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# Endogenous Pricing to Market and Financing Costs 

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ABSTRACT

This paper studies the endogenous determination of pricing to market, in a model with time dependent transportation costs, where the future terms of trade are random. Allowing time dependent transportation costs adds a dimension of investment to the pre-buying of imports, implying that financial considerations determine the frequency of pricing to market, and the deviations from relative PPP. If the expected discounted cost of last minute delivery is higher than pre-buying, one exercises the option of spot market imports if the realized terms of trade are favorable enough. Pricing to market is observed in countries characterized by low terms of trade volatility and low financing costs. In these circumstances, imports are pre-bought, and the spot market for imports is inactive. In countries where the financing costs and the terms of trade volatility are high, few imports are pre-bought, the price of imports is determined by the realized real exchange rate, and a version of relative PPP holds. With an intermediate level of terms of trade volatility and of financing costs, a mixed regime is observed, and some imports are pre-bought. If the realized real exchange rate is favorable enough, more imports are purchased in the spot market, the price of imports is determined by the realized real exchange rate, and the relative PPP holds. If the realized real exchange rate is weak, pricing to market would prevail, increasing consumers' welfare by shielding them from the adverse purchasing power consequences of weak terms of trade. Higher financing costs increase the cost of pre-buying imports, reducing thereby the frequency of pricing to market, increasing the expected relative price of imports, reducing the expected deviations from relative PPP, and reducing welfare.

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Endogenous pricing to market and financing costs

## 1. Introduction and summary

The puzzling lack of a tighter association between goods' prices and the exchange rate is one of the intriguing observations in International Economics. Following Krugman (1987), pricing in domestic currency and pricing to market (PTM) have provided an interpretation to this puzzle. ${ }^{1}$ A direct implication of the PTM hypothesis is the low pass-through from the exchange rate to prices, and the resultant failure of the relative PPP to hold in the short and intermediate-runs. While the empirical literature confirmed these predictions, it also detected a systematic heterogeneity of the patterns of PTM across various goods. ${ }^{2}$ Recent studies also suggest that the relative PPP holds better in emerging markets. ${ }^{3}$ An important unresolved question concerns the conditions under which PTM is endogenously chosen by the producers, and when should we expect the relative PPP to hold more tightly. Addressing these questions is crucial for a better understanding of issues like the welfare implications of exchange rate volatility, the incidence of protective policies, and the welfare ranking of fixed versus flexible exchange rate regimes.

The purpose of this paper is to provide a framework where the degree of pricing to market is endogenously determined, as part of the problem of balancing the benefits of pre-set prices with the costs of managing the delivery system needed to support rigid prices. This paper is motivated by the inherent trade-off between price and quantity adjustments, where pricing in local currency requires that quantities should be plentiful to fulfill the demand at the pre-set price. Hence, pricing in local currency

1 See Obstfeld and Rogoff (1996, Chapters 9, 10) for an overview of the low association between goods' prices and the exchange rate, and Obstfeld and Rogoff (2000) for a recent overview of puzzles in International Economics (including the pricing puzzle), highlighting the relevance of transportation costs. See Goldberg and Knetter (1997) for a comprehensive review of the empirical literature that followed Krugman's study.

2 See Isard (1977) and Wei and Parsley (1996) for empirical studies of (deviations from) the law of one price. See Marston (1990) and Kenetter (1993) for studies of pricing to market, and Rogoff (1996) for an overview of the PPP puzzle.

3 See Hausmann, Panizza and Stein (1999) and Calvo and Reinhart (2000).
and pricing to market may involve complex issues of delivery management. In such a system, the degree of local currency pricing is impacted by the financial costs of timely delivery of goods, as well as by the transportation costs associated with timely re-supply of inventories. ${ }^{4}$

The paper departs from the pervious pricing to market literature by allowing time dependent transportation costs. The presumption is that the cost of delivering a good ordered ahead of time is lower than the cost of a last minute delivery, as pre-buying would allow to find the cheapest means of transportation, even if it would require more time to deliver [see Carlton (1979) for a pioneering analysis on costly delivery lags]. ${ }^{5}$ We model the implications of time dependent transportation costs on the pricing and the delivery of imports in a 2 period, 2 goods model, where the second period terms of trade are random. Allowing time dependent transportation costs adds a dimension of investment to the pre-buying of imports. With uncertain future terms of trade, spot market imports resembles an option -- one exercises the option of last minute imports if the realized terms of trade are favorable enough.

The above suggests a simple way of modeling endogenously the switch from pricing to market to a flexible price environment. Assuming that the expected discounted cost of last minute delivery is higher than pre-buying, it follows that in countries where the terms of trade volatility is small, most imports are pre-bought, and the spot market for imports is inactive. In these circumstances the prices of importables are delinked from the realized terms of trade, as is the case in the pricing to market (PTM) regime. Greater volatility induces more frequent realizations of relatively high and low values of the real exchange rate. For terms of trade volatility high enough, it would make sense to scale down the pre-buying, in order to exploit the "good tail" of the real

4 These issues were sidestepped by most of the literature by assuming instant delivery of traded goods. 5 An example of these considerations is the pricing of heating oil to consumers, where the "pre-buy protection plan" allows consumers to purchase forward the desired amount of heating oil at a pre-set price that is expected to be lower than the future spot market price for last minute delivery. Each spring customers in New England are advised -- "Before the weather turns colder, protect and insulate your wallet from the inevitable rising costs of home heating oil. This program (the pre-buy) allows you to lock into fuel oil prices while they are low, and pay that one low fixed price for your entire year's usage, no matter how high the prices may soar."
exchange distribution, where spot market imports are cheaper. In these circumstances, we will obverse a mixed regime -- if the realized real exchange rate is favorable enough, imports are purchased in the spot market, the price of imports is determined by the realized real exchange rate, and a version of relative PPP holds.

Otherwise, the pricing to market will prevail. As is the case with options, the value of the option of spot market imports increases with the volatility, implying that the frequency of pricing to market tends to be lower in more volatile economies.

Another implication of time dependent transportation costs is that financial considerations determine the frequency of pricing to market, and the deviations from relative PPP. Specifically, higher financing costs would increase the cost of pre-buying, encouraging spot market imports, reducing the frequency of pricing to market, increasing the tendency of relative PPP to hold, and increasing the expected price of imports. The net effect is welfare reducing as the PTM shields consumers from the adverse purchasing power effects of weak terms of trade. This result is of special relevance for emerging markets, where limited financial depth and costly credit encourage spot market trade, and discourage pre-buying. It suggests 2 channels explaining why relative PPP may hold better for emerging markets -- first the volatility, and second the financing costs. Both imply less frequent pricing to markets, and greater association between the exchange rate, the prices of imports, and the volume of trade. While the 2 channels reinforce each other in reducing the incidence of PTM, they have different welfare implications. Our discussion shows that, for a given real interest rate, higher terms of trade volatility tend to increase welfare. Higher financing costs are always welfare reducing, and are associated with lower imports. ${ }^{6}$

The implications of pricing to market on the desirable exchange rate flexibility have been studied by comparing the behavior of the nominal exchange rate and prices in regimes with polar pricing rules for imports. In the first, import prices are set in producer's currency, as has been the traditional assumption in the MundellFleming open economy macro models. In the second system, import prices are set in consumer currencies, in line with the pricing to market literature. ${ }^{7}$ These studies pointed out that the welfare ranking of fix versus

6 These effects are potentially large, and provide an interpretation for the potential use of cutting trade credit as a means of inducing borrowers to service their debt.
$7 \quad$ See Devereux and Engel (1998) and Betts and Devereux (2000).
flexible exchange rate regimes and the dynamics of output and consumption hinge on the pricing rule. For example, Devereux and Engel (1998) report that "When prices are set in producer's currency, as in the traditional framework, we find that there is a trade-off between floating and fixed exchange rates. Exchange rate adjustment under floating rates allows for a lower variance of consumption, but exchange rate volatility itself leads to a lower average level of consumption. When prices are set in consumer's currency, floating exchange rates always dominate fixed exchange rates." Our findings imply that the pre-setting prices in consumer's currency would not characterize emerging markets, and would be observed more frequently in the OECD countries. This finding, combined with the Devereux and Engel (1998) results, may provide another explanation for the "fear to float" by emerging market economies.

Section 2 describes the model, and characterizes the impact of a higher discount rate and higher terms of trade volatility for the case of linear intertemporal preferences. Section 3 investigates the welfare consequences of higher financing costs. Section 4 concludes. The Appendix extends the model to risk averse agents.

## 2. The model

Assume a small economy producing a domestic traded good, and consuming both domestic and foreign goods (denoted by $x$ and $y$, respectively). Imports are associated with transportation costs that depend on the delivery lag. We focus on the simplest version of the model - a 2 period endowment model, where the supply of the domestic good in period i is $\bar{x}_{i}(\mathrm{i}=1,2)$. The domestic and the foreign markets are geographically separated. Imports of the foreign goods are subject to time dependent transportation costs, assumed to be higher for last minute delivery. ${ }^{8}$ Consider the case where the consumer's utility $H$ is the discounted value of temporal utilities $v_{t}(\mathrm{t}=1,2)--9$

$$
\text { (1) } H=v_{1}+\frac{v_{2}}{1+\rho} \quad \text { where } v_{t}=\left\{\begin{array}{l}
v_{1}=x_{1} \quad \text { for } t=1 \\
v_{2}=x_{2}+\frac{\theta}{\beta}\left[Y_{2}\right]^{\beta} \quad \text { for } t=2
\end{array} \quad, 0<\beta<1 ; 0<\theta .\right.
$$

For simplicity of presentation, we assume that imports are consumed only in the second period. We normalize the first period prices of the domestic good to 1 . Consumers can pre-buy imports in the first period for a scheduled delivery in the second period. The "pre-buying" price of $y_{2}$ is $1+t$ units of $x_{1}(t$ stands for the transportation costs, where the implicit cost of $y_{2}$, net of transportation and financing, is normalized to 1 ). Consumers may postpone buying y to the second period, relying on the spot market. The spot market price of $y_{2}$ is random

8 Pre-buying may be cheaper also if production costs are lower when producers have more lead time [see Carlton (1979) for further discussion of this possibility]. The logic of our analysis applies also to the case where production costs are time dependent.

9 With utility (1) the demand for imports is likely to be independent from income, simplifying the analytical discussion. It can be shown that the main results of the paper are applicable to other utilities, though the analytical discussion is more involved.
(2) $\frac{1+\tilde{t}}{1+\varepsilon}$
where $\tilde{t}$ is the transportation cost for spot "last minute" deliveries, and $\varepsilon$ is a random shock determining the second period international relative price of the domestic good. We denote by $f(\varepsilon)$ the corresponding p.d.f. of the "external terms of trade," defined in the interval $\underline{\varepsilon} \leq \varepsilon \leq \bar{\varepsilon}$, where $-1<\underline{\varepsilon}$. We denote by $y_{2}^{p}$ the prebuying of imports contracted for second period delivery, and by $y_{2}^{s}$ the spot market imports. The opportunity costs of imports $y_{2}^{p} ; y_{2}^{s}$ in terms of exports sold in period 1 and 2 are denoted by $x_{1}^{p} ; x_{2}^{s}$, respectively, where

$$
\begin{align*}
& x_{1}^{p}=y_{2}^{p}(1+t) \\
& x_{2}^{s}=y_{2}^{s} \frac{1+\tilde{t}}{1+\varepsilon} \tag{3}
\end{align*}
$$

The consumption of x and y are characterized by

$$
\begin{align*}
& x_{1}=\bar{x}_{1}-x_{1}^{p}-s \\
& x_{2}=\bar{x}_{2}+s\left(1+r^{*}\right)-x_{2}^{s} ; \quad y_{2}=\frac{x_{1}^{p}}{1+t}+\frac{x_{2}^{s}(1+\varepsilon)}{1+\tilde{t}}, \tag{4}
\end{align*}
$$

where $s$ is the first period saving, yielding a real interest rate $r *$ (defined in terms of the domestic good).
To avoid a corner solution stemming from the linearity of the intertemporal utility we assume first that $\rho=r^{*}$. The drawback of using the linear intertemporal utility is that it does not allow us to investigate fully the effects of changing the financing cost on the optimal patterns of pre-buying. We will address these issues in the Appendix, where we illustrate how to extend the analysis to allow for risk averse consumers. The consumer problem is to determine the optimal pair $\left\langle x_{2}^{s} ; x_{1}^{p}\right\rangle$. We solve it backwards -- first we find the optimal spot market trade in the second period. Next, applying this solution we construct the expected utility in the first period. Finally, we find the pre-buying that maximizes this expected utility.

The consumer determines the second period consumption plan by finding the spot market imports that would maximize

$$
\begin{align*}
& \operatorname{MAX}\left[\bar{x}_{2}+\left(1+r^{*}\right) s-x_{2}^{s}+\frac{\theta}{\beta}\left[\frac{(1+\varepsilon) x_{2}^{s}}{1+\tilde{t}}+\frac{x_{1}^{p}}{1+t}\right]^{\beta}\right]  \tag{5}\\
& x_{2}^{s}, \quad \text { S.T. } \quad x_{2}^{s} \geq 0
\end{align*}
$$

The solution of which implies that the consumer will buy $y_{2}$ in the spot market only if the realized terms of trade are favorable enough, $\varepsilon>\varepsilon$ *, where
(6) $\varepsilon^{*}=\left(\frac{x_{1}^{p}}{1+t}\right)^{1-\beta} \frac{1+\tilde{t}}{\theta}-1$,
and the optimal second period spot market exports (used to finance spot market imports) are

$$
x_{2}^{s}= \begin{cases}0 & \text { if } \varepsilon \leq \varepsilon *  \tag{7}\\ {\left[\left(\frac{(1+\varepsilon) \theta}{1+\tilde{t}}\right)^{1 /(1-\beta)}-\frac{x_{1}^{p}}{1+t}\right] \frac{1+\tilde{t}}{1+\varepsilon}} & \text { if } \varepsilon>\varepsilon^{*}\end{cases}
$$

The resultant second period utility is
$v_{2}= \begin{cases}\tilde{v}_{2}\left(\varepsilon^{*}\right)=\bar{x}_{2}+s\left(1+r^{*}\right)+\frac{\theta}{\beta}\left[\frac{x_{1}^{p}}{1+t}\right]^{\beta} & \text { if } \varepsilon \leq \varepsilon^{*} \\ \tilde{v}_{2}(\varepsilon)=\bar{x}_{2}+s\left(1+r^{*}\right)+\frac{x_{1}^{p}(1+\tilde{t})}{(1+\varepsilon)(1+t)}+\left(\frac{1}{\beta}-1\right) \theta^{1 /(1-\beta)}\left(\frac{1+\varepsilon}{1+\tilde{t}}\right)^{\beta /(1-\beta)} & \text { if } \varepsilon>\varepsilon^{*}\end{cases}$

It is easy to confirm that for $\varepsilon^{*}<\varepsilon, \tilde{v}_{2}\left(\varepsilon^{*}\right)<\tilde{v}_{2}(\varepsilon)$. The first period utility is

$$
\begin{equation*}
v_{1}=\bar{x}_{1}-s-x_{1}^{p} . \tag{9}
\end{equation*}
$$

The consumer's expected utility is

$$
\begin{align*}
& E(U)=v_{1}+\frac{1}{1+\rho}\left[\tilde{v}_{2}\left(\varepsilon^{*}\right) \int_{\varepsilon}^{\varepsilon *} f(\varepsilon) d \varepsilon+\int_{\varepsilon^{*}}^{\bar{\varepsilon}} \tilde{v}_{2}(\varepsilon) f(\varepsilon) d \varepsilon\right]=  \tag{10}\\
& v_{1}+\frac{1}{1+\rho}\left[\tilde{v}_{2}\left(\varepsilon^{*}\right)+\int_{\varepsilon^{*}}^{\bar{\varepsilon}}\left\{\tilde{v}_{2}(\varepsilon)-\tilde{v}_{2}\left(\varepsilon^{*}\right)\right\} f(\varepsilon) d \varepsilon\right]
\end{align*}
$$

The last line in (10) implies that the consumer will exercise the option of spot market imports in the second period only if the terms of trade were favorable enough -- if $\varepsilon^{*}<\varepsilon$. Otherwise (when $\varepsilon^{*}>\varepsilon$ ), the second period supply of imports is determined by the pre-buying of y . In these circumstances the prices of imports are delinked from the realized terms of trade, as is the case in the pricing to market (PTM) regime.

Equation (6) indicates that the choice of the optimal pre-buying determines also the range where PTM applies, occurring with probability $F\left(\varepsilon^{*}\right)=\int_{\varepsilon}^{\varepsilon^{*}} f(\varepsilon) d \varepsilon$. It implies that increasing the pre-buying of imports reduces the range where the option of spot market imports would be exercised, increasing the frequency of PTM. This suggests that the optimal pre-buying of imports tends to be lower the greater the value of the flexibility associated with the option of using the spot market is, as will be the case when the terms of trade volatility go up. Pre-buying implies also implicit saving, hence the opportunity cost of pre-buying increases with the discount rate. This suggests that a higher discount rate would reduce the pre-buying, thereby increasing the range where the option of imports via the spot market is exercised.

We verify these claims by studying the first order condition determining the optimal pre-buying

$$
\text { (11) }-1+\frac{1}{1+\rho}\left[\int_{\varepsilon}^{\varepsilon^{*}} \frac{\partial \tilde{v}_{2}\left(\varepsilon^{*}\right)}{\partial x_{1}^{p}} f(\varepsilon) d \varepsilon+\int_{\varepsilon^{*}}^{\varepsilon} \frac{\partial \tilde{v}_{2}(\varepsilon)}{\partial x_{1}^{p}} f(\varepsilon) d \varepsilon\right]=0
$$

$$
\begin{equation*}
-1+\frac{1}{1+\rho}\left[\int_{\underline{\varepsilon}}^{\varepsilon^{*}} \frac{\theta}{1+t}\left[\frac{x_{1}^{p}}{1+t}\right]^{\beta-1} f(\varepsilon) d \varepsilon+\int_{\varepsilon^{*}}^{\varepsilon} \frac{1+\tilde{t}}{(1+\varepsilon)(1+t)} f(\varepsilon) d \varepsilon\right]=0 \tag{12}
\end{equation*}
$$

The expected gain from pre-buying is the discounted expected marginal utility induced by pre-buying minus the opportunity cost of pre-buying. Optimality requires this gain to be zero at the margin. Applying (6) to (12), we can rewrite the first order condition as

$$
\begin{equation*}
-1+c\left[\frac{1}{1+\varepsilon^{*}} \int_{\varepsilon}^{\varepsilon^{*}} f(\varepsilon) d \varepsilon+\int_{\varepsilon^{*}}^{\bar{\varepsilon}} \frac{1}{1+\varepsilon} f(\varepsilon) d \varepsilon\right]=0 \text { where } c=\frac{1+\tilde{t}}{(1+\rho)(1+t)} \tag{12'}
\end{equation*}
$$

The term $c$ is the expected relative intertemporal cost of the spot market to forward imports. ${ }^{10}$ Henceforth we will assume that $c>1$, as will be the case if last minute delivery is relatively costly (Section 4 reviews the results for the case when $c<1$ ). The equilibrium relative price in the second period is determined by the ratio of the marginal utilities of the 2 goods --

$$
\begin{equation*}
\frac{p_{y, 2}}{p_{x, 2}}=\frac{d v_{2} / d y_{2}}{d v_{2} / d x_{2}}=\theta\left[y_{2}\right]^{\beta-1} \tag{13}
\end{equation*}
$$

Applying (5) - (7) it follows that

10 Equation (12') has a simple interpretation. With optimal pre-buying, the discounted expected effective relative price of future imports equals the transportation costs ratio, $\frac{1+t}{1+\tilde{t}}$, where the effective future relative price of imports is defined net of transportation costs, and is given by $\min \left[\frac{1}{1+\varepsilon^{*}} ; \frac{1}{1+\varepsilon}\right]$.

$$
\frac{p_{y, 2}}{p_{x, 2}}=\min \left[\frac{1+\tilde{t}}{1+\varepsilon} ; \frac{1+\tilde{t}}{1+\varepsilon^{*}}\right]=\left\{\begin{array}{ll}
\frac{1+\tilde{t}}{1+\varepsilon} & \text { if } \varepsilon>\varepsilon^{*}  \tag{14}\\
\frac{1+\tilde{t}}{1+\varepsilon^{*}} & \text { if } \varepsilon \leq \varepsilon^{*}
\end{array}\right. \text {. }
$$

Hence, if the realized terms of trade are favorable enough ( $\varepsilon>\varepsilon^{*}$ ), the spot market for imports is active. In these circumstances, the price of imports is determined by the realized real exchange rate, $\frac{p_{y, 2}}{p_{x, 2}}=\frac{1+\tilde{t}}{1+\varepsilon}$, and the relative PPP holds (adjusted for transportation costs). We refer to this regime as the flexible price regime, and denote it by FL.

If the realized terms of trade are weak $(\varepsilon<\varepsilon *)$, no spot market trade will take place, and the PTM regime will prevail, $\frac{p_{y, 2}}{p_{x, 2}}=\frac{1+\tilde{t}}{1+\varepsilon^{*}}$. Hence, optimal pre-buying of imports shields the consumer from the "bad tail" of the terms of trade distribution. Recalling (13), the corresponding imports' relative price is

$$
\begin{equation*}
\left.\frac{p_{y, 2}}{p_{x, 2}}\right|_{P T M}=\theta\left(\frac{x_{1}^{p}}{1+t}\right)^{\beta-1} . \tag{15}
\end{equation*}
$$

The relative price in the PTM regime is determined by the first period pre-buying of the imports, $x_{1}^{p}$, which is the outcome of maximizing the first period expected utility. Applying (15) to the first order condition, (12), it follows that

$$
\begin{equation*}
\frac{p_{y, 2}}{p_{x, 2}} \left\lvert\, P T M=\frac{1}{F\left(\varepsilon^{*}\right)}\left[(1+t)(1+\rho)-\int_{\varepsilon^{*}}^{\varepsilon} \frac{1+\tilde{t}}{1+\varepsilon} f(\varepsilon) d \varepsilon\right] .\right. \tag{16}
\end{equation*}
$$

As the probability of PTM approaches one, $\varepsilon^{*} \rightarrow \bar{\varepsilon}, F\left(\varepsilon^{*}\right) \rightarrow 1$ and

$$
\begin{equation*}
\left.\frac{p_{y, 2}}{p_{x, 2}}\right|_{P T M} \xrightarrow[\varepsilon^{*} \rightarrow \varepsilon]{ }(1+t)(1+\rho) . \tag{17}
\end{equation*}
$$

Hence, the relative price of imports in the PTM regime increases with the discount factor $\rho$. This suggests that a higher discount factor will reduce the optimal pre-buying, and will increase the frequency of PTM.

Proposition 1 summarizes the resulting pricing system:

## Proposition 1

a. Higher discount rate, higher pre-buying transportation cost, and lower transportation cost of imports purchased in the spot market reduce the first period pre-buying. This in turn implies that the frequency of PTM drops and the frequency of flexible prices (FL regime) increase.
b. Low volatility economies are characterized by PTM. Spot market imports would be observed if the volatility of the terms of trade exceeds a threshold. Above that threshold, higher volatility will reduce the frequency of PTM, increasing spot market imports. A low enough probability of PTM may reverse the impact of volatility.

## Proof

Proposition 1a follows from the observation that a lower $c$ would reduce the valuation of the expected gain from pre-buying, reducing the optimal pre-buying of imports (recall that $\left.c=\frac{1+\tilde{t}}{(1+\rho)(1+t)}\right)$. We denote the LHS of (12') by $L, L=-1+c\left[\frac{1}{1+\varepsilon^{*}} \int_{\underline{\varepsilon}}^{\varepsilon *} f(\varepsilon) d \varepsilon+\int_{\varepsilon^{*}}^{\varepsilon} \frac{1}{1+\varepsilon} f(\varepsilon) d \varepsilon\right]$. Note that $\operatorname{sgn} \frac{d F}{d c}=\operatorname{sgn} \frac{d \varepsilon^{*}}{d c}$. The first order condition determining $\varepsilon *$ is $L=0$. Hence $\frac{d \varepsilon^{*}}{d c}=-\frac{\partial L / \partial c}{\partial L / \partial \varepsilon^{*}}$. The second order condition for maximization implies that $\partial L / \partial \varepsilon *<0$, hence $\operatorname{sgn} \frac{d F}{d c}=\operatorname{sgn}[\partial L / \partial c]=\operatorname{sgn}\left[\frac{1}{1+\varepsilon^{*}} \int_{\varepsilon}^{\varepsilon^{*}} f(\varepsilon) d \varepsilon+\int_{\varepsilon^{*}}^{\bar{\varepsilon}} \frac{1}{1+\varepsilon} f(\varepsilon) d \varepsilon\right]>0$.

This situation is summarized in Figure 1, where the downwards sloping curves depict the marginal benefit of pre-buying imports [LHS of (12')] for varying values of the $\varepsilon *$ (note that (6) implies that higher $\varepsilon$ * corresponds to a higher pre-buying of imports). The optimal threshold $\varepsilon *$ (and the corresponding optimal prebuying of imports) is determined at $\mathrm{L}=0$, at point A for curve ZZ . Increasing $c$ shifts the curve upwards,
increasing thereby the optimal pre-buying of imports, and the frequency of PTM. For a high enough $c$ we converge to a pure PTM pricing system, where spot market imports would be too expensive for all possible future terms of trade realizations. A lower c shifts this curve downwards, reducing thereby the optimal threshold $\varepsilon^{*}$.


Figure 1 -- Marginal Benefit of Pre-buying imports
The Figure traces the dependency of $L$ on $\varepsilon *$, for a uniform distribution, where $f(\varepsilon)=1 /(2 \bar{\varepsilon})$, and $\bar{\varepsilon}=0.4$. The solid, dashed and dotted curves correspond to $c=1.2,1.3,1.4$, respectively.

The intuition for result 1 b is that in the absence of any volatility, $c>1$ implies that pre-buying is cheaper, hence the PTM regime will prevail. Greater volatility induces more frequent realizations of relatively high and low real exchange rates. For volatility high enough, it would make sense to scale down the pre-buying, in order to exploit the "good tail" of the real exchange distribution, where spot market imports are cheaper.

We illustrate the impact of higher volatility by considering a uniform distribution of the terms of trade shock, where $\bar{\varepsilon}=-\underline{\varepsilon} .{ }^{11}$ In these circumstances the first order condition determining the frequency of PTM, (12'), can be reduced to

$$
\begin{equation*}
0=-1+\frac{0.5 c}{\bar{\varepsilon}}\left[\frac{2 F \bar{\varepsilon}}{1+\bar{\varepsilon}(2 F-1)}+\ln \frac{1+\bar{\varepsilon}}{1+\bar{\varepsilon}(2 F-1)}\right] \tag{18}
\end{equation*}
$$

This condition implies that the spot market imports will be exercised (and hence $F<1$ ) only if the volatility is high enough, so that $\bar{\varepsilon}>c-1 .{ }^{12}$ Applying (18) we infer that

$$
\begin{equation*}
\operatorname{sgn} \frac{d F}{d \bar{\varepsilon}}=\operatorname{sgn}\left\{\frac{1}{\bar{\varepsilon}}\left[\frac{1}{1+\bar{\varepsilon}}-\frac{2 F-1}{1+\bar{\varepsilon}(2 F-1)}\right]-\frac{1}{(\bar{\varepsilon})^{2}} \ln \frac{1+\bar{\varepsilon}}{1+\bar{\varepsilon}(2 F-1)}-\frac{2 F(2 F-1)}{[1+\bar{\varepsilon}(2 F-1)]^{2}}\right\} \tag{19}
\end{equation*}
$$

Hence,
(20) $\left.\operatorname{sgn} \frac{d F}{d \bar{\varepsilon}}\right|_{F \rightarrow 1}=\operatorname{sgn}\left\{-\frac{2}{[1+\bar{\varepsilon}]^{2}}\right\}<0$

11 Similar results apply if the terms of trade follow a truncated normal distribution, or if the log of the terms of trade follows the normal distribution.
12 This result is obtained by evaluating $0=-1+\frac{0.5 c}{\bar{\varepsilon}}\left[\frac{2 F \bar{\varepsilon}}{1+\bar{\varepsilon}(2 F-1)}+\ln \frac{1+\bar{\varepsilon}}{1+\bar{\varepsilon}(2 F-1)}\right]$ at $\mathrm{F}=1$.

When the probability of PTM is low enough, further increase in volatility may reverse the association between volatility and the probability of PTM. ${ }^{13}$

Figure 2 depicts the dependency of the frequency of PTM on the terms of trade volatility, for different levels of $c$. Above the threshold $c-1$, a higher term of trade volatility reduces the frequency of the PTM regime. Lower c shifts the curve leftwards and downwards. For a small enough $c$ and for volatility high enough, the association between volatility and pricing to market may be reversed -- higher volatility increases F. ${ }^{14}$


Figure 2 -- Volatility and Pricing to Market incidence
The Figure traces the dependency of the PTM probability $F$ on the volatility, for a uniform distribution, where $f(\varepsilon)=1 /(2 \bar{\varepsilon})$, and $c=1.03,1.1,1.15$.

13 To confirm this, note that
$\left.\operatorname{sgn} \frac{d F}{d \bar{\varepsilon}}\right|_{F \rightarrow 0}=\operatorname{sgn}\left\{\frac{1}{\bar{\varepsilon}}\left[\frac{1}{1+\bar{\varepsilon}}+\frac{1}{1-\bar{\varepsilon}}\right]-\frac{1}{(\bar{\varepsilon})^{2}} \ln \frac{1+\bar{\varepsilon}}{1-\bar{\varepsilon}}\right\}=\operatorname{sgn}\left\{\frac{2 \bar{\varepsilon}}{1-\bar{\varepsilon}^{2}}-\ln \frac{1+\bar{\varepsilon}}{1-\bar{\varepsilon}}\right\}>0$. The last inequality follows from the observations that $d\left\{\frac{2 \bar{\varepsilon}}{1-\bar{\varepsilon}^{2}}-\ln \frac{1+\bar{\varepsilon}}{1-\bar{\varepsilon}}\right\} / d \bar{\varepsilon}>0$ for $\bar{\varepsilon}>0$, and $\frac{2 \bar{\varepsilon}}{1-\bar{\varepsilon}^{2}}-\ln \frac{1+\bar{\varepsilon}}{1-\bar{\varepsilon}}=0$ for $\bar{\varepsilon}=0$.

14 See Aizenman (1984) for a model of deviations from PPP in a one good world, where the domestic and the foreign markets are separated by time independent transportation costs. It is shown there that even in that simpler world, the association between volatility and deviations from PPP may be reversed for a high enough volatility.

## 3. Financing costs, volatility and welfare

The empirical literature suggests that import prices are more responsive to the exchange rate in emerging markets. Our discussion in the previous section suggests two independent channels explaining this finding volatility and financing costs. The purpose of this section is to investigate the welfare cost of these channels, and to identify which of the two has a greater impact on the overall welfare of a country. One way to address these issues with linear, risk neutral preferences is to assume that the interest rate exceeds the discount factor, and all pre-buying is financed by credit, as would be the case if $\bar{x}_{1}=0$, and $r^{*}>\rho$. This is a special example of the model studied in Section 1, corresponding to $s=-x_{1}^{p} ; \quad v_{1}=0$. The first order condition determining the optimal level of pre-buying in these circumstances is ${ }^{15}$

$$
(21)-1+c^{*}\left[\frac{1}{1+\varepsilon^{*}} \int_{\underline{\varepsilon}}^{\varepsilon^{*}} f(\varepsilon) d \varepsilon+\int_{\varepsilon^{*}}^{\varepsilon} \frac{1}{1+\varepsilon} f(\varepsilon) d \varepsilon\right]=0 ; \text { where } c^{*}=\frac{1+\tilde{t}}{\left(1+r^{*}\right)(1+t)}
$$

We assume that the interest rate and the terms of trade volatility are not too high, resulting in an internal equilibrium where some pre-buying takes place, and $0<F<1$. Proposition 2 summarizes the resulting pricing system:

## Proposition 2

a. Higher financing costs increase the expected price of imports, reducing the expected deviations from relative PPP, and reducing welfare.
b. For a given real interest rate, higher terms of trade volatility tend to increases welfare.

15 This condition is obtained from (10), noting that with $r^{*}>\rho, v_{1}=0$, and
$\tilde{v}_{2}\left(\varepsilon^{*}\right)=\bar{x}_{2}-x_{1}^{p}\left(1+r^{*}\right)+\frac{\theta}{\beta}\left[\frac{x_{1}^{p}}{1+t}\right]^{\beta} ; \tilde{v}_{2}(\varepsilon)=\bar{x}_{2}-x_{1}^{p}\left(1+r^{*}\right)+\frac{x_{1}^{p}(1+\tilde{t})}{(1+\varepsilon)(1+t)}+\left(\frac{1}{\beta}-1\right) \theta^{1 /(1-\beta)}\left(\frac{1+\varepsilon}{1+\tilde{t}}\right)^{\beta /(1-\beta)}$.

## Proof

a. Recalling that optimal pre-buying is determined by $\frac{\partial E(U)}{\partial x_{1}^{p}}=0$, the envelope theorem implies that the welfare effect of the higher interest rate is

$$
\begin{equation*}
\frac{d E(U)}{d r^{*}}=\frac{\partial E(U)}{\partial r^{*}}+\frac{\partial E(U)}{\partial x_{1}^{p}} \frac{d x_{1}^{p}}{d r^{*}}=-\frac{x_{p}^{1}}{1+\rho} . \tag{22}
\end{equation*}
$$

Recalling (14), the expected relative price of imports is

$$
\begin{equation*}
E\left(\frac{p_{v, 2}}{p_{x, 2}}\right)=(1+\tilde{t})\left[\int_{\varepsilon}^{\varepsilon_{\varepsilon}^{*}} \frac{1}{1+\varepsilon^{*}} f(\varepsilon) d \varepsilon+\int_{\varepsilon^{*}}^{\varepsilon} \frac{1}{1+\varepsilon} f(\varepsilon) d \varepsilon\right] \tag{23}
\end{equation*}
$$

Applying (6) and (21) to (23) we infer that

$$
\begin{equation*}
\frac{\partial E\left(\frac{p_{y, 2}}{p_{x, 2}}\right)}{\partial r^{*}}=1+t>0 \tag{24}
\end{equation*}
$$

Hence, higher financing costs increase the expected relative price of imports by the transportation cost of spot market imports.

We turn now to assess the expected deviations from PPP, adjusted for transportation costs. The actual relative price of imports is provided by (14). The spot market relative price of imports is $\left.\frac{p_{y, 2}}{p_{x, 2}}\right|_{\text {spot }}=\frac{1+\tilde{t}}{1+\varepsilon}$. A measure of the deviation from the relative PPP is the percentage gap between the relative price for spot market imports and the relative price observed in the domestic market,

$$
\begin{equation*}
1-\frac{p_{y, 2} / p_{x, 2}}{p_{y, 2} /\left.p_{x, 2}\right|_{\text {spot }}} \tag{25}
\end{equation*}
$$

Applying (14), if follows that

$$
\begin{equation*}
E\left(1-\frac{p_{y, 2} / p_{x, 2}}{p_{y, 2} / p_{x, 2} \mid s p o t}\right)=\int_{\underline{\varepsilon}}^{\varepsilon^{*}} \frac{\varepsilon^{*}-\varepsilon}{1+\varepsilon^{*}} f(\varepsilon) d \varepsilon>0 \tag{26}
\end{equation*}
$$

Hence, the expected deviations from PPP is proportional to the expected gap between the PTM relative price and the flexible price terms of trade, evaluated along the "bad tail" of the terms of trade distribution. Higher financing costs imply that

$$
\begin{equation*}
\frac{\partial}{\partial r^{*}}\left[E\left(1-\frac{p_{y, 2} / p_{x, 2}}{p_{y, 2} / p_{x, 2} \mid s p o t}\right)\right]=\int_{\varepsilon}^{\varepsilon^{*}} \frac{1+\varepsilon}{\left(1+\varepsilon^{*}\right)^{2}} f(\varepsilon) d \varepsilon \frac{\partial \varepsilon^{*}}{\partial r^{*}}<0 . \tag{27}
\end{equation*}
$$

Recalling that higher financing costs reduce the pre-buying, it follows that it also reduces the expected deviations from PPP.
b. We illustrate the welfare effects associated with greater volatility by considering a uniform distribution, where $f(\varepsilon)=\frac{1}{2 \bar{\varepsilon}}$. In these circumstances, $E(U)=\frac{1}{1+\rho} \frac{1}{2 \bar{\varepsilon}}\left[\tilde{v}_{2}\left(\varepsilon^{*}\right) \int_{-\bar{\varepsilon}}^{\varepsilon^{*}} d \varepsilon+\int_{\varepsilon^{*}}^{\varepsilon} \tilde{v}_{2}(\varepsilon) d \varepsilon\right]$. Hence

$$
\begin{aligned}
& \frac{\partial E(U)}{\partial \bar{\varepsilon}}=-\frac{E(U)}{\bar{\varepsilon}}+\frac{1}{1+\rho} \frac{1}{2 \bar{\varepsilon}}\left[\tilde{v}_{2}\left(\varepsilon^{*}\right)+\tilde{v}_{2}(\bar{\varepsilon})\right]= \\
& \frac{1}{1+\rho} \frac{1}{\bar{\varepsilon}}\left\{\frac{\tilde{v}_{2}\left(\varepsilon^{*}\right)+\tilde{v}_{2}(\bar{\varepsilon})}{2}-\frac{1}{2 \bar{\varepsilon}}\left(\tilde{v}_{2}\left(\varepsilon^{*}\right) \int_{-\bar{\varepsilon}}^{\varepsilon^{*}} d \varepsilon+\int_{\varepsilon^{*}}^{\bar{\varepsilon}} \tilde{v}_{2}(\varepsilon) d \varepsilon\right)\right\}
\end{aligned}
$$

Applying (8), it follows that for most plausible parameters, $\tilde{v}_{2}(\varepsilon)$ is a convex function of $\varepsilon$ for $\varepsilon^{*}<\varepsilon$. ${ }^{16}$ Hence $\frac{1}{2 \bar{\varepsilon}} \int_{\varepsilon^{*}}^{\varepsilon} \tilde{v}_{2}(\varepsilon) d \varepsilon<\frac{\bar{\varepsilon}-\varepsilon^{*}}{2 \bar{\varepsilon}} 0.5\left[\tilde{v}_{2}\left(\varepsilon^{*}\right)+\tilde{v}_{2}(\bar{\varepsilon})\right]=0.5(1-F)\left[\tilde{v}_{2}\left(\varepsilon^{*}\right)+\tilde{v}_{2}(\bar{\varepsilon})\right]$. Applying this result to (28) we infer that

$$
\begin{aligned}
& \frac{\partial E(U)}{\partial \bar{\varepsilon}}=\frac{1}{1+\rho} \frac{1}{\bar{\varepsilon}}\left\{\frac{\tilde{v}_{2}\left(\varepsilon^{*}\right)+\tilde{v}_{2}(\bar{\varepsilon})}{2}-\frac{1}{2 \bar{\varepsilon}}\left(\tilde{v}_{2}\left(\varepsilon^{*}\right) \int_{-\bar{\varepsilon}}^{\varepsilon^{*}} d \varepsilon+\int_{\varepsilon^{*}}^{\overline{\bar{v}}} \tilde{v}_{2}(\varepsilon) d \varepsilon\right)\right\}> \\
& \frac{1}{1+\rho} \frac{1}{\bar{\varepsilon}}\left\{\frac{\tilde{v}_{2}\left(\varepsilon^{*}\right)+\tilde{v}_{2}(\bar{\varepsilon})}{2}-\left(\tilde{v}_{2}\left(\varepsilon^{*}\right) F+0.5(1-F)\left[\tilde{v}_{2}\left(\varepsilon^{*}\right)+\tilde{v}_{2}(\bar{\varepsilon})\right]\right)\right\}= \\
& \frac{1}{1+\rho} \frac{0.5 F}{\bar{\varepsilon}}\left\{\tilde{v}_{2}(\bar{\varepsilon})-\tilde{v}_{2}\left(\varepsilon^{*}\right)\right\}>0
\end{aligned}
$$

(28')


Figure 3
Volatility and welfare
The intuition of this result is traced with the help of Figure 3, reporting the dependency of the second period utility on the realized terms of trade. Higher volatility spreads the probability weights to the right and the left.

16 Applying (6) and (8) it follows that for all probability distribution functions $\tilde{v}_{2}(\varepsilon)$ is a convex for $\varepsilon^{*}<\varepsilon$ if and only if $2\left[\frac{1+\varepsilon^{*}}{1+\varepsilon}\right]^{\frac{1}{1-\beta}}+\frac{\beta}{1-\beta}>1$. This condition holds for all p.d.f. if the demand for imports is elastic (i.e., if $0.5<\beta$ ). It can be verified that for a uniform distribution this condition holds for a wide range of elasticities - if $2\left[\exp \left(1-\frac{2}{c^{*}}\right)\right]^{\frac{1}{1-\beta}}+\frac{\beta}{1-\beta}>1$.

Equation (28) implies that the net effect is proportional to the difference between the average utility at the tail points (A and C , equal to the height of point H ) and the expected utility across the entire support. The convexity of curve ABC implies that this difference is positive. Consequently, a greater terms of trade volatility increases the expected utility. ${ }^{17}$

We illustrate the quantitative nature of these results with the help of a simulation. The bold curve in figure 4A traces the dependency of the probability of the PTM regime on the interest rate. The bold curve in Figure 4B traces the dependency of the probability of the PTM regime on volatility. The contours in both panels trace the welfare relative to the benchmark at points Q and $\mathrm{Q}^{\prime}$, respectively, where $F=1$. Note that increasing the interest rate from zero to about 0.3 eliminates the pre-buying, and induces a welfare drop of about $6.5 \%$. This welfare drop is due to the elimination of gains from the pre-buying -- gains attributed to the ability to protect the purchasing power against weak future terms of trade. It can be shown that the welfare drop is associated with a large drop in average imports [from 0.44 at point Q , to 0.29 at point M ]. Volatility by itself, however, enhances welfare, as is shown in Figure 4B.
$17 \quad$ Figure 3 assumes a uniform distribution. Similar logic implies that if the demand for imports is elastic (i.e., if $0.5<\beta$ ), a mean preserving increase in the terms of trade volatility would increase welfare for all probability distribution functions.


Figure 4A - Interest rate and PTM

## Financing costs, volatility and welfare

The Figures trace the probability of PTM $(F)$ for a uniform distribution, where
$f(\varepsilon)=1 /(2 \bar{\varepsilon}) ; \quad \theta=0.75 ; \quad \beta=0.5 ; \quad t=0.1 ; \quad \tilde{t}=0.4 ; \quad \rho=0$. The bold line traces the dependency of F on the interest rate (Figure 4A) and volatility (Figure 4B), respectively. Figure 4A assumes $\bar{\varepsilon}=0.25$, Figure 4 B assumes $\mathrm{r}=.25$. The contours trace the welfare relative to the benchmark at point Q and $\mathrm{Q}^{\prime}$ in Figure 4A and 4 B , respectively.

## 4. Concluding remarks

Our discussion provides two possible explanations for why relative PPP holds better for emerging markets. Higher volatility of terms of trade and higher financial costs in emerging markets should induce tighter association between the exchange rate and imports prices. While we treated the interest rate and the volatility as independent, the two are likely to be positively correlated. First, if the domestic capital market in emerging markets is segmented from the global market, one expects credit to be financed by risk averse agents who would demand a higher interest rate to compensate for the higher volatility. Second, the literature on costly state verification pointed out that volatility tends to be associated with higher financial costs [see Townsend (1979)]. Hence, one expects that higher terms of trade volatility will increase the cost of credit. Applying this association, one may combine Figures 4A and 4B together, tracing the combined effects of the volatility and the interest rate channels. In these circumstances, one expects that the net welfare effect of volatility will be negative -- the higher cost of credit will terminate PTM, eliminating any welfare gains from higher volatility. We illustrate this in Figure 5, which assumes a linear association between terms of trade volatility and financing costs. The bold line outlines the dependency of the probability of the PTM regime on the volatility. The contours trace the welfare, relative to the benchmark at point L , where $F=1$.

A key assumption of our paper is that pre-buying is cheaper -- the discounted expected cost of spot market imports exceeds the expected cost of pre-buying imports. One expects this assumption to hold better for goods and commodities with high freight/value ratio (oil, cars, etc.). Our methodology is applicable even if this assumption is violated. ${ }^{18}$ To simplify, we assumed competitive pricing. The analysis can be readily

18 The discounted expected cost of spot market imports is below the expected cost of pre-buying if the interest rate is high enough, or if the transportation cost of last minute delivery is not significantly higher than the transportation cost of pre-buying. In these circumstances, if the volatility of the terms of trade is low, no imports are pre-bought [hence $F=0$ ]. For high enough volatility, some imports are pre-bought, at a level that is determined by (12), and $F>0$. Even in these circumstances, proposition 2 continues to apply -- a higher interest rate would reduce the probability of PTM, would increase the expected price of importables, and would reduce the expected deviations from relative PPP.
extended to allow for monopolistic competitive pricing, where the market power would impact on the pricing rule.

Our paper suggests that in the circumstances facing emerging markets, pricing to market would be observed less frequently than in the OECD countries. Hence, in evaluating the choice of exchange rate regimes for emerging markets, assuming relative PPP is likely to describe better the economic environment. This in turn suggests a bias towards lower flexibility of the exchange rate in emerging markets. First, lower flexibility of the exchange rate may reduce the real interest rate in segmented capital markets (see Aizenman and Hausmann (2000), where this channel is modeled). Second, as Devereux and Engel (1998) showed, pricing to market biases the choice in favor of a flexible exchange rate by the resultant delinking of domestic prices from the exchange rate, a bias that would not hold for emerging markets where the pricing to market is not a viable option.


Figure 5

## Financing costs and volatility

The Figure traces the dependency of the PTM probability $F$ on the volatility, for a uniform distribution, where $f(\varepsilon)=1 /(2 \bar{\varepsilon}) ; \quad \theta=0.75 ; \quad \beta=0.5 ; \quad t=0.1 ; \tilde{t}=0.4 ; \quad \rho=0$, and where the interest rate increases with volatility, $r=0.5 \bar{\varepsilon}$. The bold line depicts the dependency of F on the volatility. The contours trace the welfare relative to the benchmark at point L , where $F=1$.

## Appendix A

The paper assumed risk neutral agents, with intertemporal linear preferences. While these assumptions simplified the analysis, the linear model has a limited ability to account for the impact of a higher interest rate. This Appendix illustrates how to extend our analysis to the case of risk averse consumers. Specifically, suppose that with the exception of preferences, all the assumptions of the paper hold. The consumers maximize the expected value of

$$
H=U\left(v_{1}\right)+\frac{1}{1+\rho} U\left(v_{2}\right) \quad ; \text { where } U(v)= \begin{cases}\frac{[v]^{1-\phi}}{1-\phi} & \text { for } \phi \neq 1  \tag{A1}\\ \ln v & \text { for } \phi=1\end{cases}
$$

Hence, the consumer's expected utility is

$$
\begin{align*}
& E(H)=U\left(v_{1}\right)+\frac{1}{1+\rho}\left[U\left[\tilde{v}_{2}\left(\varepsilon^{*}\right)\right] \int_{\varepsilon}^{\varepsilon *} f(\varepsilon) d \varepsilon+\int_{\varepsilon^{*}}^{\bar{\varepsilon}} U\left[\tilde{v}_{2}(\varepsilon)\right] f(\varepsilon) d \varepsilon\right]=  \tag{A2}\\
& U\left(v_{1}\right)+\frac{1}{1+\rho}\left[U\left[\tilde{v}_{2}\left(\varepsilon^{*}\right)\right]+\int_{\varepsilon^{*}}^{\bar{\varepsilon}}\left\{U\left[\tilde{v}_{2}(\varepsilon)\right]-U\left[\tilde{v}_{2}\left(\varepsilon^{*}\right)\right]\right\} f(\varepsilon) d \varepsilon\right]
\end{align*}
$$

The consumer's problem is to determine the optimal triplet $\left\langle x_{2}^{s} ; x_{1}^{p} ; s\right\rangle$. We solve it backwards -- first we find the optimal spot market trade in the second period $\left(x_{2}^{s}\right)$. Next, applying this solution we construct the expected utility in the first period. Finally, we find the pre-buying and the saving $\left(x_{1}^{p} ; s\right)$ that maximizes this expected utility. Note that the first part of the solution (optimal $\left(x_{2}^{s}\right)$ ) is identical to the one in Section 2, because it deals with the patterns of consumption in the second period, after the uncertainty of the terms of trade has been resolved. Hence, equation (5) - (9) continue to hold. Applying these conditions to (A2), we infer that the first order conditions determining the optimal saving and the optimal pre-buying are

$$
\text { (A3) } \begin{cases}a . & -U^{\prime}\left(v_{1}\right)+\frac{1+r^{*}}{1+\rho}\left[U^{\prime}\left[\tilde{v}_{2}\left(\varepsilon^{*}\right)\right] \int_{\underline{\varepsilon}}^{\varepsilon *} f(\varepsilon) d \varepsilon+\int_{\varepsilon^{*}}^{\varepsilon} U^{\prime}\left[\tilde{v}_{2}(\varepsilon)\right] f(\varepsilon) d \varepsilon\right]=0 \\ b . & -U^{\prime}\left(v_{1}\right)+\frac{1+\tilde{t}}{(1+\rho)(1+t)}\left[\frac{U^{\prime}\left[{\tilde{v_{2}}}_{2}\left(\varepsilon^{*}\right)\right]^{\varepsilon *}}{1+\varepsilon^{*}} \int_{\underline{\varepsilon}}^{\varepsilon} f(\varepsilon) d \varepsilon+\int_{\varepsilon^{*}}^{\bar{\varepsilon}} \frac{U^{\prime}\left[\tilde{v}_{2}(\varepsilon)\right]}{1+\varepsilon} f(\varepsilon) d \varepsilon\right]=0\end{cases}
$$

where $U^{\prime}=\partial U / \partial x$ is the marginal utility of $x$ (the domestic good). There are two ways to transfer purchasing power from the first to the second period -- saving and pre-buying. The first order conditions in (A3) imply that intertemporal arbitrage exhausts the utility gain from intertemporal trade. Optimal saving is reached when the first period marginal utility of $x$ equals the interest rate times the expected marginal utility of the second period consumption, discounted by the subjective rate of time preference (see (A3a)). Similarly, optimal pre-buying equates the marginal utility of the first period consumption of $x$ ( $=$ the opportunity cost of x in the first period) to the discounted expected marginal utility induced by pre-buying $1 /(1+\mathrm{t})$ units of second period $y$ (see (A3b)).

The impact of higher financing costs are summarized in the following proposition:

## Proposition A1

Higher interest rate reduces the first period pre-buying for small savings. This in turn implies that the frequency of PTM goes down.

## Proof

A higher interest rate would increase the relative price of pre-buying. This would lead to a substitution away from pre-buying to spot market delivery, reducing the incidence of pricing to market. The assumption that the net saving is small implies that the induced income effect due to the interest rate change is small, and hence the substitution effect would dominate.

We illustrate it for the case where $\phi=1$ (hence $U(v)=\ln v$ ). Similar methodology applies for the case where $\phi \neq 1$. The first order conditions can be rewritten as

$$
\text { (A4) } \begin{cases}a . & -\frac{1}{v_{1}}+\frac{1+r *}{1+\rho}\left[\frac{1}{\tilde{v}_{2}\left(\varepsilon^{*}\right)} \int_{\underline{\varepsilon}}^{\varepsilon *} f(\varepsilon) d \varepsilon+\int_{\varepsilon^{*}}^{\varepsilon} \frac{1}{\tilde{v}_{2}(\varepsilon)} f(\varepsilon) d \varepsilon\right]=0 \\ b . & -\frac{1}{v_{1}}+\frac{1+\tilde{t}}{(1+\rho)(1+t)}\left[\frac{1}{\tilde{v}_{2}\left(\varepsilon^{*}\right)\left(1+\varepsilon^{*}\right)} \int_{\underline{\varepsilon}}^{\varepsilon^{*}} f(\varepsilon) d \varepsilon+\int_{\varepsilon^{*}}^{\bar{\varepsilon}} \frac{1}{\tilde{v}_{2}(\varepsilon)(1+\varepsilon)} f(\varepsilon) d \varepsilon\right]=0\end{cases}
$$

We denote these first order conditions by
(A5) $\begin{cases}a . & L_{1}=0 \\ b . & L_{2}=0\end{cases}$

Using equation (8), (9), and (A1) we infer that the impact of changing the interest rate on saving and pre-buying is summarized by

$$
\left(\begin{array}{ll}
\frac{\partial L_{1}}{\partial s} & \frac{\partial L_{1}}{\partial x_{1}^{p}}  \tag{A6}\\
\frac{\partial L_{2}}{\partial s} & \frac{\partial L_{2}}{\partial x_{1}^{p}}
\end{array}\right)\left[\begin{array}{c}
d s \\
d x_{1}^{p}
\end{array}\right]=-d r *\left[\begin{array}{c}
\frac{\partial L_{1}}{\partial r^{*}} \\
\frac{\partial L_{2}}{\partial r^{*}}
\end{array}\right]
$$

where

$$
\begin{aligned}
& \frac{\partial L_{1}}{\partial s}=-\frac{1}{\left(v_{1}\right)^{2}}-\frac{\left(1+r^{*}\right)^{2}}{1+\rho}\left[\frac{1}{\left[\tilde{v}_{2}\left(\varepsilon^{*}\right)\right]^{2}} \int_{\varepsilon}^{\varepsilon^{*}} f(\varepsilon) d \varepsilon+\int_{\varepsilon^{*}}^{\bar{\varepsilon}} \frac{f(\varepsilon)}{\left[\tilde{v}_{2}(\varepsilon)\right]^{2}} d \varepsilon\right]<0 \\
& \frac{\partial L_{1}}{\partial x_{1}^{p}}=-\frac{1}{\left(v_{1}\right)^{2}}-\frac{1+r^{*}}{1+\rho}\left[\frac{1}{\left[\tilde{v}_{2}\left(\varepsilon^{*}\right)\right]^{2}} \frac{\theta}{1+t}\left[\frac{x_{1}^{p}}{1+t}\right]^{\beta-1 \varepsilon^{*}} \int_{\varepsilon}^{\varepsilon} f(\varepsilon) d \varepsilon+\int_{\varepsilon^{*}}^{\varepsilon} \frac{1+\tilde{t}}{(1+\varepsilon)(1+t)} \frac{f(\varepsilon)}{\left[\tilde{v}_{2}(\varepsilon)\right]^{2}} d \varepsilon\right]<0 \\
& \frac{\partial L_{2}}{\partial s}=-\frac{1}{\left(v_{1}\right)^{2}}-\frac{(1+\tilde{t})\left(1+r^{*}\right)}{(1+\rho)(1+t)}\left[\frac{1}{\left[\tilde{v}_{2}\left(\varepsilon^{*}\right)\right]^{2}\left(1+\varepsilon^{*}\right)} \int_{\underline{\varepsilon}}^{\varepsilon^{*}} f(\varepsilon) d \varepsilon+\int_{\varepsilon^{*}}^{\varepsilon} \frac{f(\varepsilon)}{\left.\tilde{v}_{2}(\varepsilon)\right]^{2}(1+\varepsilon)} d \varepsilon\right]<0
\end{aligned}
$$

$$
\begin{aligned}
& \frac{\partial L_{2}}{\partial x_{1}^{p}}=-\frac{1}{\left(v_{1}\right)^{2}}-\frac{1+\tilde{t}}{(1+\rho)(1+t)}\left[\frac{1}{\left[\tilde{v}_{2}\left(\varepsilon^{*}\right)\right]^{2}\left(1+\varepsilon^{*}\right)} \frac{\theta}{1+t}\left[\frac{x_{1}^{p}}{1+t}\right]^{\beta-1_{\varepsilon}^{* *}} \int_{\varepsilon} f(\varepsilon) d \varepsilon+\int_{\varepsilon^{*}}^{\bar{\varepsilon}} \frac{1+\tilde{t}}{(1+\varepsilon)^{2}(1+t)} \frac{f(\varepsilon)}{\left[\tilde{v}_{2}(\varepsilon)\right]^{2}} d \varepsilon\right]<0 \\
& \frac{\partial L_{1}}{\partial r^{*}}=\frac{1}{v_{1}\left(1+r^{*}\right)}-s \frac{1+r^{*}}{1+\rho}\left[\frac{1}{\left[\tilde{v}_{2}\left(\varepsilon^{*}\right)\right]^{2}} \int_{\underline{\varepsilon}}^{\varepsilon^{*}} f(\varepsilon) d \varepsilon+\int_{\varepsilon^{*}}^{\varepsilon} \frac{f(\varepsilon)}{\left[\tilde{v}_{2}(\varepsilon)\right]^{2}} d \varepsilon\right] \\
& \frac{\partial L_{2}}{\partial r^{*}}=-s \frac{1+\tilde{t}}{(1+\rho)(1+t)}\left[\frac{1}{\left[\tilde{v}_{2}\left(\varepsilon^{*}\right)\right]^{2}\left(1+\varepsilon^{*}\right)} \int_{\underline{\varepsilon}}^{\varepsilon^{*}} f(\varepsilon) d \varepsilon+\int_{\varepsilon^{*}}^{\bar{\varepsilon}} \frac{f(\varepsilon)}{\left.\tilde{v}_{2}(\varepsilon)\right]^{2}(1+\varepsilon)} d \varepsilon\right]
\end{aligned}
$$

Hence, for $s=0$, the signs of (A6) can be summarized by
(A7) $\left[\begin{array}{ll}(-) & (-) \\ (-) & (-)\end{array}\right]\left[\begin{array}{c}d s \\ d x_{1}^{p}\end{array}\right]=-d r *\left[\begin{array}{c}(+) \\ 0\end{array}\right]$
Note that the second order conditions for maximization imply $\left(\begin{array}{cc}\frac{\partial L_{1}}{\partial s} & \frac{\partial L_{1}}{\partial x_{1}^{p}} \\ \frac{\partial L_{2}}{\partial s} & \frac{\partial L_{2}}{\partial x_{1}^{p}}\end{array}\right)>0$, from which we infer that

$$
\begin{align*}
& \operatorname{sgn} \frac{d x_{1}^{p}}{d r^{*}}=\operatorname{sgn} \frac{(-)}{(+)}<0  \tag{A8}\\
& \operatorname{sgn} \frac{d s}{d r^{*}}=\operatorname{sgn} \frac{(+)}{(+)}>0
\end{align*}
$$

Hence, when the income effect associated with changing the interest rate is small, higher interest rate will reduce the pre-buying of imports, and will increase saving.

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