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Energy analysis of a passive solar system

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Abstract—This work aims at presenting the numerical solution to a natural convection problem concerning the use of a passive solar system for building heating purpose. The system consists of a modification of the well-known Trombe–Michel passive system. The main differences consist of thermal insulation on the southern wall surface, the presence of two solar ducts separated by a thin metallic plate with collector function, and a thermal storage over the ceiling of the heated rooms. The numerical solution to the simple mathematical model – based on energy and mass conservation equations – is achieved by a finite difference method, which allows to determine both the time-dependent temperature profile on each component of the system and the air flow pattern in the solar ducts. A comparison between the numerical results and some experimental data is reported: it shows a very satisfactory agreement. At last, the hour by hour energy fluxes are shown in some graphs. © Elsevier, Paris

passive solar system / natural convection / time dependent temperature distribution

Résumé — Analyse énergétique d'un système solaire passif. Ce travail présente une solution numérique au problème de la convection naturelle dans le cas du chauffage de bâtiments par un système solaire passif. Le système étudié est une variante du système Trombe–Michel. La principale différence consiste en une isolation de la paroi sud, deux «canaux» solaires séparés par une plaque métallique mince, jouant le rôle de collecteur, et un stockage thermique au-dessus du plafond des pièces chauffées. La résolution numérique du modèle simple basé sur les équations de conservation de la masse et de l'énergie est réalisée par la méthode des différences finies. Elle permet d'obtenir, en fonction du temps, les profils de température de chaque composant du système, ainsi que les vitesses d'air dans les canaux solaires. Une comparaison du modèle avec des données expérimentales est réalisée. Elle fait apparaître un accord très satisfaisant. Les flux énergétiques en fonction du temps sont également fournis pour le système étudié, et comparés à ceux correspondant au cas du système Trombe–Michel. © Elsevier, Paris

système solaire passif / convection naturelle / distribution temporelle de la température

Nomenclature

a_w	inside heat transfer coefficient by radiation and convection	$W \cdot m^{-2} \cdot K^{-1}$
c	specific heat	$J \cdot kg^{-1} \cdot K^{-1}$
D	hydraulic diameter of the ceiling channel	m
f	friction factor	
g	gravity acceleration	$m \cdot s^{-2}$
Gr	Grashof number	
H	solar vertical duct height	m
h_{G1}	heat transfer coefficient by convection between the transparent cover system and the external environment	$W \cdot m^{-2} \cdot K^{-1}$
h_{G2}	heat transfer coefficient by convection in the solar duct	$W \cdot m^{-2} \cdot K^{-1}$
h_{P1}	air–metal heat transfer coefficient by convection in the GP duct	$W \cdot m^{-2} \cdot K^{-1}$
h_{P2}	air–metal heat transfer coefficient by convection in the PW duct	$W \cdot m^{-2} \cdot K^{-1}$
h_{INF}	heat transfer coefficient by convection (inferior side) for air in the ceiling horizontal duct	$W \cdot m^{-2} \cdot K^{-1}$
h_{SUP}	heat transfer coefficient by convection (superior side) for air in the ceiling horizontal duct	$W \cdot m^{-2} \cdot K^{-1}$
I_β	total solar radiation at slope β	$W \cdot m^{-2}$
L	wall width	m
Pe	Peclet number ($Pe = Re\ Pr$)	
Pr	Prandtl number	
r	pressure drop coefficient	
Ra	Rayleigh number ($Ra = Gr\ Pr$)	
Re	Reynolds number	
s	width	m

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T	temperature	K
T_A	ambient temperature outside the building	K
T_{sky}	sky temperature	K
v	wind velocity	$m \cdot s^{-1}$
V	air velocity	$m \cdot s^{-1}$
x, z	transverse and longitudinal coordinates	m

Greek symbols

$(\tau\alpha)$	glass transmittance–plate absorbance product	
α	absorbance	
β	coefficient of cubical thermal expansion of air	K^{-1}
ε	emittance	
λ	thermal conductivity	$W \cdot m^{-1} \cdot K^{-1}$
ψ	rate of solar energy concerning the first node in the cover system	
ρ	density	$kg \cdot m^{-3}$
θ	aspect ratio of the rectangular duct ($\theta < 1$)	
σ	Stefan–Boltzmann constant	$W \cdot m^{-2} \cdot K^{-4}$
τ	time	s

Subscripts

1	outer surface
2	inner surface
a	air in the horizontal duct
G	transparent cover system
GP	air between the cover system and the metal plate
in	inlet
P	thin metallic absorber plate
PW	air between the metal plate and the building wall
W	building wall

1. INTRODUCTION

Passive solar heating of buildings is one of the most effective means of using solar energy and offers a simple and economical tool, suitable to a wide range of latitudes. Passive design requires a profound knowledge of energy and heat flow behaviour in every detail and component of the considered structure. The design of solar systems for building heating requires an accurate selection of the proper simulation method among the available ones, in order to rely on a suitable and validated model [1–2].

In order to determine the energy performance of buildings, the most reliable mathematical models are based on time-dependent solutions of equations describing mass and energy balance of the structure elements.

This work aims at presenting a simple model giving time-dependent solutions of the temperature distributions in two different passive solar systems (winter situation). *The former is a classical Trombe–Michel system, the latter is a Trombe–Michel wall coupled with a storage ceiling structure (Barra–Costantini system [3]). A large number of papers can be found in literature, devoted to the analysis of the Trombe–Michel system; many buildings all over the world have been equipped with this passive system. On the contrary, only a few prototypes of the Barra–Costantini system have been built, even if this heating system seems to be much more efficient; this paper represents the first contribution in the analysis and modeling of the Barra–Costantini system.*

Differential equations have been solved through the finite-difference method, taking into account complex boundary conditions and geometry. Then, the energy fluxes are easily determined. The results refer to the passive heating system of a building near Rome (Italy), the performance of wich, with solar radiation and ambient temperature, has been experimentally measured and recorded.

2. THE PASSIVE SOLAR SYSTEM

The passive system displayed in *figure 1*, proposed by Barra and Costantini [3], is composed by a southward thermally insulated wall, and a transparent cover system. A thin metal plate (absorbing solar energy) is placed between these two components, it gives two parallel and independent vertical ducts, where an air flow is operated by natural convection. Air flows by thermosiphoning in the solar duct, removing heat on both sides of the metal plate (which works as a solar energy collector); through the openings C (*figure 1*) it reaches the horizontal ducted ceiling, storing heat in the

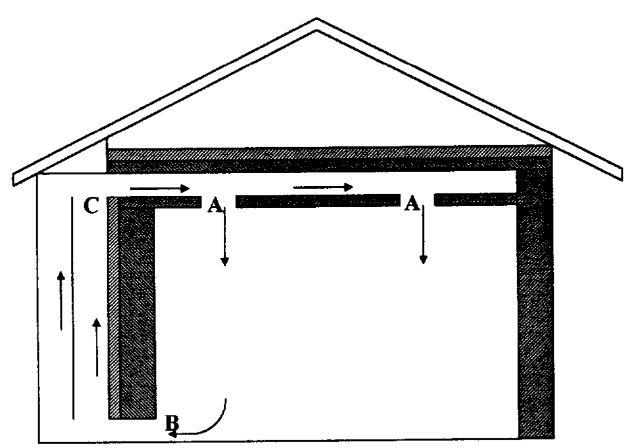

Figure 1. Air flow in the Barra–Costantini system.

ceiling structure. Then, through the openings A, air is mixed in the room; at last, through the openings B in the southern wall, it enters the solar duct. Operable panels or dampers are located in the inside face of the openings in order to operate automatic changeover between night and day functions. Undesired reverse air flow during the night is thus prevented. In this system, as warm air rises in the vertical ducts, it enters the rooms through the openings on the ceiling, while simultaneously cool air comes from the rooms through the openings in the bottom of the wall. The warmed air from the solar duct stores heat in the ceiling and then mixes with the colder indoor air.

3. MODELLING AND METHOD OF SOLUTION

The solution to the energy balance equation allows to calculate the air average velocity in both ducts, under the usual assumption that the density and temperature of the air in the gap varies linearly with height [1]:

$$V_{\mathrm{GP}} = \left(\frac{2\,g\,H\,\beta\,(T_{\mathrm{GP}} - T_{\mathrm{in}})}{\sum_i r_{\mathrm{GP,i}}} \right)^{\frac{1}{2}}$$

$$V_{\mathrm{PW}} = \left(\frac{2\,g\,H\,\beta\,(T_{\mathrm{PW}} - T_{\mathrm{in}})}{\sum_i r_{\mathrm{PW,i}}} \right)^{\frac{1}{2}} \qquad (1)$$

where the term $\sum_i r_i$ represents the sum of pressure drop coefficients of the previously described air paths. This term is linked to the friction factor, which depends on the velocities; for this reason, the calculation of Equation (1) is carried out through an iterative procedure. The friction factor is given by the Blasius correlation for turbulent flow ($f = 0.3164\ Re^{-0.25}$); for laminar flow, it is deduced by the correlation [4]:

$$f = \frac{96}{Re}(1 - 1.20244\,\theta + 0.88119\,\theta^2 + 0,88819\,\theta^3 - 1.69812\,\theta^4 + 0.72366\,\theta^5) \qquad (2)$$

The transparent cover system is described in the model by two nodes; it is characterized by the absorption of solar radiation, conduction, convection to the external environment and to the first duct, radiation towards the external environment and the metal plate. The time-dependent temperature of the transparent cover system, between the external environment and the air flow in the solar duct, is given by the solution to the energy balance equations, considering the single nodes on the inner and outer surfaces:

$$\begin{cases} \left(\frac{\rho\,c\,s}{2}\right)_{\mathrm{G}} \frac{\mathrm{d}T_{\mathrm{G,1}}}{\mathrm{d}\tau} = h_{\mathrm{G1}}(T_{\mathrm{A}} - T_{\mathrm{G,1}}) \\ + \frac{\lambda_{\mathrm{G}}}{s}(T_{\mathrm{G,2}} - T_{\mathrm{G,1}}) + \psi\,\alpha_{\mathrm{G}} I_{\beta} + \sigma\,\varepsilon_{\mathrm{G}}\,(T_{\mathrm{sky}}^4 - T_{\mathrm{G,1}}^4) \\ \left(\frac{\rho\,c\,s}{2}\right)_{\mathrm{G}} \frac{\mathrm{d}T_{\mathrm{G,2}}}{\mathrm{d}\tau} = h_{\mathrm{G2}}(T_{\mathrm{GP}} - T_{\mathrm{G,2}}) \\ + \frac{\lambda_{\mathrm{G}}}{s}(T_{\mathrm{G,1}} - T_{\mathrm{G,2}}) + (1-\psi)\,\alpha_{\mathrm{G}}\,I_{\beta} + \sigma\,\frac{T_{\mathrm{P}}^4 - T_{\mathrm{G,2}}^4}{\varepsilon_{\mathrm{P}}^{-1} + \varepsilon_{\mathrm{G}}^{-1} - 1} \end{cases} \qquad (3)$$

The convective heat transfer coefficient between cover system and outside environment is given in [1] as $h_{\mathrm{G1}} = 5.7 + 3.8$ v, and the sky temperature is $T_{\mathrm{sky}} = T_{\mathrm{A}}$-6 (Whillier correlation). The convective heat transfer coefficient in the solar duct [1] for the air flowing on the transparent cover system is:

$$h_{\mathrm{G2}} = \begin{cases} \frac{\lambda_{\mathrm{GP}}}{s_{\mathrm{PG}}}\left[0.01711\,(Gr_{\mathrm{GP}}\,Re_{\mathrm{GP}})^{0.29}\right] & \text{if } V_{\mathrm{GP}} \approx 0 \\ \frac{\lambda_{\mathrm{GP}}}{s_{\mathrm{PG}}}\left[4.9 + \frac{0.0606\,X^{-1.2}}{1 + 0.0856\,X^{-0.7}}\right] & \text{laminar flow} \\ \frac{\lambda_{\mathrm{GP}}}{s_{\mathrm{PG}}}(0.0158\,Re_{\mathrm{GP}}^{0.8}) & \text{turbulent flow} \end{cases}$$

where the factor X is given by:

$$X = \frac{H\,(1+L)}{2\,Pe_{\mathrm{GP}}\,L\,s_{\mathrm{GP}}} \qquad (5)$$

The bulk air temperature T_{GP} in the solar duct, between the transparent cover and the metal plate, is deduced by the partial differential equation for energy conservation:

$$V_{\mathrm{GP}}\frac{\partial T_{\mathrm{GP}}}{\partial z} + \frac{\partial T_{\mathrm{GP}}}{\partial \tau} = \frac{1}{(\rho\,c\,s)_{\mathrm{GP}}}\left[h_{\mathrm{P1}}\,(T_{\mathrm{p}} - T_{\mathrm{GP}}) - h_{\mathrm{G2}}\,(T_{\mathrm{GP}} - T_{\mathrm{G,2}})\right] \qquad (6)$$

The coefficient h_{P1}, analogous to h_{G2}, is determined as indicated in the equations (4). The thin metal plate is characterized by radiative heat transfer with both transparent cover and building wall, convection with two air streams, absorption of the solar radiation. Neglecting the conduction in the thin metal width, assuming grey-body approximation, the plate temperature is given by the solution to the balance equation:

$$(\rho\,c\,s)_{\mathrm{P}}\frac{\partial T_{\mathrm{P}}}{\partial \tau} = (\tau\,\alpha)\,I_{\beta} + h_{\mathrm{P1}}\,(T_{\mathrm{GP}} - T_{\mathrm{P}}) + h_{\mathrm{P2}}\,(T_{\mathrm{PW}} - T_{\mathrm{P}}) + \sigma\,\frac{T_{\mathrm{G,2}}^4 - T_{\mathrm{P}}^4}{\varepsilon_{\mathrm{P}}^{-1} + \varepsilon_{\mathrm{G}}^{-1} - 1} + \sigma\,\frac{T_{\mathrm{W,1}}^4 - T_{\mathrm{P}}^4}{\varepsilon_{\mathrm{P}}^{-1} + \varepsilon_{\mathrm{W}}^{-1} - 1} \qquad (7)$$

The total effective transmittance–absorption product $\tau\alpha$ [1] is calculated according to the hour, day, value of direct and diffuse solar radiation.

An energy balance for the vertical wall allows to obtain the following equation:

$$\int_0^{s\mathrm{W}} (\rho c)_\mathrm{W} \frac{\partial T_\mathrm{W}(x)}{\partial \tau} \, \mathrm{d}x = a_\mathrm{W} \, (T_\mathrm{in} - T_{\mathrm{W},2}) - h_{\mathrm{W}1} \, (T_{\mathrm{W},1} - T_\mathrm{PW}) - \sigma \frac{T_{\mathrm{W},1}^4 - T_\mathrm{P}^4}{\varepsilon_\mathrm{P}^{-1} + \varepsilon_\mathrm{W}^{-1} - 1} \quad (9)$$

The temperatures in the single nodes discretizing the wall are calculated by solving the time-dependent Fourier equation, considering the different materials in the structure layout. The heat transfer coefficient for the indoor air flowing in the vertical wall, of height H, is [5]:

$$\begin{aligned} a_\mathrm{W} &= 1.39 \left(\frac{T_\mathrm{in} - T_{\mathrm{W},2}}{H}\right)^{0.25} && \text{if } H\,(T_\mathrm{in} - T_{\mathrm{W}2}) < 1 \\ a_\mathrm{W} &= 1.54 \,(T_\mathrm{in} - T_{\mathrm{W},2})^{0.33} && \text{if } H\,(T_\mathrm{in} - T_{\mathrm{W}2}) > 1 \end{aligned} \quad (10)$$

In the horizontal duct, in the ceiling, the air temperature is calculated with an equation similar to (6), where the convective heat exchange coefficients are:

$$h_\mathrm{SUP} = 0.58 \, \frac{\lambda_\mathrm{a}}{D} \, Ra^{0.2} \qquad \textit{for any } Ra \quad (11)$$

$$\begin{cases} h_\mathrm{INF} = 0.13 \, \dfrac{\lambda_\mathrm{a}}{D} \, Ra^{0.333} & Ra \leqslant 2 \cdot 10^8 \\ h_\mathrm{INF} = 0.16 \, \dfrac{\lambda_\mathrm{a}}{D} \, Ra^{0.333} & Ra > 2 \cdot 10^8 \end{cases} \quad (12)$$

The computer code can be easily applied to the Trombe–Michel system too, by considering one single solar duct without the thin metal plate and neglecting the ducted ceiling. The model of the transparent cover remains unchanged, whereas radiative exchange inside the duct is operated directly between the wall and the cover.

4. RESULTS AND EXPERIMENTAL VALIDATION

The comparison between numerical results and tested values shows an excellent prediction capability of the simple model described in the previous section. For this reason, the use of this method gives a satisfactory solution both to the problem of sizing the components of the system, and to the evaluation of its energy performance at different climatic and design conditions.

Reported experimental values have been collected by ENEA from 1980 to 1982, in a building prototype in Salisano, located 50 kilometers from Rome, at 42° 15′ latitude, 300 m above the sea level [3]. *Any detail concerning the geometry, the experimental data acquisition, the instruments and their precision can be found in [3].* The constructive data of the prototype are summarized as follows:

– transparent cover: one plate of polyester enforced with glass fiber, 1.524 mm thickness, $\lambda = 0.1$ W·m^{-1}·K^{-1};

– thin metal plate: one alluminium plate 0.2 mm thick, 3.13 m high, 2.85 m wide, $\lambda = 200$ W·m^{-1}·K^{-1};

– vertical solar ducts: 0.06 m wide;

– channels inside the ceiling: five parallele channels, 0.6 m pitch, delimited by 0.04 m thick upper and lower walls, 2.05 m wide, 0.145 high, 6.50 m long;

– southward wall: blocs of Lecabloc of laterice, without plaster, 0.25 m thick, $\lambda = 0.4$ W·m^{-1}·K^{-1};

– thermal insulation: high density expanded polystirene, 0.08 m thick, $\lambda = 0.04$ W·m^{-1}·K^{-1}.

Experimental values recorded hour by hour during a day of February, used as input data for the numerical simulation, are reported in *table 1*. *Figures 2* and *3* show the good agreement between the experimental and the numerical results; *figure 2* shows the time-dependent mean air velocity in the ceiling duct, the air flow occurs only in the sunny hours, the maximum velocity value is 0.56 m·s^{-1}, the highest difference

TABLE I
Experimental hour data during a day of February

Solar hour	Ambient temperature (°C)	Indoor temperature (°C)	Solar radiation (W·m^2)
1.00	6.0	18.5	0
2.00	5.0	18.5	0
3.00	5.0	18.0	0
4.00	5.0	18.0	0
5.00	5.0	17.0	0
6.00	5.0	16.5	0
7.00	5.5	16.0	0
8.00	6.0	16.0	200
9.00	9.0	18.0	450
10.00	10.0	19.0	650
11.00	12.0	20.0	750
12.00	14.0	21.0	800
13.00	13.5	22.0	780
14.00	16.0	22.5	700
15.00	13.5	22.0	450
16.00	13.5	21.5	350
17.00	12.0	21.0	100
18.00	10.0	20.5	0
19.00	9.5	20.5	0
20.00	8.0	20.0	0
21.00	7.5	20.0	0
22.00	8.5	19.0	0
23.00	8.5	19.0	0
24.00	6.0	19.0	0

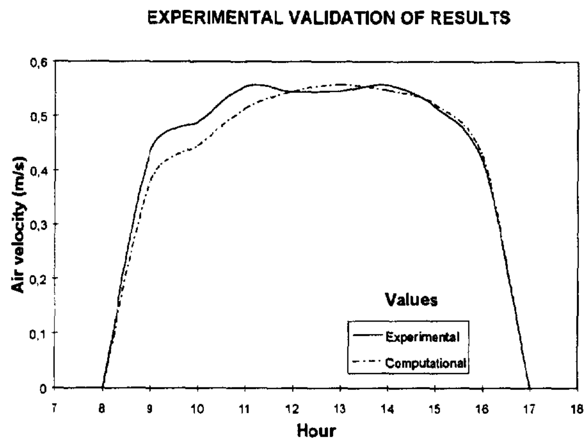

Figure 2. Mean air velocity in the ceiling horizontal duct.

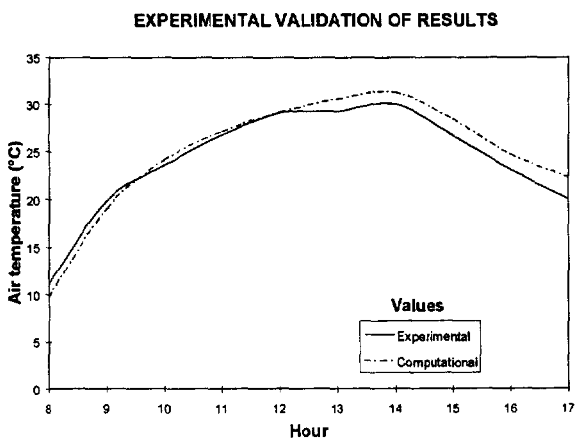

Figure 3. Air bulk temperature at the top of the solar duct.

between the experimental data and the numerical results is 5 cm·s^{-1} (during the morning hours). In *figure 3* the time-dependent bulk air temperature T_{GP} at the top of the solar duct is shown; at dawn it is 10 °C, then it increases reaching 31.7 °C at 13.50 h, the maximum shifting between experimental data and numerical results is 1.7 K. The model also allows to evaluate the energy performance of both passive solar systems (Barra–Costantini and Trombe–Michel) in time-dependent conditions. The comparison is made considering the same building in the same location, the same transparent cover system, the same width of the solar ducts (so that the air steam has the same cross section), the same wall (insulated only in the Barra–Costantini system). The hour by hour energy fluxes for both systems are presented in *figures 4–6*, during a day of February.The solid line, in *figures 4* and *5*, refers to the power transferred to the warm air, from the solar ducts, to the indoor air; the dashed line refers to the power transferred from the wall inner surface (considering a 1 m large wall, with heigth H) to the

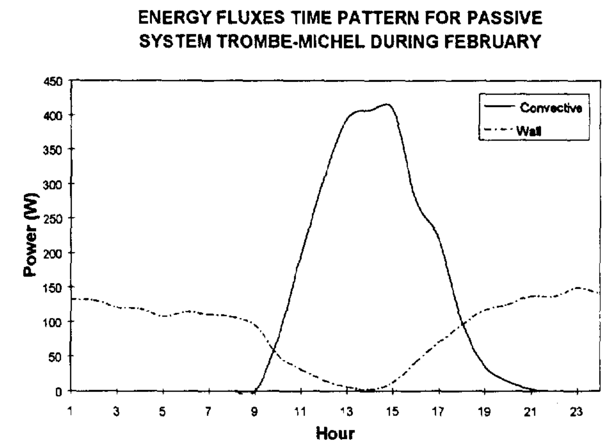

Figure 4. Energy fluxes for the Trombe–Michel system.

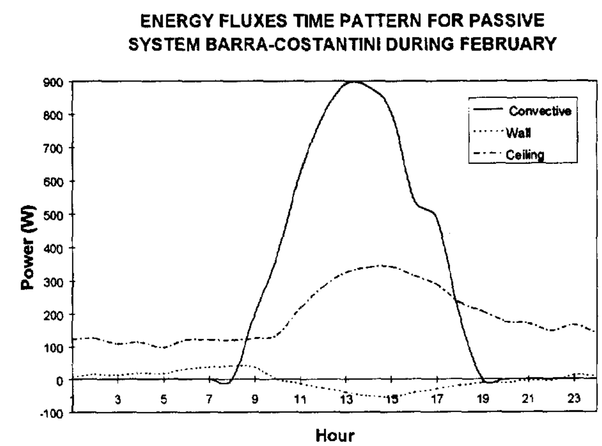

Figure 5. Energy fluxes for the Barra–Costantini system.

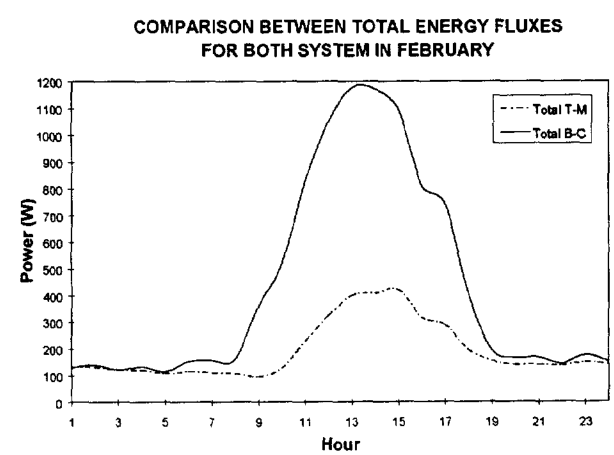

Figure 6. Total energy fluxes for both systems (Barra–Costantini and Trombe–Michel).

air room. In *figure 5*, the pointed line refers to the power transferred by the ceiling storage to the ambient room. *The effect of the ceiling storage is quite effective in the Barra–Costantini system, mainly in the afternoon and evening The heat transferred to the room by the air stream is much greater in the Barra–Costantini system, as well as the heat received from the storage.* As proved in *figure 6* too, during the night, the two different passive systems have a similar behaviour; the wall of the Trombe–Michel systems is equivalent to the insulated wall and the ceiling of the Barra–Costantini system. On the contrary, during the sunny hours, the metal absorber in the double solar duct of the Barra–Costantini system allows to obtain a greater power from the air entering the ambient rooms through the openings.

A careful analysis, carried out for the days of a whole year, puts in evidence how the system described in *figure 1* offers a better performance than the classical Trombe–Michel system for the passive heating of a building. This is a clear consequence of the different structural features which allow the first system to quickly remove heat from the metal plate and to reduce thermal losses thanks to the insulating material of the southward wall.

In conclusion, the Barra–Costantini system allows to obtain a remarkable energy gain; at present its diffusion is rather scanty in our countries because it requires a more peculiar architecture and a more expensive investment, but it represents a reliable passive tool to reduce the energy consumption in building heating.

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