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ENERGIZATION OF CHARGED PARTICLES TO HIGH ENERGIES BY AN INDUCED SUBSTORM ELECTRIC FIELD WITHIN THE MAGNETOTAIL

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Abstract

Observations have shown that electrons and protons are energized up to at least 1 MeV in the magnetotail during substorms. This magnitude cannot be explained by the cross-tail electrostatic field, which typically has a modest potential difference of only 50 kV. A rotational electric field induced by a time-dependent magnetic field does not have such a limitation. In order to evaluate its capabilities a simple model of a localized, growing disturbance in the neutral sheet current was used to calculate perturbation magnetic and electric fields; the model includes the formation of X and 0 type neutral lines. Plasma sheet test particles were followed in these time dependent fields using the full relativistic equation of motion. The most efficient energizing mechanism is a two step process, with an initial linear acceleration along a neutral line up to moderate energies, followed by betatron acceleration. The former imparts a large magnetic moment $\mu = W_{\perp}/B$ to the particle as it begins to gyrate and drift in the magnetic field, $W_{\perp}$ being the transverse kinetic energy. During this drift into regions of stronger magnetic field there is a large increase in $W_{\perp}$, the relativistic invariant $\gamma\mu$ remaining constant. Because of the rotational property of the induced electric field it is possible for a particle to gain energy even when the drift motion by itself would cause a loss. The model also includes acceleration along magnetic field lines in the plasma sheet, with moderate energy gain. A cyclic pattern of electron and proton precipitation is predicted, such as is observed during auroral breakup.

1. Introduction

Numerous observations have been reported in the literature of bursts of energetic particles, electrons and protons with energies up to one MeV or more, generally in association with magnetospheric substorm activity (Hones *et al.*, 1976a; Sarris *et al.*, 1976a,b; Terasawa and Nishida, 1976; Kirsch *et al.*, 1977; Baker and Stone, 1976; and their references). The most intense proton bursts occur in the dusk half of the plasma sheet, but weaker proton as well as energetic electron bursts do not show such a dawn-dusk asymmetry. The intense proton fluxes usually have a component of flow toward the dusk side of the magnetotail as well as either toward or away from the earth. Often the flow is directed more or less across magnetic field lines. Field-aligned fluxes have also been observed, especially in short lived bursts lasting only some tens of seconds. Sometimes oppositely directed electron and proton streams occur simultaneously, but at other times only one sign of particle is present. Inverse time dispersion has been observed, with low energy protons arriving before the high energy ones. On the basis of these and other characteristics of the bursts the various authors have concluded that the particles are energized within the magnetotail, possibly in association with the formation of a new neutral line during the development of a substorm. The particles then propagate throughout the magnetosphere (except for the high latitude lobes of the magnetotail), and even through its boundaries. These detailed recent observations were preceded by many years of earlier observations of energetic particle fluxes, and of related phenomena. Nevertheless, this hard fact of life, that charged particles are accelerated to high energies within the magnetosphere, remains unexplained. In this paper we consider this important question, and describe a mechanism that appears capable of providing an explanation for the observations.

There is not much doubt that the energy must come from the electromagnetic field in the magnetotail; furthermore an *electric* field must be the immediate cause of the particle energization, no matter what the detailed mechanism may be, since the magnetic Lorentz force does not affect the speed of the particle. We therefore need to focus our attention on the electric field, and endeavour to understand its nature and causes. In particular, we need to note that there can be two kinds of electric fields, as specified by their divergence and curl.

Much of the magnetospheric literature has been concerned with the electrostatic field. It is produced by a distribution of charge, for example by polarization of plasma, as described by Poisson's equation. Such a conservative field is limited in its ability to provide high energies to charged particles, since the highest possible energy gain is that available from the maximum potential difference. For example, the potential difference across the magnetotail is typically some tens of kilovolts, with the highest value ever deduced from observations being only 240,000 volts (Gurnett, 1972). That is clearly inadequate to explain particle energies as high as a few million electron volts.

Locally strong electric fields are associated with substorm development. However, only a region of limited extent within the magnetotail is involved; this spatial limitation presents further difficulties, in that it implies proportionately higher electric fields for a given voltage difference. For example, Terasawa and Nishida (1976) estimated that a field of 80 mV/m acting over a distance of one earth radius R_e was implied by their observation of 0.5 MeV electrons. However, there is some reason to doubt the reality of such a very strong electric field in view of the effects it would produce on the local plasma. For example, convection would be forced at the velocity $\underline{V}_E = \underline{E} \times \underline{B}/B^2$, except in some limited regions where the guiding center approximation might be invalid. Table 1 shows the magnitude V_E for various combinations of the electric and magnetic field strengths. It also shows the Alfvén speed for a typical plasma sheet density of 0.3 protons cm^{-3}. One would not expect the convection velocity to be super-Alfvênic, and thus only the combinations in the upper right-hand corner of the table seem reasonable. These are in line with observed flow velocities, deduced from the first velocity moment of the measured proton distribution function, which generally do not exceed 1000 km/second (Hones *et al.*, 1976b; Frank *et al.*, 1976). Thus the energization of charged particles to very high energies is probably not due to extremely large electric fields. With a limited spatial extent, only moderate potential differences can be contemplated.

In view of these difficulties with electrostatic fields we are led to consider an electric field of the second kind, namely one that is induced by explicit time-dependence of the magnetic field, as described by Faraday's law. Such a field is rotational, and has a finite electromotive force $\varepsilon=\oint \underline{E}\cdot\underline{dl}$. In contrast to the case with an electrostatic field, there is no well-defined limit to the energy that a particle can gain from an induced field. There is of course some limit, for example one imposed by the magnetic field if the first invariant is constant; however, that limit would be one on the ratio of final to initial energies, and it does not depend on the absolute value of the energy, except indirectly through excessive increase in the gyroradius of the particle relative to the size of the accelerating region. If a particle can gyrate in the presence of a rotational electric field it can gain energy, no matter what energy it may already have received from that field. The well-known betatron accelerator mechanism works in just this way.

Only the time-independent case of betatron acceleration has been considered for the magnetosphere (Northrup, 1962; Axford, 1969; Roederer, 1970). That arises from the transverse compression associated with the large scale convection of plasma within the magnetotail, the increase in transverse energy $W_\perp$ being in general proportional to the increase in magnetic field strength B along the convection path. The energy comes from the cross-tail electrostatic field (Hines, 1963), and the process thus faces the sharp limit discussed above. Although the steady state betatron process may be important for lower energy ring current and auroral particles, it cannot explain the very high energies observed in the bursts. We are thus led to consideration of the time-dependent problem of betatron energization in an induced electric field as a possible mechanism for producing the bursts of very energetic particles.

Ideally, to understand this possibility we should begin by considering the plasma physics of instabilities that might lead to substorm development. Unfortunately that is very difficult and complex, and it is not understood at present. Heikkila and Pellinen (1977) have suggested that this complication might be sidestepped temporarily by starting with an ad hoc model based directly on some relevant observations; the problem then becomes one in electromagnetism rather than in plasma physics. In particular, they noted the significance of the fact that the magnetic field in the magnetotail does change during a substorm: a locally induced electric field results. To facilitate consideration of the induced electric field using the vector potential they suggested consideration of the responsible disturbance in the cross-tail current, often called the neutral sheet current. To a high approximation the fluctuations are divergence-free, and they are therefore meanders. Those that are in the plane of the neutral sheet can be described as positive or negative, with magnetic moments respectively parallel or antiparallel to the normal or z-component of the magnetic field (see Figures 1 and 2). With observed values of the quantities involved the induced electric field can be quite large. Because of topological considerations it cannot be cancelled by polarization of the plasma, although it can be redistributed. It was suggested that this induced electric field should have several important consequences for substorm development, including that of energizing charged particles.

In order to do calculations it is necessary to adopt a specific mathematical model of the perturbation. Pellinen and Heikkila (1976) have presented some calculations on one such model; the details will be published elsewhere. Here we show a few key results in order to demonstrate the possible energizing effects on charged particles at various points within the perturbation. The transverse component can indeed energize charged particles by a betatron process up to several 100 keV with no sharp upper limit. The parallel component of the induced electric field can produce moderately energetic field-aligned fluxes of both electrons and protons, streaming in opposite directions in a precipitation pattern such as is observed during auroral breakup.

2. Description of the Model and Calculations

We have chosen a simple model of a current perturbation confined within an expanding area of the neutral sheet, as shown in Figure 1 and 2. We adopt a coordinate system x'y'z' parallel to the GSM system (Ness, 1965) but displaced so as to have the origin at the middle of the disturbance. For simplicity we omit the primes, as the choice of origin is obvious from the context of each discussion. The cross-tail component is assumed to vary linearly with x, the distance along the magnetotail, the density being represented by the spacing of current lines. This choice leads to equations that are analytically integrable, similar to those in the work of Murphy *et al.* (1975). The perturbation is assumed to grow with time, as specified by a strength parameter j(t). Similarly the parameters a(t) and b(t) specifying the half-widths of the disturbed area are also time dependent. For simplicity in the calculations we assumed linear growth rates in all three parameters. Thus the cross-tail component of the perturbation is given by the relations

$$J_{y+} = J_o + \frac{jx}{a} - j, \qquad 0 \le x \le 2a \quad |y| \le b$$

$$J_{y-} = J_o - \frac{jx}{a} - j, \qquad -2a \le x \le 0 \quad |y| \le b$$

$$J_{y} = J_o \qquad |x| \ge 2a \text{ or } |y| > b$$

Current continuity at the edges of the disturbance $y = \pm b(t)$ is satisfied by means of narrow concentrated currents in the x-direction of width δ, a feature suggested by the observed topology of Birkeland currents during auroral breakup (Bonnevier et al, 1970). These x-directed portions of the perturbation current are given by

$$J^{\pm}_{x+} = \mp \frac{jx}{\delta} \left(1 - \frac{x}{2a}\right), \qquad 0 \le x \le 2a$$

$$J^{\pm}_{x-} = \mp \frac{jx}{\delta} \left(1 + \frac{x}{2a}\right), \qquad -2a \le x \le 0$$

The induced electric field is the negative partial time-derivative of the vector potential, using the Coulomb gauge to separate out the instantaneous electrostatic field. The disturbance magnetic field is the curl of the vector potential; this disturbance field was added to a model field. The model needs to be one that corresponds to the stressed conditions typical of the magnetotail just before the expansion phase (Fairfield and Ness, 1970), Unfortunately, most of the detailed models based on measurements are not valid at large distances down the tail, or else are appropriate to quiet conditions; we therefore used the simple analytic model of Bird and Beard (1972), with a neutral sheet current whose strength varies slowly with distance down the tail. We have placed the center of the disturbance at 30 R_e behind the earth.

The assumption of an infinitely thin neutral sheet current involves an unphysical discontinuity at $z = 0$. Following Murphy et al (1975) we avoid this technical difficulty by doing calculations only for finite z. In order to draw conclusions about processes within the neutral sheet we take calculated values at $z = 0.1\ R_e$, where R_e is one earth radius, neglecting the x and y components of the magnetic field. This separation is much smaller than the dimensions of the disturbance at all times for which we do calculations.

In our calculations we have taken the negative and positive meanders to be of equal strength. In reality, we expect the negative meander to be the important one, and the positive one to be a reaction to it, of somewhat weaker strength. In any case the positive meander has rather mild effects (mostly on the precipitation pattern of electrons and protons due to the parallel component of the induced electric field) and it is the negative one that is important.

Using recent observations as a guide (Hones et al., 1976b; Frank et al., 1976; Sarris et al.,1976a) we have chosen expansion velocities $v_x = 1000$ km/sec and $v_y = 500$ km/sec, giving a square shape for each loop at all times. After 15 seconds when we calculate the effect on particles the disturbance has grown to $-2.36\ R_e<x<2.36R_e$ and $-1.18R_e<y<1.18R_e$. We have taken the current density j(t) to change 1.3 mA/m sec corresponding to a rate of change of the magnetic field at the middle of the disturbance of 10 nT/minute = 0.17 nT/sec. At this time the current density in the middle, at x = 0, has dropped from 26 mA/m to 6.5mA/m. The corresponding magnetic and induced electric fields at $z = 0.1R_e$ are shown in Figure 3a and b, for the negative meander only. There is a region of negative B_z surrounded by the positive B_z characteristic of the magnetotail; the boundary between the two is a continuous neutral line or ring. In the region of decreasing current (near x = -30) it is an X-type neutral line, while the other three sides of the pattern in the region of increasing current have an 0-type neutral line. There are magnetic field line loops around the local 0-line.

The induced electric field has x and y components, with a rotational character as shown in Figure 1 and Figure 2. Near the X-line the field is in the dawn-to-dusk sense, with $\underline{E}\cdot\underline{J}>0$; it has a value of about 6 mV/m roughly constant over a large area. Near the O-line it has the opposite or dusk-dawn direction with a strong gradient in the field strength, and moderate values (≈2mV/m) near the O-line.

This model was chosen to represent only the essential features at the beginning of a substorm. After 20 seconds the current in the middle of the disturbance vanishes, and then reverses direction. Whether or not this is physically realistic is an open question, one that needs to be faced in the further development of the model. It is possible, for example, that the relative importance of time development and motion may change, but an induced electric field would nevertheless remain very much as pictured here. The electrons we consider reach high energies before this current reversal; the protons take a somewhat longer time, but they do not remain in the region near the reversing current, and so the results we present should still be physically meaningful.

Since occasionally the magnetic field has been observed to change very rapidly we have done some calculations with the 15 times faster rate of change $dB/\partial t = 2.5$ nT/sec (see e.g. Kirsch <u>et al</u>, 1977) but with the same propagation velocities as stated above.

In view of the high rates of change of the magnetic and electric fields, in both time and space, there is a possibility that the particle motions may not be such as to conserve the magnetic moment. We therefore have not used the guiding center approximation, but rather the complete relativistic equation of motion for the particles,

$$\frac{d}{dt}\,(m\,\dot{\underline{r}}) = q\underline{E}\,(\underline{r},\, t) + q\dot{\underline{r}} \times \underline{B}\,(\underline{r},\, t)$$

This was split into the component forms

$$\dot{x} = u, \quad \dot{y} = v, \quad \dot{z} = v$$

$$\begin{aligned} \dot{u} &= \dot{u}(x,\, y,\, x,\, u,\, v,\, w,\, t) \\ \dot{v} &= \dot{v}(x,\, y,\, z,\, u,\, v,\, w,\, t) \\ \dot{w} &= \dot{w}(x,\, y,\, z,\, u,\, v,\, w,\, t) \end{aligned}$$

which were numerically solved by means of the fourth order Runge--Kutta method. Time dependence was taken into account by calculating new field values for each point along the particle trajectory, with a basic time interval of 10^{-4} seconds; the time interval was decreased in the relativistic region to keep the numerical method valid.

The important and difficult question of possible modification of the electric field by the plasma has been ignored. Discussion and rationalization of this philosophy has been presented briefly by Heikkila and Pellinen (1977), and is pursued further below in the concluding section.

3. Transverse Energization

To look for possible large increase in transverse energy we have followed typical plasma sheet test particles inserted at various locations in the time-dependent fields. In figure 4, two trajectories are shown for electrons with initial energies of 1 keV, and two for protons with 10 keV, all launched at $t = 15$ seconds after the beginning of the disturbance. Pitch angles of 90^o throughout the motion were assumed, the path being in the plane of the neutral sheet (in the manner described in part 2 above). It was found that considerable increases in energy could be imparted to charged particles starting in limited regions near the X-type neutral line, and near the O-line in the negative meander. Only moderate increases resulted for particles in the positive meander, largely because the z-component of the magnetic field is increased rather than decreased within it.

Consider first the proton path shown by the dashed curve on the right hand side, starting near the O-line at $y = 0$. Figure 3b shows that this proton first experiences some linear acceleration by the $-y$ directed induced electric field. However, this acceleration is rather small because the field strength is low. Soon the proton is deflected toward the left by the earthward directed induced electric field, as well as by the southward magnetic field. Here it is slightly decelerated after it crosses the zero line of E_y, and the radius of curvature of the path decreases rapidly. After further magnetic deflection the proton finds itself in an accelerating electric field, and the energy increases to 36 keV as it heads off to the right, away from the earth. The subsequent path is a gyration plus a drift. However the proton does not actually leave the disturbance, since that too is, by assumption, growing in size. The solid curve shows the same proton path in normalized coordinates, obtained by dividing the actual distances by the size parameters a(t) and b(t). Thus, the proton gyrates around a fixed point within the disturbance, with a decreasing radius of curvature as the magnetic field strength increases. Here the fact that it is indeed betatron acceleration is evident. The energy increased to 56 keV in about 17 seconds.

A rather different behaviour was found for the second proton, launched near the X-type neutral line. Here the y-directed induced electric field is considerably stronger; furthermore, for this trajectory the combination of electric and magnetic effects were such as to allow a direct linear acceleration right out of the disturbance region, toward the dusk side, in 3.5 seconds with 70 keV energy. This is quite a healthy increase, especially when it is noted that only a moderately slow rate of change of the magnetic field was assumed. Furthermore, two factors that were neglected here would both enhance this increase. First, the dawn-dusk cross-tail electrostatic field is also in this same direction, but of course it is weak and not very effective. A more significant increase could result from a new polarization field produced by the plasma in response to the induced electric field, as discussed by Heikkila and Pellinen (1977); a factor of 2 could result, and thus 140 keV for the proton energy as it leaves the disturbance.

We do not show the continuation of this path beyond this point, $t = 5$ seconds, since it is not far from the region where the J_x component of current becomes negative. However, we were tempted to follow the proton further, and found that it did find a region where it experienced betatron energization, and it reached 750 keV after 26.5 seconds, having reached $x = -32.8$, $y = +4.5$ R_e. Thus, it seems possible that very high energies can be imparted to protons; furthermore, the model predicts a dusk-side preference for the very energetic bursts, in good agreement with observations.

Similar results were found for electrons, except that there is little difference between the X and O regions. In both cases shown in Figure 4 the electron experienced an initial linear acceleration up to several keV, followed by a gradient drift toward regions of higher magnetic field strength, near the edges of the disturbance. Only 3.5 seconds is needed for the energy to reach 75 keV near the O-line, and 1.1 seconds to 100 keV near the X-line. The importance of the rotational character of the electric field is shown by the fact that the energy gain over the straight line path shown dotted would be only 5 keV.

In order to stress that the energy gain is dependent on the assumed rate of change of the magnetic field we show in Figure 5 one case of electron energization with the very large rate of 2.5 nT/s, a value actually observed on at least one occasion (Kirsch et al., 1977, reporting data obtained by Ness). Here the electron energy reached 1 MeV in about 3 seconds. In the initial linear acceleration the energy rose to 50 keV at point A. (See the insert in Figure 5.) During the subsequent gradient drift AB the energy steadily increased on each gyration, and the relativistic invariant $\gamma\mu$ was constant (see Roederer, 1970, p. 21). Over the portion BC the drift velocity was so high, about 10% of the total, that allowance needs to be made for it, as shown by the dashed line, since the magnetic moment is evaluated in the drifting coordinate system. Between C and D the energy increases at a much slower rate; this is due to a combination of two opposing factors. The electron drift here is in the direction of the electric field, with a resultant loss in energy amounting to 120 keV. At the same time the electron gains 278 keV due to its gyrational motion, about 0.4 keV average over 706 gyrations, leaving a net gain of 158 keV as shown in Figure 5. The total path over CD alone is 65 R_e, within the confines of a region with dimensions of about 1 R_e. This particular result demonstrates clearly the capabilities of a rotational electric field, which are quite different from those of an electrostatic field. After point D the electron again gains energy from both the drift and gyrational motions.

4. Field-aligned acceleration

To get an estimate of the longitudinal energization by the induced electric field (again neglecting all other fields), we have followed electrons along magnetic field lines starting from various locations in the disturbance area. Here too the initial pitch angle was taken to be 90°, but in the tail field away from the neutral sheet, motion along field lines was included. When started in regions where the induced E_x component is large, for example near the edges of the disturbance, an electron is very effectively accelerated along the magnetic field lines. Gradient and curvature drifts are generally not important. The energy gained on one such trajectory is shown in Figure 6. It first reaches nearly 50 keV over the negative meander on the evening side; however, with our assumption that the positive meander is equally strong the electron encounters some deceleration before it leaves the region of the disturbance toward the earth. Not all of the energy is lost, however, because of the divergence of the magnetic field lines away from the neutral sheet, and the consequent attentuation of the induced electric field. In this example the electron reaches the plane at $x = -20\ R_e$ (from the earth) with 21 keV. Contours of equal electron energy in this plane are shown in Figure 7. It may be noted that the electron beam has a rather large cross section, with the electron energy decreasing only slowly with distance from the center of the beam. As pointed out by Heikkila and Pellinen (1977), this is a consequence of the weak 1/r dependence of the vector potential on distance from the disturbance element. From this cross-sectional area we can get an estimate of the size of the precipitation area, and the auroral form that would be produced. Since the magnetic field strength decreases by a factor of about 2000, the flux tube containing electrons with energy greater than 5 keV would decrease to about $200 \times 400\ \mathrm{km}^2$. Such a form would almost fill an all-sky photograph, in good agreement with observations of auroral breakup. These electrons come from the region beside and beyond the negative meander, since the magnetic field line loops around the O-line are not connected to the earth.

Rather than follow protons in a similar manner, we have simply calculated the line integral of the induced electric field along the magnetic field lines, beginning in regions where the field points toward the earth (over the positive meander). Here the induced electric field would, if it persists, accelerate protons up to several tens of keV. However, the travel times would be much longer than for the electrons, and it is likely that the parallel component of the net electric field would be reduced by the electron component of the local plasma before much proton precipitation could result. Thus these figures should be viewed with more caution than the ones for electrons. There is of course also a time delay between the electron and proton precipitation.

Precipitation along these flux tubes, as well as along the complementary ones on the morning side (shown in Figure 1) produces a cyclic pattern (Figure 1) as observed during auroral breakup (Montbriand, 1971; Fukunishi, 1975). Electrons are precipitated at the higher latitudes on the evening side: this location corresponds to the westward travelling surge, which is indeed a pure electron precipitation event. A corresponding proton precipitation area appears on the morning side, with some time delay and possible attentuation. The two precipitation areas at the lower latitudes have opposite polarities; this again is in agreement with the observations, although these features are not as striking as the higher latitude ones. Further analysis of such observations should help to determine the correct relative sizes and strengths of the negative and positive meanders.

5. Discussion

Faced with a stubborn problem, we have adopted drastic measures for its solution, taking the induced electric field to be all important. Happily, these have yielded promising new results, which success constitutes justification of sorts for the modus operandi. However, some objective comments are in order.

We did not begin at the beginning, but rather we jumped right into the middle of the problem. This does not necessarily imply any reservations about the work, since the model was chosen on the basis of direct observations. It does mean that the basic physics of the plasma processes leading to the disturbance remain a problem to be tackled. Analysis of ad hoc models should help in that endeavor, since it provides some insight as to the significance of various features such as topological considerations in three dimensions , and explicit time-dependence.

Our ad hoc model is perhaps simplistic, and it is not even self-consistent since the electromagnetic and velocity fields were not matched. Eventually a more physically realistic model needs to be found, either by means of a satisfactory theory, or empirically on the basis of more detailed observations. Nevertheless, we believe that our model does include the essential features, and all that better models are likely to do is provide more accurate quantitative detail. In view of the great variability of the actual events being studied, the quantitative aspect is not as important as the qualitative, at this stage.

Our most questionable stroke is undoubtedly the neglect of all other electric field besides the induced one. In broad terms this neglect is justified by the logic that if indeed it is the induced electric field that is the essence of the problem (Heikkila and Pellinen, 1977), then all other fields must be of secondary importance. They should then be just perturbations that can be considered later, after the first order analysis has been done.

However, a few further comments can be made. Neglect of the cross-tail electrostatic field E^m is surely safe at this stage of substorm development, since it is weaker by at least an order of magnitude. On the other hand, the local plasma will undoubtedly produce a new polarization electric field E^p that is comparable with the induced electric field, and this secondary field cannot be ignored in the continued development of the substorm. In particular, the plasma is likely to respond so as to cancel the parallel component of the induced electric field. But here too observations pose a serious problem for the conventional wisdom, that parallel electric fields are small: intense fluxes of energetic electrons and protons streaming in opposite directions along magnetic field lines (Kirsch et al., 1977) are difficult to explain in any other way than by means of a strong parallel component of the electric field, one existing in the very region where the observations were made. Thus more and more observations are beginning to indicate that the plasma is rather helpless against invasion by the induced electric field. Even if only transient, a parallel component $E_\parallel$ does appear to exist for at least a few minutes. Since electrons and protons can be energized in a few seconds or tens of seconds, such a transient aspect can be ignored. It would of course be more important for protons, whose travel times are longer.

As far as the transverse component is concerned the situation seems considerably safer, for two reasons. First, as the plasma is able to reduce $E_\parallel$ it must increase $E_\perp$, in order to leave the emf $\varepsilon = \oint \underline{E}\cdot\underline{dl}$ unchanged. Thus, if anything, features of importance to the transverse energization process, such as the large initital linear acceleration, and the large gradients in the electric field, may be enhanced. Second, since the closed-circuit emf cannot be affected by charge polarization, the true time-dependent betatron process remains unaltered. There may be some modification due to the convection of the particle to different regions of the disturbance where the emf can be different; this is likely to mean a change in the locations of the best source regions, but no basic change in the broad picture.

Turning now to the positive aspects, a possible mechanism for energizing electrons and protons to high energies during magnetospheric substorms has been demonstrated. Given the small size of the region in which energization is found to occur, and the limitations on the electric field imposed by the plasma sheet medium, it is difficult to conceive of any other process than betatron acceleration that would be adequate for the task. Even the betatron mechanism has its limitations, largely those associated with the near constancy of the magnetic moment; our calculations show that the linear acceleration in the region near a neutral line is important, in that it can provide the initial high energy that is necessary so that the betatron mechanism can boost the energy into the Mev range. This initial linear acceleration bears some resemblance to the discharge in a neutral point region first proposed by Giovanelli (1947), Hoyle (1948), and Dungey (1953). It is regrettable that the idea of a time-dependent discharge was not followed up, but was replaced by studies of a time-independent process. Since the essential element , the induced electric field, was thereby left out , it is no wonder that none of the later theories has been able to explain the observations.

Particle energies up to about one MeV or more, such as have been observed in the bursts, and such as are typical of the storm time particles in the trapped radiation belts, can be produced with electric and magnetic fields of the magnitude found in the magnetotail. Propagation of the particles from their place of origin within the magnetotail to other parts of the magnetosphere is a further problem to be tackled, but that is one that does not appear quite as formidable as the production process itself has been up to now.

As might have been expected, the amount of energy gain by a particle is quite sensitive to its initial location and direction of motion within the disturbance. We need to consider whether enough particles can be so energized to explain the observations.

A typical flux might be taken as $10^5 cm^{-2} sec^{-1}$, but this should be reduced by say a factor of 10 because gyration brings each particle many times to some one location. The flux has been observed over a region with area of some 10 R_e^2, implying a total production rate of something like $10^{23} sec^{-1}$ of both protons and electrons. This is a reasonable value, being only 10^{-3} of the estimated supply of particles into the plasma sheet (Hill, 1974). With an average energy of 100 keV these particles would carry 10^9 watts of energy, which is only a few percent of the energy dissipated during a substorm. These particles could come from a region of the plasma sheet (density of 0.3 cm^{-3}) only 10^{-3} R_e^3, or some 1000 km to a side. Thus there is no reason to doubt the capability to produce the high fluxes observed.

The next step is to take a new look at the various relevant observations, including ionospheric absorption events seen with riometers, relativistic electron precipitation events, X-ray events, and various aspects of substorm development. On the basis of these, with the greater detail and coverage offered by the International Magnetospheric Study, it should be possible to adjust and improve the model of the disturbance. Such an improved model should then provide some direction to the search for the plasma processes responsible for the disturbance.

In view of the acknowledged similarity between magnetospheric substorms and solar flares, it seems likely that these same considerations will lead to an understanding of the production of energetic solar flare particles. Perhaps even the production of cosmic rays may turn out to be due to the betatron process operating in galactic space, in addition to or as an alternative to, the Fermi process.

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Table 1

CONVECTION SPEED E/B and ALFVEN SPEED V_A

Electric Field E	Magnetic Field B				
	1	3	10	30	nT
0.3 mV/m	300	100	30	10	km/s
1	1000	300	100	30	
3	3000	1000	300	100	
10	10000	3000	1000	300	
30	30000	10000	3000	1000	($<V_A$)
100	100000	30000	10000	3000	($>V_A$)
Alfvén speed for 0.3 proton/cm^3	40	120	400	1200	km/s

Figure Captions

Figure 1. Positive and negative meanders of the neutral sheet current, with magnetic moments respectively parallel and anti-parallel to the z-component of the magnetic field, induce rotational electric fields as shown. The components parallel to the magnetic field lines in the plasma sheet produce a cyclic precipitation pattern of electrons and protons, such as is observed during auroral breakup.

Figure 2. In the mathematical ad hoc model of the disturbance a rectangular shape is assumed, with linearly increasing amplitude and size.

Figure 3a. Perturbation magnetic and electric field strengths in the region of the negative current meander, at $t = 15$ seconds and for $\partial B/\partial t = 0.17$ nT/s. The region with negative B_z is surrounded by a neutral line, with X type geometry near $x = -30$ where the current strength is decreasing, and 0 type on the other three sides where the current is increasing.

Figure 3b. The components of the induced electric field in the plane of the neutral sheet, relative to the neutral line in the negative meander.

Figure 4. Paths of two protons and two electrons, starting with typical plasma sheet energies near the 0 and X type neutral lines. The solid curves are normalized by the size of the growing disturbance. The straight line path shown dotted for one electron would involve an energy gain of only 5 keV, as compared with the actual gain of 75 keV for the tortuous path; the difference is due to the electromotive force of the induced electric field.

Figure 5. Kinetic energy and the first relativistic invariant for an electron starting near the 0 line, along the path shown in the inset, for a much faster rate of change of the magnetic field of 2.5 nT/s. The shaded band indicates the range of variation during gyration.

Figure 6. Kinetic energy gained by an electron due to the parallel component of the induced electric field in the plasma sheet above and below the neutral sheet. The initial gain in energy is due to the negative meander, and the subsequent loss is due to the positive one, on the dusk side.

Figure 7. Energy contours for electrons and protons heading toward the earth at $x = -20\ R_e$. The particle shown in Figure 6 passes through the 20 keV contour. This energized electron flux would produce the pure electron excited aurora in the westward travelling surge in an auroral breakup. Protons may achieve the high energies shown only if the positive meander is as strong as the negative one, and if the time scale for reduction of $E_{\parallel}$ by the plasma is sufficiently long.

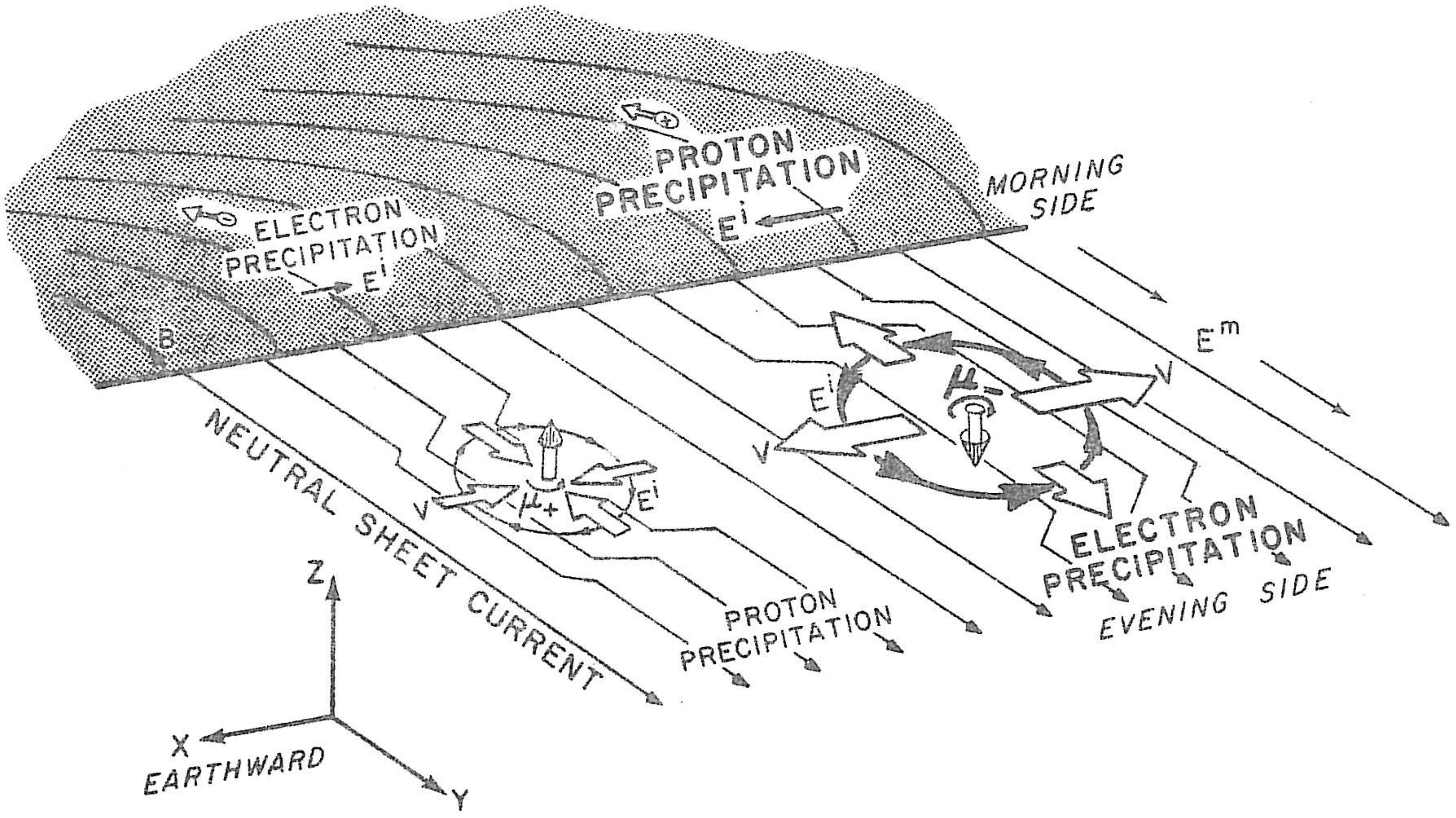
PROTON
PRECIPITATION
E^i
MORNING
SIDE
ELECTRON
PRECIPITATION
E^i
B
E^m
V
E^i
μ^-
V
μ^+
E^i
V
NEUTRAL SHEET CURRENT
ELECTRON
PRECIPITATION
EVENING SIDE
PROTON
PRECIPITATION
Z
X
EARTHWARD
Y

Figure 1

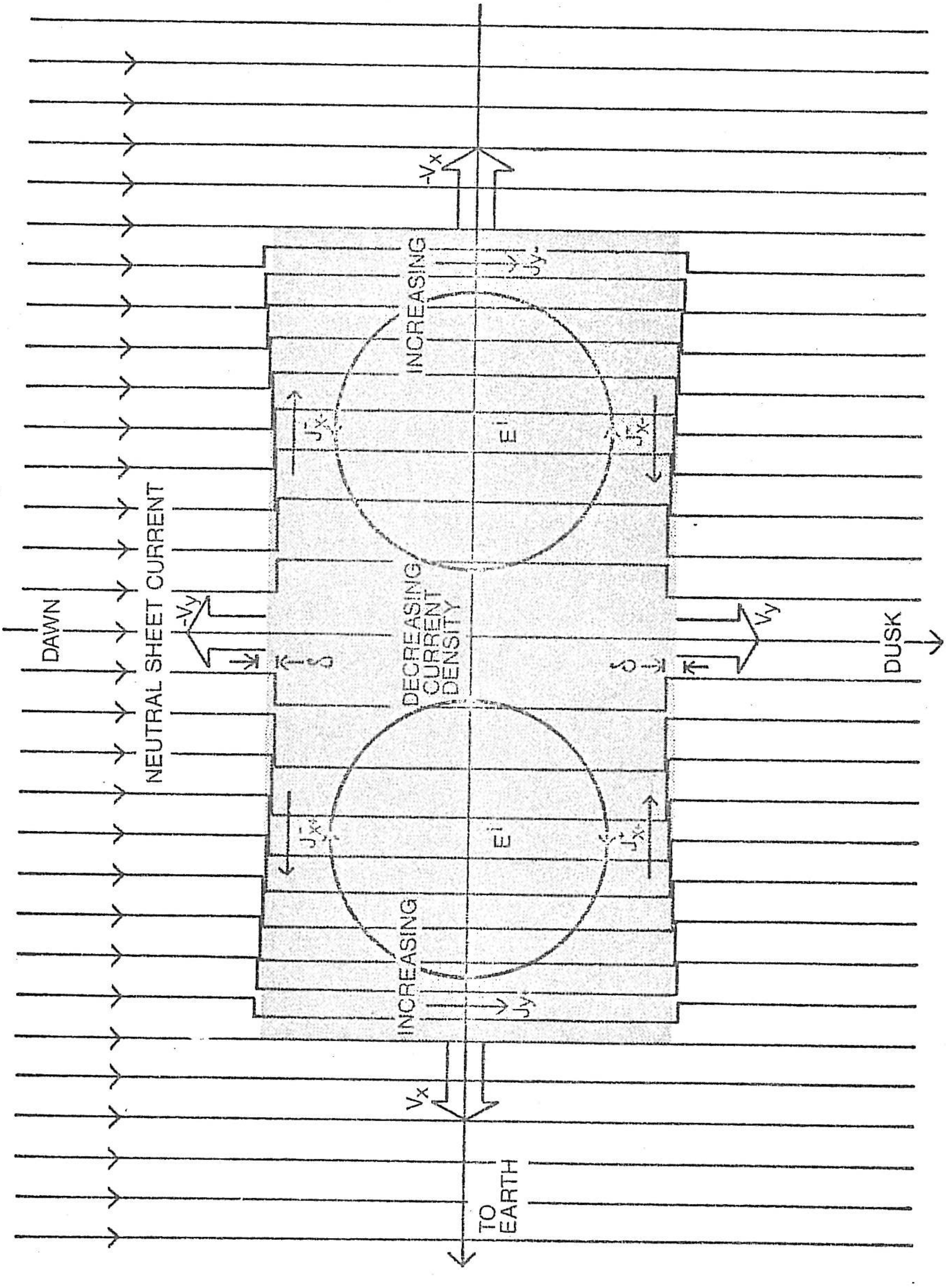
DAWN
DUSK
NEUTRAL SHEET CURRENT
$-V_x$
V_x
$-V_y$
V_y
INCREASING
INCREASING
DECREASING CURRENT DENSITY
J_{y+}
J_{y+}
J_{x-}
J_{x+}
E'
E'
δ
δ
TO EARTH

Figure 2

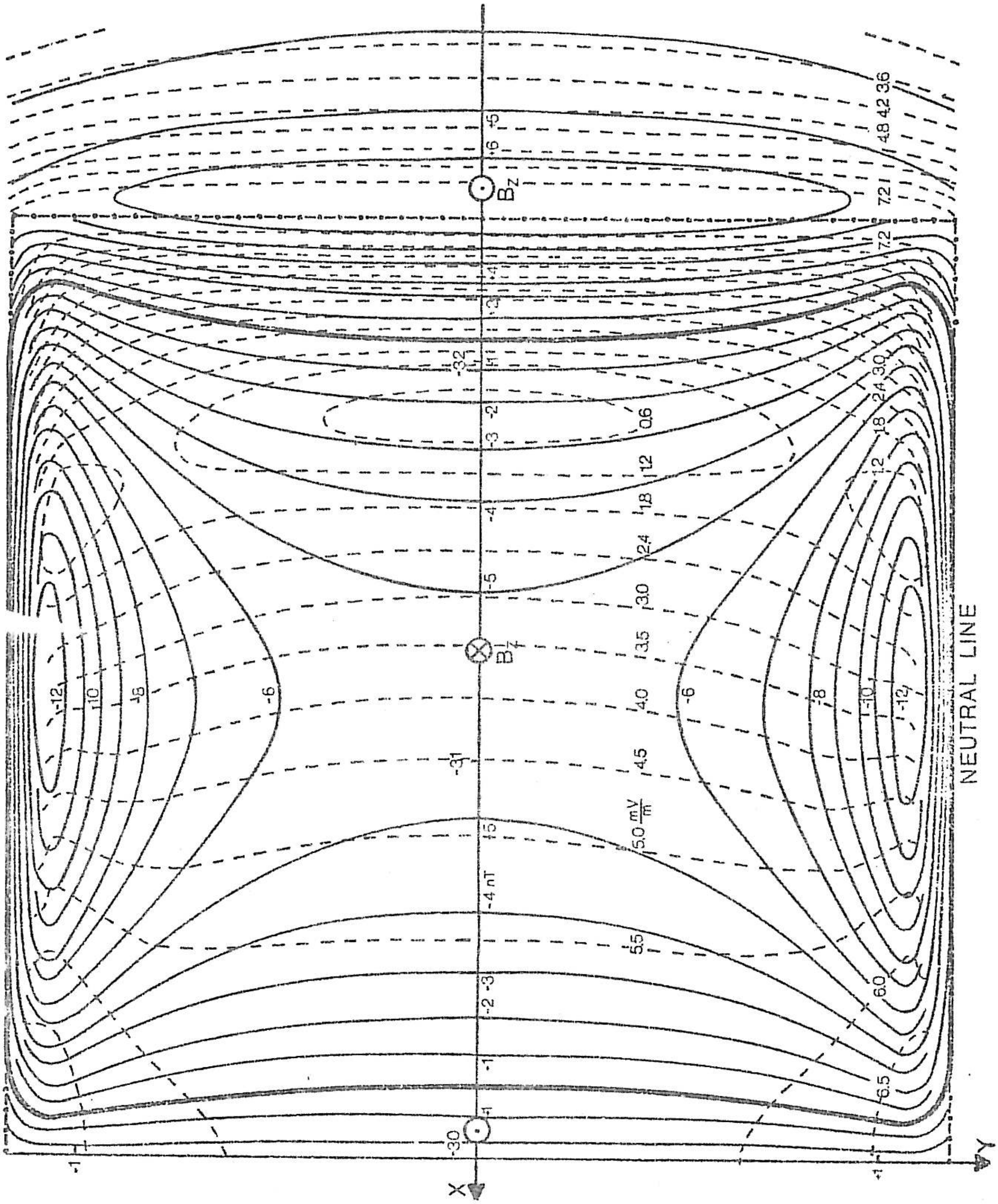
NEUTRAL LINE
Bz
X
Y
5.0 mV/m
-4 nT
-30
-31
-32
-12
-10
-8
-6
0.6
1.2
1.8
2.4
3.0
3.5
4.0
4.5
5.5
6.0
6.5
7.2
7.21
4.8
4.2
3.6

Figure 3a

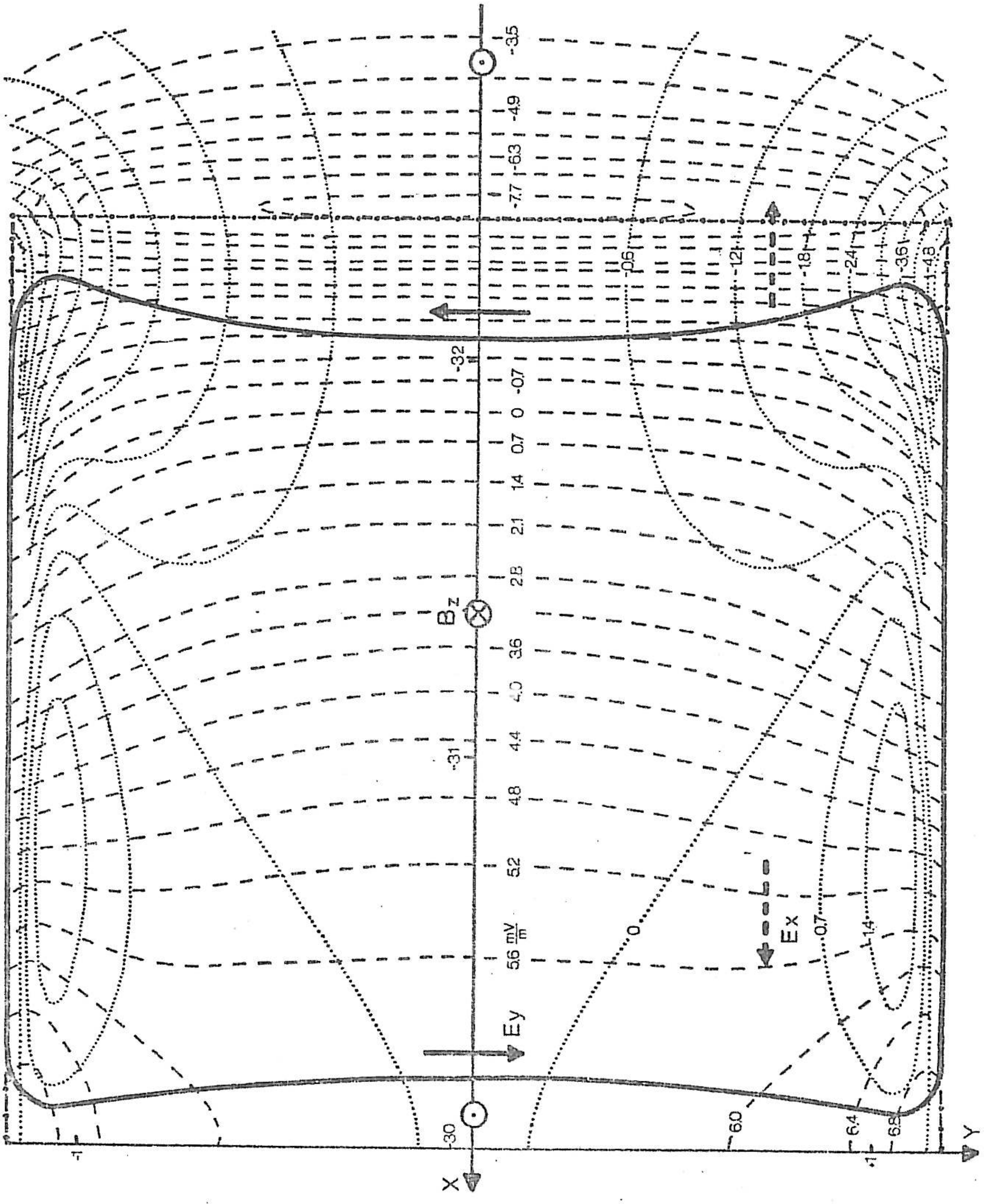
-3.5
-4.9
-6.3
-7.7
-0.6
-1.2
-1.8
-2.4
-3.6
-4.8
-3.2
-0.7
0
0.7
1.4
2.1
2.8
B_z
3.6
4.0
4.4
-3.1
4.8
5.2
5.6 $\frac{mV}{m}$
0
E_x
0.7
1.4
E_y
-3.0
6.0
6.4
6.8
-1
1
X
Y

Figure 3b

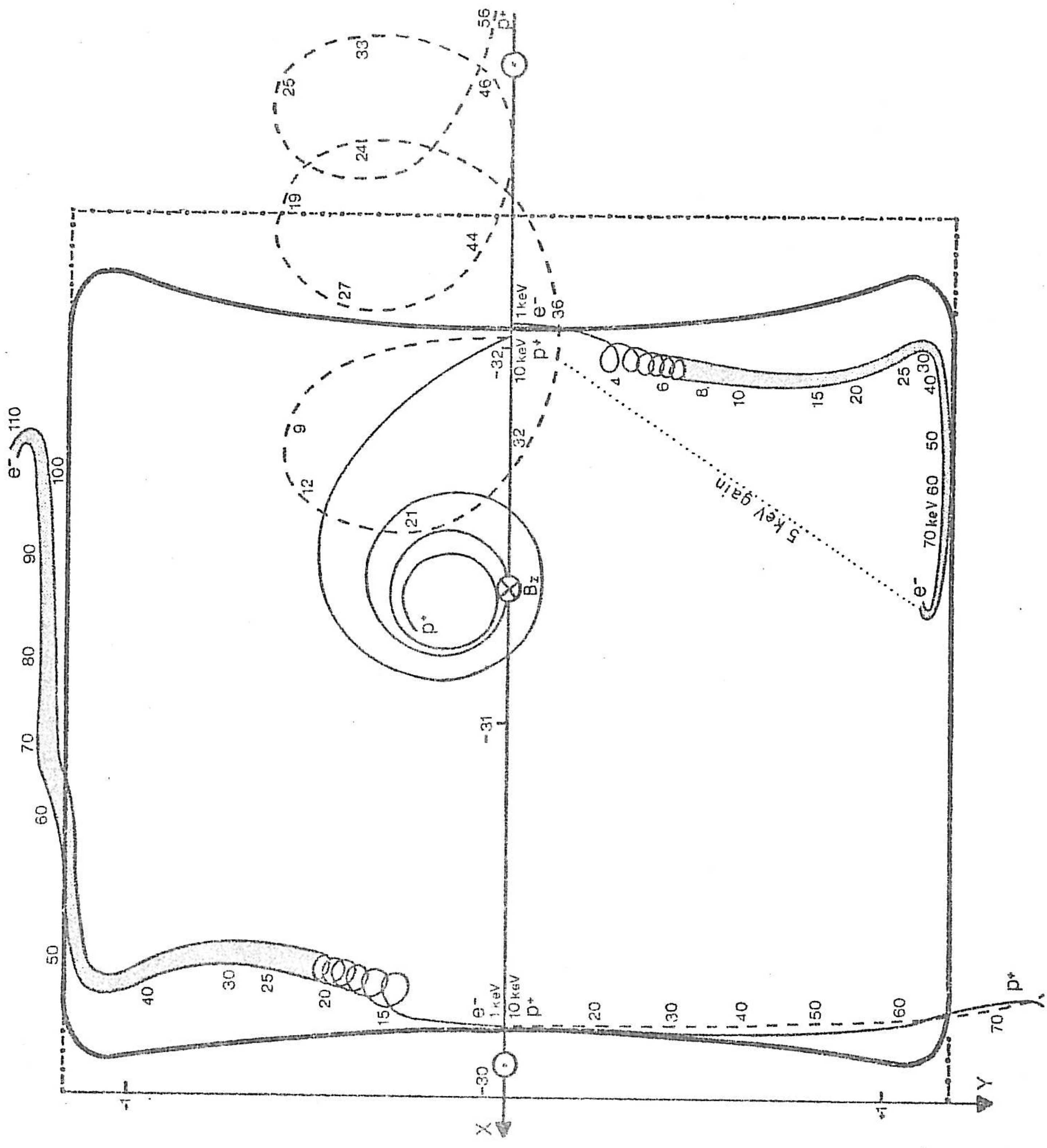
e⁻
110
100
90
80
70
60
50
40
30
25
20
15
e⁻
1 keV
10 keV
p⁺
20
30
40
50
60
70
p⁺
-30
X
Y
-1
+1
-31
-32
32
36
1 keV
e⁻
10 keV
p⁺
4
6
8
10
15
20
25
30
40
50
60
70 keV
e⁻
5 keV gain
B_z
p⁺
9
12
21
19
27
44
24
25
33
46
56
p⁺

Figure 4

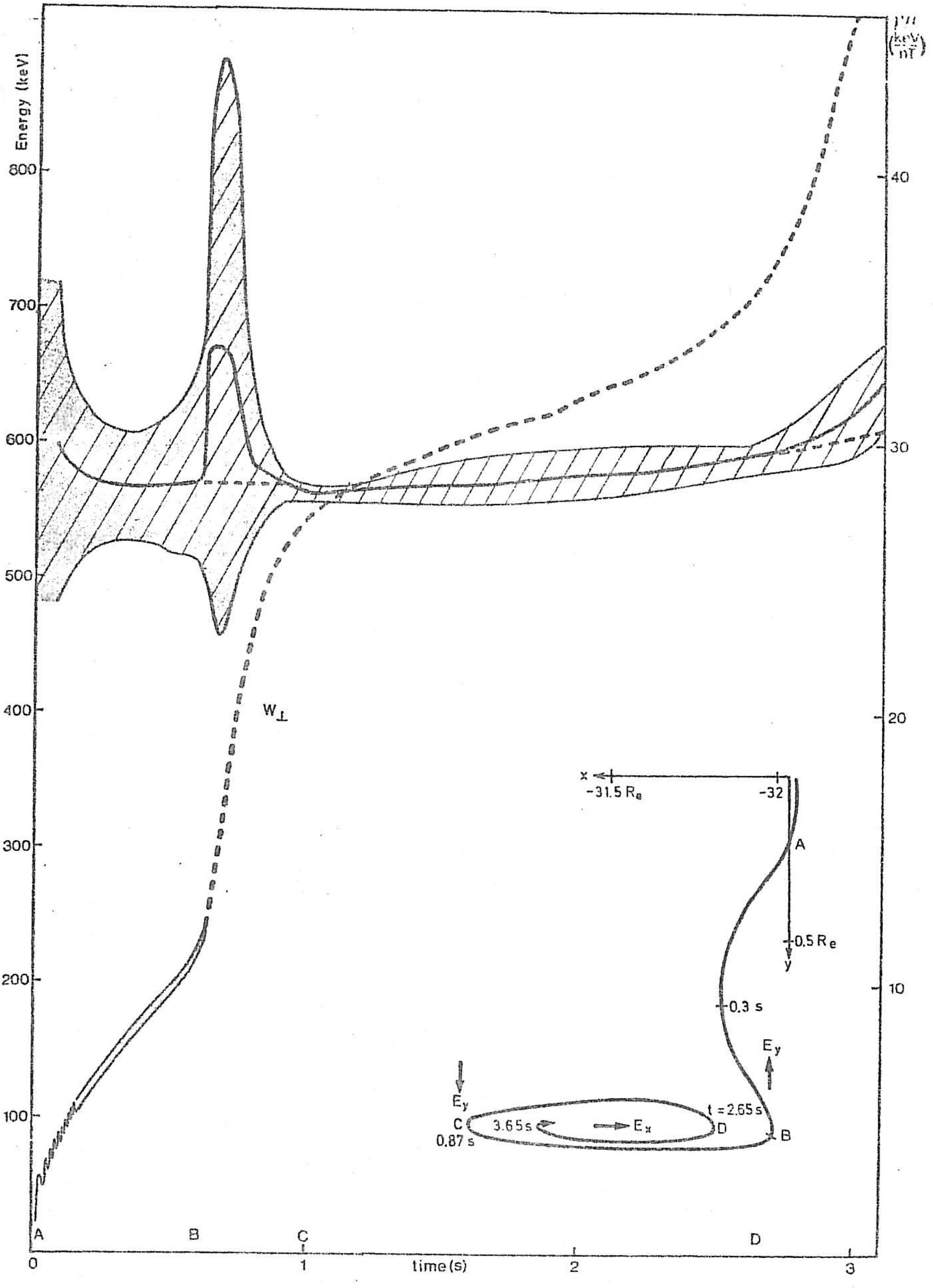
Energy (keV)
800
700
600
500
400
300
200
100
(keV/nT)
40
30
20
10
$W_\perp$
x
-31.5 R_e
-32
A
0.5 R_e
y
0.3 s
E_y
E_y
C
3.65 s
E_x
t = 2.65 s
D
B
0.87 s
A
B
C
D
0
1
time(s)
2
3

Figure 5

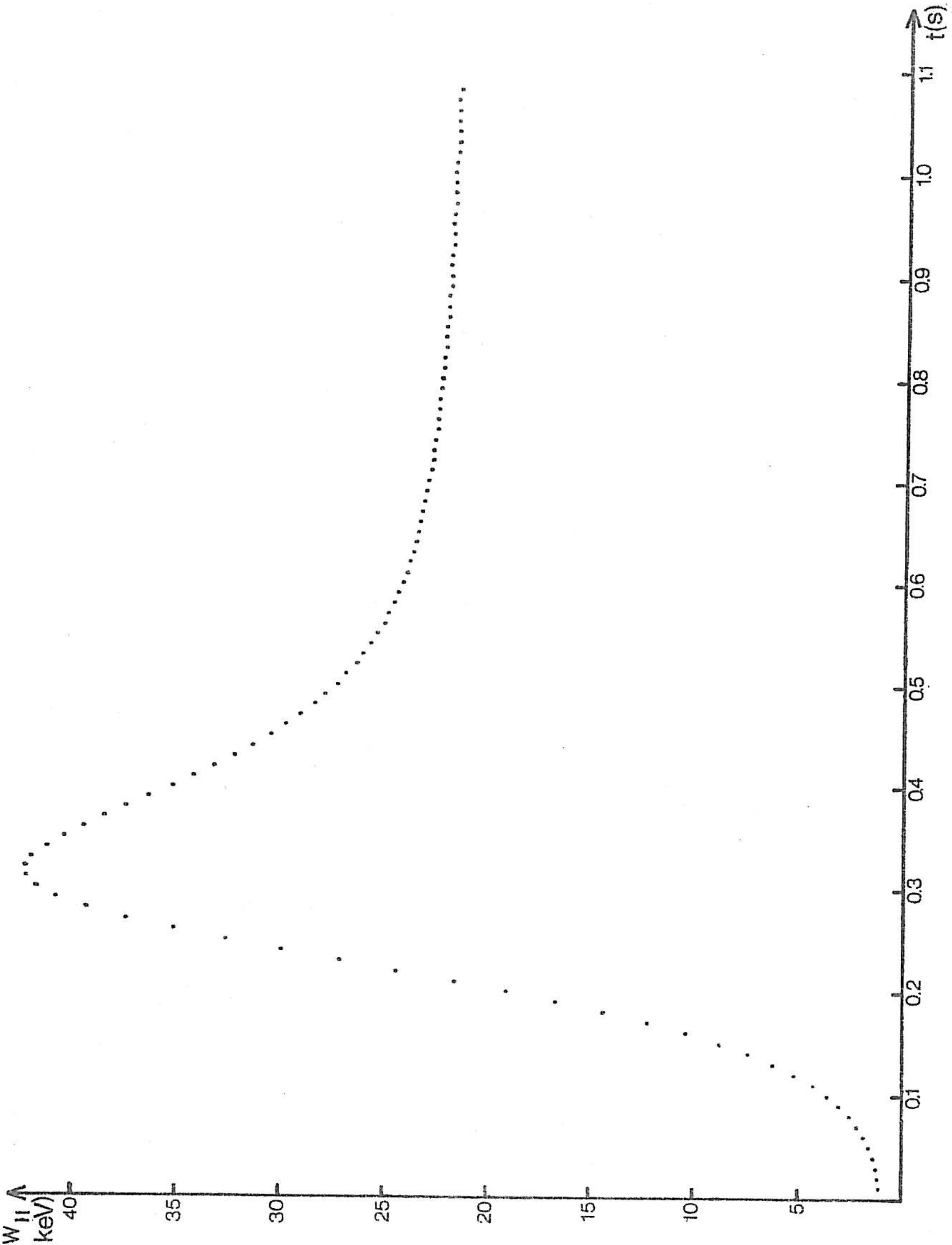
$W_{\parallel}$ (keV)
40
35
30
25
20
15
10
5
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
t(s)

Figure 6

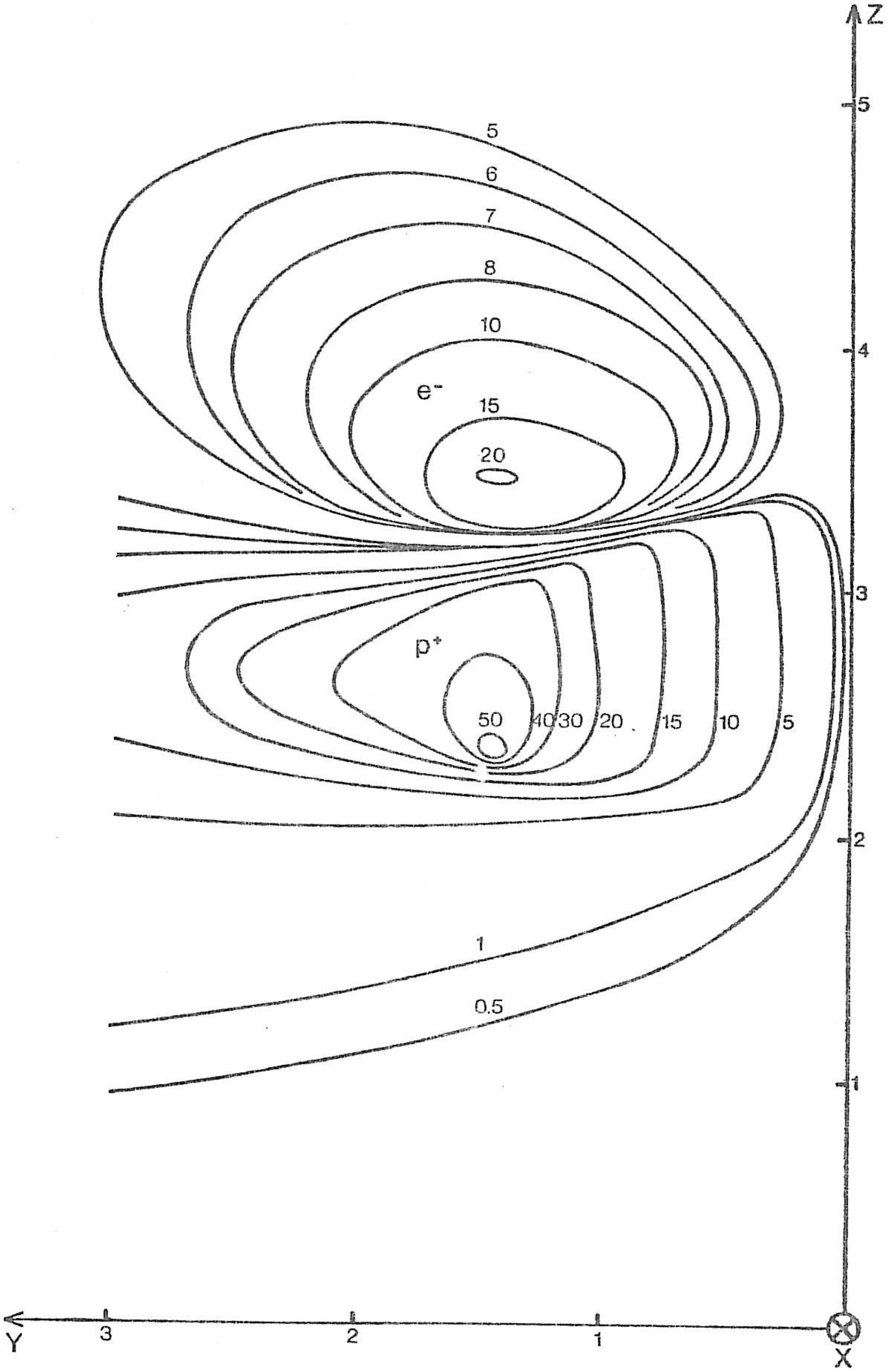

Figure 7

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ENERGIZATION OF CHARGED PARTICLES TO HIGH ENERGIES BY AN INDUCED SUBSTORM ELECTRIC FIELD WITHIN THE MAGNETOTAIL

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Observations have shown that electrons and protons are energized up to at least 1 MeV in the magnetotail during substorms. This magnitude cannot be explained by the cross-tail electrostatic field, which typically has a modest potential difference of only 50 kV. A rotational electric field induced by a time-dependent magnetic field does not have such a limitation. In order to evaluate its capabilities a simple model of a localized, growing disturbance in the neutral sheet current was used to calculate perturbation magnetic and electric fields; the model includes the formation of X and O type neutral lines. Plasma sheet test particles were followed in these time dependent fields using the full relativistic equation of motion. The most efficient energizing mechanism is a two step process, with an initial linear acceleration along a neutral line up to moderate energies, followed by betatron acceleration. The former imparts a large magnetic moment $\mu = W_{\perp}/B$ to the particles as it begins to gyrate and drift in the magnetic field, $W_{\perp}$ being the transverse kinetic energy. During this drift into regions of stronger magnetic field there is a large increase in $W_{\perp}$, the relativistic invariant $\gamma\mu$ remaining constant. Because of the rotational property of the induced electric field it is possible for a particle to gain energy even when the drift motion by itself would cause a loss. The model also includes acceleration along magnetic field lines in the plasma sheet, with moderate energy gain. A cyclic pattern of electron and proton precipitation is predicted, such as is observed during auroral breakup.

Key words: Induced electric field, betatron acceleration, linear acceleration, energetic particles bursts, van Allen belt particles, magnetospheric substorms.