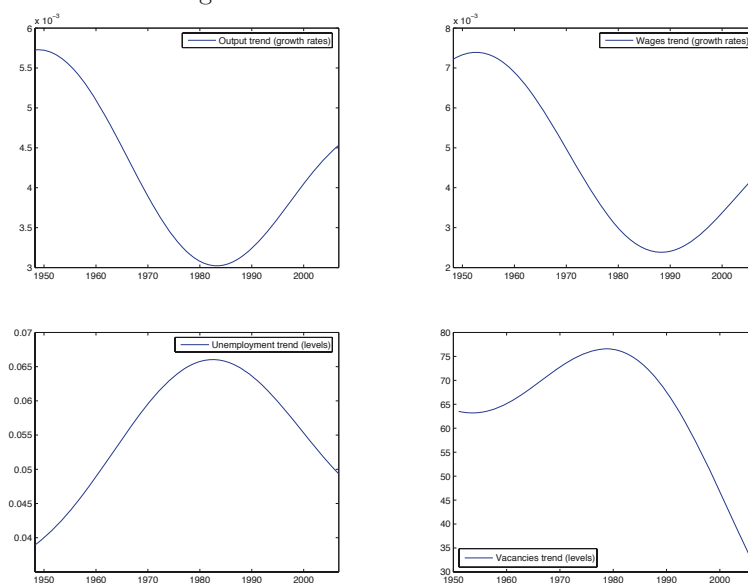
source: own calculations

cies). Next observation worth noting is the fact that both components of the medium term cycle seem to be correlated - there is a lot of comovement between different frequencies. It suggests that it could be possible that both kind of fluctuations share the common sources.

Figure 3 (constructed in the similar fashion for variables measuring the cycle in goods - and to some extent capital - market) shows that substantial medium term variation in the data is also present in other markets of the US economy, namely in goods and capital markets. The observations made for the labor markets are also valid here. Even more to say, in the case of GDP and its components (and likewise wages), the variation of the medium term component of the cycle seems to be larger than the variation of the higher frequency component of the cycle. Interest rate seems to be more volatile at business cycle frequencies, although the extent of medium term variation is also substantial. So, the medium term fluctuations are present in the whole US economy and it seems to be well justified not to treat them as a concept distinct from the business fluctuations.

Figure 4 shows the trends of unemployment, vacancies (both in terms of levels), output and wages (both in terms of growth rates) that are left after the filtering of the raw data series and will not be explained by the macroeconomic model presented in next chapters. The first thing worth noting is the smoothness of the trends presented. There is some variation in the behavior of trends, but these movements are very slow and last for many years. Although the trend behavior is going to be outside the focus of

Figure 4: Trends of selected variables



Output and wages - trends of variables are presented as quarterly growth rates

Unemployment and vacancies - trends of variables are presented in levels

source: own calculations

this study, one need to admit, that the amplitudes of trend movements are quite substantial (e.g. growth of trend output varies from 0.3% to 0.57%, on quarterly basis or the level of trend unemployment varies from 4% to 6.5%). The trends of output and wages behave almost identically, with a steady decline from 50-ties to 80-ties and a rise thereafter. Additionally, the trend in wages seems to lag a trend in output by about 3-4 years. Unemployment trend is almost a mirror image of the evolution of output trend, without any time shifts. The trend in vacancies seems to behave a bit differently. Its shape is similar to the one exhibited by unemployment, with quite pronounced lead. The trend in vacancies increases in 60-ties and 70-ties, then declines from the late 70-ties towards the end of the sample period.

The observations and intuition gained from the visual inspection of Figures 2 and 3 are summarized in Table 1, which shows selected moments of output, consumption, investments, interest rate, wages, unemployment, employment, vacancies, labor share and the job finding probability, calculated for the separate periods of data availability. The moments presented include:

- volatility (standard deviations),
- comovement (correlation with output),
- persistence (autocorrelations),

calculated for the medium term cycle and its higher and medium term frequency components separately. Additionally, Table 1 also presents volatility

of a medium term cycle of a given variable in relation to the volatility of a medium term cycle of output (GDP).

The volatility of the medium frequency component of the cycle in GDP is substantial and over twice as large as the volatility of the higher frequency component of the cycle<sup>33</sup>. Also the persistence of both components of the cycle is large, especially in case of the medium frequency component of the cycle, where the autocorrelation is almost 1. The volatility of consumption within the whole medium term cycle is about 75% of the volatility of GDP (the lower volatility of consumption in relation to GDP is well documented in the business cycle literature, see e.g. King, Plosser, and Rebelo (1988) or King and Rebelo (1999) and the references therein). Consumption exhibits far more volatility over medium term frequencies, than GDP (see also upper right panel of Figure 3). This observation is consistent with the Permanent Income - Life Cycle theory of consumption (see e.g. Hall 1978) which predicts the consumption smoothing behavior of households. Consumption smoothing, in turn, implies that consumption should exhibit relatively lower volatility in higher frequencies, consistently with the evidence presented in Table 1. Additionally, consumption comoves with output - and more importantly this comovement is more pronounced in medium term frequencies. Also the persistence of consumption is large, especially in medium frequencies (the correlation of higher frequency component is somewhat lower).

Consistently with the RBC literature and data evidence presented therein (see e.g. Plosser 1989), investments are far more volatile than output - in our case investments are 2.6 times as volatile as output. In case of investments, the relative volatility of the higher and medium frequency components of the cycle is relatively balanced, which is also consistent with the erratic behavior of investments in the US manufacturing (documented e.g. in Caballero, Engel, and Haltiwanger 1995), that predicts more volatility of investments over short time horizons. Also the comovement of investment with output is less pronounced than in case of consumption. Additionally, this comovement (contrary to the behavior of consumption) is more pronounced in higher frequencies, associated with the business cycle. The persistence of investments is also smaller than that of GDP and consumption, but still the persistence over medium term frequencies is substantial.

Interest rate is much less volatile than output, although its volatility is not negligible (see lower right panel of Figure 3). What is more important,

<sup>33</sup>The calculation of the relative importance of the two components of the medium term cycle is somewhat troublesome, due to the fact that these components are not orthogonal and there is a non zero covariation between them that cannot be attributed to either of the components. In the context of the calculated moments of the filtered variables it is better to look at the relative volatility of the medium versus higher frequency components of the cycle. The relative importance of either of the components can be calculated using directly spectral decomposition of the time series and then integrating the periodogram over relevant frequencies to obtain the share of a given component in overall variation of a given variable. This results of this exercise will be presented in section 2.4.

Table 1: Selected moments of the medium term cycle (and its higher and medium frequency components) in the US economy

	Standard deviations			Correlation with output			Autocorrelation			
	cycle [2,200]	high [2,32]	medium [32,200]	relative to output	cycle [2,200]	high [2,32]	medium [32,200]	cycle [2,200]	high [2,32]	medium [32,200]
Output	0.039	0.016	0.035	1.00	1.00	1.00	1.00	0.96	0.81	0.99
Consumption	0.028	0.008	0.027	0.73	0.88	0.76	0.91	0.98	0.74	0.99
Investments	0.103	0.069	0.076	2.66	0.73	0.84	0.74	0.91	0.81	1.00
Interest rate	0.005	0.003	0.004	0.14	-0.33	-0.15	-0.39	0.93	0.79	0.99
Wages	0.034	0.010	0.032	0.89	0.81	0.77	0.82	0.97	0.73	0.99
Unemployment	0.225	0.145	0.168	5.82	-0.84	-0.82	-0.88	0.94	0.88	0.99
Vacancies	0.208	0.131	0.172	5.63	0.53	0.83	0.45	0.96	0.90	0.99
Employment	0.013	0.008	0.010	0.34	0.84	0.84	0.87	0.95	0.88	0.99
Labor share	0.014	0.008	0.012	0.37	0.15	-0.23	0.26	0.87	0.61	0.99
Job finding probability	0.063	0.039	0.050	1.62	0.74	0.77	0.75	0.92	0.81	0.99

source: own calculations



interest rates also fluctuate substantially in medium term frequencies - the volatility of medium frequency component of the cycle is higher than the volatility of the higher frequency component of the cycle. Interest rate is slightly countercyclical, at all frequencies considered, but the bulk of this negative correlation is concentrated in medium term frequencies. Additionally, the persistence of interest rate is relatively modest at business cycle frequencies.

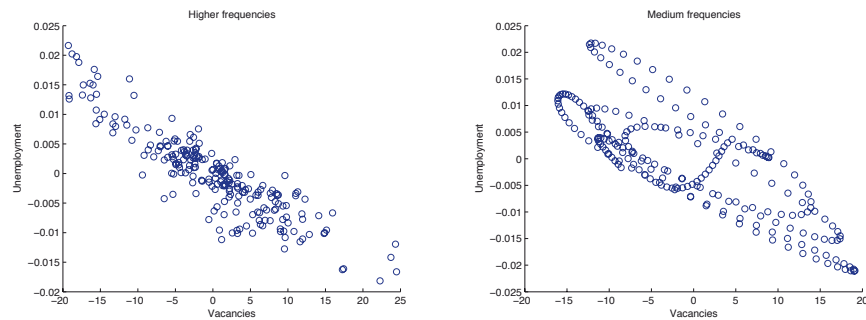
The volatility of wages is only a little lower than the volatility of output and, more importantly, medium term fluctuations of wages are much higher than the business cycle fluctuations. According to the data used in this exercise, wages are relatively procyclical, especially at medium term frequencies and are rather highly persistent (as in the other variables studied in this exercise the persistence of the medium frequency component of the cycle is very large).

The volatility of unemployment is very high, when compared to the volatility of output - unemployment fluctuates almost 6 times more than output. This fact is widely known in the literature<sup>34</sup> and is mainly due to the fact that unemployment changes quite rapidly over the cycle (e.g. increase in unemployment rate of 3 – 4 pp. are not unusual during recession, whereas the average unemployment level amounts to 5.6%, so the overall volatility of unemployment rate is substantial). Fluctuations in unemployment occur both at higher and medium frequencies, although the volatility of both these components is quite balanced, with a little more mass concentrated in medium term frequencies. Unemployment comoves negatively with output, not only in business cycle frequencies, but also in medium term frequencies. The latter negative correlation is even more pronounced than usually measured (in business cycle frequencies) countercyclicality of unemployment. Unemployment displays moderate degree of persistence, which is (in the case of the higher frequency component of the cycle), comparable to the persistence of employment and vacancy rates. The behavior of the employment rate is a mirror image of the behavior of unemployment, with almost the same degree of comovement with output (procyclicality, in this case) and persistence. The relative volatility is smaller than that of unemployment rate, but is also an mirror image of the unemployment volatility (with the average level of the employment rate of 94.4%, the fluctuations of amplitude 3 – 4pp are relatively small).

Vacancies are only a little less volatile than unemployment. Additionally, they are relatively more volatile in medium term frequencies. Their correlation with output is positive, but it is relatively weak (especially in medium term frequencies; in higher frequencies, procyclicality of vacancies is quite strong and similar to the one exhibited by investment). Vacancies exhibit

<sup>34</sup>For further reference and discussion on the measured volatility of unemployment rate, see e.g. Costain and Reiter (2003), Merz (1995) or Gomes, Greenwood, and Rebelo (2001)

Figure 5: Beveridge Curve in US labor market



source: own calculations

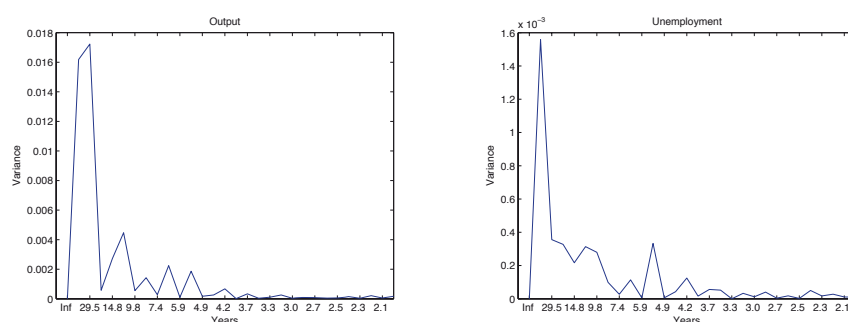
substantial persistence.

Labor share is, by definition, a product of wages, employment and the inverse of output, so its behavior is determined by these three factors. The volatility of labor share is similar to the volatility of employment and is about 3 times smaller than the volatility of output. The volatility of labor share is present at all frequencies, although the volatility of medium term component seems to play a greater role in shaping the overall volatility of labor share. The evidence on the nature of cyclical behavior of labor share is mixed and labor share seems to be acyclical. In higher frequencies, labor share is slightly negatively correlated with output, whereas in medium frequencies, labor share is slightly procyclical. Labor share is the least persistent series in our data set, especially in business cycle frequencies (compare also the middle right panel of Figure 2). The average of labor share over the sample is equal to 0.57.

Job finding probability (as calculated by Shimer 2007) is quite volatile - much more volatile than output, but not as much as unemployment or vacancies. It also displays more volatility in medium term frequencies. Job finding probability is relatively strongly procyclical at all frequencies considered and displays moderate degree of persistence.

Figure 5 shows the Beveridge curve (the negative comovement of unemployment and vacancies) for the US data, for the period 1951 – 2006. The left panel reproduces the unemployment-vacancies pairs for the business cycle frequencies, whereas the right panel shows the pairs for the medium frequencies only. The overall unemployment-vacancies correlation, calculated for the whole medium term cycle amounts to  $-0.81$  and is higher in absolute terms for the higher frequency component of the cycle (and amounts to  $-0.9$ ) than for the medium term frequencies ( $-0.8$ ). What is more important, The Beveridge curve seems to be an important ingredient of the labor market behavior in all frequencies considered. One fact worth mentioning (that will not be addressed in this study, but seems to be an interesting area for future

Figure 6: Periodogram of the selected time series



source: own calculations

research) are the longer term shifts of the Beveridge curve, apparent from the right panel of Figure 5 (present in the data, although as was mentioned, the correlation calculated over the whole sample is relatively high in absolute terms). These kind of shifts are absent in higher frequencies, where the Beveridge curve seems to be a very regular phenomenon. This feature of the Beveridge curve have inspired one of the specification of the model that we are considering in section 5.2.

## 2.4 The role of the medium term component in overall fluctuations

In order to assess the relative importance of the medium term component of the cycle, one could directly use the spectral analysis tools and compute the periodogram of the underlying series<sup>35</sup>. Then, it is possible to integrate the periodogram over relevant frequencies in order to get the mass of variance in the specified frequency range. The calculation of the exact integral of the periodogram over the specified frequency range is impossible due to the limited sample size. But it is possible to calculate the approximate contribution of the specified frequency range to the overall volatility of a given time series.

Figure 6 shows the periodogram of output and unemployment rate<sup>36</sup>, for the ease of exposition, plotted only for frequencies above 2 years. As spectral methods can be applied only to stationary data (with zero mean), we demeaned the stationary series (e.g. unemployment rate) and extracted the linear trend from the log levels of nonstationary variables (e.g. output)

<sup>35</sup>For further reference and description of spectral methods see e.g. Sargent (1987) or Hamilton (1994). The periodograms presented here were computed using the procedures available for Matlab (namely: fft routine).

<sup>36</sup>Periodograms for other variables analyzed are relatively similar to the ones presented in Figure 6 and, although there are some differences across series, the overall conclusions drawn from limited set of variables are valid for all variables considered.

Table 2: Importance of medium frequency fluctuations - evidence from spectral methods

	Share of specific frequency components in overall variation		
	Higher [2, 32]	Medium [32, 200]	Low [200, ∞]
Output	0.16	0.52	0.31
Consumption	0.06	0.43	0.51
Investments	0.41	0.49	0.10
Interest rate	0.31	0.48	0.20
Wages	0.05	0.22	0.73
Unemployment	0.25	0.38	0.37
Vacancies	0.26	0.33	0.41
Employment	0.25	0.38	0.37
labor share	0.11	0.37	0.52
Job finding probability	0.32	0.55	0.14

source: own calculations

and then applied the Fourier transform to convert the covariances present in the data into the distribution of variance in the frequency domain.

The left panel of Figure 6 shows the periodogram of output. It is clearly visible that although output exhibits quite a lot of volatility in frequencies up to 8 years, the bulk of the variance (of the stationary series, so without the log-linear trend) is concentrated in medium frequencies. The peak of the periodogram (so the frequency of the important fluctuations) occurs for frequencies between 30 and 50 years. There is also a smaller peak in the periodogram in frequencies between 10 and 12 years, which is, according to our definition, attributed to the medium term component of the cycle. The other peaks occur in business cycle frequencies, namely for the frequency of 8 years, 6 – 7 years and 5 years.

The periodogram of unemployment rate, depicted in right panel of Figure 6 is quite similar to the one calculated for the deviations of output from linear trend. The peak with the highest variance occurs in frequencies between 30 and 40 years, but a substantial part of the overall variance of the unemployment rate occurs also in frequencies from 10 to 30 years, suggesting that the portion of the overall variance, that can be attributed to the medium frequency component of the cycle should be substantial. In business cycle frequencies, peaks in periodogram occur at frequencies 6 – 7 years, 5.5 years and about 4 years, with some additional variance distributed over higher frequencies.

Table 2 shows the last piece of evidence on the importance of medium frequency component of the fluctuations. It shows the results of periodogram integration over specified frequency ranges<sup>37</sup>. Table 2 presents the share of

<sup>37</sup>The calculation takes into account a proper correction imposed of the first and the last frequency, described e.g. in chapter 6 of Hamilton (1994). As the sample length of

variance of higher, medium and low (trend) frequency components of the data in the overall variance of the time series<sup>38</sup>. The first observation worth emphasizing is the important role played by medium frequency fluctuations in the overall variation of time series - an observation coherent with the evidence presented earlier. Additionally (and also coherently with the previous evidence), the volatility of medium frequency component of the data seems to be higher than the volatility of higher (business cycle) frequencies alone. What is more important from the perspective of this study (the unified treatment of higher and medium frequencies component), in most cases the components of raw data that we are going to study cover over half of the overall fluctuations. The exceptions are consumption, wages and labor share, where the volatility of low frequency components (trends) seems to dominate the overall volatility.

Summarizing the results presented in this chapter, one may conclude that medium term fluctuations are relatively substantial in the US data (with respect to their variance). Medium term components of the cycle are often more variable than the usually analyzed business cycle fluctuations alone. Additionally, the pattern of comovement of various variables with output at medium frequencies is comparable to the pattern observed for higher frequencies which suggest that both these kinds of fluctuations could be analyzed jointly and could be a result of the same driving forces.

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the data is far from infinity, the integration performed here is only approximate. So, one should treat the evidence presented in Table 2 with caution, but nevertheless it shows quite convincingly the data properties with regard to the decomposition of the variance of the cycle into medium term and higher term frequencies.

<sup>38</sup>As indicated before, the raw data were demeaned and in the case of nonstationary data - also detrended.



## Chapter 3

# Model specification

This chapter presents and discusses a theoretical model that is constructed in order to explain the medium term cycle present in the data, as documented in chapter 2. First section gives a short and non-mathematical overview of the model structure and dependencies. Next sections concentrate on a detailed model description, including: household behavior and decisions, labor market developments, decisions made by producers of different goods (like factor demands and wage setting framework) or R&D producers and the behavior of the government. Next, we define the symmetric equilibrium in this economy and discuss aggregate model equations, together with steady state considerations.

### 3.1 Overview

Consider a discrete-time economy consisting of four type of agents, interacting with each other: households, final good producers, intermediate good producers and innovation (R&D) producers. There are markets for final goods, intermediate goods, capital and labor.

Households consume final goods or invest them in physical capital. They also invest a portion of their income in the R&D sector, yielding a market interest rate. They pay lump-sum taxes. Households supply inelastically labor to intermediate goods producers and negotiate wages with them. They also rent their stock of capital to intermediate goods producers and receive a rental price on it. When a given member of the household becomes unemployed, he or she receives an unemployment benefit.

Final good producers buy intermediate goods from intermediate goods producers and combine them into a final good, using a constant elasticity of substitution technology. The number of intermediate goods produced changes over time and depends on the level of activity in the R&D sector. Perfect competition in the sector of final good producers drives down their profits to zero.

Intermediate good producers manufacture a variable number of inter-

mediate goods. A producer of a given intermediate good buys a blueprint (giving him the monopoly right to produce this good) from an R&D firm that invented this good in exchange for all future profits it generates. An intermediate good firm faces a downward sloping demand for its output from a final good firm. The intermediate good producer combines labor and capital (rented from the household) using a constant returns to scale production function. When changing its employment, a given firm needs to post a vacancy first (and pay an associated cost) and faces a given probability of filling it. Vacancies are filled by unemployed members of households who are searching for available job offers. The matching of unemployed with vacancies is summarized by a constant returns to scale matching function. In a given period a constant, but random and exogenously given part of the existing employment relationships dissolves, which diminishes the stock of employed people in a given firm. When matched, a worker starts producing next period. At the end of the period, both intermediate firms and workers negotiate over wages and split the surplus from the existing match using a Nash sharing rule. Intermediate firms also set the price of their products as a constant markup over marginal costs.

R&D producers borrow funds from households (at a market interest rate) and transform them into new intermediate goods. Specifically, each innovator conducts R&D by using the final good as input into developing new products. The intensity of transformation into new intermediate goods depends on aggregate conditions and is taken as given by an innovator. The technology of production of new intermediate goods assures that there is a positive spillover of the aggregate stock of innovations, as well as the congestion effect, raising the costs of developing new products as the aggregate level of R&D expenditures increases. Additionally, in each period, a constant fraction of the stock of innovations becomes obsolete and is no longer demanded and produced.

Next sections describe our model economy in a more detailed fashion.

## 3.2 Households

Households maximize lifetime utility, derived from consumption of final goods, subject to intertemporal budget constraint<sup>39</sup>. The instantaneous utility of a household is given by  $u(C_t) = \frac{C_t^{1-\varsigma}-1}{1-\varsigma}$ , where  $C_t$  is consumption<sup>40</sup>. We ab-

<sup>39</sup>For a comprehensive discussion on different theories of consumption, their implications and empirical testing, see Bukowski (2005) or Kula (2006), the latter in the context of households' retirement decisions. Additionally, Liberda (1996) discusses the implications of different consumption theories for the behavior of savings.

<sup>40</sup>We want to keep the structure of the model as simple as possible, so when specifying the details of the economy, we chose to abstract from many additional small enhancement of the model. This is the reason why we do not apply e.g. the habit (see e.g. Abel 1990, Boldrin, Christiano, and Fisher 2001) in the specification of the utility function, although the literature suggest that habit persistence brings the model closer to the data



stract from the decision on labor supply, so households supply a fixed amount of labor, normalized to one. We also defer from the intensity of labor utilization (average working time), so the model describes labor adjustments on extensive margin only. The inclusion of the intensive margin in labor adjustments would change the very short run properties of the model (see e.g. Trigari 2006). With the model focused on explaining the medium run characteristics of the data, the incorporation of hours in addition to the number of workers seems unnecessary. It also allows us to concentrate on labor demand issues and to analyze a relatively simple model.

Households derive their income from renting labor to intermediate firms,  $W_t N_t$ , unemployment benefits,  $b_t U_t$ , interest on renting capital to intermediate firms  $r_t K_t$  and total profits<sup>41</sup>  $\Pi_t$ , collected from: 1) final goods producers, 2) intermediate goods producers, 3) R&D producers. Households also borrow  $L_t$  to firms in the innovative sector (invest in development of new products) and these loans pay a gross interest rate  $R_t$ . Households spend their income on consumption, investments in physical capital  $I_t$  and pay lump-sum taxes  $T_t$  levied by the government. Households also face the law of motion of the physical capital stock, given by  $K_{t+1} = (1 - \delta)K_t + I_t$ , where  $\delta$  is the depreciation rate of capital.

As we normalize the size of the labor force to one, all relevant variables are expressed in per labor force terms. This normalization is coherent with our choice to abstract from households decisions on joining or exiting the labor force.

We concentrate on the problem of a representative household<sup>42</sup>, which is to choose the path of consumption  $C_t$ , investments  $I_t$  and loans to innovative firms  $L_t$  that solve the following problem:

$$\max_{C_t, S_t, I_t, K_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\varsigma} - 1}{1-\varsigma} \right) \quad (1)$$

subject to:

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and solves some problems with asset prices. For further references on this issue, see the discussion in Francis and Ramey (2005) or Nason and Kano (2004). The same line of reasoning explains our abstracting from investments adjustments costs in the capital accumulation (for further discussion on this issue, see e.g. Christiano, Eichenbaum, and Evans 2005, Smets and Wouters 2003).

<sup>41</sup>In equilibrium, total profits will be equal to zero. Perfect competition in both final goods sector and R&D sector assures that the profits of these sectors are zero. Intermediate goods producers operate in monopolistic competition environment and incur fixed costs of production (the purchase of blueprint that gives an intermediate firms access to technology of producing intermediate goods), which assures zero-profits in equilibrium.

<sup>42</sup>The introduction of unemployment into the equilibrium generates heterogeneity in the model in the sense that each individuals' labor income depends on his or her labor status. Thus an individual's allocation is a function of his or her entire employment history. As the distributional issues of households wealth are not the purpose of this study, we follow Merz (1995) and Andolfatto (1996) in assuming a large number of members of families who perfectly insure each other against fluctuations in income. This assumption enables us to focus on the allocation of a representative agent.

$$C_t + I_t + L_t + \Pi_t + T_t = w_t N_t + b_t U_t + r_t K_t + R_t L_{t-1} \quad (2)$$

$$K_{t+1} = (1 - \delta)K_t + I_t \quad (3)$$

When we denote  $\lambda_t$  and  $\omega_t$  as Lagrange multipliers on the budget constraint and on the capital accumulation equation respectively, the first order condition (FOC) for consumption can be expressed as follows:

$$\lambda_t = u'(C_t) = C_t^{-\varsigma} \quad (4)$$

The FOC with respect to investments equates both Lagrange multipliers (recall that capital and consumption goods are both final goods, so their price in terms of utility need to be the same). Then, the time path of capital satisfies the following equation:

$$\lambda_t = \beta E_t \lambda_{t+1} (1 + r_{t+1} - \delta) \quad (5)$$

Borrowing to the innovative sector is governed by the following equation:

$$\lambda_t = \beta E_t \lambda_{t+1} R_{t+1} \quad (6)$$

Combining equations (5) and (6) yields the no arbitrage condition between investing in physical capital and in R&D activity, stating that the gross returns on both types of investments must be equal in equilibrium:

$$E_t \lambda_{t+1} R_{t+1} = E_t \lambda_{t+1} (1 + r_{t+1} - \delta) \quad (7)$$

Transforming equation (6) and using (4) gives:

$$1 = \beta E_t \frac{u'(C_{t+1}) R_{t+1}}{u'(C_t)} \quad (8)$$

which is the standard Euler equation. It shows how households evaluate a one dollar claim to a consumption good (in utility terms) today and tomorrow.

### 3.3 Labor market

Individual workers are employed by intermediate good firms. When a given intermediate good firm chooses to increase its employment, it posts vacancies. Both workers and vacancies are indexed by  $i$  - the identifier of a given intermediate firm. As there are  $A_t$  firms operating in the economy in period  $t$ , the total number of vacancies and people employed is the sum of individual quantities and is given by  $V_t = \int_0^{A_t} v_t(i) di$  and  $N_t = \int_0^{A_t} n_t(i) di$ .

The number of new matches in the economy  $M_t$  is a constant returns

to scale (specifically, Cobb-Douglas<sup>43</sup>) function of aggregate unemployment (denoted by  $U_t$ ) and vacancies  $V_t$ :

$$M_t = m(U_t, V_t) = \sigma_m U_t^\sigma V_t^{1-\sigma} \quad (9)$$

The probability  $q_t$  that a given firm fills an open vacancy in period  $t$  is given by:

$$q_t = \frac{M_t}{V_t} \quad (10)$$

so it is given by the number of successful matches (filled vacancies) per total number of vacancies posted. On the other side of the labor market, we may specify the probability  $s_t$  that any worker looking for a job is matched with an open vacancy at time  $t$ . The vacancy filling probability<sup>44</sup> is given by the number of new matches (workers who find the job) per total number of workers looking for a job:

$$s_t = \frac{M_t}{U_t} \quad (11)$$

Both these probabilities are known to the individual agents in the economy (workers and intermediate firms) and, due to the large number of agents, are exogenous to them (when making his or her decisions, each individual does not take into account his or her own impact on the probability).

When one introduces the labor tightness index<sup>45</sup>

$$\theta_t = \frac{V_t}{U_t} \quad (12)$$

then the probabilities of filling a vacancy and finding a job can be expressed as:  $q_t = q(\theta_t) = \frac{m(U_t, V_t)}{V_t} = m\left(\frac{U_t}{V_t}, 1\right) = \sigma_m \theta^\sigma$  and  $s_t = s(\theta_t) = \frac{m(U_t, V_t)}{U_t} = m\left(1, \frac{V_t}{U_t}\right) = \sigma_m \theta^{\sigma+1}$ , respectively.

Each intermediate good firm exogenously separates a fraction  $\rho$  of its workforce<sup>46</sup>, so the law of motion for the total number of employed persons

<sup>43</sup>The assumption of constant returns to scale matching function is justified on the basis of the literature survey performed by Petrongolo and Pissarides (2001), where they conclude that most empirical estimates of the matching functions cannot reject the null hypothesis of constant returns. Additionally, there are numerous examples in the literature using the Cobb-Douglas specification. For the reference, see e.g. Pissarides (1985) or Shimer (2005).

<sup>44</sup>The proper definition of both probabilities,  $s_t$  and  $q_t$  should be  $\min\left\{\frac{M_t}{V_t}, 1\right\}$  and  $\min\left\{\frac{M_t}{U_t}, 1\right\}$ , respectively. The calibration strategy applied to the model assures that both measures are proper probabilities, i.e. they are no greater than 1.

<sup>45</sup>The tightness of the labor market rises either when there are more vacancies competing for a given number of unemployed or when there are less persons seeking for a given number of vacancies. In other words, a tight labor market means that it is hard for an employer to find an employee.

<sup>46</sup>According to Hall (2005c) job destruction is relatively constant over the business cycle, so without loss of plausibility, one can assume a constant job destruction rate. Also, Shimer (2005) shows the evidence on relatively low variation in separation rate during the business cycle. The assumption of a constant separation rate is commonly used in the

in any period is given by the number of employed workers from the previous period, that have not separated plus an inflow of newly matched workers from the previous period<sup>47</sup>:

$$N_t = (1 - \rho)N_{t-1} + M_{t-1} \quad (13)$$

It follows, that in the analyzed framework the fluctuations in the number of people employed, and thus the number of unemployed persons (as the labor force size is constant and there is no endogenous labor supply), is due to cyclical variation in hiring of new workers, and not due to separations.

The labor force is normalized to one in each period (in the steady state of the model, the number of unemployed and workers are assumed to be stationary). Then, the number of unemployed at the beginning of a given period  $1 - N_t$ . This is different from the searching workers in period  $t$ ,  $U_t$ , as a measure of  $\rho N_t$  workers discontinue their match and search for new jobs in the same period:

$$U_t = 1 - N_t + \rho N_t = 1 - (1 - \rho) N_t \quad (14)$$

### 3.4 Final good producers

There are  $A_t$  different varieties of intermediate goods in each period  $t$ , indexed by  $i$ ,  $y_t(i)$ . These are combined by final good producers into a single final good  $Y_t$  according to a CES technology of the form:

$$Y_t = \left( \int_0^{A_t} y_t(i)^{\frac{1}{\mu}} di \right)^{\mu} \quad (15)$$

Final good producers operate in a perfectly competitive environment. Each of them maximizes profits, subject to production technology (15) and faces exogenous unit costs of production (intermediate goods prices  $p_t(i)$ ), sale price (the final good price  $P_t$ ) and the number of intermediate goods available for production purposes<sup>48</sup>  $A_t$ . Thus, the problem of a representative final good producer can be summarized as follows:

$$\max_{y_t(i)} P_t Y_t - \int_0^{A_t} p_t(i) y_t(i) di \quad (16)$$

subject to (15).

recent general equilibrium models with search on the labor market. For reference, see e.g. Christoffel and Linzert (2005), Gertler and Trigari (2006) or Trigari (2006). The models with endogenous separation rate include, among others: Trigari (2004), Krause and Lubik (2007).

<sup>47</sup>The convention used here is that it takes one period for the newly matched person to start working in a given firm. This assumption is widely used in the search literature (see e.g. Pissarides 2000) and can be justified by the fact that it takes some time for a newly employed worker to become fully productive in a new firm, due to e.g. a training period.

<sup>48</sup>Determination of the number of intermediate goods will be described in the section 3.6.

The demand of each final goods producer for the output of  $i$ -th intermediate good producer is given by the solution to the above problem, which takes the form:

$$y_t(i) = \left( \frac{p_t(i)}{P_t} \right)^{\frac{-\mu}{\mu-1}} Y_t \quad (17)$$

As there is an infinite number of firms in the sector, and entry and exit of new firms into the production of final goods drives down any potential profits to zero, the aggregate price level  $P_t$  assures that each firm earns no profits:

$$P_t = \left( \int_0^{A_t} p_t(i)^{\frac{1}{1-\mu}} di \right)^{1-\mu} \quad (18)$$

So the aggregate price index is a weighted average of the prices of intermediate goods and depends on the number of products available.

## 3.5 Intermediate goods producers

### 3.5.1 Factor demands and intermediate good prices

Each intermediate good producer  $i$  (as in each period  $t$  there are  $A_t$  intermediate goods available for production, so  $i \in [0, A_t]$ ) rents labor  $n_t(i)$  and capital  $k_t(i)$  from the households and combines it into an intermediate good<sup>49</sup> according to the constant returns to scale production function. We specify the production technology in a convenient and widely used Cobb-Douglas form:

$$y_t(i) = f(Z_t, k_t(i), n_t(i)) = Z_t k_t(i)^\alpha n_t(i)^{1-\alpha} \quad (19)$$

where  $Z_t$  represents the stationary technological innovation, common to all intermediate good producers. As the intermediate good sector is imperfectly competitive (we assume monopolistic competition in this sector), producers set their prices as a constant markup over their marginal costs.

The law of motion of employment at a producer  $i$  is the counterpart of the aggregate employment equation (13) and is given by the formula:

$$n_t(i) = (1 - \rho)n_{t-1}(i) + q_{t-1}v_{t-1}(i) \quad (20)$$

where the number of new matches of the  $i$ -th producer is expressed as  $m_t(i) = q_t v_t(i)$ , reflecting that information set of a given producer includes the vacancy filling probability it faces on the labor market.

Let  $w_t(i)$  be the wage rate, and  $r_t(i)$  - the rental price of capital (both

<sup>49</sup>Each producer of intermediate good  $i$  purchases, in exchange for his profits, a blueprint from the innovative firm that invented  $i$ -th intermediate good, that gives him the access to production technology of this good.

expressed in terms of final goods) faced by firm  $i$ . As all firms operating in the economy are owned ultimately by households, firms discount the future streams of funds with the discount factor, consistent with household's preferences. So the present (time  $t$ ) value of a claim worth one dollar in period  $t + s$  is given by  $\Lambda_{t,t+s} = \beta^s E_t \frac{u'(C_{t+s})}{u'(C_t)}$ . The firm treats the discount factor as given.

Adjusting employment is subject to real costs, being a function of vacancies and wages  $c(v_t(i), w_t(i))$ . The adjustment costs are increasing in the number of posted vacancies  $v_t(i)$  and in the current wage rate<sup>50</sup>  $w_t(i)$ , so  $c'_v > 0$  and  $c'_w > 0$ .

The value of the  $i$ -th firm real profits  $\Pi_t(i)$  (expressed in terms of a final good) discounted for period 0 is given by:

$$\Pi_0(i) = \sum_{t=0}^{\infty} \Lambda_{0,t} \left[ \begin{array}{l} \frac{p_t(i)}{P_t} y_t(i) - w_t(i) n_t(i) \\ -c(v_t(i), w_t(i)) - r_t(i) k_t(i) \end{array} \right] \quad (21)$$

In each period, an intermediate firm  $i$  decides on employment  $n_t(i)$ , the number of vacancies  $v_t(i)$ , its demand for capital  $k_t(i)$  and sets the prices of its products  $p_t(i)$  to maximize the present discounted value of its real profits (21), subject to the employment evolution equation (20), the technology of production (19) and the demand it faces (17) from the final goods producers. The Lagrange multiplier  $J_t(i)$  on the employment constraint gives the current period asset value (expressed in terms of final goods) of a worker to the firm. The multiplier on the production function,  $mc_t(i)$ , gives the contribution of an additional unit of output to the firm's real revenues and hence (taking into consideration that in equilibrium marginal costs are equal to marginal revenues) the firm's real marginal cost (expressed in terms of final goods).

The problem of the  $i$ -th producer can be stated as follows:

<sup>50</sup>The commonly used assumption concerning labor adjustments costs is that they are a linear function of vacancies only, so  $c'_v = 0$  (see e.g. Mortensen and Pissarides 1994, Shimer 2005, Krause, Lopez-Salido, and Lubik 2007). Part of the recent literature (for the reference, see e.g. Christoffel and Linzert 2005, Trigari 2006) also assume that adjustments costs are function of only the number of posted vacancies, but additionally expresses them in utility terms, so the costs in terms of final goods are given by the function:  $c(v_t(i)) = \frac{\kappa v_t(i)}{\lambda_t}$  in our notation. On the other hand, Gertler and Trigari (2006) assumes that  $c(v_t(i)) = \frac{\kappa(q_t v_t(i))^2}{n_t(i)}$ , so hiring costs are a quadratic function of the hiring rate  $\frac{q_t v_t(i)}{n_t(i)}$ . All these functional forms are valid for stationary models. In order for our model to have a steady state consistent with a balanced growth path, hiring costs need to depend on wages (or, when assuming that employment, unemployment, vacancies are stationary, hiring costs could also depend on output) in order to assure that the structure of GDP and of intermediate producers' profits are constant in the steady state.

$$\max_{\substack{n_t(i), v_t(i), \\ k_t(i), p_t(i)}} \sum_{t=0}^{\infty} \Lambda_{0,t} \left[ \begin{array}{l} \frac{p_t(i)}{P_t} \left( \frac{p_t(i)}{P_t} \right)^{\frac{-\mu}{\mu-1}} Y_t - w_t(i) n_t(i) \\ - c(v_t(i), w_t(i)) - r_t(i) k_t(i) \\ + mc_t(i) \left( z_t k_t(i)^\alpha n_t(i)^{1-\alpha} - \left( \frac{p_t(i)}{P_t} \right)^{\frac{-\mu}{\mu-1}} Y_t \right) \\ + J_t(i) ((1-\rho)n_{t-1}(i) + q_{t-1}v_{t-1}(i) - n_t(i)) \end{array} \right] \quad (22)$$

The first order condition for employment yields the following equation (for the reference, see e.g. Krause and Lubik 2007):

$$J_t(i) = mc_t(i) f_{n,t}(i) - w_t(i) + E_t \Lambda_{t,t+1} (1-\rho) J_{t+1}(i) \quad (23)$$

where  $f_{n,t}(i) = \frac{\partial y_t(i)}{\partial n_t(i)} = (1-\alpha) \frac{y_t(i)}{n_t(i)}$  is the marginal product of labor. It states that the asset value of additional worker  $J_t(i)$  is given by his or her contribution to the producer's profits  $mc_t(i) f_{n,t}(i)$ , corrected by the cost of employing him or her - the wage rate  $w_t(i)$  - with the additional continuation value. The continuation value equals to the asset value of a worker in the next period, discounted by the stochastic discount factor  $\Lambda_{t,t+1}$  and conditional on the worker not being separated from the producer (a worker survives in a firm with a probability  $1 - q_t$ ). Equation (23) utilizes the fact that the asset value of an unfilled vacancy is equal to 0.

Optimization over the number of vacancies gives the following formula:

$$\frac{c'_v(v_t(i), w_t(i))}{q_t} = E_t \Lambda_{t,t+1} J_{t+1}(i) \quad (24)$$

It states that the marginal cost of posting a vacancy (as the vacancy costs are incurred in each period, when the vacancy is opened and not filled, the total costs are multiplied by the expected time to fill a vacancy  $\frac{1}{q_t}$ ) should be equal to the expected marginal benefit of an additional worker. The alternative interpretation of this condition is that firms post vacancies up to the point where the asset value of an open vacancy is zero (see the corresponding equation (16) on page 9 in Trigari 2006).

Combining equations (23) and (24) yields the job creation condition:

$$\frac{c'_v(v_t(i), w_t(i))}{q_t} = E_t \Lambda_{t,t+1} \left[ \begin{array}{l} mc_{t+1}(i) f_{n,t+1}(i) - w_{t+1}(i) \\ + (1-\rho) \frac{c'_v(v_{t+1}(i), w_{t+1}(i))}{q_{t+1}} \end{array} \right] \quad (25)$$

as a first-order difference equation in  $q_t$ .

The first order condition for capital yields the standard demand for capital equation, given by:

$$r_t(i) = mc_t(i) f_{k,t}(i) \quad (26)$$

where  $f_{k,t}(i) = \frac{\partial y_t(i)}{\partial k_t(i)} = \alpha \frac{y_t(i)}{k_t(i)}$  is the marginal product of capital. It states that the marginal cost of renting capital from households  $r_t(i)$  should be equal to

the marginal benefit, given by the marginal product of capital, multiplied by the marginal contribution of an additional unit of production to the firm's profits (marginal revenue, which in equilibrium is equal to  $mc_t(i)$ ).

Optimization over prices gives the following formula:

$$\left(\frac{-1}{\mu-1}\right) \left(\frac{p_t(i)}{P_t}\right)^{\frac{-1}{\mu-1}-1} \frac{Y_t}{P_t} - mc_t(i) \frac{-\mu}{\mu-1} \left(\frac{p_t(i)}{P_t}\right)^{\frac{-\mu}{\mu-1}-1} \frac{Y_t}{P_t} = 0$$

which, after rearranging:

$$\frac{y_t(i)}{P_t(\mu-1)} [p_t(i) - \mu mc_t(i)P_t] = 0$$

collapses to a standard markup equation, stating that intermediate goods producers set their prices  $p_t(i)$  by charging a constant markup  $\mu$  over their nominal marginal costs (expressed in terms of a final good):

$$p_t(i) = \mu \cdot mc_t(i)P_t \quad (27)$$

In order to infer the marginal costs from the previous equations, one could use the first order condition for employment (23), capital (3) and apply the Euler theorem for homogeneous functions<sup>51</sup>, which after rearranging yields:

$$mc_t(i) = \frac{r_t(i)k_t(i) + w_t(i)n_t(i)}{y_t(i)} + \frac{n_t(i) \left[ J_t(i) - (1-\rho) \frac{c'_t(v_t(i), w_t(i))}{q_t} \right]}{y_t(i)} \quad (28)$$

The changes in marginal costs arise from two sources. The first one is standard and is due to the average cost of labor and capital. The second one arises from the presence of labor market frictions and is a function of a difference between the value of a worker for the firm and the cost of posting a vacancy, corrected for labor productivity.

### 3.5.2 Wage setting

This section describes determination of wages in the economy. We apply a standard approach in the search literature and introduce a Nash bargain between firms and workers over wages. The joint surplus of a successful match is given by  $J_t(i) + W_t(i) - U_t$ , where  $J_t(i)$  is the the asset value of a job for the firm (recall that the asset value of an open vacancy is zero),  $W_t(i)$  is the worker's asset value of being employed and  $U_t$  is the worker's outside option - the asset value of being unemployed. Notice that due to zero asset value of an open vacancy, there is no outside option for the firm in the bargaining considered here.

<sup>51</sup>  $f_{k,t}(i)k_t(i) + f_{n,t}(i)n_t(i) = y_t(i)$



The value of a job for a firm (in terms of a final good) is given by equation (23), rewritten here for convenience:

$$J_t(i) = mc_t(i)f_{n,t}(i) - w_t(i) + E_t\Lambda_{t,t+1}(1 - \rho)J_{t+1}(i) \quad (29)$$

From a worker's perspective, one can define the value of a job  $W_t(i)$  as a combination of compensation for work and the continuation value:

$$W_t(i) = w_t(i) + E_t\Lambda_{t,t+1} [(1 - \rho)W_{t+1}(i) + \rho U_{t+1}] \quad (30)$$

The last term reflects the expected future state of a worker, which is a combination of a possibility of staying in the firm (with asset value  $W_{t+1}(i)$  and probability  $1 - \rho$ ) and a possibility of being fired (with asset value  $U_{t+1}$  and probability  $\rho$ ).

When a person is unemployed, the asset value of being in this state  $U_t$  is given by unemployment benefit  $b_t$  an unemployed person receives and a continuation value (recall that there is no disutility of work in household's preferences):

$$U_t = b_t + E_t\Lambda_{t,t+1} [s_t W_{t+1}(i) + (1 - s_t)U_{t+1}] \quad (31)$$

The last term of equation (31) reflects the possibility of being employed (and receiving the asset value of  $W_{t+1}(i)$  with probability  $s_t$ ) and the possibility of staying unemployed (with asset value of  $U_{t+1}$  and probability  $1 - s_t$ ).

Firms and workers negotiate over wages to maximize the joint surplus from a match and divide the surplus according to their relative bargaining power, so they maximize the so-called Nash product:

$$\max_{w_t(i)} [J_t(i)^{1-\eta} (W_t(i) - U_t)^\eta]$$

where  $\eta$  is the relative bargaining power of workers. The micro-foundations of this two parties cooperative game can be found in a seminal paper written by John Nash (see Nash 1953). The solution to the Nash bargaining problem is given by the well-known formula:

$$(1 - \eta) (W_t(i) - U_t) = \eta J_t(i) \quad (32)$$

Substituting the expressions for  $W_t(i)$ ,  $U_t$  and  $J_t(i)$  yields:

$$\begin{aligned} w_t(i) - b_t + E_t\Lambda_{t,t+1}(1 - \rho - s_t) (W_{t+1}(i) - U_{t+1}) &= \\ = \frac{\eta}{1 - \eta} [mc_t(i)f_{n,t}(i) - w_t(i) + E_t\Lambda_{t,t+1}(1 - \rho)J_{t+1}(i)] \end{aligned}$$

$$w_t(i) = (1 - \eta) b_t + \eta [mc_t(i)f_{n,t}(i) + s_t E_t\Lambda_{t,t+1} J_{t+1}(i)]$$

Using (24) and the definition of  $\theta_t$  (see equation (12)) yields the following formula for negotiated wages:

$$w_t(i) = (1 - \eta) b_t + \eta [mc_t(i) f_{n,t}(i) + \theta_t c'_v(v_t(i), w_t(i))] \quad (33)$$

Thus, the negotiated wage is increasing in unemployment benefits, marginal costs, labor tightness index and the cost of posting a vacancy. Relative bargaining power changes the weight of factors attributable to workers' or employers' gains from a successful match.

### 3.5.3 Vacancy posting costs

The vacancy posting costs are specified as:  $c(v_t(i), w_t(i)) = \kappa w_t(i) v_t(i)$ . In order to post a vacancy, an intermediate good producer has to pay a unit cost, proportional to the negotiated wage rate<sup>52</sup>,  $\kappa w_t(i)$ . It follows (given that in the symmetric equilibrium the relationship  $V_t = \int_0^{A_t} v_t(i) di = A_t v_t$  holds) that the aggregate hiring costs are given by:

$$C(V_t, w_t) = \int_0^{A_t} c(v_t(i), w_t(i)) di = \kappa A_t w_t v_t = \kappa w_t V_t$$

## 3.6 R&D sector

The formulation of the R&D sector adopted here follows the important contribution into the endogenous growth theory developed by Romer (1990), but also takes into account the Jones's critique of Romer's model (see Jones 1995). The details of the model follow the application of Comin and Gertler (2006), but since our intention was to keep our model relatively simple, we decided not to treat invention and adoption as separate processes<sup>53</sup>. The formulation assumes, as in Romer (1990), that technological change arises, in large part, because of intentional actions of people who respond to market incentives.

Let us assume that there is an infinite number of innovative firms operating in the R&D sector. Each innovative firm  $p$  borrows  $L_t(p)$  of final goods from the households and uses it to invent new intermediate goods, of the measure  $A_{t+1} - A_t$ . After inventing a new good, an innovative firm sells a blueprint to an intermediate good producer in exchange for expected profits from production of this new good. Each innovative firm must pay

<sup>52</sup> Most models describing labor market are stationary and vacancy posting costs are usually constant. In the model described here, vacancy posting costs need to be growing in line with wages or product in order for the steady state (consistent with a balanced growth path) to exist.

<sup>53</sup> The separation of adoption process, incorporated in Comin and Gertler (2006), aims at accounting for delays between the invention of new goods and their ultimate incorporation into the productive use. This argument is supported by the evidence on slow diffusion of new technologies, discussed e.g. in Rotemberg (2003) or Comin and Hobijn (2004). We abstract from this extension and one may treat the invention process that we apply here as a reduced form, incorporating both of these stages.

back  $R_{t+1}L_t(p)$  to the household next period. Innovative firms are owned by households, so when evaluating future flows of funds, they use the pricing kernel that is consistent with households preferences  $\Lambda_{t,t+s} = \beta^s E_t \frac{u'(C_{t+s})}{u'(C_t)}$ . There is perfect competition among firms in the R&D sector, so there are no additional profits from innovation activity.

Let  $\pi_t$  be a measure of profits of an individual firm in the intermediate sector in the symmetric equilibrium (due to symmetry, each intermediate firm will earn the same profits in a given period), expressed in terms of final goods. So  $\pi_t$  is given by (see equation (21)):

$$\pi_t = \frac{p_t}{P_t} y_t - w_t n_t - c(v_t, w_t) - r_t k_t$$

The value of a unit of a new intermediate good is equal to present discounted value (adjusted by the possibility of the product to become obsolete) of all future profits that the production of this new good is expected to generate. Thus, it is given by the following formula:

$$v_t^I(p) = \pi_t + (1 - \phi) E_t \Lambda_{t,t+1} v_{t+1}^I(p) \quad (34)$$

where  $\phi$  is the rate of product obsolescence.

The production technology of a new intermediate good is given by:

$$A_{t+1}(p) - (1 - \phi) A_t(p) = \varphi_t L_t(p) \quad (35)$$

where the intensity of transforming the borrowed funds into new intermediate goods,  $\varphi_t$  is exogenous to the innovative firm and given by the formula:

$$\varphi_t = \chi \frac{A_t}{K_t} \left( \frac{L_t}{K_t} \right)^{\psi-1} \quad (36)$$

This formulation of the intensity of innovative activity follows the convention used by Comin and Gertler (2006) and embodies three properties. First, it includes a positive spillover effect of the current stock of innovations on the creation of new products -  $\varphi_t$  increases with  $A_t$ . Second, the formulation assures that there is an aggregate congestion effect, introduced via the factor  $\left( \frac{L_t}{K_t} \right)^{\psi-1}$ . Third, the scaling factor  $K_t$  assures that in equilibrium the growth rate of new intermediate goods, and so the growth rate of the economy, is stationary.

The representative R&D producer maximizes his profits subject to the technology of production (35), so his problem can be stated as follows:

$$\begin{aligned} \max_{A_{t+1}(p), L_t(p)} & \left[ E_t \Lambda_{t,t+1} (1 - \phi) v_{t+1}^I(p) [A_{t+1}(p) - (1 - \phi) A_t(p)] - L_t(p) \right] \\ & - \xi_t [A_{t+1}(p) - (1 - \phi) A_t(p) - \varphi_t L_t(p)] \end{aligned} \quad (37)$$

where  $\xi_t$  is the Lagrange multiplier associated with the technology constraint. The solution to the problem can be characterized by the following FOCs:

$$E_t \Lambda_{t,t+1} (1 - \phi) v_{t+1}^I(p) = \xi_t$$

$$\xi_t \varphi_t = 1$$

which can be combined into:

$$(1 - \phi) E_t \Lambda_{t,t+1} v_{t+1}^I(p) = \frac{1}{\varphi_t} \quad (38)$$

Free entry into the innovative sector ensures that in equilibrium profits are driven down to zero, so  $\pi_t^I = 0$ , which gives:

$$E_t \Lambda_{t,t+1} (1 - \phi) v_{t+1}^I(p) [A_{t+1}(p) - (1 - \phi) A_t(p)] = L_t(p) \quad (39)$$

### 3.7 Government

Government in this economy levies lump sum taxes on households and uses them to finance unemployment benefits. We assume that there is no possibility to accumulate debt by the government, so the government budget constraint is given by:

$$b_t U_t = T_t \quad (40)$$

The government conducts social policy by setting the unemployment benefit  $b_t$ . We assume that the government policy is passive in the sense that unemployment benefits grow at their steady state rate of growth (equal to the steady state growth of wages):

$$b_{t+1} = \gamma_b b_t \quad (41)$$

where  $\gamma_b = \frac{\bar{b}_t}{\bar{b}_{t-1}}$  and bars over variables denote steady state values.

### 3.8 Other issues

#### 3.8.1 Resource constraint

The resource constraint in the final goods market requires that the production of final goods must be used either for consumption or for investment in physical capital, investment in R&D or for adjusting employment by intermediate good firms:

$$Y_t = C_t + I_t + \int_0^1 L_t(p) dp + \int_0^{A_t} c(v_t(i), w_t(i)) di \quad (42)$$

### 3.8.2 Normalization

The average price level was chosen as a numeraire in this economy, so all prices are expressed in terms of aggregate final good prices. As the model describes real values and relative prices, we normalize the index of final goods price  $P_t$  at unity, so:

$$P_t = 1 \quad (43)$$

### 3.8.3 Technological progress

The economy is hit by a stationary technological innovation, persistent over time. The law of motion for technological progress  $Z_t$  is given by:

$$Z_t = \bar{Z} + \rho_Z (Z_{t-1} - \bar{Z}) + \zeta_t$$

where  $\zeta_t$  is a zero mean technological innovation and  $\rho_Z$  measures the auto-correlation of technological progress. When expressed in terms of deviations from stationary equilibrium  $\hat{Z}_t = \frac{Z_t}{\bar{Z}}$ , it can be reformulated as:

$$\hat{Z}_{t+1} = (1 - \rho_Z) + \rho_Z \hat{Z}_t + \epsilon_{t+1} \quad (44)$$

where technological innovation  $\epsilon_t = \frac{1}{\bar{Z}} \zeta_t$  is an iid process with zero mean and standard deviation  $\sigma_Z$ .

## 3.9 Definition of equilibrium

A symmetric equilibrium for this economy is:

allocation for households:

$$\{C_t, L_t, I_t, K_{t+1}\},$$

allocation in the final goods sector:

$$\{Y_t, y_t(i)\},$$

allocation and prices in the intermediate goods sector:

$$\{k_t(i), n_t(i), v_t(i), p_t(i), r_t(i), w_t(i), mc_t(i), \pi_t(i)\},$$

allocation and values in the R&D sector:

$$\{A_{t+1}(p), L_t(p), v_t^I(p)\},$$

average prices:

$$\{P_t, R_t, r_t, w_t\},$$

aggregate labor market variables:

$$\{M_t, U_t, V_t, N_t, q_t, s_t, \theta_t\},$$

and government allocation:

$$\{b_t, T_t\},$$

that satisfy the following conditions:

- the consumer allocation solves the household's problem (1),

- aggregate labor market variables satisfy equations: 9-14,
- final goods producers solve the problem (16),
- intermediate goods producers solve the problem (22),
- wages are negotiated by intermediate firms and households according to equation (33),
- R&D firms solve the problem (37),
- government policy is pursued according to (41),
- average and individual factor prices coincide:  $r_t(i) = r_t$  and  $w_t(i) = w_t$ ,
- labor market clears:  $V_t = \int_0^{A_t} v_t(i) di$  and  $N_t = \int_0^{A_t} n_t(i) di$ ,
- market for physical capital clears:  $K_t = \int_0^{A_t} k_t(i) di$ ,
- market for loans clears:  $L_t = \int L_t(p) di$ ,
- free entry condition (18) in the final goods sector holds,
- free entry condition (39) in the R&D sector holds,
- the government budget is balanced - equation (40) holds,
- and the aggregate resource constraint (42) is satisfied.

### 3.10 Relationships of the model

Let us consider the relations of the symmetric equilibrium of the described economy. We denote aggregate quantities and prices with capital letters and sectoral, symmetric quantities and prices with small letters.

Consumption:

$$\lambda_t = C_t^{-\varsigma}$$

Capital accumulation:

$$K_{t+1} = (1 - \delta) K_t + I_t$$

Savings (return on loans to the innovative firms)

$$1 = E_t \beta \frac{R_{t+1} \lambda_{t+1}}{\lambda_t}$$

No arbitrage condition, that relates the gross return on investment in firms equity with the gross return on capital investments (in household utility terms):

$$E_t \lambda_{t+1} R_{t+1} = E_t \lambda_{t+1} (1 + r_{t+1} - \delta)$$

Unemployment:

$$U_t = 1 - (1 - \rho) N_t$$

The number of new matches between workers and intermediate firms:

$$M_t = \sigma_m U_t^\sigma V_t^{1-\sigma}$$

The probability that a firm fills an open vacancy:

$$q_t = \frac{M_t}{V_t}$$

The probability that a job-seeking person finds a new job:

$$s_t = \frac{M_t}{U_t}$$

labor market tightness:

$$\theta_t = \frac{V_t}{U_t}$$

The evolution of employment:

$$N_t = (1 - \rho)N_{t-1} + M_{t-1}$$

Aggregate output:

$$Y_t = \left( \int_0^{A_t} y_t^\mu di \right)^\mu = A_t^\mu y_t$$

which, given  $y_t = Z_t k_t^\alpha n_t^{1-\alpha}$  and  $K_t = \int_0^{A_t} k_t di = A_t k_t$  and  $N_t = \int_0^{A_t} n_t di = A_t n_t$ , collapses to:

$$Y_t = A_t^{\mu-1} Z_t K_t^\alpha N_t^{1-\alpha}$$

The price of a unit of a final good:

$$p_t = A_t^{\mu-1}$$

Intermediate goods prices (markup condition):

$$p_t = \mu \cdot mc_t$$

Job creation condition (given that  $f_{n,t} = (1 - \alpha) \frac{y_t}{n_t} = (1 - \alpha) A_t^{-\mu} \frac{Y_t}{N_t} A_t = (1 - \alpha) A_t^{1-\mu} \frac{Y_t}{N_t}$  and in equilibrium marginal costs can be related to the number of firms operating in the intermediate sector:  $mc_t = \frac{1}{\mu} p_t = \frac{1}{\mu} A_t^{\mu-1}$ ):

$$\frac{\kappa w_t}{q_t} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left[ \frac{1 - \alpha}{\mu} \frac{Y_{t+1}}{N_{t+1}} - w_{t+1} + (1 - \rho) \frac{\kappa w_{t+1}}{q_{t+1}} \right]$$

Demand for capital:

$$r_t = mc_t f_{k,t}$$

which, given  $f_{k,t} = \alpha \frac{y_t}{k_t} = \alpha A_t^{1-\mu} \frac{Y_t}{K_t}$ , collapses to:

$$\mu \cdot r_t = \alpha \frac{Y_t}{K_t}$$

which states that each firm adjusts demand for capital to the point where the marginal product of capital equals the markup  $\mu$  over the rental price of capital  $r_t$ .

Wages:

$$w_t = (1 - \eta) b_t + \eta \left[ \frac{1 - \alpha}{\mu} \frac{Y_t}{N_t} + \theta_t \kappa w_t \right]$$

which collapses to:

$$w_t = \frac{(1 - \eta)}{1 - \eta \kappa \theta_t} b_t + \frac{\eta}{1 - \eta \kappa \theta_t} \frac{1 - \alpha}{\mu} \frac{Y_t}{N_t}$$

Profits of an intermediate good firm expressed in terms of a final good:

$$\pi_t = A_t^{-1} \left[ \left( \frac{\mu - \alpha}{\mu} \right) Y_t - w_t (N_t + \kappa V_t) \right]$$

Value of a new intermediate good:

$$v_t^I = \pi_t + (1 - \phi) E_t \Lambda_{t,t+1} v_{t+1}^I$$

When we introduce the discounted streams of total profits of intermediate goods firms  $V_t^I$  (so  $V_t^I = A_t v_t^I$ ), then:

$$V_t^I = \left[ \left( \frac{\mu - \alpha}{\mu} \right) Y_t - w_t (N_t + \kappa V_t) \right] + (1 - \phi) E_t \Lambda_{t,t+1} \frac{A_t V_{t+1}^I}{A_{t+1}}$$

Production of new intermediate goods:

$$\frac{A_{t+1}}{A_t} = 1 + \chi \left( \frac{L_t}{K_t} \right)^\psi - \phi$$

Zero-profit condition for entry in the innovative sector:

$$(1 - \phi) E_t \Lambda_{t,t+1} \frac{V_{t+1}^I}{A_{t+1}} [A_{t+1} - (1 - \phi) A_t] = L_t$$

Aggregate resource constraint:

$$Y_t = C_t + I_t + L_t + \kappa w_t V_t$$



### 3.11 Steady state

The detailed derivation of the steady state relationships, properties and restrictions of the balanced growth path are enclosed in Appendix A. Summarizing the results of these calculations, the steady state of the model that is consistent with the balanced growth path, apart from specific forms of the relations between variables, imposes some restrictions on the comovement of various variables. Thus, the steady state implies that some variables need to be growing at different rates (we denote a growth rate of a given variable  $X_t$  with  $\gamma_X$ ):

- stationary variables:  $R, r, q, s, \theta, Z$ ;
- variables growing at a rate  $\gamma_Y$ :  $Y, C, V^I, L, K$ ;
- variables growing at a rate  $\gamma_N$ :  $U, N, V, M$ ;
- variables growing at a rate  $\gamma_w$ :  $w, b$ ;
- variables growing at a rate  $\gamma_A$ :  $A$ ;
- variable growing at a rate  $\gamma_A^{\mu-1}$ :  $p$ ;

The consistency of the steady state with balanced growth facts also implies some restrictions on the growth rates of variables:

- $\gamma_A = \chi \left( \frac{\bar{S}}{\bar{K}} \right)^\psi + 1$ ,
- $\gamma_Y^{1-\alpha} = \gamma_A^{\mu-1} \gamma_N^{1-\alpha}$ ,
- $\gamma_w \gamma_N = \gamma_Y$ .

So, the steady state consistent with a balanced growth path is a set of paths of variables that grow at specific rates outlined above and that satisfy restrictions on growth rates given above.

When one assumes that the population, along with the labor force, is a stationary variable, then it follows that restrictions on growth rates collapse to:

$$\gamma_Y = \gamma_A^{\frac{\mu-1}{1-\alpha}}$$

$$\gamma_w = \gamma_Y$$



## Chapter 4

# Calibration and estimation strategy

The model was calibrated to match the dynamic properties of the US economy. The period length of the model was set to one quarter. The model describes the medium-term properties of the data, so, on the one hand, the natural choice for the frequency of the model is one year, as was chosen e.g. by Comin and Gertler (2006). But on the other hand, the flows in labor market occur in much higher frequencies and usually the period length is set to be a quarter (see e.g. Shimer 2005). In order to calibrate the labor market in detail, I follow the latter convention<sup>54</sup>. As in the model, the labor force is normalized to one, we also normalize the relevant data counterparts by the size of the labor force.

The elasticity of output with respect to capital  $\alpha$  was set to 0.33. This value is commonly used in the literature concerning the business cycle fluctuations (see e.g. Prescott 1986, Gertler and Trigari 2006). In some papers the values of  $\alpha$  are somewhat different, e.g. 0.4 in Cooley and Prescott (1995) for the U.S. economy or 0.3 used in the calibration in Smets and Wouters (2003) for the model of the Euro Zone. However, these calibrations rely on the labor share in total income. In the models with search and wage bargaining, the calibration of the labor share could be done using other parameters (e.g. relative bargaining power of workers vs. employers or replacement ratios), so it is possible to calibrate  $\alpha$  basing on technological considerations, as it is done in the RBC literature.

Based on evidence presented in Basu and Fernald (1997), the average markup in the goods market  $\mu$  was set to 0.10. The elasticity of intertemporal substitution was set to  $\zeta = 2$ . This choice is often used in the literature, e.g. in Fuhrer (2000) or Greenwood, Hercowitz, and Huffman (1988).

The steady state investment share in GDP ( $\frac{\bar{I}}{\bar{Y}}$ ) was calibrated at 21.1%,

<sup>54</sup>Additionally, Costain and Reiter (2003) stress that using the Cobb-Douglas specification of the matching function, like the one adopted here, creates some problems in the model behavior when periods are so long that the transition probabilities (between e.g. employment and unemployment) are near one.

based on annual National Income and Product Accounts (NIPA) data and R&D data from the Bureau of Economic Analysis for the period 1960-2002. The definition of physical investments used in the calibration procedure include: gross private domestic investments (less total R&D investments, taken from Sumiye Okubo and Sliker (2006)), net exports of goods and services and personal consumption expenditures on durable goods<sup>55</sup>. The share of R&D investments to GDP ( $\frac{\bar{I}}{\bar{Y}}$ ) was set to 2.57%, based on calculations made by BEA (see Sumiye Okubo and Sliker 2006). The steady state capital to output ratio ( $\frac{\bar{K}}{\bar{Y}}$ ) was calibrated from the NIPA (with physical capital measure approximated by the sum of private fixed assets and the stock of consumer durable goods) which gives the value of 2.49, in annual terms. The steady state growth of output (i.e. GDP per labor force) was calibrated on the basis of GDP data from NIPA and Labor Force Statistics from the Current Population Survey, published by the Bureau of Labor Statistics (BLS). It follows, that  $\gamma_Y = 1.0044$ .

The calibrated values of  $\alpha$ ,  $\mu$  and output growth imply (conditional on the assumption of no population growth  $\gamma_N = 1$ ) that the growth of technology is  $\gamma_A = 1.0296$  (see section 3.11). The other parameters controlling the technology growth were calibrated on the basis of the evidence discussed in Comin and Gertler (2006). The parameter controlling the strength of congestion effect in R&D production technology was calibrated at  $\psi = 0.95$ <sup>56</sup> and the rate of obsolescence was set to 0.03 annually, which yields the quarterly value of  $\phi = 0.0076$ .

Based on these observations, the quarterly depreciation rate was set on the basis of the steady state properties of the model  $\delta = \frac{\bar{I}}{\bar{K}} - (\gamma_Y - 1)$ , which gives 1.69%, a value very close to the one commonly used in the literature (see e.g. Comin and Gertler 2006, Cooley and Prescott 1995), where the annual depreciation is close to 8%. The return on capital was also calibrated from the steady state relationship  $\mu\bar{r} = \alpha\frac{\bar{Y}}{\bar{K}}$ , which yields the value of  $\bar{r} = 3\%$ . This implies the steady-state value of  $\bar{R}$  to be equal to 1.0133. The implied value of  $\beta$ , the household's discount factor is 0.995, very close to the value of 0.99, suggested by Prescott (1986) for the quarterly specification of households preferences.

The elasticity of new matches with respect to unemployment ( $\sigma$ ) was calibrated at 0.45, on the basis of the evidence presented by Mortensen and Nagypal (2005). The value of  $\sigma$  used in our study is in the middle of the ones reported in the literature<sup>57</sup> and seems the most reasonable, taking into

<sup>55</sup>The inclusion of net exports in the definition of investments when calibrating closed economy models (and assigning government expenditures to consumption in the models with no government) follows Cooley and Prescott (1995).

<sup>56</sup>We have performed additional exercises in the estimation procedure and estimated this coefficient together with the stochastic parameters of the technological process. We have obtained almost the same value of  $\psi$ .

<sup>57</sup>Literature reports values of 0.24, as in Hall (2005b), 0.4, as in Blanchard and Diamond (1989), Andolfatto (1996) and Merz (1995). The value higher than 0.5 is reported in an

consideration the discussion in Mortensen and Nagypal (2005).

The exogenous employment exit rate  $\rho$  is calibrated on the basis of the work done by Robert Shimer, documented in Shimer (2007). He calculated the monthly time series of the employment exit rate, dating from 1947 onwards. Recalculation of the monthly probability of job exit to quarterly terms, using the formula  $p_{quarterly} = 1 - (1 - p_{monthly})^3$ , and averaging across time, gives the value of  $\rho = 0.0997$ . It is very close to the evidence for the period 1972-1982 presented by Abowd and Zellner (1985).

The steady state unemployment rate (not taking into account workers that are being matched this period, but are to start working next period) was set to 5.6%, which reflects the average unemployment rate for the period 1948-2006. The employment rate was set correspondingly. The parameter  $\bar{M}$  was calibrated using the steady state relationships, and the calibrated values for  $\rho$  and  $\gamma_n$ , which gives  $\bar{M} = 0.094$ . So, on average, 9.4% of labor force is changing its status on the labor market from unemployment to employment during a quarter. Thus, the sum of unemployed and matched workers (both per labor force), which is the unemployment rate in the model context, is equal to 15%. The steady state properties of the model imply that the probability of finding a job takes the value of  $\bar{s} = 0.63$ . It is relatively close to the job finding probability calculated by Shimer (2007), which, after recalculating to the quarterly frequency, amounts to 0.83.

The probability of filing a vacancy was calibrated at  $\bar{q} = 0.7$ , basing on the evidence reported by Cooley and Quadrini (1999) and den Haan, Ramey, and Watson (2000), used also by Trigari (2006). Such a calibration strategy<sup>58</sup> allows to recover the steady state number of vacancies (in relation to the labor force) of  $\bar{V} = 0.13$ . The implied value of  $\sigma_m$ , the parameter governing the matching technology, is equal to 0.67. The calibration strategy pins down the steady state labor tightness index  $\bar{\theta}$  at the value of 0.895.

The steady state level of wages  $\bar{w}$  was calibrated to match the steady state labor share in income ( $\frac{\bar{n}\bar{w}}{\bar{y}}$ ), which is 57% in the data (the average value of the ratio of compensation of employees to the GDP for the period 1948-2006, taken from NIPA). The steady state level of employment implies that  $\bar{w} = 0.6$ .

The calibration of the steady state labor share pins down the steady state vacancy posting costs ( $\kappa\bar{w}\bar{V}$ ), which amount to 3.6% of output. There is no direct evidence on this number, but the value used in our study seems reasonable (e.g. in the calibration exercise done by Gertler and Trigari (2006), the adjustments costs were set at 1% of GDP). The resulting steady state consumption-output ratio  $\frac{\bar{C}}{\bar{Y}}$ , after accounting for labor adjustments costs and other uses of output, is 73%. Pinning down vacancy posting costs allows

important contribution of Shimer (2005). Mortensen and Nagypal (2005) argue with the value used by Shimer.

<sup>58</sup>We have chosen this calibration strategy as the vacancy data are relatively unreliable, as they are not covering the whole US economy.

also to infer the steady state relation of profits to output  $\frac{\bar{\pi}}{\bar{Y}}$ . It amounts to 9.4% and is very close to the calibrated markup of prices over marginal costs.

The rest of the calibration strategy depends on the chosen value of replacement ratio  $\frac{\bar{b}}{\bar{w}}$ . As the calibration of the replacement ratio is somewhat controversial<sup>59</sup>, different values of this ratio (in the range [0.05, 0.95]) were considered (see subsection 5.1.1 for more details). For each value of  $\frac{\bar{b}}{\bar{w}}$ , the steady state relationships were applied to pin down the value of the relative bargaining power of workers  $\eta$ .

Our strategy to determine the stochastic properties of the underlying shocks that govern the stochastic properties of the economy diverges from the one commonly used in the RBC literature (see e.g. Cooley and Prescott 1995, Prescott 1986), which utilizes the information from the empirical definition of the Solow residual. The model counterpart of the Solow residual includes not only the exogenous technology process, but also endogenous R&D process, so it is somewhat harder to calibrate the properties of exogenous technology in this way. Instead, we use directly information from GDP data<sup>60</sup> for the US economy and estimate the stochastic properties of the model<sup>61</sup>, conditional on all other calibrated parameters. The estimated parameters are the stochastic properties of the underlying technology process (so, the estimated parameters are: the autocorrelation  $\rho_Z$  and standard deviation  $\sigma_\epsilon$  of the technology shock). An additional advantage of estimation over calibration is that it is possible to recover the values of the shocks that led to the observed fluctuations in output and compute the time series of model variables that can be directly compared to their data counterparts. So, the procedure applied here allows not only to compare the moments (standard deviations, correlations and autocorrelations) of the variables implied by the model with the data counterparts, but also to investigate more closely the behavior of the model generated data.

The model parameters along with shocks hitting the economy was estimated using the Dynare toolbar developed by Michel Juillard and his coau-

<sup>59</sup>The average ratio of total unemployment benefits paid (data taken from The Economic Report of the President) per unemployed person to the total compensation of employees (from NIPA accounts) per number of employed from CPS for the period 1960-2006 is 0.11. It is also very close to the average replacement ratio for the period 1961-2003, taken from OECD data, which amounts to 0.12. But the values used in the literature to calibrate the search models range from 0.4 used in Hall (2005c), Shimer (2005) or Gertler and Trigari (2006) to values of 0.8 used in Costain and Reiter (2003). There is a problem with the calibration of this parameter, as it covers not only unemployment benefits, but also the value of all non-work activities, including e.g. home production. The discussion on this issue will be continued in the next chapter.

<sup>60</sup>The series for GDP was normalized by the size of the labor force and then filtered using the Band Pass filter developed by Christiano and Fitzgerald (1999), with the trend defined as fluctuations in frequencies of more than 50 years, so consistently with the discussion in section 2.1. Additionally, to be consistent with the definitions of variables used in the model, the data used in estimation is GDP in relation to its trend, normalized to take an average value of one in the sample.

<sup>61</sup>The model, estimated and calibrated with the Dynare toolbox, was written in a stationary form, obtained by expressing all variables relative to their steady state paths. The details of the derivation of the stationary form of the model can be found in Appendix B.

thors<sup>62</sup>. The software uses the application of the perturbation theory (see e.g. Judd 1996) to simulate the model<sup>63</sup>. The estimation is done by expressing the model in a state space form and then applying nonlinear Kalman filtering technique (see e.g. Rubio and Fernandez-Villaverde 2005) to recover the parameters of the model and the underlying shocks. The estimation procedure used here applies the csminwel procedure developed by Chris Sims to optimize the likelihood function<sup>64</sup>. The estimators obtained are the point Maximum Likelihood estimates<sup>65</sup>.

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<sup>62</sup>See e.g. Juillard (1996) or Collard and Juillard (2001). One can get more details on the current version of the software on the Dynare project web page <http://www.ceprenap.cnrs.fr/juillard/mambo/index.php>.

<sup>63</sup>See also Stokey, Lucas, and Prescott (1989), Ljungquist and Sargent (2000) or Klima (2006) for a comprehensive studies of issues in dynamic programming and various methods of solving dynamic recursive economic models.

<sup>64</sup>The Sims procedure uses a quasi-Newton method with BFGS (Broyden-Fletcher-Goldfarb-Shanno) update of the estimated inverse hessian. It is robust against certain pathologies common for likelihood functions. It attempts to be robust against "cliffs", i.e. hyperplane discontinuities.

<sup>65</sup>It is also possible to consider Bayesian techniques and Metropolis-Hastings algorithms to obtain the estimates of the parameters governing the technology process. Since the distributional issues of parameters are not the subject of this study, we used a simplified version of the estimation procedure.





# Chapter 5

## Results of the simulations

### 5.1 The basic model

#### 5.1.1 Calibration issues

This section describes the results of the procedure described in the section 4. The standard model of the search in the labor market, embedded in the general equilibrium framework, underestimates the variation of unemployment and vacancies. This shortcoming of the search-matching model is discussed e.g. in Hall (2005b), Shimer (2004) or Costain and Reiter (2003). In order to check whether this feature is also present in the general equilibrium model with endogenous growth, we investigated the properties of the labor market variables against the evidence from the US economy.

The controversy in the literature associated with the choice of replacement ratio (see footnote 59) have motivated us to check the properties of our model as a function of the replacement ratio. We decided to concentrate our analysis on selected (and very important) features of the US labor market:

- **Variation of unemployment**, measured as a relative standard deviations of unemployment and output ( $\frac{\sigma_U}{\sigma_Y}$ ). The US data estimate for the period 1948-2006 (variables are defined as deviations from the long run trend) is  $\frac{\sigma_U}{\sigma_Y} = 5.8$ . It follows that unemployment is very volatile - almost 6 times as output. For further discussion on the large volatility of unemployment see section 2 or the evidence presented and discussed e.g. in Costain and Reiter (2003), Merz (1995) or Gomes, Greenwood, and Rebelo (2001)).
- **Variation of wages**. The relative standard deviations of wages and output in the US economy, calculated for the period 1948-2006 is  $\frac{\sigma_w}{\sigma_Y} = 0.89$ . Thus, wages are almost as volatile as output.
- **Beveridge curve** - the negative correlation between unemployment and vacancies  $\sigma(U, V)$ . It is documented e.g. by Blanchard and Diamond (1989) for the US economy and by Nickell, Nunziata, Ochel, and

Quintini (2001) for OECD countries. The unemployment-vacancies correlation in our database (calculated for the period 1951-2006) is  $\sigma(U, V) = -0.81$ , so it is very pronounced in the data. Shimer (2005) reports this correlation  $\sigma(U, V) = -0.9$ , while Costain and Reiter (2003) report the value  $-0.933$ . Both these values are very close to our estimate, which includes also medium term component of the cycle.

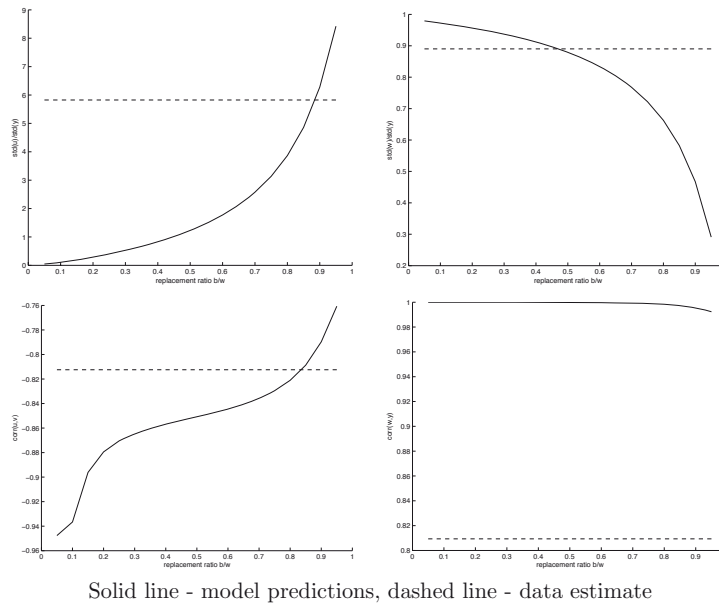
- **Instantaneous comovement of wages and output**, measured by the correlation of wages with output  $\sigma(w, Y)$ . The data estimate for the period 1948-2006 is 0.81.

Solid lines on Figure 7 shows these moments of variables, generated by our model, as a function of replacement ratio. The Figure also reproduces the data estimated, which are marked with dashed lines. For each value of the replacement ratio, the model was recalibrated and the new steady state was found. Then, we estimated the parameters of the model together with shocks<sup>66</sup> (for details see chapter 4). The moments were calculated using the simulated values of the variables generated by the model with the estimated stochastic shock series.

The upper left panel of Figure 7 shows the behavior of relative standard deviations of unemployment and output. It is apparent that the model underestimates the volatility of unemployment. Only for very high values of replacement ratio, the volatility of unemployment generated by the model coincides with the data estimate. This shortcoming of the standard search-matching model is widely addressed in the recent literature. The papers of Hall (2005b), Shimer (2004), Costain and Reiter (2003) and Gertler and Trigari (2006) are only examples of the discussion on this issue. In a standard model, one of the ways to get the volatility of the model generated unemployment close to the data, is to assume (also consistently with our results) that the replacement ratio is close to one (see the discussion in Costain and Reiter 2003). It means either that the disutility from hours spent on market activities or the extend of home production or the income that a worker can receive when unemployed need to be relatively large in comparison to the gain that the worker achieves when engaging in market activities. Many economists believe that the opposite is true - alternative activities for most workers, including unemployment compensation, are worth far less than the

<sup>66</sup>With our calibration strategy, some of the steady state parameters of the model depend on the replacement ratio, so it is very hard to estimate the replacement ratio directly from the data. Instead, we adopt a different approach - we perform the grid search over replacement ratios and for each iteration we recalculate the steady state of the model, given the chosen value of the replacement ratio. Then, we estimate the stochastic parameters of the model, together with the model variables (which are consistent with the evolution of output in US economy and estimated series of technological shocks) and calculate the moments of these series. This approach is a bit more extensive, but simpler to perform than direct estimation of the replacement ratio. Additionally, it gives you more information on the model properties, as depicted e.g. in Figure 7.

Figure 7: Properties of the basic model - moments of variables.



source: own calculations

worker's gain from the market production (for a discussion on this issue, see Hall 2005c). We share the same belief, so we rather expect that the replacement ratio is close to 0.5.

The volatility of wages declines monotonically with the replacement ratio and is close to the data estimate for the values of replacement ratio between 0.4 and 0.5. For high values of the replacement ratio, close to 0.9 the volatility of the wages drops substantially (compare the upper right panel of Figure 7) to the levels close to 0.3 - 0.4 of the volatility of output - well below the volatility observed in the data. Our model generates the negative correlation of unemployment and vacancies (the Beveridge curve) for the whole range of values of the replacement ratio, but for the replacement ratio close to 0.85 the levels of unemployment-vacancies correlation is very close to the data estimate (see lower left panel of Figure 7). The correlation between wages and output is very high (well above the data counterpart) for all values of the replacement ratio with a tendency to decline for high values of the replacement ratio.

So, the most striking discrepancy between the model predictions and the data is the volatility of unemployment. Why do the standard search models (for a reasonable value of the replacement ratio) cannot match the behavior of one of its central elements - the unemployment? The literature (see e.g. Hall 2003, Costain and Reiter 2003, Hall 2005b, Shimer 2004) points out, that search theory determines only the bargaining set, describing the range of feasible wages and not the wage itself. The upper bound of the bargaining

set - worker's reservation wage  $\bar{w}$  equates the unemployment value  $U_t$  to the employment value at the reservation wage  $V_t(\bar{w})$ . The upper bound of the feasible wage range - an employer's reservation wage  $\underline{w}$  is the entire anticipated surplus from the match  $J_t$ . Any wage within the bargaining set  $[\bar{w}, \underline{w}]$  is efficient, in the sense that it does not distort the individual decisions of agents and leads to a successful job contract. But simultaneously, the way that the negotiated wage splits the joint surplus from a match between the two parties, matters for the vacancy posting activity of employers and thus for the aggregate situation in the labor market. In this way, it affects the recruiting effort of employers and the behavior of vacancies and unemployment. If the negotiated wage closely follows the changes of the employer's reservation wage  $\underline{w}$  (e.g. as a consequence of a productivity shock that changes directly the marginal product of labor and an employer's perceived gain from a match) then the employer's recruiting effort responds only slightly to changes in the economic environment.

This mechanism is also at work in the setup presented here and is responsible for the underestimation of the unemployment variability that the model predicts. It is apparent when one takes into account the very high correlation between wages and output (which is closely related to labor productivity and ultimately to a product of labor). This correlation is high for each level of the replacement ratio considered in the analysis. The observation from Figure 7, that the volatility of unemployment rises with the replacement rate can also be explained by the behavior of the wage within the bargaining set. When the replacement ratio is high relative to the wage (and labor productivity) then the bargaining set narrows as the worker's reservation wage increases. With relatively small joint surplus from a match, even a tiny variation in labor productivity affect strongly the surplus and then even small deviations of the negotiated wages and productivity could induce a more pronounced reaction of hirings, thus leading to a higher volatility of vacancies and unemployment. The latter property of the search model with Nash bargaining was exploited e.g. by Merz (1995) or Costain and Reiter (2003) to make the unemployment series volatile in their simulations.

One of the ways to deal with this issue is to consider a different (than the Nash bargaining used here) method to pin down the wage within the bargaining set. In other words, it is worth exploring the consequences of other wage determination schemes as equilibrium selection mechanisms. Additionally, one may consider the introduction of other shocks, affecting the labor market directly, i.e. disentangling the relation between wages, recruiting effort and employment. This kind of solution was proposed e.g. by Pissarides (2007). The discussion and the results of these extensions in the context of our study is the subject of the next sections of this chapter.

We now go back to the discussion of the results of simulations performed on the basic version of the model. Taking into account the analysis depicted

in Figure 7 we choose the value of replacement ratio  $\frac{\bar{b}}{\bar{w}} = 0.5$ . This value matches the volatility of wages and the Beveridge curve to their data counterparts. The chosen value of the replacement ratio is consistent with the view that value of non work activities is far below what workers gain in market activities. Values in the neighborhood of 0.4 – 0.5 are also used in the literature (for the reference, see Hall (2005c), Gertler and Trigari (2006) or Shimer (2005)).

When we determine the replacement ratio, it is possible to calibrate the rest of the model parameters. Given replacement ratio, we set the bargaining power of workers  $\eta$ , that is consistent with observed labor share. It follows that  $\eta = 0.52$ . Notice that this value is relatively close to the elasticity of new matches with respect to unemployment  $\sigma = 0.45$ . When these two parameters coincide then the so called "Hosios rule" is fulfilled (see the results in Hosios (1990) or in Shimer (2005) for the extension for the stochastic environment). The equality  $\sigma = \eta$  assures that the inefficiency introduced by the decentralization of the search process in the presence of matching rigidities is ruled out. As the difference between these parameters in our calibration is moderate, one also should expect that the extent of inefficiency introduced into economy by decentralized matching process (stemming from the lack of coordination of individual actions) is small.

The estimates of parameters of the stochastic process governing the evolution of production technology of intermediate goods are as follows: the autocorrelation of technology shock is  $\rho_Z = 0.947$  and the standard deviation of innovation is  $\sigma_Z = 0.01$ . Both these parameters are somewhat different from the usual values used in the RBC literature. For example, Cooley and Prescott (1995) uses the values of 0.007 for the standard deviations of technology shock and 0.95 for the persistence (autocorrelation) of shocks. Values similar to Cooley and Prescott's are also applied by Prescott (1986) and King and Rebelo (1999) (in the latter, the persistence was set at 0.98).

At the first look, it seems that the internal propagation and amplification mechanism of the model is quite weak - the volatility of underlying shocks is quite large, when compared to basic RBC models and the persistence of shocks is close to the standard values. But our measure of cycle in the data contain also variation in medium term frequencies, so the overall variation of the cycle is much higher than in standard measures used in the RBC literature. With our definition of the cycle, its standard deviation is 0.039, whereas e.g. in King and Rebelo (1999) volatility of output is 0.0181. A better measure of internal amplification mechanism is the relation of volatility of model generated output and volatility of the underlying shocks, which is 1.9 for King and Rebelo (1999) model specification and 3.9 for our basic version of the model. So, the model we are considering has a strong internal propagation mechanism. The same is true for persistence - the relation of model generated autocorrelation of output and autocorrelation of underlying

shocks is 0.73 in basic RBC model and 1.01 for our model. The reason for much stronger internal propagation mechanism in the model considered here is the inclusion of semi-endogenous<sup>67</sup> growth component. The failure of standard RBC models in reproducing the realistic propagation mechanism is well known in the literature (see e.g. Cogley and Nason 1995, Jones, Manuelli, and Siu 2005). The literature also suggests some other extensions to overcome this issue e.g. the inclusion of learning by doing, like in Chang, Gomes, and Schorfheide (2002) or the inclusion of search on the labor market<sup>68</sup> - see in this context the work of den Haan, Ramey, and Watson (2000).

In our model, the variation in output is generated not only by variation in capital, labor and exogenous technology process, but also from variation in the number of products. This additional source of variation diminishes the role of exogenous shocks needed to account for the volatility of output observed in reality. Additionally, the fact that changes in the number of products available are rather slow-moving, they introduce a mechanism that propagates the influence of short-living shocks further in time. Thus, also the persistence of exogenous shocks can be smaller in the model considered here compared to the basic RBC model in order to reflect the volatility of the US economy. So the model considered in this study allows for much more realistic and richer propagation mechanism than the one generated by the standard RBC model.

### 5.1.2 Impulse response functions

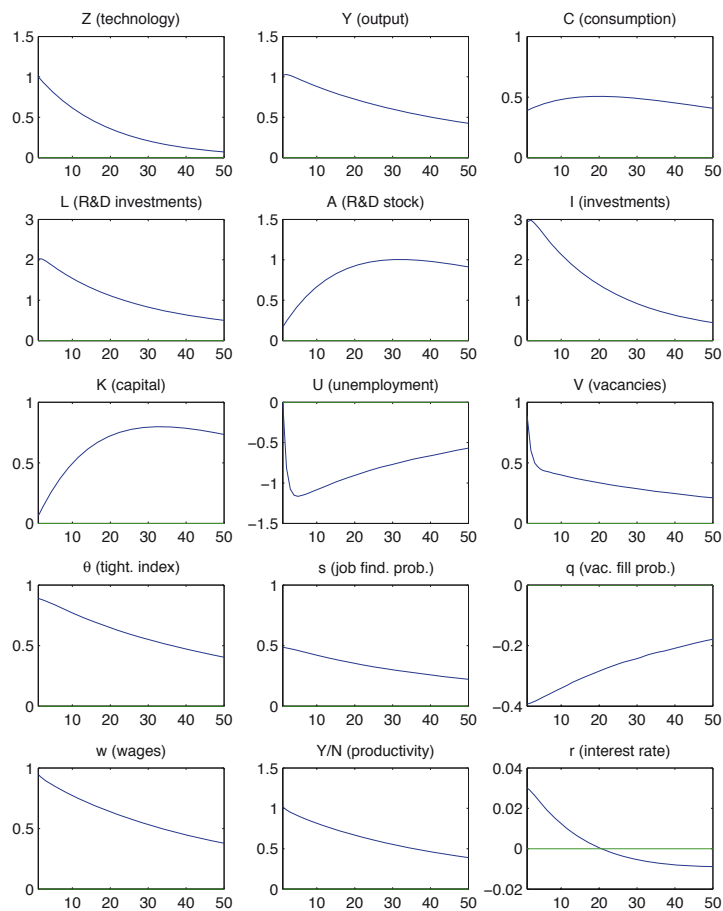
The best way to investigate the dynamic properties of the model is to compute its impulse response functions (IRF). The IRF inform about the response of a model economy to a given shock. Being more precise, one can define the response in period  $t + s$  of variable  $X$  to an impulse  $\epsilon$  occurring at time  $t$  as:  $IRF(X, s) = \frac{dX_{t+s}}{d\epsilon_t}$ . Figure 8 shows the impulse responses of selected variables of the model to a 1% shock (innovation) in an intermediate goods technology  $\epsilon_t$ .

After a positive technology shock in the intermediate goods sector, the economy experiences a prolonged period of growth. Output increases, along with consumption, but the consumption rises less than output. The increase of productivity rises also the marginal product of capital and thus the interest rate and return on capital (not shown in the graph). Higher return on capital rises in turn the profitability of investing in physical capital and the model economy experiences a substantial rise in investments. Additionally, the

<sup>67</sup>The endogenous growth component of the model should be rather called semi-endogenous, as it assumes that short run changes in the economic environment affect only growth rates in short and medium term, and do not affect the steady state (like e.g. in the case of AK family of endogenous growth models).

<sup>68</sup>Introducing a search rigidity into the RBC model helps to generate more internal propagation, but alone is insufficient to induce fluctuations in the medium term, see chapter 5.4 for further discussion.

Figure 8: Selected impulse response functions of the basic model to a 1% technology shock



source: own calculations

higher market interest rate implies that the future value of new products rises. This effect, combined with the expectations of better economic conditions and higher profits in the future induce the increased effort of innovative firms to invent and introduce new products into the markets. Thus, the households' investments in the R&D sector rises (in general, investments in the R&D sector mimic the behavior of investments in physical capital, although they are less volatile), as they give the same expected yield as investments in physical capital (due to the no arbitrage condition). So, with both types of investments rising, households' savings also rise and the increase of consumption is smaller than output and income.

When the efficiency of production activity is higher than expected, employers react by exercising more recruiting effort. The number of vacancies posted by employers rises as well as the number of new matches. Thus, we observe an increase in job finding probability and consequent decline in unemployment. Workers are also more interested in looking for a job, as the negotiated wage rises. The rise in wages is a consequence of both rising marginal product of labor and the fact, that the labor market becomes more tight (the labor market tightness index  $\theta$  rises). So, with the reservation wage of workers almost unchanged (note that the unemployment benefits, which are the major determinant of the workers reservation wage, are assumed to grow at their steady state rate, lower than negotiated wages in the simulation).

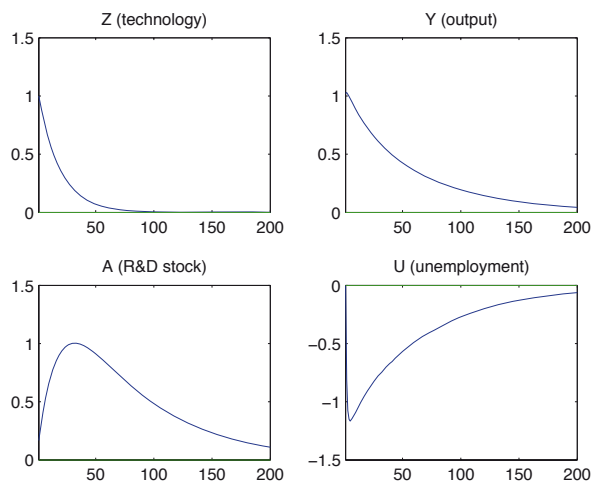
More tight labor market (higher  $\theta$ ) affects additionally the hiring decisions of employers. More posted vacancies and less workers looking for jobs makes it harder for an employer to fill a vacancy (probability of filling a vacancy declines). Additionally, tight labor market exercises upwards pressure on negotiated wages, diminishing the employer's surplus from a match and thus reducing the employer's incentive to hire. So, after three or four quarters the number of posted vacancies, although still higher than in the baseline, declines relative to the first period peak by over a half. This fact translates into unemployment gradually reverting to the baseline level.

After initial peaks, most variables steadily revert to their steady state levels. This tendency is apparent both in variables reflecting the stance of the labor market and the goods market. It is worth to note that consumption experiences a long lasting hump with a peak about five years after a shock and then steadily reverts to the steady state. Investments in both physical capital and innovative activity behave differently. The peak is reached within a year and then both investments outlays revert to the long-run trend. This behavior of investment translates into a long-lasting increase of both capital and R&D stocks, which reach their peaks after about 8 years and then gradually decumulate.

On feature of the impulse response functions of the model, not visible in Figure 8 is the fact, that it takes quite a long time for the model economy



Figure 9: Impulse responses of a basic model - longer time horizon



source: own calculations

to revert to the steady state. In order to emphasize this point, we decided to show also IRFs for longer time period (50 years) for a small subset of variables described by the model. These are depicted in Figure 9. What is worth noting is the substantial propagation mechanism of the model. The shock that induced the fluctuations in economic activity dies out completely after 20 years, but the large internal propagation mechanism, due to the endogenous technology component of the model, induces the economy to deviate from the steady state for a long period of time. The long-lasting reactions of output (which die out after about 40 years) are the consequence of a long period of both R&D and physical capital adjustments. Both of these last long (up to 50 years), as changes in economic activity in reaction to a technology shock, also induce long lasting adjustment of interest rate<sup>69</sup>. The adjustments in goods market translate additionally into gradual shifts occurring in the labor market, resulting e.g. in medium term fluctuations of the unemployment rate. It is also worth noting, that most of the variables display hump shaped impulse responses to a technology shock - a feature that is one of the 'stylized facts' of US economy (for a reference, see e.g. Cogley and Nason 1995, Blanchard and Quah 1989).

The impulse response functions show the model's ability to generate medium term cycle in both goods and labor markets. In order to check whether the model is able to track closely the behavior of the US economy, one needs to look into moments of the data generated by the model and, given the estimates of the shocks affecting the economy, also the time paths of variables. These issues are the subject of the next subsection.

<sup>69</sup>The medium run swings in interest rate are one of the stylized facts discussed previously in this study.

### 5.1.3 Comparison with US economy behavior

In this section, we look more closely into the details of the model economy and compare them with the behavior of the US economy. The usual way to examine model properties against data, introduced into the applied macroeconomics by a seminal contribution by Kydland and Prescott (1982) is to compare the second moments of variables (standard deviations, correlations, autocorrelations) generated by a given model with their data counterparts<sup>70</sup>. On these grounds one may assess whether the model replicates the volatility, comovement and persistence of different variables reasonably close to the ones observed in reality. The results of this exercise are depicted in Table 3. But the additional advantage from the estimation procedure applied here is the ability to look into the time series generated by the model, given the estimated series of exogenous shocks affecting the economy. As the model was estimated on the output data, one may interpret the time series of the other variables generated by the model as the behavior consistent with development of output, given that the model (data generating process) is true. The results of the latter exercise are depicted in Figure 10 (the graphs show the evolution of selected variables, as observed for the US economy<sup>71</sup> and the evolution of the variables generated by the model; the graph for output is omitted as the procedure applied here assures that in case of output both time series perfectly coincide).

Table 3 shows standard deviations, correlations with output and autocorrelation properties of the US data and the model economy. All the moments are calculated for the medium term cycle, for the high-frequency component of the cycle (frequencies in the range [2, 32] quarters, as in the usual business cycle analysis) and for the medium term component (frequencies in the range [32, 200] quarters, so it covers variation of variables between 8 and 50 years). For the US economy, we use the same procedure as described in section 2, so it applies the band-pass filter of Christiano and Fitzgerald (1999) to US data series. For the model economy, the procedure is somewhat different. As the model is expressed in relation to the steady state, we recalculate the levels of

<sup>70</sup>The measure of output used for the US economy is GDP (chained 2000 dollars, from NIPA) divided by the size of labor force (the latter data from CPS); consumption is measured by real personal consumption expenditure on nondurables and services per labor force; investments is measured by real gross private domestic investments plus real consumption expenditures on durables; interest rates are measured by yields on 1-year treasury securities with constant maturity, deflated by next period inflation of consumption expenditures; wages and labor share are measured respectively as real compensation of employees per employment (from CPS) or per real GDP; unemployment and employment are measured respectively as the number of unemployed and employed per labor force (all data from CPS); vacancies are measured by the index of Help Wanted Advertising from the Conference Board; job finding probability is taken from the work done by Robert Shimer (for details, see Shimer 2007).

<sup>71</sup>For these series to be comparable, we express the data for the US economy as in the model. We calculate the cyclical component of the data using the band-pass filter of Christiano and Fitzgerald (1999). Then we calculate deviations of the data from the trend (defined as resulting component of the time series, covering the frequencies above 200 quarters), and normalize them, so as they equal 1 on average.

Table 3: Selected moments of the US economy and basic model economy

	A. US Economy												
	Standard deviations					Correlation with output					Autocorrelation		
	cycle [2,200]	high [2,32]	medium [32,200]	relative to output	cycle [2,200]	high [2,32]	medium [32,200]	cycle [2,200]	high [2,32]	medium [32,200]	cycle [2,200]	high [2,32]	medium [32,200]
Output	0.039	0.016	0.035	1.00	1.00	1.00	1.00	0.96	0.81	0.99	0.96	0.81	0.99
Consumption	0.028	0.008	0.027	0.73	0.88	0.76	0.91	0.98	0.74	0.99	0.98	0.74	0.99
Investments	0.103	0.069	0.076	2.66	0.73	0.84	0.74	0.91	0.81	1.00	0.91	0.81	1.00
Interest rate	0.005	0.003	0.004	0.14	-0.33	-0.15	-0.39	0.93	0.79	0.99	0.93	0.79	0.99
Wages	0.034	0.010	0.032	0.89	0.81	0.77	0.82	0.97	0.73	0.99	0.97	0.73	0.99
Unemployment	0.225	0.145	0.168	5.82	-0.84	-0.82	-0.88	0.94	0.88	0.99	0.94	0.88	0.99
Vacancies	0.208	0.131	0.172	5.63	0.53	0.83	0.45	0.96	0.90	0.99	0.96	0.90	0.99
Employment	0.013	0.008	0.010	0.34	0.84	0.84	0.87	0.95	0.88	0.99	0.95	0.88	0.99
Labor share	0.014	0.008	0.012	0.37	0.15	-0.23	0.26	0.87	0.61	0.99	0.87	0.61	0.99
Job finding probability	0.063	0.039	0.050	1.62	0.74	0.77	0.75	0.92	0.81	0.99	0.92	0.81	0.99
<b>B. Model Economy</b>													
	Standard deviations					Correlation with output					Autocorrelation		
	cycle [2,200]	high [2,32]	medium [32,200]	relative to output	cycle [2,200]	high [2,32]	medium [32,200]	cycle [2,200]	high [2,32]	medium [32,200]	cycle [2,200]	high [2,32]	medium [32,200]
	Output	0.039	0.016	0.035	1.00	1.00	1.00	1.00	0.96	0.81	0.99	0.96	0.81
Consumption	0.023	0.006	0.022	0.59	0.91	0.99	0.91	0.98	0.81	0.99	0.98	0.81	0.99
Investments	0.094	0.046	0.080	2.42	0.93	1.00	0.92	0.95	0.82	0.99	0.95	0.82	0.99
Interest rate	0.001	0.000	0.001	0.03	0.49	0.97	0.38	0.95	0.80	0.99	0.95	0.80	0.99
Wages	0.036	0.014	0.032	0.92	0.99	1.00	0.98	0.96	0.80	0.99	0.96	0.80	0.99
Unemployment	0.050	0.018	0.047	1.30	-0.90	-0.73	-0.93	0.98	0.87	0.99	0.98	0.87	0.99
Vacancies	0.039	0.010	0.038	1.06	0.01	0.84	-0.07	0.95	0.62	0.97	0.95	0.62	0.97
Employment	0.003	0.001	0.003	0.08	0.90	0.74	0.93	0.98	0.87	0.99	0.98	0.87	0.99
Labor share	0.003	0.001	0.003	0.08	-0.45	-0.98	-0.37	0.97	0.76	1.00	0.97	0.76	1.00
Job finding probability	0.017	0.008	0.015	0.44	0.96	1.00	0.95	0.96	0.80	0.99	0.96	0.80	0.99

source: own calculations

variables multiplying the model generated variables by the trend component, as extracted from the data<sup>72</sup> (recall that in the model  $\hat{x} = \frac{x_t}{\bar{x}}$ , so  $x_t = \hat{x}\bar{x}$ ). This way of obtaining the levels of variables makes them comparable to the observed data. Next, we filter the model generated variables using the same procedures as in the case of US data.

We can use Table 3 and Figure 10 to assess the properties of the model economy. The behavior of consumption is generally well reproduced by the model (see the upper left panel of Figure 10). The model underpredicts a bit the volatility of consumption<sup>73</sup>, especially for the higher frequency component (but the volatility of medium term component seems to be reproduced reasonably well, a feature also apparent in the Figure 10 as longer term swings in consumption). Also, the correlation with output is comparable to the one observed in the data - the model predicts that correlation of consumption with output is lower for the higher frequency component and higher for the medium frequency component. The same is true for the pattern of first-order autocorrelation, although the model predicts too high autocorrelation for the higher frequency component.

The model slightly underpredicts the overall volatility of investments in the data. It reflects relatively well the volatility pattern for the two components of the medium term cycle in investments - the volatility of the medium term component of investments is a bit higher than the higher frequency component, exactly as in the data. The correlation of investments with output and its pattern is also generally well reproduced, although the model overestimates these correlations. The autocorrelation pattern is almost exactly matched.

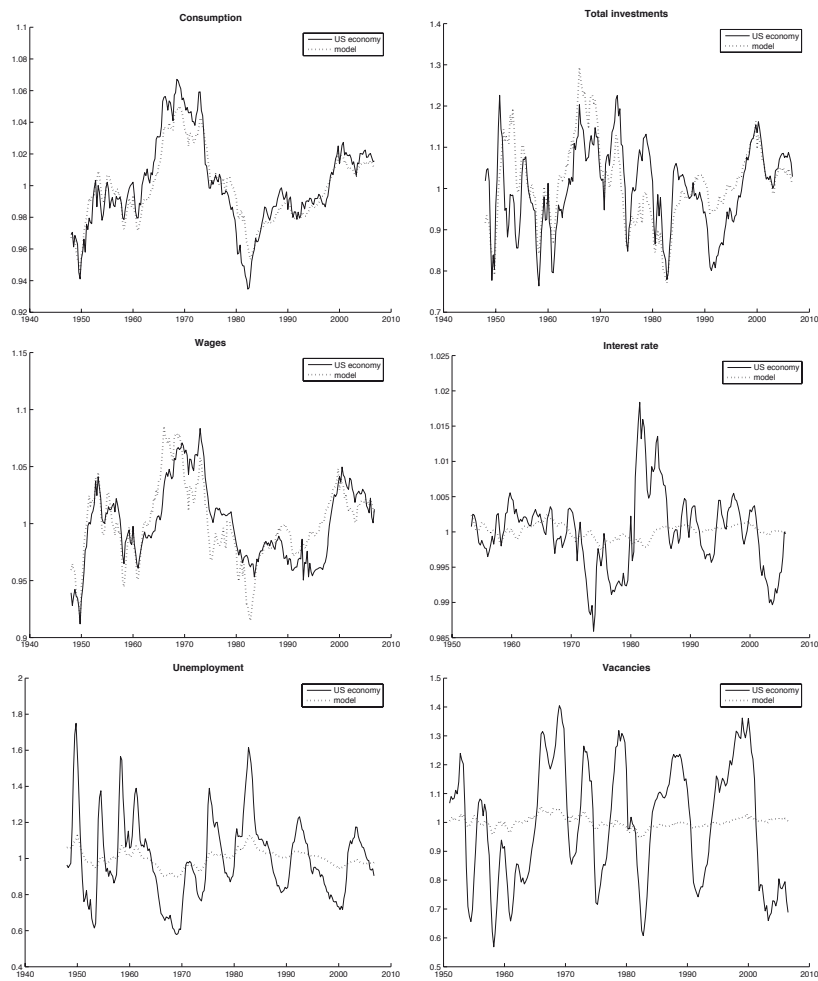
The volatility pattern of wages is also very well reproduced by the model, with the caveat that the model puts a little more emphasis on the volatility of the higher frequency component of the cycle in wages. It also tracks well the pattern of persistence in wages. But the model highly overestimates the correlation of wages and output in full range of frequencies considered<sup>74</sup> (which are almost one in the model, whereas in the data, these correlations are closer to 0.8). These findings are confirmed by the middle left panel of

<sup>72</sup>In the model economy, the steady state describes constant paths or paths with constant growth. In the US economy, there are slow movements in the trends, which are not accounted for by the model. So the procedure applied here assures comparability between moments of US and model economy.

<sup>73</sup>As our modeling strategy was to keep the model relatively simple in order to emphasize the main mechanisms of the interaction of search and endogenous growth, we do not include in the model additional extensions that would improve the model fit to the data. In case of consumption, the introduction of habit persistence (see e.g. Abel 1990) would smooth the reaction of consumption and diminish its volatility. The same applies in case of investments, where introduction of capital adjusting costs (see e.g. Christiano, Eichenbaum, and Evans 2005, Smets and Wouters 2003, Bayer 2006) or variable capital utilization (see e.g. Greenwood, Hercowitz, and Huffman 1988, King and Rebelo 1999) would improve the model fit with respect to the investment behavior.

<sup>74</sup>Recall the discussion in chapter 5.1.1 on the calibration issues and its impact for the behavior of wages and labor market variables.

Figure 10: Time series of US economy and data generated by basic model



source: own calculations

Figure 10, which shows that the behavior of wages is generally well simulated by the model.

The visual inspection of the middle right panel of Figure 10 and relevant rows of Table 3 shows that the model has problems with reproducing the behavior of the interest rate. First, it predicts too little variability of interest rate for the whole range of frequencies considered (the overall variation in the data is over 4 times higher than the one generated by the model). Additionally, the model predicts procyclicality of the interest rate in the case of both the high and the medium frequency component, whereas in the data, the interest rate is rather countercyclical (but the negative correlation of the interest rate and output is not too high). On the bright side, the model reproduces the pattern of persistence of the interest rate correctly. The problem with interest rate is an apparent shortcoming of the model, but as King and Rebelo (1999) have noticed, a number of modern macroeconomic models are unable to match the behavior of this variable (see also the discussion in King and Watson 1996). In order for the model to match the interest rate behavior, it should be enhanced in the direction shown by the recent asset prices literature (e.g. by expressing preferences similar to the one introduced by Epstein and Zin (1991) or allowing for other extensions described e.g. in Campbell (2002) on consumption based asset pricing models). Our intention was to keep the model relatively simple in order to highlight the main mechanisms of endogenous growth and search in the labor market and our main concern is not the interest rate. Thus, we do not consider further enhancement of the model in this direction<sup>75</sup>.

The model generated unemployment is too smooth, when compared to its data counterpart. The model generated unemployment rate should be 4 times more volatile to match the volatility observed in the US economy. This applies to both higher frequency and medium frequency component of the cycle in unemployment. The other moments and their frequency patterns, reflecting the nature of the cyclical and persistence of unemployment are roughly matched. The model predicts countercyclical unemployment, as in the data. The same remarks are valid for the behavior of employment rate. The model has problems with matching the volatility of vacancies and their correlation with output. It underestimates both these moments for all frequency range

<sup>75</sup>It could be an interesting route of further research, as interest rate behavior affects the hiring decisions of employers by changing the expected asset value of a job. The relations of interest rates and unemployment in the context of their comovement in medium term cycle are also described by Hall (2005d) and Hall (2005c). We have conducted some exercises on this issue in a reduced form way by enhancing the model with the interest rate shock (introduced into the producers FOC with respect to capital, see equation 26) and estimating its stochastic properties (volatility and persistence) using data on output and market real interest rate. Although the changed model reproduces the behavior of the interest rate, its ability to reproduce labor market facts improves very slightly. But this line of research is still open and accounting for a proper description of interest rate behavior by incorporating more elaborate extension of interest rate determination could have a potential to add more elaborate and complex explanation of labor market behavior, which includes e.g. the monetary policy.

considered<sup>76</sup>. Better picture emerges when comparing the persistence patterns, as the model reproduces them quite well. The model problems with reproducing the behavior of the labor market variables is also apparent in the bottom panel of Figure 10, which clearly shows the low volatility of the model generated data.

The problems with matching moments of employment imply that model generated labor share properties differ from the data on US economy. As the model predicts the right volatility of output and wages, the low labor share volatility generated by the model is due to the problems with reflecting the true extent of volatility in employment. In consequence, the standard deviation of the model generated labor share is much lower than its data counterpart. It also applies to the medium and higher frequency components of the cycle. Labor share is acyclical in the US economy (there is small positive correlation with output for the whole cycle and its medium frequency component and slightly negative correlation in the case of higher frequency component). In the model economy, labor share is rather countercyclical, especially for the high frequency component. Additionally, the persistence of labor share seems to be overestimated by the model.

The problems in reflecting the volatility of unemployment and vacancies implies the underestimation of the volatility of job finding probability, that the model predicts (for both the higher and the medium frequency components). Additionally, the job finding probability generated by the model comoves too much with output, when compared to the data, but its persistence seems to be well matched.

As was discussed in the section 5.1.1 and depicted in Figure 7, the model generated Beveridge curve (the negative relation of unemployment and vacancies) amounts to  $-0.853$  and is very close to the data estimate ( $-0.812$ ). This correlation is unaffected by the volatility issues of unemployment and vacancies, as the relation of standard deviations of these variables is very close to the data estimate.

Summing up, the stochastic properties of the model share the volatility problems, discussed in section 5.1.1. The model generates the right behavior of investments, consumption and, to a certain extent, wages with volatility and persistence of shocks that are relatively small, compared to the values usually used in the RBC literature. But the model underestimates the volatility of labor market variables, especially unemployment and vacancies, which translates into the model's problems of matching moments of other variables that are observed in the data (e.g. labor share, job finding probability). What is worth emphasizing, the model generates the volatility pattern consistent with the one observed in the US economy. Namely, it predicts that for

<sup>76</sup>One should have in mind, that the proxy for the vacancies, that we use as characterizing the US economy (Help Wanted Advertising, published by the Conference Board) is a rough measure of this economic concept.

all variables considered, the volatility of the medium frequency component of the cycle is higher than the volatility of the higher frequency component. So the model is dealing relatively well with reproducing the medium term properties of the data. But still, the basic model need to be extended in the direction that improves its fit in reproducing variables describing labor market. It follows then, that the endogenous growth component of the model is working relatively well, but the model share problems with describing the labor market that are common the ones described by the search literature.

These extensions are the subject of the next parts of this study. We consider two routes to enhance the model's description of the labor market (we will leave the underestimation of the interest rates unsolved, as this shortcoming influence the main results only slightly). First, we study the impact introducing additional shock to the matching technology. This is the subject of section 5.2. Second, we extend the wage setting mechanism, along the lines suggested by the recent literature, namely by introduction of the wage norm. This second route is based on the specification with only technological shock and is discussed in section 5.3.

## 5.2 The model with technology and matching shocks

In this section we consider the two-shock version of the model. One of the shock is inherited from the basic version of the model - the technology shock being the main driving force of the business cycle fluctuations, at least as regarded by the advocates of the real business cycle paradigm. The second shock we are going to consider is a shock to the matching technology, as reflected by the matching function (9). So, the  $\sigma_t^m$  is now given by the following first order autoregressive process (expressed as deviations from the steady state  $\bar{\sigma}^m$ ):

$$\hat{\sigma}_t^m = (1 - \rho_m) + \rho_m \hat{\sigma}_{t-1}^m + \epsilon_t^m$$

where  $\rho_m$  measures the persistence of the shock to the matching technology  $\epsilon_t^m$ .

Shocks to the matching technology reflect changes in the efficiency of the matching process. Namely, we assume that there are periods when, for given levels of unemployment and vacancies, the number of successful matches that realize in the economy vary. The introduction of the shocks to the matching technology (shifters of the Beveridge curve) can be justified by visual inspection of the right panel of Figure 5. It is apparent that the Beveridge curve exhibits shifts that are very pronounced in medium term frequencies. By adding the shock to the matching function we tried to mimic these kind of movements and analyze their consequences for our results, with



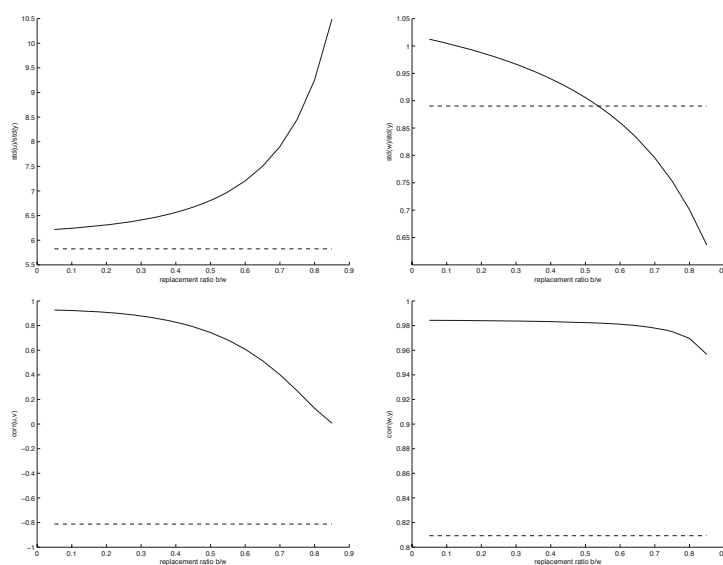
a special emphasis on labor market volatility issues.

We treat the matching technology shock as a reduced form shock, without any specific interpretation (very similar to the intermediate goods technology shock). The question is what could lead to these kind of changes in the functioning of the labor market? There are several kinds of disturbances that induce changes in the efficiency of the matching process:

- government policies and changes in legal regulations, such as firing restrictions or changes in the tax code that make it more or less costly for the producer to fire a worker. When the producer decides to employ more workers, he or she posts the number of vacancies that he or she needs, but simultaneously and he or she knows that e.g. firing the worker in the future will be more costly, so he or she tries to find a worker who is best suited for the job offer. Thus, he or she probably spends more time on searching and the observed efficiency of matching technology declines.
- changes in the composition of the labor force, with respect to e.g. skills, earnings expectations, working conditions, etc. These kind of changes in the environment disharmonize labor supply and labor demand, leading to the decline in the efficiency of matching. These kind of changes could be due to the changes in the demographic composition or educational attainment of the labor force, from which we abstract in the macroeconomic model.
- changes in the structure of the production (e.g. shifts from manufacturing towards services) could also lead to this kind of short run inefficiencies in the matching process.
- heterogeneity among workers and firms, e.g. induced by idiosyncratic characteristics (see e.g. Smith and Zenou 2003) of a given match. These kind of changes could also be induced by changes in the parameters of the distributions of individual characteristics, that Hall (2005c) suspects could be the additional source of labor market fluctuations, distinct from the variation in efficiency of production.

In order to estimate the model parameters (we follow the convention applied earlier and estimate only the stochastic properties of the shocks) of the model with two shocks, one needs to provide two time series of variables described by the model. We have decided to base the estimation of the model parameters and underlying shocks on the information contained in the time series of output and labor share (all data taken from NIPA accounts) for the quarterly period 1948-2006. The choice of output is natural and is consistent with approach applied in the analysis of the basic version of the model. The choice of labor share aims at providing the model with indirect information

Figure 11: Properties of the matching model - moments of variables



Solid line - model predictions, dashed line - data estimate

source: own calculations

on both the dynamics of wages and employment, thus making it relatively easy for the estimation procedure to account for the volatility of both of these variables<sup>77</sup>.

We follow the same convention as in section 5.1 when analyzing the basic version of the model. So, we discuss briefly the additional calibration issues with the choice of the replacement ratio and then concentrate on the behavior of the model against the data.

### 5.2.1 Calibration issues

The calibration of the two-shock model (for simplicity, we name this version of the model as the matching model) is the same as before. We perform the same kind of grid search over different values of replacement ratio and estimate the stochastic properties of both shocks (volatility and persistence) for each iteration. Figure 11 depicts the four moments that we are focusing on (relative variation of unemployment and wages as well as cyclicalities of unemployment and wages).

The volatility of unemployment rises with replacement ratio, as depicted in upper-left panel of Figure 11. The unemployment volatility generated by the model is higher than the data estimate for the whole range of the replacement ratios considered, so the inclusion of the shock to the matching technology indeed increased the unemployment volatility. The volatility of

<sup>77</sup>Although one cannot exclude the possibility that the estimation procedure will drive down the volatility of either wages or employment to small values in order to account for the overall volatility of labor share.

wages is decreasing in the replacement ratio, as reflected in the upper right panel of Figure 11 (the same observation was valid in the case of the basic version of the model). The model fits the data estimate of wages volatility for replacement ratios between 0.5 and 0.6. But the problematic issue is the Beveridge curve (unemployment-vacancies correlation, depicted in the lower-left panel of Figure 11), which is positive for all values of the replacement ratios considered. The minimum value of the Beveridge curve, which is close to 0, is generated for the values of replacement ratio close to 0.9 - far above the values, that we consider plausible. The matching model also overestimates the wages-output correlation. For all values of the replacement ratio considered, this correlation is well above the data estimate. The latter correlation is declining in replacement ratio, like in the basic version of the model.

So, the introduction of the shock to the matching technology increased the volatility of unemployment, but simultaneously destroyed the Beveridge curve and did not helped with wages-output correlation. Having considered the above properties, we choose to pick the value of the replacement ratio as in the basic model, so  $\frac{b}{w} = 0.5$ . The resulting value of workers bargaining power  $\eta = 0.52$  is again very close to the elasticity of new matches with respect to unemployment ( $\sigma = 0.45$ ), so the “Hosios rule” almost holds in the case of the matching model.

The estimates of the stochastic properties of the underlying shocks are as follows:

- volatility of the intermediate goods production technology shocks  $\sigma_Z = 0.011$ ,
- persistence of the intermediate goods production technology shocks  $\rho_Z = 0.906$ ,
- volatility of the matching technology shocks  $\sigma_m = 0.048$ ,
- persistence of the matching technology shocks  $\rho_m = 0.72$ .

The volatility of the intermediate goods technology shock is a bit higher than in the case of the basic model and its persistence is much lower, not only when compared to the basic model, but also when compared to the RBC literature (see e.g. Prescott 1986). The estimated volatility of the matching technology shocks is quite large, with relatively low persistence.

### 5.2.2 Comparison with US economy behavior

Figure 12 and Table 4 show, respectively, time series of the variables generated by the matching model and their moments. As in subsection 5.1.3, Table 4 shows standard deviations, correlations with output and autocorrelations for the medium term cycle (within the periodicity of [2, 200] quarters)

and for both the high frequency component (periodicity ranging from 2 to 32 quarters) and the medium frequency component (periodicity in the range of [32, 200] quarters).

The matching model, as well as the basic model, correctly mimics the patterns of consumption and investments - the overall volatility of both of these variables matches their data counterparts. The same applies when considering the medium and the high frequency components - the model rightly predicts that volatility of medium term component is relatively higher, especially in case of consumption. The model underpredicts slightly the volatility of the higher frequency component of both consumption and investments. In the case of the medium frequency component, the matching model underestimates its volatility in case of consumption and overestimates it in case of investments. All these observations are confirmed visually in the upper panel of Figure 12. Additionally, the model generated investments are too procyclical for all frequencies, but the comovement of consumption and output seems to be correctly reproduced.

The extension of the model was introduced mainly to enhance the model ability to replicate the dynamics of the labor market. We have seen that shocks to matching technology hardly affected the behavior of the goods market, as reflected by consumption, investments and the real interest rates (especially, it has not improved the behavior of the latter, see the middle left panel of Figure 12). So, the question is: whether the extension of the model indeed improved the model's ability to replicate the US labor market behavior?

The estimation procedure applied here assures that the time series generated by the model (almost) exactly reproduce the behavior of labor share (the lower panel of Table 4 assures that it is indeed the case). But how does the model disentangle the variation in labor share between wages and employment (and thus - unemployment)? Closer look into the Table 4 assures that model correctly replicates the overall volatility of both these variables. The matching model overestimates slightly the volatility of both wages and unemployment. The model reproduces also the correct pattern of volatility of the medium vs. the high frequency component of the cycle, although it amplifies a bit the high frequency variation of wages and the medium frequency variation of employment. The former observation is confirmed in the middle-left panel of Figure 12, with the medium term cycle in wages roughly matched and too high short-term volatility. The model underestimates the extent of cyclicity of employment (especially for the higher frequency component, when the correlation becomes close to 0, but negative) and underestimates the procyclicality of wages. The autocorrelation pattern of both these variables is, roughly speaking, correctly reflected by the model.

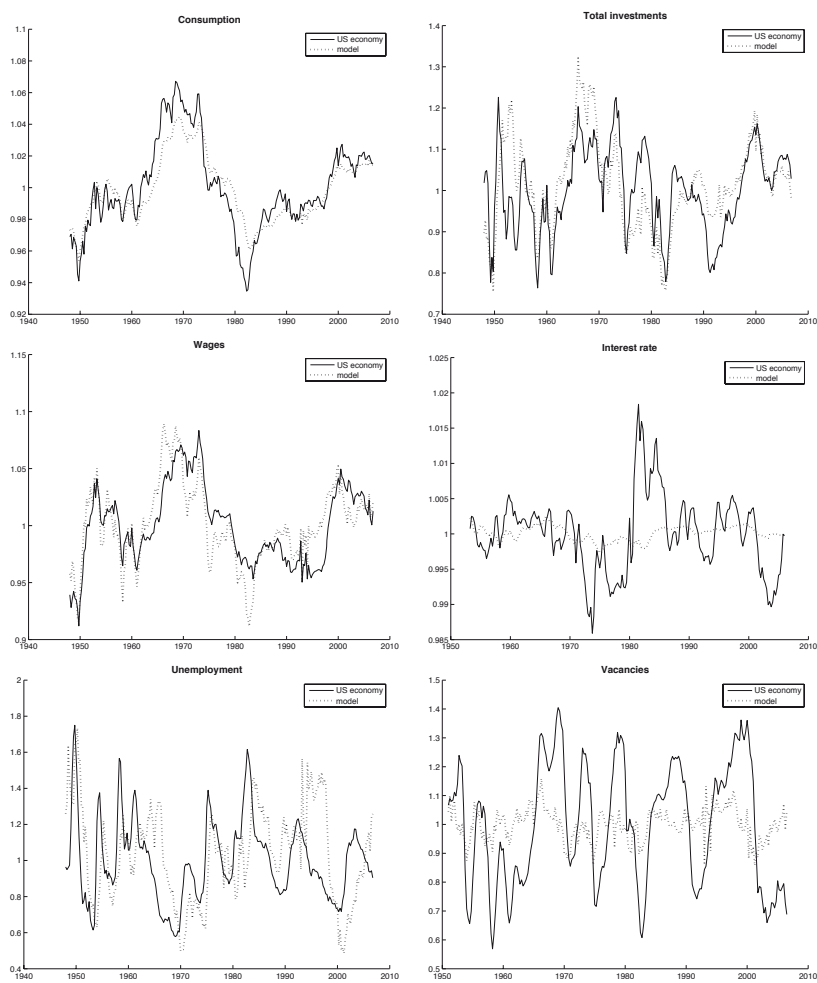
Given the behavior of employment, the volatility of unemployment generated by the model is higher than the data counterpart. The model roughly

Table 4: Selected moments of the US economy and matching model economy

	A. US Economy														
	Standard deviations					Correlation with output					Autocorrelation				
	cycle [2,200]	high [2,32]	medium [32,200]	relative to output	cycle [2,200]	high [2,32]	medium [32,200]	cycle [2,200]	high [2,32]	medium [32,200]	cycle [2,200]	high [2,32]	medium [32,200]		
Output	0.039	0.016	0.035	1.00	1.00	1.00	1.00	0.96	0.81	0.99	0.96	0.81	0.99		
Consumption	0.028	0.008	0.027	0.73	0.88	0.76	0.91	0.98	0.74	0.99	0.98	0.74	0.99		
Investments	0.103	0.069	0.076	2.66	0.73	0.84	0.74	0.91	0.81	1.00	0.91	0.81	1.00		
Interest rate	0.005	0.003	0.004	0.14	-0.33	-0.15	-0.39	0.93	0.79	0.99	0.93	0.79	0.99		
Wages	0.034	0.010	0.032	0.89	0.81	0.77	0.82	0.97	0.73	0.99	0.97	0.73	0.99		
Unemployment	0.225	0.145	0.168	5.82	-0.84	-0.82	-0.88	0.94	0.88	0.99	0.94	0.88	0.99		
Vacancies	0.208	0.131	0.172	5.63	0.53	0.83	0.45	0.96	0.90	0.99	0.96	0.90	0.99		
Employment	0.013	0.008	0.010	0.34	0.84	0.84	0.87	0.95	0.88	0.99	0.95	0.88	0.99		
Labor share	0.014	0.008	0.012	0.37	0.15	-0.23	0.26	0.87	0.61	0.99	0.87	0.61	0.99		
Job finding probability	0.063	0.039	0.050	1.62	0.74	0.77	0.75	0.92	0.81	0.99	0.92	0.81	0.99		
<b>B. Model Economy</b>															
	Standard deviations					Correlation with output					Autocorrelation				
	cycle [2,200]	high [2,32]	medium [32,200]	relative to output	cycle [2,200]	high [2,32]	medium [32,200]	cycle [2,200]	high [2,32]	medium [32,200]	cycle [2,200]	high [2,32]	medium [32,200]		
	Output	0.039	0.016	0.035	1.00	1.00	1.00	1.00	0.96	0.81	0.99	0.96	0.81	0.99	
Consumption	0.022	0.004	0.021	0.56	0.83	0.95	0.84	0.99	0.82	0.99	0.99	0.82	0.99		
Investments	0.106	0.051	0.091	2.74	0.93	0.98	0.91	0.94	0.74	0.99	0.94	0.74	0.99		
Interest rate	0.001	0.000	0.001	0.03	0.44	0.97	0.33	0.95	0.80	0.99	0.95	0.80	0.99		
Wages	0.036	0.016	0.032	0.94	0.97	0.92	0.98	0.94	0.74	0.99	0.94	0.74	0.99		
Unemployment	0.264	0.155	0.236	6.83	-0.40	0.09	-0.52	0.93	0.75	0.99	0.93	0.75	0.99		
Vacancies	0.075	0.043	0.062	2.03	-0.08	0.41	-0.22	0.77	0.29	0.99	0.77	0.29	0.99		
Employment	0.016	0.008	0.014	0.42	0.40	-0.09	0.52	0.93	0.71	0.99	0.93	0.71	0.99		
Labor share	0.014	0.008	0.012	0.37	0.15	-0.23	0.26	0.87	0.61	0.99	0.87	0.61	0.99		
Job finding probability	0.108	0.061	0.086	2.80	0.42	0.11	0.53	0.82	0.42	0.99	0.82	0.42	0.99		

source: own calculations

Figure 12: Time series of US economy and data generated by matching model



source: own calculations

matches the business cycle volatility of unemployment and overstates the medium term variability. This is also apparent from the lower-left panel<sup>78</sup> of Figure 12. Additionally, the countercyclicality of unemployment is understated by the model (it especially applies to the higher frequency component of the cycle, similarly to the employment case), when compared to the data counterpart. On the contrary, the model predicts the correct pattern of persistence of unemployment.

In case of vacancies, the model underestimates its volatility, for both the higher and the medium frequency component of the cycle (see the lower right panel of Figure 12) and it correctly predicts that the medium term volatility is relatively higher than the business cycle volatility. In the data, vacancies are rather procyclical, whereas the model generated vacancies seem to be acyclical. The model also understates the persistence of vacancies (mainly in case of the higher frequency component).

The job finding probability, as predicted by the model, is relatively volatile, when compared the time series, estimated by Shimer (2007). It applies to volatility of both the medium frequency and the higher frequency components. The persistence of job finding probability is correctly reproduced by the model for its medium frequency component, but the model heavily underestimates the persistence of higher frequency component.

As it is apparent from the lower-left panel of Figure 11, the two shock model have problems with reproducing the Beveridge curve. It predicts that the correlation between unemployment and vacancies is highly positive (and amounts to  $+0.76$ ), whereas the data estimate suggest that the relationship is strongly negative, amounting to  $-0.81$ . The reason for this is easy to understand. The data suggest that the US economy for most of the time moves along the Beveridge curve. Shocks to the matching technology act as shifters of the Beveridge curve. The strong negative correlation observed in the data suggest that these kind of shifts do not occur frequently (see also Figure 5, which shows that the shifts occur mostly in medium term frequencies). The similar conclusion was drawn by Shimer (2005), who introduced shocks to the separation rate in his model, in addition to productivity shocks. Changes to the separation rate are very similar to the shocks to matching technology considered here, and they also flatten the Beveridge curve. As the Beveridge curve seems to be an important empirical property of labor markets in many countries (see e.g. Nickell, Nunziata, Ochel, and Quintini 2001, Layard, Nickell, and Jackman 1991), this extension does not seem very promising.

Summing up, the two shock model amplifies the volatility of labor market

<sup>78</sup>Additionally, a bit closer insight into this Figure reflects that unemployment generated by the model lags the true development of unemployment for the US economy by 2-3 quarters (e.g. the peaks or troughs of business cycle do not exactly coincide). The model has also problems with matching the decline of unemployment in the first half of the 90ties and predicts the decline should have occurred in the 2nd half of the 90ties.

variables (especially unemployment and job finding probability), without any consequences for the variability of goods market. But this virtue comes at a cost of the destruction of the Beveridge curve in the model generated data. Additionally, the model variables (especially in case of employment and unemployment) exhibit too little comovement with output, when compared to their data counterparts. The inclusion of the matching technology shocks did not break the strong link between wages and output, that the model predicts. So this route of extending the model, although partly successful, is not without a flaw. Thus, to deal with the volatility of labor market variables, it is worth considering an extension of the model in a different direction.

The next step of the analysis deals directly with the wage determination problem, in order to bring down the procyclicality of wages to the levels observed in the data. As was mentioned in section 5.1.1, the recent literature on unemployment volatility in search-matching models argues that the standard Nash bargaining concept of wage negotiations could be the main reason for the volatility problems. We elaborate on this issue and investigate the model properties in the following section.

### 5.3 Real wage rigidity

The results from the previous chapters confirm that not only the basic model, but also the model extended with matching technology shocks, have problems with properly reflecting the labor market phenomena. In the recent literature, a lot of attention was paid to the issue of wage determination. Standard DMP literature employs the Nash bargaining as a simple and efficient wage determination scheme (for more details, see the chapter 3.5.2 or Pissarides (2000)). The first who stressed the role of wage determination in explanation of observed fluctuations in unemployment was Veracierto (2002) and Shimer (2003).

Robert E. Hall (see Hall 2003, Hall 2005b) noticed that the search-matching theory itself determines only the bargaining set (the range of feasible wages) and not the wage itself. The upper bound of the bargaining set - worker's reservation wage  $\bar{w}$  - equates the unemployment value  $U_t$  to the employment value at the reservation wage  $V_t(\bar{w})$ . The upper bound of the feasible wage range - an employer's reservation wage  $\underline{w}$  is the entire anticipated surplus from the match  $J_t$ . Any wage within the bargaining set  $[\bar{w}, \underline{w}]$  is efficient, in the sense that both worker and employer benefit from the match in the sense of receiving match values, that are at least as large as their respective outside option values.

The Nash wage determination scheme sets the wage that splits proportionally the joint surplus from a match. But then, in the case of e.g. productivity shock, the negotiated wage changes almost proportionally to the change of the employer's reservation wage. When the joint surplus from a match is



relatively large (most economists assume that it is true), then the employer's surplus from a match is almost unchanged and an employer has little incentives to engage in recruiting effort. Thus, even with relatively volatile labor productivity shifts, the volatility of employment and unemployment is small. Additionally, wages follow closely productivity and output. This is exactly what happens in both the basic and the two shock version of our model.

Hall proposed different equilibrium selection rule to pin down wage within the bargaining set<sup>79</sup>. The basic idea is that the previous period's wage sets the norm for this period's wage. This type of norm was discussed by Akerlof, Dickens, Perry, Gordon, and Mankiw (1996) in the context of downward wage rigidity. Hall sets his wage determination rule in a way that do not permit the norm to lie outside the bargaining set. He rationalizes this rule in terms of the aggregation of the individual wage decisions, each perturbed by a match-specific random component, that shifts the bargaining boundaries. If the random component generates the bargaining set that do not contain the norm, the wage is reset to the nearest boundary of bargaining set, otherwise the wage is set according to the norm (the previous wage). This wage selection rule, although it introduces wage rigidity (the equilibrium wage depends on the previous period equilibrium wage), does not distort the formation of efficient matches and it assures that wage lie within the bargaining set. Thus, inefficient separations cannot occur and this kind of wage stickiness is immune to the Barro's critique<sup>80</sup>. Additionally, there is a vast literature on the existence and the nature of wage rigidity (inertia) in price and wage determination, that starts from seminal papers of Friedman (1968) and Phelps (1967)<sup>81</sup>, so the introduction of wage rigidity into the model economy is well supported by the literature<sup>82</sup>.

In order to introduce the Hall's concept of wage norm into the model considered here, one needs to change the model only a bit. Define  $w_t^n$  as a solution to the wage bargaining problem - equation (33). The wage, corrected for the wage norm (the previous average wage  $w_{t-1}$ ), is given by:

$$w_t = \alpha_w w_t^n + (1 - \alpha_w) w_{t-1} \quad (45)$$

<sup>79</sup>The contributions of Hall (2003) and Hall (2005b) was also employed recently in the model developed by Blanchard and Gali (2006).

<sup>80</sup>Barro (1977) criticized sticky wage models, in the spirit of Calvo (1983) (commonly used later), for introducing arbitrary restrictions that intelligent agents could easily avoid. In the case of time-dependent wage stickiness, this equilibrium inefficiency could be easily overcome when agents negotiate over wages in each period. In the case of rigidity introduced by Hall (2003), there are no inefficiencies associated with wage stickiness, so the friction is immune to the Barro's critique.

<sup>81</sup>Majority of the models in Keynesian tradition employ some kind of wage and/or price rigidity to assure inefficient economic fluctuations (see e.g. Smets and Wouters 2003, Christiano, Eichenbaum, and Evans 2005)

<sup>82</sup>This way of introducing wage rigidity into the DSGE model, while being a shortcut to a micro founded wage rigidity, is often used in the literature. For further reference, see e.g. Krause and Lubik (2007), Blanchard and Gali (2006) or Christoffel and Linzert (2005). See also Danthine and Kurmann (2004) for some remarks on the possible microfoundations of the wage norm.

where  $\alpha_w$  is the degree of real wage rigidity<sup>83</sup>. In the steady state, the both wages change proportionally ( $\bar{w} = \frac{\alpha_w \gamma_w}{\gamma_w + \alpha_w - 1} \bar{w}^n$ , where  $\gamma_w$  is the growth rate of wages). It follows that the wage equation, expressed as deviations from the steady state takes the form:

$$\hat{w}_t = \left( \frac{\gamma_w + \alpha_w - 1}{\gamma_w} \right) \hat{w}_t^n + \left( \frac{1 - \alpha_w}{\gamma_w} \right) \hat{w}_{t-1}$$

so the wage is a weighted average of current period's negotiated wage (from the Nash problem) and previous period's wage (the norm).

### 5.3.1 Calibration issues

We follow the same procedure as in the previous sections and present selected moments of the model economy (volatility of unemployment and wages, correlation of wages with output and the Beveridge curve) as functions of the model parameters, as implied by the estimation procedure<sup>84</sup>. Since there is also a considerable uncertainty in the choice of the degree of real wage rigidity,  $\alpha_w$ , we decided to analyze simultaneously the proprieties of the model as a function of both replacement ratio and the degree of wage rigidity. The results of this exercises are depicted in Figure 13. The rest of the model parameters are kept at the values discussed in section 4.

Volatility of unemployment is increasing in both the replacement ratio and the degree of wage rigidity (which itself is linearly decreasing in  $\alpha_w$ ). When the replacement ratio is very high, the unemployment volatility is close to the data estimate for all values of  $\alpha_w$  considered. The same is true for large degree of wage rigidity. For the value of the replacement ratio, that we have chosen ( $\frac{\bar{b}}{\bar{w}} = 0.5$ ), the unemployment volatility roughly matches its data estimate for values of  $\alpha_w$  close to 0.1.

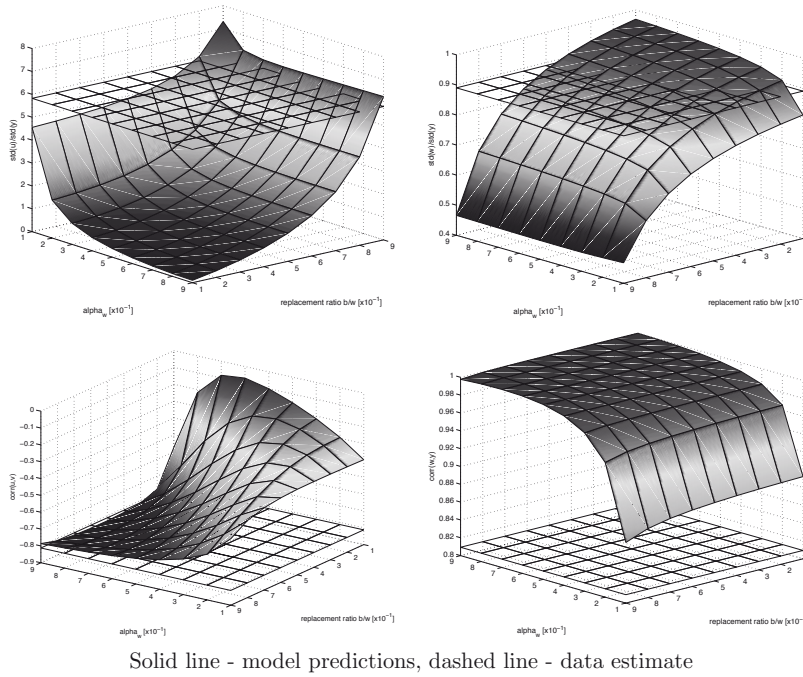
The volatility of wages (relative to GDP) is not lower than 0.45 in our parameter space. For a given  $\alpha_w$  the volatility of wages declines sharply with a replacement ratio. For a given replacement ratio, the volatility of wages declines with the degree of wage stickiness (although very slightly). For replacement rate close to 0.5 volatility of wages is little below the data estimate and matches the estimate almost exactly for high values of  $\alpha_w$ .

The Beveridge curve generated by the model is negative for all values of replacement ratios and degrees of wage stickiness considered. For a given  $\alpha_w$  the unemployment-vacancies correlation declines with the replacement ratio. For high values of the replacement ratio, the unemployment-vacancies correlation declines with  $\alpha_w$ . For lower values of the replacement ratio, the

<sup>83</sup>The calibration of Hall (2003) implies very small value (lower than 10%) of this parameter. Some calculations of Blanchard and Gali (2006) suggest that this parameter is higher, but still quite small (between 0.3 and 0.4).

<sup>84</sup>As in section 5.1.1, we estimate only the volatility and persistence of the intermediate goods technology process.

Figure 13: Properties of the model with rigid wages - moments of variables.



source: own calculations

Beveridge curve first increases with  $\alpha_w$  and then declines. The Beveridge curve generated by the model is close to the data estimate for high values of the both replacement ratio and  $\alpha_w$ . For  $\frac{\bar{b}}{\bar{w}} = 0.5$ , the Beveridge curve is closer to the data estimate for lower degree of stickiness, although it is still substantially negative even for small  $\alpha_w$ .

The wages-output correlation is relatively high for all values of the parameters considered and never reaches the data estimate. It is very slightly decreasing in the replacement ratio. The real wages procyclicality decreases with the degree of wage stickiness. For the replacement ratio of 0.5 wages-output correlation gets closer to the data counterpart for smaller values of  $\alpha_w$ .

Taking into account the above considerations, we chose the replacement ratio of  $\frac{\bar{b}}{\bar{w}} = 0.5$ , as in the previous versions of the model. We also calibrate  $\alpha_w$  to reproduce the wages-output correlation, as estimated from the US data. This gives the calibrated value<sup>85</sup> of  $\alpha_w = 0.086$ . For these values of parameters we get the following stochastic properties of the productivity process:

- volatility of technology shocks  $\sigma_Z = 0.01$ , so it is very close to the

<sup>85</sup>As was mentioned in footnote 83, the literature suggest rather small values of this parameter for a model to be consistent with the data. Additionally, Christoffel and Linzert (2005) claim, that small values of  $\alpha_w$ , between 0.03 and 0.10 are needed for a model with labor market in a DMP spirit to generate substantial inflation persistence.

value estimated in the case of the basic version of the model and implies relatively large internal propagation mechanism of the model with rigid wages, when compared to the standard RBC model.

- persistence of technology shocks  $\rho_Z = 0.87$ , which is much lower than in both the basic version of the model and the RBC model. It also acknowledges the model considerable ability to propagate shocks, without the need for their large persistence.

The value of  $\eta$  implied by the chosen values of both the replacement ratio and degree of wage rigidity is relatively high ( $\eta = 0.78$ ). In particular, it is much higher than the value of  $\sigma = 0.45$ , so the “Hosios” rule (see Hosios 1990) is far from being satisfied for this parametrization of the model. It implies that the US economy may exhibit nontrivial degree of inefficiency associated with decentralization of the search process, but this kind of conclusions need to be more thoroughly investigated. We do not deal with this issue in this study.

### 5.3.2 Impulse response functions

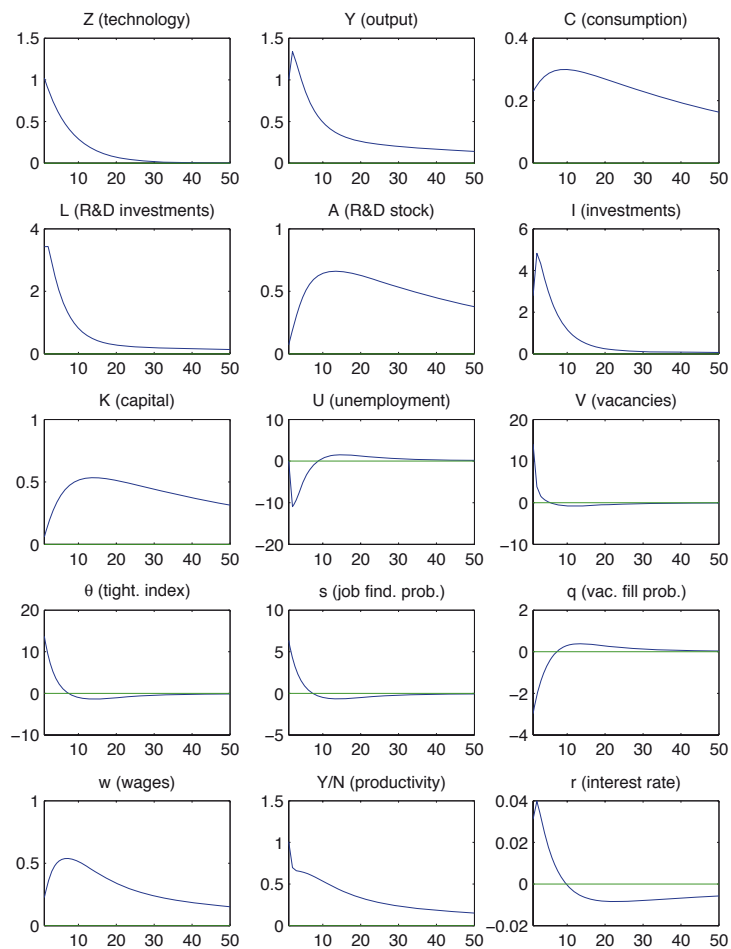
The inclusion of wage rigidity changes the structure of the model, so let us compare the impulse responses of the model with rigid wages (see Figure 14) with the impulse responses of the basic version of the model (Figure 8).

After an initial 1% shock to the technology of production of intermediate goods, output rises more than proportionally and exhibits a hump 2 quarters after the shock. At the maximum, the increase in output is equal to 1.2%. The first periods response in output in the basic version of the model is lower and amounts to a very small increase above 1% induced by the shock itself (since the adjustment of both capital and labor takes at least one quarter). So, the introduction of wage rigidity generates greater amplification of the shocks by the model. After quite rapid initial increase in output, its reaction halves within 2 – 3 years and then output slowly goes back to the steady state path.

The reaction of consumption is also more pronounced when compared with the basic version of the model. At impact, consumption rises by about 0.2% and its reaction increases within the next 3 years (to a maximum of 0.3% and then reverts slowly to the steady state path. The reaction of consumption in the basic version of the model is higher on impact and its time profile is more smooth - the maximum reaction of consumption occurs after about 5 years and amounts to 0.5%.

The investments in R&D sector rise substantially (by 3.4%) on impact. The peak lasts 2 quarters and then the investments in innovative activity slowly revert to the trend. The described reaction of R&D investments translates into an initial rise of the stock of new products, which reaches its peak

Figure 14: Selected impulse response functions of a model with rigid wages to a 1% technology shock



source: own calculations

after about 4 years. Thereafter, the R&D investments are lower than the rate of product obsolescence and the stock of available products steadily reverts to its long-run trend. The reaction of R&D investments in the basic version of the model is less pronounced - the peak occurs on impact and amounts to 2%. R&D investments, after an initial peak, steadily revert to the steady state, which translates into longer period of accumulation of the stock of new products.

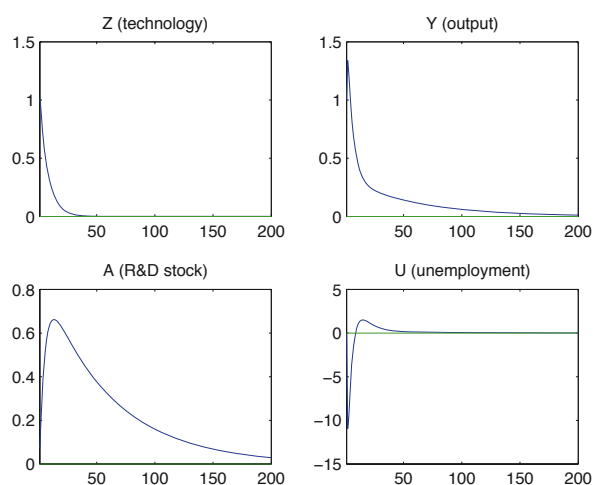
The investments in physical capital rise on impact by almost 3% and reach a peak of 4.8% after 2 quarters. Thereafter, physical investments revert to the trend and after 6 years are very close to the steady state path<sup>86</sup>. The reaction of the capital stock is similar to adjustment of the stock of new products. In the basic version of the model, investments in physical capital react less rapidly but simultaneously go back to the steady state more slowly, which translates into a smoother adjustment of the capital stock.

The wage rigidity makes the adjustment of wages to the technology shock relatively smooth. On impact, wages rise moderately (by about 0.22%). For a couple of quarters wages continue to rise and reach a peak of 0.54% in 2 years after the shock. Then, wages gradually revert to the steady state path. The reaction of wages is very different from the basic version of the model, where wages reach a peak of almost 1% on impact and then revert very slowly to the trend, following closely productivity. In the model with wage rigidity, wages increase much more slowly than productivity (see the lower panel of Figure 14), which implies that the employers' surplus from a match increases (leaving aside the additional effect stemming from a future asset value of a job contract). It translates into more vacancy postings by employers (on impact vacancies rise by about 14%) and lower unemployment (after the initial period of no adjustments of unemployment, it drops by over 9%). The rise in vacancies and drop in unemployment is considerably larger than in the model without wage rigidity, reflecting the important role of wage setting mechanism in volatility of both of these variables. The period of higher vacancy posting is quite short - it lasts about 1.5 years. This observation is consistent with the data, as vacancies exhibit substantial cyclical swings in higher frequencies, see the lower right panel of Figure 16.

The reason for the relatively short period of labor market adjustment is that with more vacancies and less workers applying for them, the labor market quickly becomes more tight (the labor market tightness index  $\theta$  rises) and the probability of filling a vacancy falls (as a mirror image, the probability of finding a job by a household member increases). It becomes harder and more costly for an employer to fill a vacancy. The expected time to find a worker increases (since it is an inverse of a probability of filling a vacancy),

<sup>86</sup>Relatively fast adjustments of investments and smooth reaction of consumption is consistent with the US data, which exhibits relatively large higher frequency variation of investments and relatively large medium frequency variation of consumption.

Figure 15: Impulse responses of a model with rigid wages- longer time horizon



source: own calculations

which translates into higher costs of employing additional worker (recall that the total cost, as perceived by an employer, is a product of one period cost of vacancy posting times the expected time to fill a vacancy). Simultaneously, more tight labor market exerts an upward pressure on negotiated wages, which explains why in the first two years after a shock, wages continue to increase, which additionally reduces the employer's surplus from a successful match and reduces the vacancy posting activity.

Figure 15 shows the impulse response functions for a longer time horizon - up to 50 years. Similarly to the version of the model without wage rigidities, the adjustment of the real variables last for a long time, in particular much longer than the life of the initial shock. The technology shock vanishes after about 10 years and it takes about 40 – 50 years for the output to settle down completely. The R&D stock also adjusts very slowly to changes in the economic environment. The reaction of unemployment is relatively short - about 12 years. This fact seems to fit the stylized facts on US economy, as the relative importance of medium term frequency component of output is greater than the relative importance of medium term component of unemployment. But the answer to the question whether the model with wage rigidity can reproduce the medium frequency variation in the data is an empirical one and will be the subject of the next section.

### 5.3.3 Comparison with US economy behavior

As in the previous sections, we present the Table with the second moments (volatility, comovement with output and persistence) of the relevant variables, as estimated from the US data (the upper panel of Table 5) and as

generated by the model (the lower panel of Table 5). These moments are calculated for the medium term cycle (in frequencies from 2 to 200 quarters), and its higher and medium frequency components. Additionally, Figure 16 shows the time series of consumption, investments, wages, interest rates, unemployment and vacancies from the US economy and the model economy.

The model with rigid wages seems to describe the US economy reasonably well. The model generated consumption is slightly smoother than in the data, like in the basic version of the model (compare also the upper left panel of Figure 16), but the model with rigid wages correctly reflects the fact, that for the US data, the volatility of the medium frequency component of the cycle is much larger than the volatility of the higher frequency component. Additionally, the model roughly matches the pattern of comovement of consumption with output, and almost matches the persistence of consumption at all frequencies.

The volatility of model generated investments exceeds slightly the relevant estimate from the US data, especially in the case of the medium frequency component of the cycle (the volatility of the higher frequency component of cycle in investments is too low), although the general pattern of volatility is well reflected by the model with rigid wages. The model also predicts too much correlation of investments with output for all frequencies considered (especially in the case of the higher frequency component), but also here the correlation pattern across frequencies is quite well reproduced. The persistence of investments is correctly reproduced by the model.

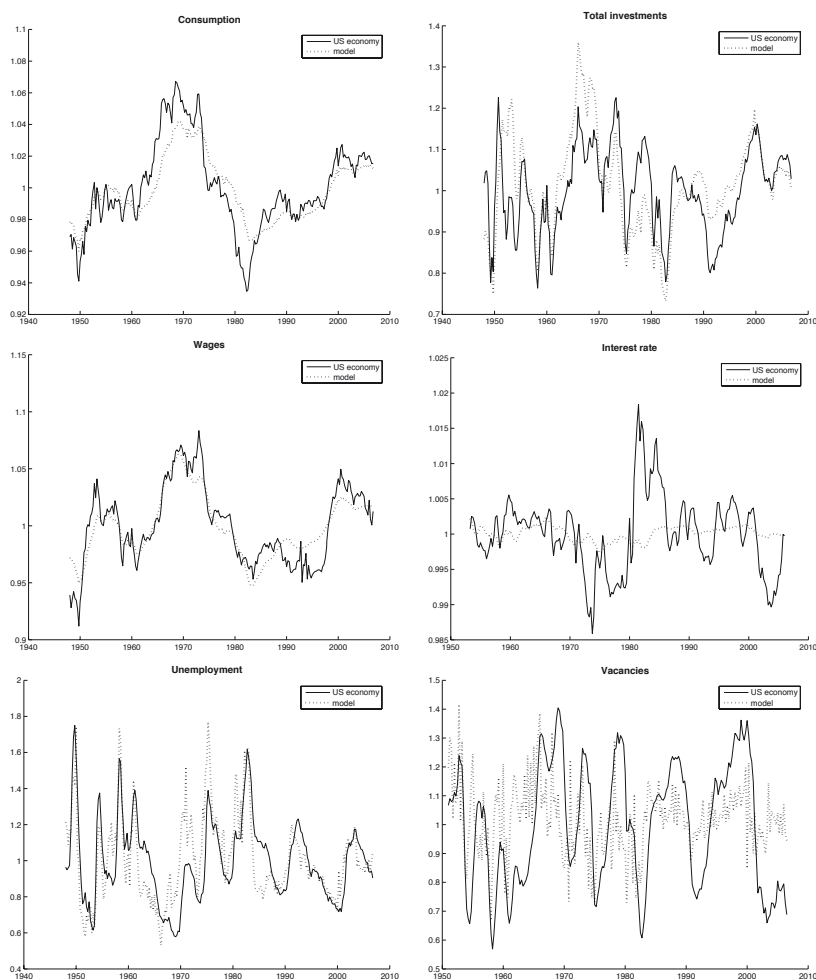
As in the other versions of the model, the interest rate is too smooth and, contrary to the data, positively correlated with output. As was mentioned before, fitting the data along this margin is quite hard for standard models of business cycle, without the specific enhancements aiming at solving this particular difficulty. As we are not concentrating on this issue, we leave intentionally this shortcoming of the model for further research. One should also mention, that despite the problems with reproduction of the volatility and comovement with output, the persistence of the model generated interest rates coincides with the data estimates.

The model with rigid wages seems to successfully reproduce the behavior of the labor market variables. The volatility of the wages is slightly smaller when compared with the US data estimate. The model correctly reflects the volatility of the medium frequency component of the data and underestimates the extent of volatility of the higher frequent component of the medium term cycle. The visual inspection of the middle-left panel of Figure 16 assures that the model generates the behavior of wages that is very close to the one observed in the US data. The procedure used to calibrate the extent of wage rigidity assures that the wages-output correlation coincides with the data estimate, although the model generates too low correlation in the higher frequency component of the cycle and slightly too high correlation





Figure 16: Time series of US economy and data generated by model with rigid wage



source: own calculations

of the medium frequency component of the cycle. The model also predicts slightly higher persistence of the higher frequency component of the cycle in wages in comparison with the data estimate. So, the introduction of wage rigidity improved the fit of the volatility of wages and its comovement with output to their data counterparts. But what about the other aspects of the labor market? Did the introduction of wage rigidity improved the model predictions along other dimensions?

For sure the answer is positive in case of unemployment. The volatility of model generated unemployment almost exactly coincides with the US data estimate. Also the volatility of both the high frequency and the medium frequency components of the cycle is matched by the model with wage rigidities. The correlation of unemployment with output is slightly higher than in the data, but it is mainly due to the countercyclicality of the medium term component of the cycle (which is less countercyclical than in the data), as the correlation of the higher frequency components of unemployment and output are almost exactly matched. The model generated persistence of unemployment is slightly lower than in the US data, especially for the higher frequency component. The good performance of the model in reproducing the unemployment behavior is confirmed by the visual inspection of the lower-left panel of Figure 16. The model correctly predicts the behavior of unemployment in the 1950s and the 1960s. There are some problems with reflecting the decline of the unemployment at the end of the 1960s. The model predicts also larger, than in reality, increase of unemployment at the beginning and in the middle of the 1970s. The model correctly reproduces the increase of unemployment at the beginning of the 1980s, but then it predicts faster decline of unemployment rate in the second half of the 1980s. The behavior of unemployment in the 1990s and latest years is generally correctly reflected by the model<sup>87</sup>.

As the employment rate mirrors the unemployment rate, all remarks made for the latter are also valid in the case of the former variable. In particular, the model generated employment rate is as volatile as in the data, comoves too little with output (especially for the medium frequency component of the cycle) and is somewhat less persistent than in the data. These affect properties of the model generated labor share. The labor share is a bit too smooth, which originates in too little volatility of the medium frequency component of the cycle. The model generated labor share is procyclical in the model, in contrast to the data, while labor share is rather acyclical or slightly countercyclical. The model with rigid wages also predicts too little

<sup>87</sup>The model with additional shocks to the Beveridge curve (see chapter 5.2 and Figure 12 therein) is better in reproducing the behavior of unemployment in the 70-ties and 80-ties, but have problems with fitting the real data in the other periods, especially in the 60-ties and 90-ties. It may suggest that in the 70-ties and 80-ties there were additional shocks hitting the economy (besides the technology shocks) that had substantial impact on the behavior of unemployment (e.g. oil crises).

persistence of the labor share. When compared with the predictions of the basic version of the model, the labor share generated by the model with wage rigidity is much more volatile, which is consistent with the data, but simultaneously it is less persistent, which is contrary to the data. In both versions of the model labor share is countercyclical, which is at odds with the data (the calibration strategy used in the model with shocks to matching technology assures perfect fit of the labor share, so it is hard to compare its predictions with respect to this variable).

The volatility of vacancies is somewhat lower than in the data and is smaller than the volatility of unemployment (in the data, both volatilities are almost the same). This prediction originates mainly from the underestimation of the volatility of the medium frequency component of the cycle. The model predicts procyclicality of vacancies, but the comovement of vacancies with output is too weak. The model with rigid wages generates too little persistence of vacancies, due to the underestimation of the persistence of the higher frequency component of the cycle. The introduction of wage rigidity increases the correlation of vacancies with output and the volatility of vacancies, but additionally - decreases its persistence.

The model generates job finding probability that is more volatile than its data counterpart, especially in the case of the higher frequency component of the cycle. The model correctly predicts the procyclicality of job finding probability, but it underestimates its persistence. The introduction of wage rigidity amplifies significantly the volatility of the job finding rate and dampens its comovement with output - both these features are supported by the data. Simultaneously, contrary to the data, wage rigidity decreases the persistence of the job finding probability.

The model generated Beveridge curve amounts to  $-0.456$ . It is a half of the data estimate, which is  $-0.81$ . In the basic version of the model, the Beveridge curve was almost matched by the model. Thus, one may conclude that the introduction of the wage rigidity flattens the Beveridge curve, but it does not destroy the Beveridge curve completely, as in the case of the model with matching technology shocks, described in the section 5.2.

Summing up, the introduction of wage rigidity improves the model performance in many dimensions with rather small costs. It only slightly affects the model's fit in the goods and capital markets, but it improves the model's fit in the labor market. Namely, the introduction of wage rigidity allows to match quite well the volatility and cyclicity of wages, unemployment, employment, vacancies and job finding probability. All the moments (standard deviations, correlations with output and autocorrelations) of the variables generated by the model roughly coincide with the data estimates, although of course the model does not exactly match all the moments. As it was mentioned before, the model is somewhat stylized and it does not have additional ingredients to improve its fit to the data. Additionally, the only driving force

of the model (following the tradition in the real business cycle literature) is a technology shock. It is hard to argue that the technology shock was the only driving force of the US economy in the whole post-war period, but the results of the model with rigid wages show that indeed, the technology shock might be a very important source of the volatility in the US economy. Furthermore, this finding applies not only to the goods or the capital markets, but also to the labor market.

## 5.4 Comparison of the model with rigid wages with the benchmark economy

In order to assess the importance of the endogenous growth component of the model, we performed a comparison of the model outcomes with the outcomes of the model without endogenous growth component - the benchmark model. As only the model with rigid wages is able to account for the volatility of labor market variables and properly reflect other aspects of the labor market, we decided to limit the comparison only to the models with wage rigidity.

The structure of the benchmark model is analogous to the model with wage rigidities, so it is a standard model of search with real rigidity. The benchmark model does not include the endogenous growth component, so in consequence:

- there is no R&D sector in this economy,
- the number of products available in the economy is constant (there is no additional variation in output due to changes in the products),
- households can only invest in physical capital,
- households receive directly profits from production of intermediate goods instead of receiving them indirectly as yields on investments in creation of new products,
- the relative price of intermediate and final good is constant.

The calibration of the benchmark model is very similar to the calibration of the model with wage rigidity, also in case of the degree of wage rigidity (parameter  $\alpha_w$ ). In order to keep the model as close as possible to the model with rigid wages, we decided to treat investments in R&D as consumption expenditures<sup>88</sup>, which results in the consumption share  $\frac{\bar{C}}{\bar{Y}} = 0.778$ . Stochastic parameters of the underlying shocks were set at the values estimated for the

<sup>88</sup>It seems quite controversial, but with the calibration strategy applied here increasing the share of investments in output without changing the capital-output ratio results in higher depreciation rate. And this implies different values of steady state gross interest rate and households' time preference  $\beta$ . Instead, we decided not to change the share of investments in output, but to increase the consumption share.

model with endogenous growth mechanism, so the volatility of productivity shocks was set at  $\sigma_Z = 0.01$  and the persistence of technology shocks was set at  $\rho_Z = 0.87$ . This calibration implies small differences in the parametrization of both models and facilitates the comparison between them.

We simulated the model using random realization of shocks and then we calculated the moments of the variables, using the procedure analogous to the one described in section 5.1.3. The results of this exercise are presented in Table 6, where the upper panel summarizes the moments of the US economy. These are limited to: 1) the standard deviations of the medium term cycle as well as its high and medium frequency components, 2) relative standard deviations of a medium term cycle of a given variable and output and 3) a correlation of a medium term cycle of a given variable and output. The upper panel of Table 6 shows these moments calculated from the US data, the middle panel reproduces the relevant moments generated by the model with real wage rigidities and the lower panel shows the results for the benchmark economy.

The benchmark model almost matches the volatility of the higher frequency component of the cycle, but it underestimates the volatility of the medium frequency component of the cycle, although the difference is rather small. The volatility of consumption generated by benchmark model is smaller than both the one observed in the US economy and for the model with endogenous growth, especially in the case of medium frequency component of the cycle (the volatility of higher frequency component of the cycle is the same as in the model with endogenous growth). The benchmark model matches the volatility of the higher frequency component of the investments cycle (the model with endogenous growth underestimated this part of overall volatility) and overstates, as well as the model with endogenous growth, the volatility of medium frequency component of the cycle. The overall volatility of the investments in the benchmark model is too high.

The benchmark model is slightly better at fitting the correlation of consumption with output but generates too much procyclicality of investments. So, summarizing the comparison for the goods market variables, one can say that the model with endogenous growth generates more medium frequency variation than the model without the endogenous growth component, but the difference is quite small. But the most interesting question, from the viewpoint of this study, is how the two models compare in case of the labor market variables?

The answer to this question is in the last rows in each panel of Table 6. They show the moments of wages, unemployment, vacancies and labor share for the US economy, for the model with endogenous growth and for the benchmark model. It is apparent that the model with endogenous growth outperforms the benchmark model in reproducing the volatility of labor market variables, as exhibited by the US economy, especially in the case of unem-

Table 6: Selected moments of the US data and models with and without endogenous growth

<b>A. US Economy</b>					
	Standard deviations				Correlation with output cycle [2,200]
	cycle [2,200]	high [2,32]	medium [32,200]	relative to output	
Output	0.039	0.016	0.035	1.00	1.00
Consumption	0.028	0.008	0.027	0.73	0.88
Investments	0.103	0.069	0.076	2.66	0.73
Wages	0.034	0.010	0.032	0.89	0.81
Unemployment	0.225	0.145	0.168	5.82	-0.84
Vacancies	0.208	0.131	0.172	5.63	0.53
labor share	0.014	0.008	0.012	0.37	0.15

<b>B. Model with rigid wages and endogenous growth</b>					
	Standard deviations				Correlation with output cycle [2,200]
	cycle [2,200]	high [2,32]	medium [32,200]	relative to output	
Output	0.039	0.016	0.035	1.00	1.00
Consumption	0.021	0.003	0.021	0.55	0.77
Investments	0.113	0.056	0.097	2.93	0.92
Wages	0.030	0.005	0.030	0.78	0.81
Unemployment	0.223	0.150	0.176	5.77	-0.65
Vacancies	0.127	0.107	0.076	3.43	0.25
Labor share	0.011	0.008	0.007	0.28	-0.58

<b>C. Benchmark model - with rigid wages and without endogenous growth</b>					
	Standard deviations				Correlation with output cycle [2,200]
	cycle [2,200]	high [2,32]	medium [32,200]	relative to output	
Output	0.036	0.017	0.031	1.00	1.00
Consumption	0.018	0.003	0.017	0.49	0.82
Investments	0.116	0.069	0.091	3.21	0.95
Wages	0.028	0.005	0.027	0.77	0.85
Unemployment	0.106	0.078	0.068	2.93	-0.67
Vacancies	0.069	0.055	0.042	1.87	0.27
Labor share	0.005	0.004	0.003	0.14	-0.46

source: own calculations

ployment, vacancies and labor share. The behavior of wages is very similar across the two models (it is not surprising as the data generating process for wages is almost the same for the two specifications). It is also true in case of the procyclicality of wages, which is very similar for both models.

The model almost matches the overall volatility of unemployment, whereas the benchmark model generates almost half of the variance of unemployment that is necessary to match the US data. Additionally, the endogenous growth model generates, in line with the data, higher relative volatility of medium frequency component of the cycle, whereas the benchmark model generates higher relative volatility of the higher frequency component of the cycle. Both models generate very similar pattern of cyclicity of unemployment, roughly in line with the data.

The same remarks apply in case of vacancies and labor share, but with the caveat that the endogenous growth model understates the volatility of vacancies and labor share and the benchmark have even greater problems with matching the data along these margins. Both models generate very similar pattern of cyclicity of vacancies and labor share.

Summing up, the introduction of endogenous growth component into the standard model with search and real rigidities greatly improves the performance of the model in the labor market. Furthermore, in many respects the model successfully reproduces the behavior of the US economy.



# Summary, conclusions and implications

This study aims at a verification of three theses:

1. Variation of economic activity in medium term frequencies is substantial and comparable to the variation in business cycle frequencies.
2. A large part of medium term fluctuations in both labor and goods markets may be explained by the same sources.
3. Endogenous growth mechanism is able to explain a large part of variation in medium term fluctuations.

After reviewing and discussing the literature that is relevant from the perspective of our study, we focus on verification of the first thesis. In order to assess the relative importance of the medium frequency component of the cycle in the overall medium term fluctuations we apply the spectral filters, developed by Christiano and Fitzgerald, in order to extract the relevant information from the data. We show that fluctuations in the medium term frequencies are substantial in the macroeconomic data. More importantly, our calculations and evidence from spectral analysis show that these kind of fluctuations exhibit substantial variance. In most of the variables checked, the medium frequency fluctuations seems to be at least as volatile as fluctuations in the usual business cycle frequencies. So, the evidence presented allows us to verify positively out first thesis, opening the floor for the next ones. Additionally, the pattern of comovement of various variables in the medium term frequencies is very similar to the pattern observed for higher frequencies which suggests that both these components could be analyzed jointly and could result from the same sources. So, our next step is to build a unified model of the medium term business cycle.

We propose a theoretical model of medium term cycle in both the goods and the labor market. We keep the structure of the model as simple as possible<sup>89</sup> in order to focus attention on the main ingredients of the model

<sup>89</sup>Additionally, due to the fact that we are trying to check whether goods and labor markets share the same sources of fluctuations, we leave labor supply considerations outside of the analysis, as these influence rather the labor market only. So, the model specification completely abstracts from labor supply decisions. Also the calibration of the model assures

- the endogenous growth mechanism and the search-matching mechanism of the labor market. These two components incorporated into the otherwise standard RBC model introduce two important channels into the model propagation mechanism. The search-matching framework allows us to introduce unemployment into the definition of equilibrium and creates a channel that propagates short run changes to economic environment (in our context: efficiency of labor and capital employment in production process) into labor market. The endogenous growth component introduces another channel that, in turn, propagates short run shocks into medium term fluctuations of economic activity via investments in R&D and introduction of new products into the market. So, the constructed model has an potential ability to generate medium term cycle in the goods and the labor markets, consistent with the data.

After providing the details on the structure of the model, we discuss our calibration strategy. Most of the model parameters are calibrated on the basis of direct evidence from the US economy, taken from the official and publicly available data sources. A small subset of the model parameters is taken from the literature on the issue. We use Maximum Likelihood estimation in order to pin down the parameters of the stochastic part of the model, extracting also a series of shocks giving rise to the observed evolution of US output. This approach additionally allows us to calculate and asses (against US data) the behavior of the time series of the model generated variables.

The assessment of the model (we refer to it as a 'basic model') is rather mixed. The model rightly predicts the relative importance of fluctuations in medium and higher frequencies. The model generates also the observed volatility of output using shocks with relatively small volatility and persistence, emphasizing the strong internal amplification and propagation mechanism. But, although the model roughly matches the behavior of variables describing the evolution of the goods market, it exhibits serious shortcomings in reproducing labor market behavior. It predicts too low volatility of most of the labor market variables, particularly in case of vacancies and unemployment. Additionally, it generates too much comovement of wages with output (almost unitary correlation, at odds with the data).

In order to perform the verification of the next theses of this study, we need to construct a model that correctly predicts the behavior of all markets in the economy. So, we introduce separately two extensions that aim at increasing the volatility of labor market. These extensions are used in the literature and bring the standard models with search-matching closer to the data.

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that labor supply considerations are left outside - all relevant variables are expressed in relation to the size of labor supply - which is treated as a normalizing variable). Allowing for endogenous labor supply decisions in the model would improve its data fit, but will not contribute to the verification of our theses.

In the first extension, we enhance the model with shocks to the efficiency of matching process (Beveridge curve shifters). This modification does not change much the model's predictions for the goods market, but it affects primarily the labor market. This shock is similar in nature to the job destruction shock, analyzed e.g. by Shimer (2005). But Shimer showed that, although the volatility of labor market rises, the shock introduces additional problems in different dimensions of model fit to the data, which proved to be true in our case. So, this additional source of volatility indeed increased the volatility of unemployment, employment and job finding probability to the values little above than the ones observed for the US economy. Thus, the matching shock changes the model predictions in the right direction, but simultaneously lowers substantially the correlation of unemployment, vacancies and job finding probability with output. And what is more important, additional shock do not fix the problem of too high procyclicality of wages, inherited from the basic version of the model. Additionally, this extension completely destroys the Beveridge curve (negative unemployment-vacancies correlation) in the model predictions. So, matching technology shock improves the model predictions in some directions, but simultaneously worsens the model fit to the data in some other dimensions, leaving us with a mixed feeling of its success in describing properly the US economy.

So, we decide to extend the model in different dimension, that is proposed in the literature. Following e.g. Hall (2005b) or Shimer (2004), we introduce wage rigidity into the model in order to break the tight link between wages and productivity. It affects the evolution of the producer's surplus from a job contract and thus - the recruiting effort and the number of vacancies, leading to the more volatile labor market. We use Robert's E. Hall idea of a wage norm as a source of rigidity, as it does not distort directly agent's decisions regarding job contracts and is immune to the critique of Barro (1977).

The introduction of wage rigidity improves the model performance in many dimensions with rather small costs. Our calibration strategy assures that the model reproduces the wage-output correlation. Simultaneously, our model relatively closely follows the data with respect to the volatility and cyclicity of unemployment, employment, vacancies and job finding probability. Additionally, it roughly reproduces the behavior of various variables at different frequencies. The only shortcoming of the results is the Beveridge curve, being about half of the data estimate, but on the bright side - still negative. What is worth emphasizing in this context - we reach a reasonable fit of the model with only one stochastic shock hitting the economy, namely a technology shock.

So, we successfully verify our second thesis - we show that it is possible to construct a unified theory, a general equilibrium model in our case, that explains a considerable part of both the higher and the medium frequencies fluctuations in both the goods and the labor markets. Additionally, we

show that our theses are true when one assumes that the most important stochastic shock affecting the economy is the technology shock. There is no doubt among economists that the technology shock is an important source of economic volatility. The importance of technology shock is one of the central elements to the proponents of the real business cycle theory, a statement being stressed and empirically proved by an important paper of Chari, Kehoe, and McGrattan (2007). It assures, that our approach to seek for a common source of medium term cycle among variables describing both goods and labor market in technology disturbances is theoretically and empirically valid.

In order to verify our third thesis, we construct a benchmark model, without the endogenous growth component, in order to assess the empirical importance of this mechanism in explaining medium term cycle. We show that the endogenous growth component generates a substantial part of the medium frequency variation in the data. It applies especially to the labor market, as the model without endogenous growth component cannot generate enough variance in medium frequencies of important labor market variables. So, we positively validate our last thesis, showing that the mixture of search and endogenous growth mechanisms performs quite well in reproducing the behavior of US economy, especially when considering its ability to generate cycles in the medium term frequencies and its ability to generate enough volatility and comovement of the labor market variables.

Apart from emphasizing the importance of the medium frequency fluctuations in the data and construction of a unified model of the medium term cycle and its application to the US economy, our study have at least two broader implications. First, in order to account for variation of economic development in medium term frequencies, there is no need to build a new generation of the macromodels. It is enough to enhance the current models with some aspects of the endogenous growth theory or maybe other mechanism yet to be discovered. Additionally, these kind of extensions ought to increase the propagation and amplification mechanism of models.

The second implication is more policy oriented. If the endogenous growth mechanism is an important ingredient to understand the medium term cycle, and it is true, that short run disturbances to the economic environment may lead to medium term fluctuations, then it follows that we should re-think our understanding of macroeconomic consequences of different policies. It especially applies to monetary policy, which is believed to affect economic activity only in the very short run. In the case of the endogenous growth model, like ours, changes to monetary policy will influence the economic fluctuations also in the medium run. The research on the consequences of the endogenous growth for the monetary policy is not a subject of this study, but it is an interesting and new avenue for future research. There are some results on the implications of changes in the number of products on the market arising from

changes in the competition for the monetary policy (see Bilbiie, Ghironi, and Melitz 2007) and these results point out that this channel significantly affects the monetary policy transmission mechanism. In our model the variation in the number of products arises from innovative activity of R&D firms. Monetary policy, affecting both the discount factor and profitability of innovative activity adds new insights into the monetary transmission mechanism, that are worth exploring more extensively in the future.



# Appendix A

## Steady state of the model

The appendix lists the relationships between variables in the model in the non-stochastic steady state. Let us denote the steady state levels<sup>90</sup> of variables with bars (e.g.  $\bar{X}$ ) and steady state growth rate (gross) of a given variable  $X$  with  $\gamma_X$ , so  $\gamma_X = \frac{\bar{X}_t}{\bar{X}_{t-1}}$ .

Consumption:

$$\bar{\lambda} = \bar{C}^{-\varsigma}$$

so  $\lambda$  grows at the rate  $\gamma_{\bar{C}}^{-\varsigma}$ .

Gross return on firms equity:

$$\bar{R}^{-1} = \beta \left( \frac{\bar{C}\gamma_C}{\bar{C}} \right)^{-\varsigma} = \beta\gamma_{\bar{C}}^{-\varsigma}$$

so  $\bar{R}$  is constant on a steady state path.

Capital accumulation:

$$\gamma_K - 1 + \delta = \frac{\bar{I}}{\bar{K}}$$

assures that capital grows at the rate of growth of investments. As the latter share the growth rate with output, it follows that capital grows at the same rate as output.

No arbitrage condition:

$$\bar{R} = 1 + \bar{r} - \delta$$

so it follows that given  $\bar{R}$  is constant, also  $\bar{r}$  is constant on a steady state path.

Unemployment:

$$\bar{U} = 1 - (1 - \rho)\bar{N}$$

<sup>90</sup>The model considered here is a non-stationary one, so there is no unique and constant level of many variables described by the model. Thus, strictly speaking,  $\bar{X}$  denote the level of a given variable in a given point in time, say  $t = 0$ ,  $\bar{X}_0$ . Then, the steady state level of a given variable  $X$  in time  $t$  is given by  $\bar{X}_t = \bar{X}_0\gamma_X^t$ . For the ease of exposition, the time subscripts will be dropped.

so the unemployed and employment share the same growth rate  $\gamma_N$ .

Matching technology:

$$\bar{M} = \sigma_m \bar{U}^\sigma \bar{V}^{1-\sigma}$$

Given that  $\gamma_V = \gamma_U = \gamma_N$ , the number of matches growth at a rate:  $\gamma_M = \gamma_U^\sigma \gamma_V^{1-\sigma} = \gamma_N$ .

The probability that a firm fills an open vacancy:

$$\bar{q} = \frac{\bar{M}}{\bar{V}}$$

is constant along the steady state path (recall that  $M$  and  $V$  share the same growth rate).

The probability that a job-seeking person finds a new job:

$$\bar{s} = \frac{\bar{M}}{\bar{U}}$$

is also constant along the steady state.

Labor market tightness:

$$\bar{\theta} = \frac{\bar{V}}{\bar{U}}$$

is constant in the steady state.

Employment:

$$\begin{aligned} \bar{N} \gamma_N &= (1 - \rho) \bar{N} + \bar{M} \\ \bar{M} &= (\rho + \gamma_N - 1) \bar{N} \end{aligned}$$

so consistently,  $N$  and  $M$  share the same growth rate.

Output:

$$\bar{Y} = \bar{A}^{\mu-1} \bar{Z} \bar{K}^\alpha \bar{N}^{1-\alpha}$$

It follows that, given that  $Z$  is a stationary variable, the growth rate of output must obey the relationship:

$$\gamma_Y = \gamma_A^{\mu-1} \gamma_K^\alpha \gamma_N^{1-\alpha}$$

Given that  $K$  must grow at the same rate as output (capital is produced using only final goods with the use of a standard technology), the growth rates of  $Y$ ,  $A$  and  $N$  must satisfy:

$$\gamma_Y^{1-\alpha} = \gamma_A^{\mu-1} \gamma_N^{1-\alpha}$$

Intermediate good prices:



$$1 = \bar{A}^{1-\mu} \bar{p}$$

so it follows that the price of intermediate good grow at a rate:

$$\gamma_p = \gamma_A^{\mu-1}$$

Job creation condition:

$$\frac{\kappa \bar{w}}{\bar{q}} = \bar{R}^{-1} \left[ \frac{1-\alpha}{\mu} \frac{\bar{Y}}{\bar{N}} \gamma_Y - \bar{w} \gamma_w + (1-\rho) \frac{\kappa \bar{w} \gamma_w}{\bar{q}} \right]$$

It follows then that:

$$\frac{\kappa \bar{w}}{\bar{q}} \left( 1 - (1-\rho) \frac{\gamma_w}{\bar{R}} \right) = \bar{R}^{-1} \left[ \frac{1-\alpha}{\mu} \frac{\bar{Y}}{\bar{N}} \gamma_Y - \bar{w} \gamma_w \right]$$

so using the property of constant labor share along the balanced growth path  $\gamma_w \gamma_N = \gamma_Y$  the following equation holds in the steady state:

$$\begin{aligned} \frac{\kappa \bar{w}}{\bar{q}} \left( 1 - (1-\rho) \frac{\gamma_w}{\bar{R}} \right) &= \frac{\gamma_w}{\bar{R}} \left[ \frac{1-\alpha}{\mu} \frac{\bar{Y}}{\bar{N}} - \bar{w} \right] \\ \frac{\kappa \bar{w}}{\bar{q}} \left( \frac{\bar{R}}{\gamma_w} - (1-\rho) \right) &= \frac{1-\alpha}{\mu} \frac{\bar{Y}}{\bar{N}} - \bar{w} \\ (1-\alpha) \frac{\bar{Y}}{\bar{N}} &= \mu \left[ \bar{w} + \frac{\kappa \bar{w}}{\bar{q}} \left( \frac{\bar{R}}{\gamma_w} - (1-\rho) \right) \right] \end{aligned}$$

which states that jobs are created up to the point where the marginal product of labor is equal to total marginal costs of labor (wages plus the term related to cost of adjusting vacancies), adjusted by a markup charged by an intermediate firm. One may reshape the steady state relation for job creation (using also our restriction of constant labor force, implying  $\gamma_w = \gamma_Y$ ) to get:

$$\frac{(1-\alpha) \frac{\bar{Y}}{\bar{N}}}{\bar{w}} = \mu \left[ 1 + \frac{\kappa}{\bar{q}} \left( \frac{\bar{R}}{\gamma_Y} - (1-\rho) \right) \right]$$

which says that without the cost of adjusting vacancies, the relation of labor product to wages is given by the markup.

Demand for capital:

$$\mu \cdot \bar{r} = \alpha \frac{\bar{Y}}{\bar{K}}$$

as  $Y$  grows at the same rate as  $K$ , so  $\gamma_r = \gamma_{mc} \gamma_A^{1-\mu}$ . We know that  $r$  must be constant along the balanced growth path, so  $\gamma_{mc} = \gamma_A^{\mu-1}$

Wages:

$$\bar{w} = \frac{(1-\eta)}{1-\eta\kappa\theta} \bar{b} + \frac{\eta}{1-\eta\kappa\theta} \frac{1-\alpha}{\mu} \frac{\bar{Y}}{\bar{N}}$$

Since  $\frac{\gamma_Y}{\gamma_N} = \gamma_w$  and  $\theta$  is a stationary variable, it follows that  $\gamma_b = \gamma_w$  must

hold. Using the steady state relationship for job creation one may get:

$$\begin{aligned}\bar{w} &= \frac{(1-\eta)\bar{b}}{1-\eta\kappa\bar{\theta}} + \frac{\eta\bar{w}}{1-\eta\kappa\bar{\theta}} \left[ 1 + \frac{\kappa}{\bar{q}} \left( \frac{\bar{R}}{\gamma_Y} - (1-\rho) \right) \right] \\ \bar{w} \left[ 1 - \eta\kappa\bar{\theta} - \eta \left( 1 + \frac{\kappa}{\bar{q}} \left( \frac{\bar{R}}{\gamma_Y} - (1-\rho) \right) \right) \right] &= (1-\eta)\bar{b} \\ \bar{w} \left[ 1 - \eta \left( 1 + \frac{\kappa}{\bar{q}} \left( \frac{\bar{R}}{\gamma_Y} - (1-\rho-\bar{s}) \right) \right) \right] &= (1-\eta)\bar{b}\end{aligned}$$

Profits of the representative intermediate good firm:

$$\bar{\pi} = \bar{A}^{-1} \left( \left( \frac{\mu-\alpha}{\mu} \right) \bar{Y} - \bar{w} (\bar{N} + \kappa\bar{V}) \right)$$

Given that  $V$  grows at the same rate as  $N$ , and  $\gamma_w\gamma_N = \gamma_Y$  along the balanced growth path, all the elements of the profits grow at the same rate, equal to  $\frac{\gamma_Y}{\gamma_A}$ .

Value of a new intermediate good (given that in equilibrium, households pricing kernel is equal to  $R_t^{-1}$ ):

$$\begin{aligned}\bar{V}^I &= \bar{A}\bar{\pi} + (1-\phi) \frac{\bar{A}\bar{V}^I\gamma_{V^I}}{\bar{A}\gamma_A\bar{R}} \\ \bar{V}^I &= \frac{\left( \frac{\mu-\alpha}{\mu} \right) \bar{Y} - \bar{w} (\bar{N} + \kappa\bar{V})}{\left( 1 - (1-\phi) \bar{R}^{-1} \frac{\gamma_{V^I}}{\gamma_A} \right)}\end{aligned}$$

So, as  $R$  is constant along the balanced growth path, it follows that:

$$\gamma_{V^I} = \gamma_Y$$

Production of new intermediate goods:

$$\gamma_A = 1 + \chi \left( \frac{\bar{L}}{\bar{K}} \right)^\psi - \phi$$

since on the balanced growth path the relation of loans to capital is constant (both quantities share the same steady state growth rates, equal to growth of output), the growth of new intermediates is also constant.

Zero-profit condition:

$$\begin{aligned}(1-\phi) \bar{R}^{-1} \frac{\bar{V}^I\gamma_{V^I}}{\bar{A}\gamma_A} [\bar{A}\gamma_A - (1-\phi)\bar{A}] &= \bar{L} \\ (1-\phi) \bar{R}^{-1} \frac{\bar{V}^I\gamma_Y}{\gamma_A} [\gamma_A - (1-\phi)] &= \bar{L}\end{aligned}$$

so as long as  $R$  is constant and  $V^I$  and  $L$  share the same growth rate (which is true on the balanced growth path), zero-profit condition is stationary.

Resource constraint:

$$\bar{Y} = \bar{C} + \bar{I} + \bar{L} + \kappa \bar{w} \bar{V}$$

It assures that  $Y$ ,  $C$ ,  $I$  and  $L$  and  $wV$  share the same growth rate.



## Appendix B

### Deviations from steady state

The Appendix B gives the details of the derivation of the model as deviations from the steady state. This representation of the model is stationary and thus is used in Dynare toolbox to solve the model. Following the convention in the literature, let us denote a deviation of a given variable  $X$  from a steady state  $\bar{X}$  in a given period  $t$  with a hat, so  $\hat{X}_t = \frac{X_t - \bar{X}}{\bar{X}}$ . Then the equations of the model can be represent as follows:

Consumption:

$$\frac{\lambda_t}{\bar{\lambda}} = \frac{(\bar{C} \frac{C_t}{\bar{C}})^{-\varsigma}}{\bar{C}^{-\varsigma}}$$

$$\hat{\lambda}_t = \hat{C}_t^{-\varsigma}$$

Savings:

$$1 = \frac{\beta \frac{R_{t+1} \lambda_{t+1}}{\lambda_t}}{\beta \frac{\bar{R} \bar{\lambda} \bar{\gamma}_C^{-\varsigma}}{\bar{\lambda}}} \gamma_C^{-\varsigma}$$

$$1 = \hat{R}_{t+1} \frac{\hat{\lambda}_{t+1}}{\hat{\lambda}_t}$$

Capital accumulation:

$$\frac{K_{t+1}}{\bar{K} \gamma_K} \gamma_K = (1 - \delta) \frac{K_t}{\bar{K}} + \frac{\bar{I}}{\bar{K} \bar{I}}$$

$$\gamma_Y \hat{K}_{t+1} = (1 - \delta) \hat{K}_t + \frac{\bar{I}}{\bar{K}} \hat{I}_t$$

No arbitrage condition:

$$\frac{\lambda_{t+1} R_{t+1}}{\bar{\lambda} \bar{R}} = \frac{\lambda_{t+1}}{\bar{\lambda}} \frac{1 - \delta + \bar{r} \frac{r_{t+1}}{\bar{r}}}{1 - \delta + \bar{r}}$$

$$\hat{\lambda}_{t+1} \hat{R}_{t+1} = \hat{\lambda}_{t+1} \left( \frac{1 - \delta}{\bar{R}} + \frac{\bar{r}}{\bar{R}} \hat{r}_t \right)$$

Unemployment:

$$\frac{U_t}{\bar{U}} = \frac{1 - (1 - \rho) N_t}{\bar{U}}$$

$$\hat{U}_t = \frac{1}{\bar{U}} - (1 - \rho) \frac{\bar{N}}{\bar{U}} \hat{N}_t$$

Matching function:

$$\frac{M_t}{\bar{M}} = \frac{\sigma_m U_t^\sigma V_t^{1-\sigma}}{\sigma_m \bar{U}^\sigma \bar{V}^{1-\sigma}}$$

$$\hat{M}_t = \hat{U}_t^\sigma \hat{V}_t^{1-\sigma}$$

The probability that a firm fills an open vacancy:

$$\hat{q}_t = \frac{\hat{M}_t}{\hat{V}_t}$$

The probability that a job-seeking person finds a new job:

$$\hat{s}_t = \frac{\hat{M}_t}{\hat{U}_t}$$

labor market tightness index:

$$\hat{\theta}_t = \frac{\hat{V}_t}{\hat{U}_t}$$

Employment:

$$\frac{N_{t+1}}{\bar{N}\gamma_N} = \frac{(1 - \rho)N_t + M_t}{\bar{N}\gamma_N}$$

$$\hat{N}_{t+1}\gamma_N = (1 - \rho)\hat{N}_t + \hat{M}_t \frac{\bar{M}}{\bar{N}}$$

$$\hat{N}_{t+1}\gamma_N = (1 - \rho)\hat{N}_t + (\gamma_N - (1 - \rho))\hat{M}_t$$

Aggregate output:

$$\frac{Y_t}{\bar{Y}} = \frac{A_t^{\mu-1} Z_t K_t^\alpha N_t^{1-\alpha}}{\bar{A}^{\mu-1} \bar{Z} \bar{K}^\alpha \bar{N}^{1-\alpha}}$$

$$\hat{Y}_t = \hat{A}_t^{\mu-1} \hat{Z}_t \hat{K}_t^\alpha \hat{N}_t^{1-\alpha}$$

Intermediate good prices:

$$\begin{aligned}\frac{p_t}{\bar{p}} &= \frac{A_t^{\mu-1}}{A^{\mu-1}} \\ \hat{p}_t &= \hat{A}_t^{\mu-1}\end{aligned}$$

Job creation condition:

$$\begin{aligned}\frac{\frac{\kappa w_t}{\bar{q}}}{\frac{\kappa \bar{w}}{\bar{q}}} &= \beta \frac{\frac{\lambda_{t+1}}{\lambda}}{\frac{\lambda_{t+1}}{\lambda}} \frac{\gamma_C^{-\varsigma}}{R} \left[ \frac{1-\alpha}{\mu} \frac{Y_{t+1}}{N_{t+1}} - \frac{w_{t+1}}{\bar{q}} + (1-\rho) \frac{\kappa w_{t+1}}{\bar{q}} \right] \\ \frac{\hat{w}_t}{\hat{q}_t} &= \frac{\hat{\lambda}_{t+1}}{\hat{\lambda}_t \bar{R}} \left[ \frac{1-\alpha}{\mu} \frac{Y_{t+1}}{\bar{Y} \gamma_Y} \frac{\bar{q} \bar{Y} \gamma_Y}{\kappa \bar{w} \bar{N} \gamma_N} - \frac{\bar{q} \gamma_w}{\kappa} \frac{w_{t+1}}{\bar{w} \gamma_w} + (1-\rho) \gamma_w \frac{\frac{w_{t+1}}{\bar{w} \gamma_w}}{\frac{\bar{q}}{\bar{q}}} \right] \\ \frac{\hat{w}_t}{\hat{q}_t} &= \frac{\hat{\lambda}_{t+1}}{\hat{\lambda}_t} \frac{\gamma_w}{\bar{R}} \left[ \frac{1-\alpha}{\mu} \frac{\bar{Y}}{\bar{w} \bar{N}} \frac{\bar{q}}{\kappa} \frac{\hat{Y}_{t+1}}{\hat{N}_{t+1}} - \frac{\bar{q}}{\kappa} \hat{w}_{t+1} + (1-\rho) \frac{\hat{w}_{t+1}}{\hat{q}_{t+1}} \right] \\ \frac{\hat{w}_t}{\hat{q}_t} &= \frac{\hat{\lambda}_{t+1}}{\hat{\lambda}_t} \frac{\gamma_Y}{\bar{R}} \left[ \left[ \frac{\bar{q}}{\kappa} + \frac{\bar{R}}{\gamma_Y} - (1-\rho) \right] \frac{\hat{Y}_{t+1}}{\hat{N}_{t+1}} - \frac{\bar{q}}{\kappa} \hat{w}_{t+1} + (1-\rho) \frac{\hat{w}_{t+1}}{\hat{q}_{t+1}} \right]\end{aligned}$$

Demand for capital:

$$\begin{aligned}\frac{\mu r_t}{\mu \bar{r}} &= \frac{\alpha \frac{Y_t}{K_t}}{\alpha \frac{\bar{Y}}{\bar{K}}} \\ \hat{r}_t &= \frac{\hat{Y}_t}{\hat{K}_t}\end{aligned}$$

Wages:

$$\begin{aligned}\frac{w_t (1 - \eta \kappa \theta_t)}{\bar{w} (1 - \eta \kappa \bar{\theta})} &= \frac{(1 - \eta)}{\bar{w} (1 - \eta \kappa \bar{\theta})} b_t + \frac{\eta \frac{1-\alpha}{\mu} Y_t}{\bar{w} (1 - \eta \kappa \bar{\theta}) N_t} \\ \frac{\hat{w}_t (1 - \eta \kappa \bar{\theta} \hat{\theta}_t)}{(1 - \eta \kappa \bar{\theta})} &= \frac{(1 - \eta) \bar{b}}{\bar{w} (1 - \eta \kappa \bar{\theta})} \hat{b}_t + \frac{\eta \frac{1-\alpha}{\mu} \frac{\bar{Y}}{\bar{N}} \hat{Y}_t}{\bar{w} (1 - \eta \kappa \bar{\theta}) \hat{N}_t} \\ \hat{w}_t (1 - \eta \kappa \bar{\theta} \hat{\theta}_t) &= (1 - \eta) \frac{\bar{b}}{\bar{w}} \hat{b}_t + \eta \frac{(1 - \alpha) \frac{\bar{Y}}{\bar{N}} \hat{Y}_t}{\mu \bar{w} \hat{N}_t} \\ \hat{w}_t (1 - \eta \kappa \bar{\theta} \hat{\theta}_t) &= (1 - \eta) \frac{\bar{b}}{\bar{w}} \hat{b}_t + \eta \left[ 1 + \frac{\kappa}{\bar{q}} \left( \frac{\bar{R}}{\gamma_Y} - (1 - \rho) \right) \right] \frac{\hat{Y}_t}{\hat{N}_t} \\ \hat{w}_t (1 - \eta \kappa \bar{\theta} \hat{\theta}_t) &= \left[ 1 - \eta \left( 1 + \frac{\kappa}{\bar{q}} \left( \frac{\bar{R}}{\gamma_Y} - (1 - \rho - \bar{s}) \right) \right) \right] \hat{b}_t \\ &\quad + \eta \left[ 1 + \frac{\kappa}{\bar{q}} \left( \frac{\bar{R}}{\gamma_Y} - (1 - \rho) \right) \right] \frac{\hat{Y}_t}{\hat{N}_t}\end{aligned}$$

Profits of a intermediate good expressed in terms of final good:

$$\begin{aligned}\frac{\pi_t A_t}{\bar{\pi} \bar{A}} &= \left( \left( \frac{\mu - \alpha}{\mu} \right) \frac{\bar{Y}}{\bar{\pi} \bar{A}} \frac{Y_t}{\bar{Y}} - \frac{\bar{w} \bar{N}}{\bar{\pi} \bar{A}} \frac{w_t N_t}{\bar{w} \bar{N}} - \kappa \frac{\bar{w} \bar{V}}{\bar{\pi} \bar{A}} \frac{w_t V_t}{\bar{w} \bar{V}} \right) \\ \hat{\pi}_t \hat{A}_t &= \left( \left( \frac{\mu - \alpha}{\mu} \right) \frac{\bar{Y}}{\bar{\pi} \bar{A}} \hat{Y}_t - \frac{\bar{w} \bar{N}}{\bar{\pi} \bar{A}} \hat{w}_t \hat{N}_t - \kappa \frac{\bar{w} \bar{V}}{\bar{\pi} \bar{A}} \hat{w}_t \hat{V}_t \right)\end{aligned}$$

Value of a new intermediate good (when employing the relation between households pricing kernel and market interest rate):

$$\begin{aligned}\frac{V_t^I}{\bar{V}^I} &= \left( 1 - (1 - \phi) \bar{R}^{-1} \frac{\gamma_{V^I}}{\gamma_A} \right) \frac{\pi_t A_t}{\bar{A} \bar{\pi}} + (1 - \phi) E_t \frac{\frac{A_t}{\bar{A}} \frac{V_{t+1}^I}{\bar{V}^I} \gamma_{V^I}}{\frac{A_{t+1}}{\bar{A} \gamma_A} \gamma_A \frac{R_{t+1}}{\bar{R}} \bar{R}} \\ \hat{V}_t^I &= \left( 1 - (1 - \phi) \bar{R}^{-1} \frac{\gamma_{V^I}}{\gamma_A} \right) \hat{\pi}_t \hat{A}_t + (1 - \phi) E_t \bar{R}^{-1} \frac{\gamma_{V^I}}{\gamma_A} \frac{\hat{A}_t \hat{V}_{t+1}^I}{\hat{A}_{t+1} \hat{R}_{t+1}}\end{aligned}$$

using the fact, that  $\gamma_{V^I} = \gamma_Y$  and  $\gamma_A = \gamma_Y^{\frac{1-\alpha}{\mu}}$  it follows that:

$$\hat{V}_t^I = \left( 1 - (1 - \phi) \bar{R}^{-1} \gamma_Y^{\frac{\mu+\alpha}{\mu-1}} \right) \hat{\pi}_t \hat{A}_t + (1 - \phi) E_t \bar{R}^{-1} \gamma_Y^{\frac{\mu+\alpha}{\mu-1}} \frac{\hat{A}_t \hat{V}_{t+1}^I}{\hat{A}_{t+1} \hat{R}_{t+1}}$$

Production of new intermediate goods:

$$\begin{aligned}\frac{\frac{A_{t+1} \gamma_A}{\bar{A} \gamma_A}}{(\gamma_A - (1 - \phi)) \frac{A_t}{\bar{A}}} - \frac{1 - \phi}{\gamma_A - (1 - \phi)} &= \frac{\chi \left( \frac{L_t}{\bar{K}_t} \right)^\psi}{\chi \left( \frac{\hat{L}_t}{\hat{K}_t} \right)^\psi} \\ \frac{\gamma_A}{\gamma_A - (1 - \phi)} \frac{\hat{A}_{t+1}}{\hat{A}_t} - \frac{1 - \phi}{\gamma_A - (1 - \phi)} &= \left( \frac{\hat{L}_t}{\hat{K}_t} \right)^\psi\end{aligned}$$

Zero-profit condition for entry in innovative sector:

$$\begin{aligned}\frac{(1 - \phi) R_{t+1}^{-1} \frac{V_{t+1}^I}{\bar{A} \gamma_A} \left[ \frac{A_{t+1}}{\bar{A} \gamma_A} \gamma_A - (1 - \phi) \frac{A_t}{\bar{A}} \right]}{(1 - \phi) \bar{R}^{-1} \frac{\bar{V}^I \gamma_Y}{\gamma_A} [\gamma_A - (1 - \phi)]} &= \frac{L_t}{\bar{L}} \\ \hat{R}_{t+1}^{-1} \frac{\hat{V}_{t+1}^I}{\hat{A}_{t+1}} \left[ \hat{A}_{t+1} \frac{\gamma_A}{\gamma_A - (1 - \phi)} - \hat{A}_t \frac{1 - \phi}{\gamma_A - (1 - \phi)} \right] &= \hat{L}_t\end{aligned}$$

Resource constraint:

$$\begin{aligned}\frac{Y_t}{\bar{Y}} &= \frac{\bar{C}}{\bar{Y}} \frac{C_t}{\bar{C}} + \frac{\bar{I}}{\bar{Y}} \left( \frac{I_t}{\bar{I}} \right) + \frac{\bar{L}}{\bar{Y}} \frac{L_t}{\bar{L}} + \kappa \frac{\bar{w} \bar{V}}{\bar{Y}} \frac{w_t V_t}{\bar{w} \bar{V}} \\ \hat{Y}_t &= \frac{\bar{C}}{\bar{Y}} \hat{C}_t + \frac{\bar{I}}{\bar{Y}} \hat{I}_t + \frac{\bar{L}}{\bar{Y}} \hat{L}_t + \frac{\kappa \bar{w} \bar{V}}{\bar{Y}} \left( \hat{w}_t \hat{V}_t \right)\end{aligned}$$



## Bibliography

- ABEL, A. B. (1990): "Asset Prices under Habit Formation and Catching Up with the Joneses," *American Economic Review*, 80(2), 38–42.
- ABOWD, J. M., AND A. ZELLNER (1985): "Estimating Gross Labor-Force Flows," *Journal of Business & Economic Statistics*, 3(3), 254–283.
- AGHION, P., AND P. HOWITT (1998): *Endogenous Growth Theory*. MIT Press, Cambridge.
- AKERLOF, G. A., W. T. DICKENS, G. L. PERRY, R. J. GORDON, AND G. N. MANKIW (1996): "The Macroeconomics of Low Inflation," *Brookings Papers on Economic Activity*, 1996(1), 1–76.
- ANDOLFATTO, D. (1996): "Business Cycles and Labor-Market Search," *American Economic Review*, 86(1), 112–132.
- ARSENEAU, D. M., AND S. K. CHUGH (2007): "Optimal fiscal and monetary policy with costly wage bargaining," International Finance Discussion Papers 893, Board of Governors of the Federal Reserve System (U.S.).
- BARRO, R. J. (1977): "Long-Term Contracting, Sticky Prices and Monetary Policy," *Journal of Monetary Economics*, 3(3), 305–316.
- BARRO, R. J., AND R. G. KING (1984): "Time-separable Preferences and Intertemporal-Substitution Models of Business Cycles," *The Quarterly Journal of Economics*, 99(4), 817–839.
- BARRO, R. J., AND X. SALA-I MARTIN (2003): *Economic Growth*, 2nd Edition. MIT Press.
- BASU, S., AND J. G. FERNALD (1997): "Returns to Scale in U.S. Production: Estimates and Implications," *Journal of Political Economy*, 105(2), 249–283.
- BASU, S., J. G. FERNALD, AND M. D. SHAPIRO (2001): "Productivity growth in the 1990s: technology, utilization, or adjustment?," *Carnegie-Rochester Conference Series on Public Policy*, 55(1), 117–165.
- BAXTER, M., AND R. G. KING (1993): "Fiscal Policy in General Equilibrium," *American Economic Review*, 83(3), 315–334.

- BAXTER, M., AND R. G. KING (1999): "Measuring Business Cycles: Approximate Band-Pass Filters For Economic Time Series," *The Review of Economics and Statistics*, 81(4), 575–593.
- BAYER, C. (2006): "Investment dynamics with fixed capital adjustment cost and capital market imperfections," *Journal of Monetary Economics*, 53(8), 1909–1947.
- BILBIE, F. O., F. GHIRONI, AND M. J. MELITZ (2007): "Monetary Policy and Business Cycles with Endogenous Entry and Product Variety," NBER Working Papers 13199, National Bureau of Economic Research, Inc.
- BLANCHARD, O. J. (1997): "The Medium Run," *Brookings Papers on Economic Activity*, 2, 89–141.
- (2006): "European unemployment: the evolution of facts and ideas," *Economic Policy*, 21(45), 5–59.
- BLANCHARD, O. J., AND P. DIAMOND (1989): "The Beveridge Curve," *Brookings Papers on Economic Activity*, 1, 1–60.
- BLANCHARD, O. J., AND J. GALI (2006): "A New Keynesian Model with Unemployment," Research Series 200610-4, National bank of Belgium.
- BLANCHARD, O. J., AND D. QUAH (1989): "The Dynamic Effects of Aggregate Demand and Supply Disturbances," *American Economic Review*, 79(4), 655–673.
- BLANCHARD, O. J., AND L. H. SUMMERS (1986): "Hysteresis and the European Unemployment Problem," in *NBER Macroeconomics Annual*, ed. by S. Fischer. Cambridge, Mass.: MIT Press.
- (1987): "Hysteresis in unemployment," *European Economic Review*, 31(1-2), 288–295.
- BOLDRIN, M., L. J. CHRISTIANO, AND J. D. M. FISHER (2001): "Habit Persistence, Asset Returns, and the Business Cycle," *American Economic Review*, 91(1), 149–166.
- BROCK, W. A., AND L. J. MIRMAN (1972): "Optimal economic growth and uncertainty: The discounted case," *Journal of Economic Theory*, 4(3), 479–513.
- BUKOWSKI, M. (2005): "Ryzyko a oszczędności, inwestycje i wzrost gospodarczy," Ph.D. thesis, Kolegium Analiz Ekonomicznych SGH.
- BUKOWSKI, M., P. KOWAL, P. LEWANDOWSKI, AND J. ZAWISTOWSKI (2005): *Struktura i poziom wydatków i dochodów sektora finansów publicznych a sytuacja na rynku pracy - doświadczenia międzynarodowe i wnioski dla Polski*. Narodowy Bank Polski.

- BURNS, A. F., AND W. C. MITCHELL (1946): *Measuring Business Cycles*. National Bureau of Economic Research.
- CABALLERO, R. J., E. M. R. A. ENGEL, AND J. C. HALTIWANGER (1995): "Plant-Level Adjustment and Aggregate Investment Dynamics," *Brookings Papers on Economic Activity*, 1995(2), 1–54.
- CALVO, G. A. (1983): "Staggered prices in a utility-maximizing framework," *Journal of Monetary Economics*, 12(3), 383–398.
- CAMPBELL, J. Y. (2002): "Consumption-Based Asset Pricing," Harvard Institute of Economic Research Working Papers 1974, Harvard - Institute of Economic Research.
- CASS, D. (1965): "Optimum Growth in an Aggregative Model of Capital Accumulation," *The Review of Economic Studies*, 32(3), 233–240.
- CHANG, Y., J. F. GOMES, AND F. SCHORFHEIDE (2002): "Learning-by-Doing as a Propagation Mechanism," *American Economic Review*, 92(5), 1498–1520.
- CHARI, V. V., P. J. KEHOE, AND E. R. MCGRATTAN (2007): "Business Cycle Accounting," *Econometrica*, 75(3), 781–836.
- CHO, J.-O., AND T. F. COOLEY (1994): "Employment and hours over the business cycle," *Journal of Economic Dynamics and Control*, 18(2), 411–432.
- CHRISTIANO, L. J., M. EICHENBAUM, AND C. L. EVANS (2005): "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, 113(1), 1–45.
- CHRISTIANO, L. J., AND T. J. FITZGERALD (1999): "The Band pass filter," Working Paper 9906, Federal Reserve Bank of Cleveland.
- CHRISTOFFEL, K., AND T. LINZERT (2005): "The role of real wage rigidity and labor market frictions for unemployment and inflation dynamics," Working Paper Series 556, European Central Bank.
- COGLEY, T., AND J. M. NASON (1995): "Output Dynamics in Real-Business-Cycle Models," *American Economic Review*, 85(3), 492–511.
- COLE, H. L., AND R. ROGERSON (1999): "Can the Mortensen-Pissarides Matching Model Match the Business-Cycle Facts?," *International Economic Review*, 40(4), 933–959.
- COLLARD, F., AND M. JUILLARD (2001): "A Higher-Order Taylor Expansion Approach to Simulation of Stochastic Forward-Looking Models with an Application to a Nonlinear Phillips Curve Model," *Computational Economics*, 17(2-3), 125–139.

- COMIN, D., AND M. GERTLER (2006): "Medium-Term Business Cycles," *American Economic Review*, 96(3), 523–551.
- COMIN, D., AND B. HOBIJN (2004): "Cross-country technology adoption: making the theories face the facts," *Journal of Monetary Economics*, 51(1), 39–83.
- COOLEY, T. F., AND E. C. PRESCOTT (1995): "Economic growth and Business Cycles," in *Frontiers of Business Cycle Research*, ed. by T. F. Cooley. Princeton University Press.
- COOLEY, T. F., AND V. QUADRINI (1999): "A neoclassical model of the Phillips curve relation," *Journal of Monetary Economics*, 44(2), 165–193.
- CORREIA, I., J. C. NEVES, AND S. REBELO (1992): "Business Cycles 1850-1950: New Facts about Old Data," *European Economic Review*, 36, 459–467.
- COSTAIN, J. S., AND M. REITER (2003): "Business Cycles, Unemployment Insurance, and the Calibration of Matching Models," Economics Working Papers 872, Department of Economics and Business, Universitat Pompeu Fabra.
- DANTHINE, J.-P., AND A. KURMANN (2004): "Fair Wages In a New Keynesian Model of the Business Cycle," *Review of Economic Dynamics*, 7(1), 107–142.
- DEN HAAN, W. J., G. RAMEY, AND J. WATSON (2000): "Job Destruction and Propagation of Shocks," *American Economic Review*, 90(3), 482–498.
- DIAMOND, P. A. (1982): "Wage Determination and Efficiency in Search Equilibrium," *Review of Economic Studies*, 49(2), 217–227.
- EPSTEIN, L. G., AND S. E. ZIN (1991): "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: An Empirical Analysis," *Journal of Political Economy*, 99(2), 263–286.
- ERCEG, C. J., D. W. HENDERSON, AND A. T. LEVIN (2000): "Optimal monetary policy with staggered wage and price contracts," *Journal of Monetary Economics*, 46(2), 281–313.
- EVANS, G. W., S. HONKAPOHJA, AND P. ROMER (1998): "Growth Cycles," *American Economic Review*, 88(3), 495–515.
- FANG, L., AND R. ROGERSON (2007): "Policy Analysis in a Matching Model with Intensive and Extensive Margins," NBER Working Papers 13007, National Bureau of Economic Research, Inc.

- FATAS, A. (2000): "Endogenous growth and stochastic trends," *Journal of Monetary Economics*, 45(1), 107–128.
- FLAIM, P. O. (1990): "Population changes, the baby boom, and the unemployment rate," *Monthly Labour Review*, 113(8).
- FRANCIS, N., AND V. A. RAMEY (2005): "Is the technology-driven real business cycle hypothesis dead? Shocks and aggregate fluctuations revisited," *Journal of Monetary Economics*, 52(8), 1379–1399.
- FRIEDMAN, M. (1968): "The Role of Monetary Policy," *American Economic Review*, 58(1), 1–17.
- FUHRER, J. C. (2000): "Habit Formation in Consumption and Its Implications for Monetary-Policy Models," *American Economic Review*, 90(3), 367–390.
- GALI, J., AND F. ZILIBOTTI (1995): "Endogenous Growth and Poverty Traps in a Cournotian Model," *Annales D'Economie aet de Statistique*, (37/38), 197–213.
- GERTLER, M., AND A. TRIGARI (2006): "Unemployment Fluctuations with Staggered Nash Wage Bargaining," *Computing in Economics and Finance* 525, Society for Computational Economics.
- GOMES, J., J. GREENWOOD, AND S. REBELO (2001): "Equilibrium unemployment," *Journal of Monetary Economics*, 48(1), 109–152.
- GOMUŁKA, S. (1998): *Teoria innowacji i wzrostu gospodarczego*. CASE.
- GÓRA, M. (2005): "Trwale wysokie bezrobocie w Polsce. Wyjasnienia i propozycje," *Ekonomista*, 1, 27–48.
- GRABEK, G., B. KŁOS, AND G. UTZIG-LENARCZYK (2007): "SOE-PL - model DSGE małej otwartej gospodarki estymowany na danych polskich," *Materiały i Studia* 217, Narodowy Bank Polski.
- GREENWOOD, J., Z. HERCOWITZ, AND G. W. HUFFMAN (1988): "Investment, Capacity Utilization, and the Real Business Cycle," *American Economic Review*, 78(3), 402–417.
- GROWIEC, J. (2005): "O modelowaniu endogenicznego wzrostu z uwzględnieniem krótkookresowych zjawisk cyklicznych," *Ekonomista*, 3, 321–343.
- GROWIEC, J., AND I. SCHUMACHER (2007): "Technological Opportunity, Long-Run Growth, and Convergence," Discussion Paper 57, Center for Operations Research and Econometrics (CORE), Universit ´e catholique de Louvain, Belgium.

- HALL, R. E. (1978): "Stochastic Implications of the Life Cycle-Permanent Income Hypothesis: Theory and Evidence," *The Journal of Political Economy*, 86(6), 971–987.
- (2003): "Wage Determination and Employment Fluctuations," NBER Working Papers 9967, National Bureau of Economic Research, Inc.
- (2005a): "Employment Efficiency and Sticky Wages: Evidence from Flows in the Labor Market," *Review of Economics and Statistics*, 87(3), 397–407.
- (2005b): "Employment Fluctuations with Equilibrium Wage Stickiness," *American Economic Review*, 95(1), 50–65.
- (2005c): "Job Loss, Job Finding, and Unemployment in the U.S. Economy Over the Past Fifty Years," in *NBER Macroeconomics Annual 2005*, ed. by M. Gertler, and K. Rogoff, chap. 2, pp. 101–137. Cambridge, Mass.: MIT Press.
- (2005d): "Separating the Business Cycle from Other Economic Fluctuations," NBER Working Papers 11651, National Bureau of Economic Research, Inc.
- HAMILTON, J. D. (1994): *Time series Analysis*. Princeton University Press.
- HANSEN, G. D. (1985): "Indivisible labor and the business cycle," *Journal of Monetary Economics*, 16(3), 309–327.
- HODRICK, R., AND E. C. PRESCOTT (1980): "Postwar U.S. Business Cycles: An Empirical Investigation," Working paper, Carnegie-Mellon University.
- HOSIOS, A. J. (1990): "On the Efficiency of Matching and Related Models of Search and Unemployment," *Review of Economic Studies*, 57(2), 279–298.
- HOUTHAKKER, H. S. (1955-1956): "The Pareto Distribution and the Cobb-Douglas Production Function in Activity Analysis," *Review of Economic Studies*, 23(1), 27–31.
- JONES, C. I. (1995): "R&D-Based Models of Economic Growth," *Journal of Political Economy*, 103(4), 759–784.
- JONES, L. E., R. E. MANUELLI, AND H. E. SIU (2005): "Fluctuations in Convex Models of Endogenous Growth II: Business Cycle Properties," *Review of Economic Dynamics*, 8(4), 805–828.
- JUDD, K. L. (1996): "Approximation, perturbation, and projection methods in economic analysis," in *Handbook of Computational Economics*, ed. by H. M. Amman, D. A. Kendrick, and J. Rust, vol. 1 of *Handbook of Computational Economics*, chap. 12, pp. 509–585. Elsevier.

- JUHN, C., AND S. POTTER (2006): "Changes in Labor Force Participation in the United States," *Journal of Economic Perspectives*, 20(3), 27–46.
- JUILLARD, M. (1996): "Dynare : a program for the resolution and simulation of dynamic models with forward variables through the use of a relaxation algorithm," CEPREMAP Working Papers (Couverture Orange) 9602, CEPREMAP.
- KEYNES, J. M. (1936): *The General Theory of Employment, Interest and Money*. Cambridge University Press.
- KING, R. G., C. I. PLOSSER, AND S. T. REBELO (1988): "Production, growth and business cycles : I. The basic neoclassical model," *Journal of Monetary Economics*, 21(2-3), 195–232.
- KING, R. G., AND S. T. REBELO (1999): "Resuscitating real business cycles," in *Handbook of Macroeconomics*, ed. by J. B. Taylor, and M. Woodford, vol. 1, chap. 14, pp. 927–1007.
- KING, R. G., AND M. W. WATSON (1996): "Money, Prices, Interest Rates and the Business Cycle," *The Review of Economics and Statistics*, 78(1), 35–53.
- KLIMA, G. (2006): "Programowanie dynamiczne i modele rekursywne w ekonomii," *Materiały i Studia* 201, Narodowy Bank Polski (NBP).
- KOKOSZCZYŃSKI, R. (2004): *Współczesna polityka pieniężna*. Polskie Wydawnictwa Ekonomiczne, Warszawa.
- KOOPMANS, T. C. (1947): "Measurement without Theory," *Review of Economics and Statistics*, 29, 161–172.
- (1965): "On the Concept of Optimal Economic Growth," in *The Econometric Approach to Development Planning*. Rand-McNally.
- KRAUSE, M. U., D. J. LOPEZ-SALIDO, AND T. LUBIK (2007): "Do Search Frictions Matter for Inflation Dynamics?," Kiel Working Papers 1353, Kiel Institute for the World Economy.
- KRAUSE, M. U., AND T. A. LUBIK (2007): "The (ir)relevance of real wage rigidity in the New Keynesian model with search frictions," *Journal of Monetary Economics*, 54(3), 706–727.
- KULA, G. J. (2006): "Optimal Retirement Decision," Research Series 329, Tinbergen Institute.
- KWIATKOWSKI, E. (2002): *Bezrobocie. Podstawy teoretyczne*. Polskie Wydawnictwo Naukowe, Warszawa.

- KYDLAND, F. E., AND E. C. PRESCOTT (1982): "Time to build and aggregate fluctuations," *Econometrica*, 50(6), 1345–1370.
- LAGOS, R. (2006): "A Model of TFP," *Review of Economic Studies*, 73(4), 983–1007.
- LAYARD, R., S. NICKELL, AND R. JACKMAN (1991): *Unemployment: macroeconomic performance and the labour market*. Oxford University Press.
- LIBERDA, B. (1996): "Oszczednosci w teoriach konsumpcji i wzrostu," *Ekonomista*, 3, 325–342.
- LJUNGQUIST, L., AND T. J. SARGENT (2000): *Recursive Macroeconomic Theory*. MIT Press.
- LJUNGQVIST, L., AND T. J. SARGENT (1998): "The European Unemployment Dilemma," *Journal of Political Economy*, 106(3), 514–550.
- LONG, JOHN B, J., AND C. I. PLOSSER (1983): "Real Business Cycles," *Journal of Political Economy*, 91(1), 39–69.
- LUCAS, R. (1988): "On the mechanics of Economic Growth," *Journal of Monetary Economics*, 22, 3–42.
- LUCAS, R. E. (1977): "Understanding Business Cycles," *Carnegie Rochester Conference Series on Public Policy*, 5, 7–46.
- MANKIW, N. G. (1985): "Small Menu Costs and Large Business Cycles: A Macroeconomic Model," *The Quarterly Journal of Economics*, 100(2), 529–538.
- MANKIW, N. G., AND D. ROMER (eds.) (1991): *New Keynesian Economics*. MIT Press.
- MCCALL, J. J. (1970): "Economics of information and job search," *Quarterly Journal of Economics*, 84, 113–126.
- MERZ, M. (1995): "Search in the labor market and the real business cycle," *Journal of Monetary Economics*, 36(2), 269–300.
- MORTENSEN, D. T. (1982): "Property Rights and Efficiency in Matching, Racing, and Related Games," *American Economic Review*, 72(5), 968–979.
- MORTENSEN, D. T., AND E. NAGYPAL (2005): "More on Unemployment and Vacancy Fluctuations," NBER Working Papers 11692, National Bureau of Economic Research, Inc.
- MORTENSEN, D. T., AND C. A. PISSARIDES (1994): "Job Creation and Job Destruction in the Theory of Unemployment," *Review of Economic Studies*, 61(3), 397–415.



- NASH, J. (1953): "Two-Person Cooperative Games," *Econometrica*, 21(1), 128–140.
- NASON, J. M., AND T. KANO (2004): "Business Cycle Implications of Habit Formation," *Computing in Economics and Finance 2004* 175, Society for Computational Economics.
- NICKELL, S., L. NUNZIATA, W. OCHEL, AND G. QUINTINI (2001): "The Beveridge Curve, Unemployment and Wages in the OECD from the 1960s to the 1990s - Preliminary Version," CEP Discussion Papers 0502, Centre for Economic Performance, LSE.
- OLSSON, O. (2005): "Technological Opportunity and Growth," *Journal of Economic Growth*, 10(1), 31–53.
- PANEK, E. (1994): "Nieklasyczne zagadnienie sterowania optymalnego wzrostem. Część II - Model z wyodrębnionymi nakładami na finansowanie postępu techniczno-organizacyjnego," *Przegląd Statystyczny*, 2, 173–185.
- PETRONGOLO, B., AND C. A. PISSARIDES (2001): "Looking into the Black Box: A Survey of the Matching Function," *Journal of Economic Literature*, 39(2), 390–431.
- PHELPS, E. (1967): "Phillips Curves, Expectations of Inflation and Optimal Unemployment over Time," *Economica*, 34(135), 254–281.
- PISSARIDES, C. A. (1985): "Short-run Equilibrium Dynamics of Unemployment Vacancies, and Real Wages," *American Economic Review*, 75(4), 676–690.
- PISSARIDES, C. A. (2000): *Equilibrium Unemployment Theory*. Cambridge, Mass.: MIT Press, 2 edn.
- (2007): "The Unemployment Volatility Puzzle: Is Wage Stickiness the Answer?," CEP Discussion Papers dp0839, Centre for Economic Performance, LSE.
- PLOSSER, C. I. (1989): "Understanding Real Business Cycles," *Journal of Economic Perspectives*, 3(3), 51–77.
- PRESCOTT, E. C. (1986): "Theory ahead of business cycle measurement," *Quarterly Review* Fall, Federal Reserve Bank of Minneapolis.
- (2004): "Why do Americans Work so Much More than Europeans?," NBER Working Papers 10316, National Bureau of Economic Research, Inc.
- RAMSEY, F. P. (1928): "A Mathematical Theory of Saving," *Economic Journal*, 38(152), 543–559.

- ROGERSON, R. (1988): "Indivisible labor, lotteries and equilibrium," *Journal of Monetary Economics*, 21(1), 3–16.
- (2004): "Two Views on the Deterioration of European Labor Market Outcomes," *Journal of the European Economic Association*, 2(2-3), 447–455.
- (2007): "Structural Transformation and the Deterioration of European Labor Market Outcomes," NBER Working Papers 12889, National Bureau of Economic Research, Inc.
- ROMER, P. M. (1989): "Human Capital And Growth: Theory and Evidence," (3173).
- (1990): "Endogenous Technological Change," *Journal of Political Economy*, 98(5), 71–102.
- ROTEMBERG, J. J. (2003): "Stochastic Technical Progress, Smooth Trends, and Nearly Distinct Business Cycles," *American Economic Review*, 93(5), 1543–1559.
- RUBIO, J. F., AND J. FERNANDEZ-VILLAYERDE (2005): "Estimating Dynamic Equilibrium Economies: Linear versus Nonlinear Likelihood," *Journal of Applied Econometrics*, 20, 891–910.
- SARGENT, T. J. (1987): *Macroeconomic Theory*. Academic Press.
- SCHUMPETER, J. A. (1939): *Business cycles: A theoretical, Historical and Statistical Analysis of the Capitalist Process*. New York and London: McGraw-Hill.
- SHIMER, R. (2003): "The Cyclical Behavior of Equilibrium Unemployment, Vacancies and Wages: Evidence and Theory," NBER Working Paper 9536, National Bureau of Economic Research.
- (2004): "The Consequences of Rigid Wages in Search Models," *Journal of the European Economic Association*, 2(2-3), 469–479.
- (2005): "The Cyclical Behavior of Equilibrium Unemployment and Vacancies," *American Economic Review*, 95(1), 25–49.
- (2007): "Reassessing the Ins and Outs of Unemployment," NBER Working Papers 13421, National Bureau of Economic Research, Inc.
- SMETS, F., AND R. WOUTERS (2003): "An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area," *Journal of the European Economic Association*, 1(5), 1123–1175.
- SMITH, A. E., AND Y. ZENOU (2003): "A Discrete-Time Stochastic Model of Job Matching," *Review of Economic Dynamics*, 6(1), 54–79.

- SOLOW, R. M. (1956): "A Contribution to the Theory of Economic Growth," *Quarterly Journal of Economics*, 70, 65–94.
- (2000): "Toward a Macroeconomics of the Medium Run," *The Journal of Economic Perspectives*, 14(1), 151–158.
- STIGLER, G. J. (1962): "Information in the labor market," *Journal of Political Economy*, 70(5), 94–105.
- STOKEY, N. L., R. E. LUCAS, AND E. C. PRESCOTT (1989): *Recursive Methods in Economic Dynamics*. Harvard University Press.
- SUMIYE OKUBO, CAROL A. ROBBINS, C. E. M., AND B. K. SLIKER (2006): "BEA 2006 Research and Development Satellite Account," *Survey on Current Business*, 86(12), 14–27.
- TOKARSKI, T. (2001): *Determinanty wzrostu gospodarczego w warunkach stałych efektów skali*. Katedra Ekonomii Uniwersytetu Łódzkiego, Łódź.
- (2005): *Wybrane modele podażyowych czynników wzrostu gospodarczego*. Wydawnictwo Uniwersytetu Jagiellońskiego, Kraków.
- TRIGARI, A. (2004): "Equilibrium unemployment, job flows and inflation dynamics," Working Paper Series 304, European Central Bank.
- (2006): "The Role of Search Frictions and Bargaining for Inflation Dynamics," Working Papers 304, IGIER (Innocenzo Gasparini Institute for Economic Research), Bocconi University.
- VERACIERTO, M. (2002): "On the Cyclical Behavior of Employment, Unemployment and Labor Force Participation," Working Paper Series WP-02-12, Federal Reserve Bank of Chicago.
- WALSH, C. E. (2003): *Monetary Theory and Policy, 2nd Edition*. MIT Press,.
- WOODFORD, M. (2003): *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press.
- WRÓBEL-ROTTER, R. (2007): "Bayesian estimation of a Dynamic General Equilibrium Model for the Polish economy," presented at the 4th Nordic Econometric Meeting in Tartu, Estonia.
- ZAJĄCZKOWSKA-JAKIMIĄK, S. (2006): "Wiedza techniczna i kapitał ludzki w teorii wzrostu gospodarczego," *Gospodarka Narodowa*, 11-12, 47–69.