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Energy and momentum in chiral theories

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Abstract. We calculate the energy momentum tensor to order E^4 in chiral perturbation theory. New terms not present in previous work enter the effective Lagrangian. We describe these and estimate the values of the new coupling constants, using the results of a dispersive analysis of the π and K energy momentum tensors and relying on tensor meson dominance for the spin two component. In addition, we compare our findings with the predictions of known scalar meson dominance models of the conformal anomaly.

I Introduction

The only rigorous technique for describing the predictions of QCD at very low energies involves the use of chiral symmetry. These predictions are best described by effective chiral Lagrangians which compactly summarize the relations between amplitudes [1, 2]. These Lagrangians can be expanded in powers of the energy, accompanied by a loop expansion of chiral perturbation theory. It is the purpose of this paper to describe the energy momentum tensor in the effective Lagrangian framework.

Despite considerable work on chiral Langrangians, the energy momentum tensor has not yet been written down at the next to leading order (called order E hereafter). It cannot be simply obtained from the presently known Lagrangian by calculating the Noether current associated with translational symmetry, because the equations of motion have been used in writing down the minimal Langrangian at order E^4. This can be remedied by repeating the construction of the minimal Lagrangian including a source which couples up to the energy momentum tensor. We will show that this produces three new terms in the Lagrangian. We then complete the renormalization of the chiral theory with the new terms and explicitly display matrix elements of the energy momentum tensor. Estimates of the new couplings are also given. Finally, we establish contact with known scalar meson dominance models of the conformal anomaly.

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Our investigation was originally motivated by the possibility that nature might choose to equip the Higgs particle with a very low mass, such that the dominant decays would be $H\rightarrow\pi\pi$ and $H\rightarrow\mu^+\mu^-$. The pion decay channel is related to $\langle\pi\pi\,|\,\theta^\mu_\mu\,|\,0\rangle$, which was studied in several papers [3, 4, 12, 13, 16]. In the meantime, the experimental bounds on the decays $Z\rightarrow H\mu^+\mu^-$ and $Z\rightarrow H\gamma$ appear to exclude this possibility. A different application concerns the decays $\psi'\rightarrow\psi\pi\pi$ and $\Upsilon'\rightarrow\Upsilon\pi\pi$. In this case a multipole expansion of the heavy quark transition produces an operator again related to the energy momentum tensor [5]. The new terms which we describe are therefore relevant for an analysis of the $\pi\pi$ spectrum in these transitions.

II Effective Lagrangian

The energy momentum tensor in QCD (or any other theory) can be identified by adding a source field $g_{\mu\nu}$ coupled to the matter fields in a generally covariant fashion, i.e., like the metric tensor of general relativity,

$$\mathscr{L}(\psi, A^{a\mu})\rightarrow\mathscr{L}(\psi, A^{a\mu}, g^{\mu\nu}). \tag{1}$$

With this substitution, the energy momentum tensor is formed by

$$\tfrac{1}{2}\theta_{\mu\nu}(x)=\frac{\partial}{\partial g^{\mu\nu}(x)}\times\sqrt{g}\,\mathscr{L}(\psi, A^{a\mu}, g^{\mu\nu})|_{g_{\mu\nu}=\eta_{\mu\nu}}. \tag{2}$$

Greens functions are found by functional differentiation of the effective action

$$e^{iZ}=\int d\psi\, d\bar{\psi}\, dA^{a\mu}\, e^{i\int dx^4\sqrt{g}\,\mathscr{L}(\psi, A^{a\mu}, g^{\mu\nu})}. \tag{3}$$

This procedure is similar to the use of external currents in order to identify current matrix elements in previous studies of chiral Lagrangians.

The effective Lagrangian which results at low energy will share the general covariance of the underlying theory. This means that it must be constructed using covariant derivatives, with indices contracted using the metric tensor $g_{\mu\nu}$, and possibly the Riemann or Ricci tensors $R^{\mu\nu\alpha\beta}$,

$R^{\mu\nu}$ or the curvature scalar R. The latter are constructed by two derivatives of the metric tensor and hence carry energy dimension E^2. All of the metric functions are scalars under chiral symmetry. The chiral symmetry aspects of the construction of the effective Lagrangian are standard and we concentrate on the new features involving the curvature.

At order E^2 in the energy expansion, there are no factors of curvature multiplied by chiral fields. The possible terms are*

$$\mathscr{L}^{(2)}=\frac{F_0^2}{4}\{g^{\mu\nu}\operatorname{Tr}(D_\mu U D_\nu U^\dagger)+\operatorname{Tr}(\chi U^\dagger+U\chi^\dagger)\} + H_0 R. \tag{4}$$

Since the chiral matrix U is a Lorentz scalar the derivatives involved are just the usual derivatives with external currents

$$D_\mu U=\partial_\mu U-\mathrm{i} l_\mu U+\mathrm{i}\, U r_\mu . \tag{5}$$

It is well known that conformal symmetry and spontaneously broken chiral symmetry are not compatible with one another [14]. The effective Lagrangian given in (4) contains a dimensionful coupling constant F_0 and the corresponding energy momentum tensor is not traceless. The standard "improvement" [6] of scalar field theory, which amounts to supplementing the kinetic energy with a term proportional to $R\phi^2$, i.e.

$$\mathscr{L}_{\text{improved}}=\tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi+\tfrac{1}{12}R\phi^2+\cdots \tag{6}$$

cannot apply here, because such a term breaks chiral symmetry. This is why the effective Lagrangian at order E^2 contains the curvature only through the term $H_0 R$ which is independent of the meson field and does not play any role in what follows. [In the context of general relativity, this term represents the Lagrangian of the gravitational field, H_0 being related to Newton's constant by $H_0=-(16\pi G)^{-1}$.]

At order E^4, there will be two classes of contributions to the effective Lagrangian.

$$\mathscr{L}^{(4)}=\mathscr{L}^{(4,g)}+\mathscr{L}^{(4,R)}. \tag{7}$$

In the first class are those formed without any factors of the curvature. These are the previously known terms [2], but with Lorentz indices raised and lowered with $g_{\mu\nu}$. In chiral $SU(3)$, this is

$$\begin{aligned}\mathscr{L}^{(4,g)} &= L_1[\operatorname{Tr}(D_\mu U D^\mu U^+)]^2\\
&+L_2\operatorname{Tr}(D_\mu U D_\nu U^+)\operatorname{Tr}(D^\mu U D^\nu U^+)\\
&+L_3\operatorname{Tr}(D_\mu U D^\mu U^+ D_\nu U D^\nu U^+)\\
&+L_4\operatorname{Tr}(D_\mu U D^\mu U^+)\operatorname{Tr}(\chi U^+ + U\chi^+)\\
&+L_5\operatorname{Tr}(D_\mu U D^\mu U^+(\chi U^+ + U\chi^+))\\
&+L_6[\operatorname{Tr}(\chi U^+ + U\chi^+)]^2+L_7[\operatorname{Tr}(\chi U^+ - U\chi^+)]^2\\
&+L_8\operatorname{Tr}(\chi U^+\chi U^+ + U\chi^+ U\chi^+)\\
&-\mathrm{i} L_9\operatorname{Tr}(F^R_{\mu\nu} D^\mu U D^\nu U^+ + F^L_{\mu\nu} D^\mu U^+ D^\nu U)\\
&+L_{10}\operatorname{Tr}(U^+ F^R_{\mu\nu} U F^{L\mu\nu})\\
&+H_1\operatorname{Tr}(F^{R\mu\nu}F^R_{\mu\nu}+F^{L\mu\nu}F^L_{\mu\nu})\\
&+H_2\operatorname{Tr}(\chi^+\chi).\end{aligned} \tag{8}$$

Here the field strength tensors $F^{L(R)}_{\mu\nu}$ are constructed with the external fields l_μ (r_μ). The last two terms are contact terms which do not contain the meson field.

The second class of contributions involve derivatives of the metric. General covariance implies that these derivatives only enter through the curvature tensor

$$R^\lambda_{\sigma\mu\nu}=\partial_\mu\Gamma^\lambda_{\nu\sigma}-\partial_\nu\Gamma^\lambda_{\mu\sigma}+\Gamma^\lambda_{\mu\alpha}\Gamma^\alpha_{\nu\sigma}-\Gamma^\lambda_{\nu\alpha}\Gamma^\alpha_{\mu\sigma}. \tag{9}$$

At order E^4, the general expression is of the form

$$\begin{aligned}\mathscr{L}^{(4,R)}&=L_{11}R\operatorname{Tr}(D_\mu U D^\mu U^+)\\
&+L_{12}R^{\mu\nu}\operatorname{Tr}(D_\mu U D_\nu U^+)\\
&+L_{13}R\operatorname{Tr}(\chi U^+ + U\chi^+)+H_3R^2+H_4R^{\mu\nu}R_{\mu\nu}\\
&+H_5R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta},\end{aligned} \tag{10}$$

where $R_{\mu\nu}$ and R are the Ricci tensor and curvature scalar, respectively

$$R_{\mu\nu}=R^\lambda_{\mu\lambda\nu};\qquad R=g^{\mu\nu}R_{\mu\nu}. \tag{11}$$

The contact terms involving the square of the curvature are the standard counter terms occurring in the quantum theory of gravity at one loop order. In the present context only those terms which contain the meson field are relevant. They involve three new low energy constants L_{11}, L_{12}, L_{13} which are not determined from previous work on effective Lagrangians.

III Renormalization

Here we identify the renormalization of the coupling constants for the one loop treatment of the effective action. This involves the calculation of the divergent terms in the one loop graphs generated by $\mathscr{L}^{(2)}$. The general chiral field U is expanded around a background field $\bar U$ which is a solution to the equations of motion in the presence of the external fields. We expand in a symmetric fashion

$$U=u\,e^{\mathrm{i}\xi}u, \tag{12}$$

where

$$\bar U=uu. \tag{13}$$

* A cosmological constant needs to be added to tune the vacuum energy to zero – we omit this term

After some algebra, we find

$$\int d^4x \sqrt{g}\, \mathscr{L}^{(2)}(U)$$
$$= \int d^4x \left\{ \sqrt{g}\, \mathscr{L}^{(2)}(\bar{U}) + \frac{F_0^2}{2}\, \xi^A D^{AB} \xi^B + \mathscr{O}(\xi^3) \right\}, \tag{14}$$

where

$$\xi^A = \tfrac{1}{2}\mathrm{Tr}(\lambda^A \xi),$$
$$D = \frac{1}{\sqrt{g}}\, d_\mu \sqrt{g} g^{\mu\nu} d_\nu + \hat{\sigma},$$
$$d_\mu^{AB} = \delta^{AB}\partial_\mu + \hat{\Gamma}_\mu^{AB},$$
$$\hat{\Gamma}_\mu^{AB} = -\tfrac{1}{4}\mathrm{Tr}([\lambda^A, \lambda^B]([u^+, \partial_\mu u] \tag{15}$$
$$- i u^+ F_\mu^R u - i u F_\mu^L u^+)),$$
$$\hat{\sigma}^{AB} = \tfrac{1}{8}\mathrm{Tr}([\lambda^A, u^+ D_\mu \bar{U} u^+][\lambda^B, u^+ D_\mu \bar{U} u^+])$$
$$+ \tfrac{1}{8}\mathrm{Tr}(\{\lambda^A, \lambda^B\}(u\chi^+ u + u^+ \chi u^+)).$$

The full one loop effective action is then obtained by integration over the quantum fluctuations

$$e^{iZ} = \int [d\xi]\, e^{i\int d^4x\, \mathscr{L}}, \tag{16}$$

resulting in

$$Z = \int d^4x \sqrt{g}\, \{\mathscr{L}^{(2)}(\bar{U})$$
$$+ \mathscr{L}^{(4)}(\bar{U})\} + \frac{i}{2}\ln\det D. \tag{17}$$

The divergences in the determinant can be worked out in various ways. The easiest method uses the heat kernel expansion, which has previously been worked out for differential operators in a background curved space [7, 15]. The divergent terms are

$$\frac{i}{2}\ln\det D = -\frac{1}{(4\pi)^2}\frac{1}{d-4}$$
$$\times \int d^4x \sqrt{g}\, [\mathrm{Tr}\{\tfrac{1}{12}\hat{\Gamma}_{\mu\nu}\hat{\Gamma}^{\mu\nu} + \tfrac{1}{2}\hat{\sigma}^2 + \tfrac{1}{6}R\hat{\sigma}\}$$
$$+ (N^2-1)(\tfrac{1}{72}R^2 - \tfrac{1}{180}R_{\mu\nu}R^{\mu\nu}$$
$$+ \tfrac{1}{180}R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}) + \ldots] \tag{18}$$

for $SU(N)$ chiral symmetry. The terms proportional to $\hat{\Gamma}_{\mu\nu}\hat{\Gamma}^{\mu\nu}$ and $\hat{\sigma}^2$ lead to the renormalization of $\mathscr{L}^{(4,g)}$ as has been given previously. The remainder, in particular the term

$$\tfrac{1}{6}R\,\mathrm{Tr}\,\hat{\sigma} = \frac{N}{12}R\,\mathrm{Tr}(D_\mu U D^\mu U^+)$$
$$+ \frac{N^2-1}{12N}R\,\mathrm{Tr}(\chi U^+ + U\chi^+), \tag{19}$$

is removed by a renormalization of the couplings occuring in $\mathscr{L}^{(4,R)}$:

$$L_i^r = L_i - \Gamma_i\lambda,$$
$$H_i^r = H_i - \Delta_i\lambda,$$
$$\lambda = \frac{\mu^{d-4}}{(4\pi)^2}\left\{\frac{1}{d-4} - \tfrac{1}{2}(\ln 4\pi + \Gamma'(1) + 1)\right\}, \tag{20}$$
$$\Gamma_{11} = \frac{N}{12}, \quad \Gamma_{12} = 0, \quad \Gamma_{13} = \frac{N^2-1}{12N},$$
$$\Delta_3 = \frac{N^2-1}{72}, \quad \Delta_4 = -\Delta_5 = -\frac{(N^2-1)}{180}.$$

The finite terms have been added to λ for later convenience.

IV The energy momentum tensor

The effective Lagrangian specified above determines the Green functions involving the energy momentum tensor (as well as vector, axial vector, scalar or pseudoscalar currents) to first nonleading order in the energy expansion. The divergences of the relevant one loop graphs are removed by the coupling constant renormalization given in (20). The resulting finite representation of the Green functions automatically obeys the Ward identities associated with the conservation of energy and momentum and of the chiral charges, because the effective theory is manifestly invariant both under coordinate transformations and under local chiral rotations.

In the framework of the effective theory, the energy momentum tensor is given by the response of the effective action to a variation of the metric. To first nonleading order, the chiral representation is of the form

$$\theta_{\mu\nu} = \theta_{\mu\nu}^{(2)} + \theta_{\mu\nu}^{(4,g)} + \theta_{\mu\nu}^{(4,R)} + \mathscr{O}(E^6). \tag{21}$$

The leading term is the energy momentum tensor of $\mathscr{L}^{(2)}$,

$$\theta_{\mu\nu}^{(2)} = \frac{F_0^2}{2}\mathrm{Tr}(D_\mu U D_\nu U^+) - g_{\mu\nu}\mathscr{L}^{(2)}. \tag{22}$$

The contribution from $\mathscr{L}^{(4,g)}$ is given by

$$\theta_{\mu\nu}^{(4,g)} = 2L_4\mathrm{Tr}(D_\mu U D_\nu U^+)\mathrm{Tr}(\chi U^+ + U\chi^+)$$
$$+ L_5\mathrm{Tr}\{(D_\mu U D_\nu U^+$$
$$+ D_\nu U D_\mu U^+)(\chi U^+ + U\chi^+)\}$$
$$- g_{\mu\nu}\mathscr{L}^{(4,g)} + \cdots, \tag{23}$$

where we have dropped terms which only contribute to matrix elements involving four or more mesons or currents and have also discarded contact contributions. Finally, the new couplings generate the term

$$\theta_{\mu\nu}^{(4,R)} = 2(g_{\mu\nu}\Box - \partial_\mu\partial_\nu)$$
$$\times \mathrm{Tr}\{L_{11}D_\alpha U D^\alpha U^+ + L_{13}(\chi U^+ + U\chi^+)\}$$
$$+ L_{12}(g_{\mu\alpha}g_{\nu\beta}\Box + g_{\mu\nu}\partial_\alpha\partial_\beta - g_{\mu\alpha}\partial_\nu\partial_\beta$$
$$- g_{\nu\alpha}\partial_\mu\partial_\beta)\mathrm{Tr}(D^\alpha U D^\beta U^+). \tag{24}$$

In the evaluation of the matrix elements to order E^4, the leading term $\theta^{(2)}_{\mu\nu}$ is needed to one loop accuracy while the remaining contributions only enter at tree level.

V Form factors

We now apply the machinery set up above to the matrix elements of $\theta_{\mu\nu}$ in the one-particle states $|\pi\rangle$, $|K\rangle$, $|\eta\rangle$. Lorentz invariance and energy-momentum conservation imply that these matrix elements are of the form

$$\begin{aligned}\langle p'|\theta_{\mu\nu}|p\rangle &= \tfrac{1}{2}(g_{\mu\nu}q^2 - q_\mu q_\nu)\theta_1(q^2) \\ &\quad + \tfrac{1}{2}P_\mu P_\nu \theta_2(q^2), \\ P_\mu &= p'_\mu + p_\mu, \\ q_\mu &= p'_\mu - p_\mu. \end{aligned} \tag{25}$$

The occurrence of two invariant form factors reflects the fact that the operator $\theta_{\mu\nu}$ contains both a scalar (spin zero) and a tensor (spin two) part. The tensor part is described by the form factor $\theta_2(q^2)$ while the matrix element of the scalar θ^μ_μ is given by the combination

$$\theta_0(q^2) = \tfrac{3}{2}q^2\theta_1(q^2) + \tfrac{1}{2}(4m^2 - q^2)\theta_2(q^2). \tag{26}$$

The idenfication of $\int d^3x\,\theta_{00}$ with the Hamiltonian gives the normalization at $q^2=0$, requiring

$$\theta_2(0) = 1. \tag{27}$$

At leading order in the energy expansion, the matrix element is given by the tree graph shown in Fig. 1a. This graph leads to $\theta_1(q^2) = \theta_2(q^2) = 1$, in accordance with the low energy theorem [5]

$$\langle p'|\theta_{\mu\nu}|p\rangle = p'_\mu p_\nu + p'_\nu p_\mu + \tfrac{1}{2}g_{\mu\nu}q^2 + \mathcal{O}(E^4) \tag{28}$$

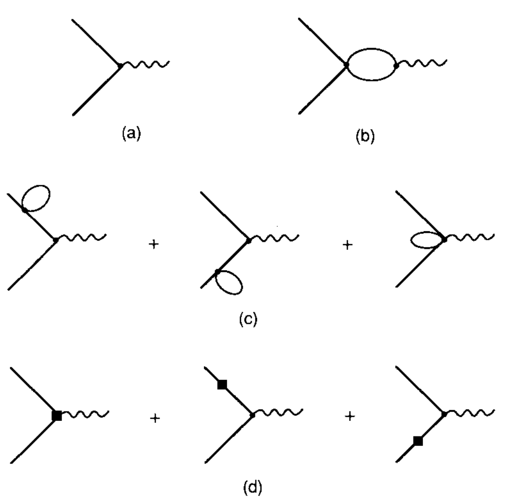

Fig. 1a–d. Chiral perturbation theory graphs relevant for the expansion of the energy momentum tensor to order E^4. The dots denote vertices of $L^{(2)}$ and $\theta^{(2)}_{\mu\nu}$ which only involve the pion decay constant, while the squares indicate vertices generated by the coupling constants $L_1, L_2, \ldots$

At the next order in the energy expansion, we need to evaluate the one loop graphs of $\theta^{(2)}_{\mu\nu}$ shown in Fig. 1b, c and the tree graphs of Fig. 1d which involve a vertex of $\theta^{(4,g)}_{\mu\nu}$, $\theta^{(4,R)}_{\mu\nu}$ or $\mathscr{L}^{(4)}$. The evaluation is straightforward. The tadpole graphs of Fig. 1c and the contributions from $\theta^{(4,g)}_{\mu\nu}$ merely renormalize the mass and the wave function – expressed in terms of the physical mass, the result becomes independent of the couplings contained in $\mathscr{L}^{(4,g)}$. The divergence contained in the one loop graph of Fig. 1b are removed by the renormalization of the coupling constants L_{11} and L_{13} given in (20). For $\theta_1(q^2)$ the result then takes the form

$$\begin{aligned}\theta^\pi_1(q^2) &= 1 + 2\frac{q^2}{F^2}(4L^r_{11} + L^r_{12}) \\ &\quad - 16\frac{m^2_\pi}{F^2}(L^r_{11} - L^r_{13}) + \frac{1}{F^2}(2q^2 - m^2_\pi)I_\pi(q^2) \\ &\quad + \frac{q^2}{F^2}I_K(q^2) + \frac{m^2_\pi}{3F^2}I_\eta(q^2), \\ \theta^K_1(q^2) &= 1 + 2\frac{q^2}{F^2}(4L^r_{11} + L^r_{12}) \\ &\quad - 16\frac{m^2_K}{F^2}(L^r_{11} - L^r_{13}) + \frac{3q^2}{4F^2}I_\pi(q^2) \\ &\quad + \frac{3q^2}{2F^2}I_K(q^2) + \frac{(9q^2 - 8m^2_K)}{12F^2}I_\eta(q^2), \\ \theta^\eta_1(q^2) &= 1 + 2\frac{q^2}{F^2}(4L^r_{11} + L^r_{12}) \\ &\quad - 16\frac{m^2_\eta}{F^2}(L^r_{11} - L^r_{13}) + \frac{m^2_\pi}{F^2}I_\pi(q^2) \\ &\quad + \frac{(9q^2 - 8m^2_K)}{3F^2}I_K(q^2) + \frac{(4m^2_\eta - m^2_\pi)}{3F^2}I_\eta(q^2), \end{aligned} \tag{29}$$

where the function $I(q^2)$ can be expressed in terms of the standard scalar one loop integral $\bar{J}(q^2)$

$$\begin{aligned}\bar{J}(q^2) &= \frac{1}{16\pi^2}\left\{\sigma \ln\frac{\sigma-1}{\sigma+1} + 2\right\}, \\ \sigma &= (1 - 4m^2/q^2)^{1/2}, \end{aligned} \tag{30}$$

as

$$\begin{aligned}I(q^2) &= \frac{1}{3q^2}(2m^2 + q^2)\bar{J}(q^2) \\ &\quad - \frac{1}{48\pi^2}\left\{\ln\frac{m^2}{\mu^2} + \tfrac{4}{3}\right\}. \end{aligned} \tag{31}$$

The function $I(q^2)$ contains a cut along the real axis, extending from $q^2 = 4m^2$ to ∞. The first two terms in the Taylor expansion in powers of q^2 are

$$I(q^2) = \frac{1}{48\pi^2}\left\{\ln\frac{\mu^2}{m^2} - 1 + \frac{q^2}{5m^2} + \mathcal{O}(q^4)\right\}. \tag{32}$$

Note that the loop contribution depends on the renormalization scale μ. A change in this scale however only adds a constant to $I(q^2)$ and one readily checks that the corresponding shift in the running couplings L_{11}^r, L_{13}^r precisely compensates for this constant, such that the result for $\theta_1(q^2)$ is scale independent.

At the order of the energy expansion we are considering here, the tensor form factor $\theta_2(q^2)$ does not receive contributions from loops and is therefore given by a polynominal of order E^2,

$$\theta_2(q^2)=1-2L_{12}^r\frac{q^2}{F^2}+\mathcal{O}(E^4). \tag{33}$$

Note that the normalization condition (27) excludes q^2-independent corrections. Furthermore, chiral symmetry requires the slopes of the form factor θ_2^π, θ_2^K and θ_2^η at $q^2=0$ to coincide in the chiral limit. This is why, to order E^2, the low energy representations of these form factors are identical. Chiral symmetry however does not determine the value of the slope. In our formalism this low energy parameter is encoded in the coupling constant L_{12}.

The structure of the above formulae is controlled by the final state interaction theorem, according to which the phase of the form factors is determined by the scattering phase shift. Chiral symmetry implies that, at low energies, the Goldstone bosons only interact weakly, the phase shift being of order E^2. Hence the leading term in the energy expansion of the form factors $\theta_1(q^2)$, $\theta_2(q^2)$ is real. Moreover, the leading term in the energy expansion of the scattering amplitude only contains *S*- and *P*-waves. Accordingly, the spin two part $\theta_2(q^2)$ picks up a phase only at order E^4 – this is why the representation (33) of this form factor does not contain contributions from one loop graphs and this also explains why L_{12} does not get renormalized. The scalar form factor $\theta_0(q^2)$, on the other hand, does pick up a phase at first nonleading order, because the final state in the matrix element $\langle p'p|\theta_\mu^\mu|0\rangle$ is in an *S*-wave configuration. One readily checks that, in the region below $K\bar{K}$ threshold, the imaginary part of the low energy representation for $\theta_0^\pi(q^2)$ which follows from (26), (29) and (33) is indeed given by $\delta_0^0\cdot(2m_\pi^2+q^2)$, where

$$\delta_0^0=\frac{1}{32\pi F^2}\times(2q^2-m_\pi^2)(1-4m_\pi^2/q^2)^{1/2}+\mathcal{O}(E^4) \tag{34}$$

is the current algebra expression for the $I=J=0$ $\pi\pi$ phase shift. We have checked that the imaginary parts of the remaining loop contributions occuring in (29) also obey the final state interaction theorem, evaluated with the current algebra predictions for the $\pi\pi\leftrightarrow K\bar{K}\leftrightarrow\eta\eta$ scattering amplitudes. Unitarity thus fixes the structure of the form factors up to a real polynomial of order E^2. In case of $\theta_1(q^2)$, this polynomial contains both a term proportional to q^2, determined by $4L_{11}+L_{12}$, and a constant term proportional to $m^2(L_{11}-L_{13})$, while in the case of $\theta_2(q^2)$, the normalization condition (27) only permits a term proportional to q^2.

VI Phenomenology of the new coupling constants

According to (33), the coupling constant L_{12} determines the slope of the tensor form factor $\theta_2(q^2)$. Since the mesons are composite, this form factor is expected to obey an unsubtracted dispersion relation

$$\theta_2(q^2)=\frac{1}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds}{s-q^2-i\varepsilon}\,\mathrm{Im}\,\theta_2(s). \tag{35}$$

The discontinuity generated by two-meson intermediate states is suppressed at low energies, because the *D*-wave phase shift is small there. The situation is similar to the case of the electromagnetic form factor where, below the resonance region, the discontinuity is also small. In that case, saturation of the dispersion relation analogous to (35) by a narrow resonance at $m_\rho\cong 770$ MeV predicts a value for the slope of the form factor which agrees with experiment. We expect the same to be true of the form factor $\theta_2(q^2)$ where the resonance in question is the f_2 (1270). Saturating the dispersion relation (35) with a narrow resonance of mass m_{f_2} and using the normalization condition (27), we obtain

$$\begin{aligned}\theta_2(q^2)&\cong m_{f_2}^2/(m_{f_2}^2-q^2)\\&=1+q^2/m_{f_2}^2+\cdots,\end{aligned} \tag{36}$$

which amounts to

$$L_{12}^r\cong-\frac{F^2}{2m_{f_2}^2}=-2{,}7\cdot 10^{-3}. \tag{37}$$

The main point here is that, at low q^2, the dispersion integral (35) receives its main contribution from rather high values of s such that the slope of the tensor form factor and hence the coupling constant L_{12} are expected to be small.

The scalar form factors $\theta_0^\pi(q^2)$ and $\theta_0^K(q^2)$ were recently analyzed in detail [16] and we now make use of this work to determine the constants L_{11} and L_{13}. The analysis of [16] is based on the Omnès-Muskhelishvili integral equations for the coupled $\pi\pi$ and $K\bar{K}$ channels and chiral symmetry is used to fix the subtraction constants occuring in these equations, resulting in a parameter free representation of the scalar form factors valid up to an energy of order 1 GeV. To compare these results with the chiral representation, we consider the Taylor series

$$\theta_0(q^2)=2m^2+q^2\dot{\theta}_0+\tfrac{1}{2}q^4\ddot{\theta}_0+\cdots. \tag{38}$$

In the chiral limit, the slope $\dot{\theta}_0$ is equal to one. The deviation from this value is controlled by the low energy constants L_{11}, L_{12}, L_{13}. In particular, the asymmetry $\dot{\theta}_0^\pi-\dot{\theta}_0^K$ is given by

$$\dot{\theta}_0^\pi-\dot{\theta}_0^K=\frac{4(m_K^2-m_\pi^2)}{F^2}\times$$

$$\times\left\{6L_{11}^r+L_{12}^r-6L_{13}^r-\frac{1}{192\pi^2}\left(\ln\frac{m_\eta^2}{\mu^2}+1\right)\right\}$$
$$-\frac{m_\pi^2}{32\pi^2F^2}\ln\frac{m_\eta^2}{m_\pi^2}. \tag{39}$$

The curvature term $\ddot{\theta}_0$, on the other hand is determined by $3L_{11}+L_{12}$,

$$\ddot{\theta}_0^\pi=\frac{8}{F^2}\left\{3L_{11}^r+L_{12}^r-\frac{3}{128\pi^2}\ln\frac{m_K^2}{\mu^2}\right\}$$
$$+\frac{1}{8\pi^2F^2}\left\{\ln\frac{m_K^2}{m_\pi^2}-\frac{8}{5}+\frac{m_\pi^2}{30m_\eta^2}\right\}. \tag{40}$$

Note that the curvature tends to infinity if m_π is put to zero. This reflects the fact that in the chiral limit, the expansion of $\theta_0^\pi(q^2)$ in power of q^2 contains a nonanalytic term proportional to $q^4\ln q^2$. In the real world, the curvature is finite, but large. The numerical value obtained in the paper referred to above is (cf. (32) and (64) of [16])

$$\ddot{\theta}_0^\pi\cong 5.9\ \mathrm{GeV}^{-2}. \tag{41}$$

This value implies

$$3L_{11}^r+L_{12}^r\cong 4.3\times10^{-3}\quad(\mu=m_\eta). \tag{42}$$

With the estimate for L_{12} given in (37), this leads to

$$L_{11}^r\cong\begin{cases}2.3\times10^{-3}, & \mu=m_\eta\\ 1.4\times10^{-3}, & \mu=1\ \mathrm{GeV}.\end{cases} \tag{43}$$

According to [16], the slopes $\dot{\theta}_0^\pi$, $\dot{\theta}_0^K$ turn out to be nearly the same. In view of (39), this amounts to

$$6L_{11}^r+L_{12}^r-6L_{13}^r\cong 0.7\times10^{-3}\quad(\mu=m_\eta). \tag{44}$$

Using the values for L_{11} and L_{12} given above, we finally obtain

$$L_{13}^r\cong\begin{cases}1.7\times10^{-3}, & \mu=m_\eta\\ 0.9\times10^{-3}, & \mu=1\ \mathrm{GeV}.\end{cases} \tag{45}$$

In summary, we note that tensor meson dominance provides an estimate for L_{12} and that the available information on the scalar form factors θ_0^π, θ_0^K then suffices to also determine L_{11} and L_{13}. With these estimates of the three new couplings, the effective Lagrangian is fixed – it allows us to calculate the Green functions of the energy momentum tensor to first non-leading order in the energy expansion in a parameter free manner.

VII Dilaton model of the conformal anomaly

Finally, we establish contact with the dilation model of the conformal anomaly described in the literature [8–11], where the matrix elements of the operator θ_μ^μ are analyzed in terms of an effective scalar field. The dilaton model is built on the fact that the trace of the energy momentum tensor represents the divergence of the dilation current $\theta_{\mu\nu}x^\nu$ and hence exhibits the breaking of conformal symmetry. In particular, θ_μ^μ receives a contribution from all of the dimensionful coupling constants occuring in the Lagrangian. In addition, dimensionless couplings also contribute, unless their β-function vanishes [10]. In the case of QCD, the Lagrangian contains a dimensionless coupling constant g whose β-function is different from zero and a set of dimensionful couplings in the form of the quark mass matrix M. Accordingly, the trace of the energy momentum tensor contains two terms

$$\theta_\mu^\mu=\frac{\beta(g)}{2g}F_{\mu\nu}^aF^{\mu\nu a}+\{1+\gamma(g)\}\bar{q}Mq. \tag{46}$$

The perturbative expansion of the functions $\beta(g)$, $\gamma(g)$ starts with

$$\beta(g)=-\beta_0\frac{g^3}{16\pi^2}+\mathcal{O}(g^5),$$
$$\gamma(g)=\frac{g^2}{2\pi^2}+\mathcal{O}(g^4). \tag{47}$$

In the case of three quark flavours, $\beta_0=9$. The first term in (46), referred to as the conformal anomaly of QCD, originates in the short distance singularities and is related to the fact that, even if the quark mass matrix is set equal to zero, the theory contains a dimensionful parameter in the form of the renormalization group invariant scale Λ_{QCD}. The second term exhibits the scale breaking generated by the quark masses. Note that their contribution to the trace of the energy momentum tensor is also affected by the short distance singularities of the theory, through the term $\gamma(g)$ which originates in the renormalization of the quark masses.

As pointed out in [17], the Green functions of θ_μ^μ obey a set of sum rules which derive from the fact that the change induced by an infinitesimal dilation of all mass scales is given by the matrix element of the operator $\int d^4x\,\theta_\mu^\mu$ (one of these sum rules will be discussed in detail below). The dilaton model results if one assumes that these sum rules are saturated by the contribution from a single scalar resonance. The model thus represents a scalar analogue of the vector meson dominance model. While in the latter, the mass of the ρ-meson and the matrix element $\langle 0|V_\mu|\rho\rangle$ which specifies the coupling of this particle to the vector current are the key parameters, the dilaton model is characterized by the mass of the σ-particle and by the matrix element $\langle 0|\theta_\mu^\mu|\sigma\rangle$. The scalar meson dominance hypothesis approximates the Green functions of θ_μ^μ by the contributions due to exchange of the σ-particle. These contributions represent the tree graphs of an effective scalar field theory, characterized by a Lagrangian of the form

$$\mathscr{L}_\sigma=\tfrac{1}{2}\partial_\mu\sigma\partial^\mu\sigma+\tfrac{1}{12}R\sigma^2-V(\sigma). \tag{48}$$

The curvature term insures that the trace of the corresponding energy momentum tensor does not contain derivatives of the field and is therefore well-behaved at high energies. The requirement that the sum rules mentioned above are saturated by the pole contributions due to σ-exchange implies that the potential $V(\sigma)$ is of the form $\lambda\sigma^4 \ln(\sigma/\mu)$. Expressing the two parameters λ and μ in terms of the position of the minimum, denoted by σ_0, and of the curvature of the potential at the minimum, i.e., by the mass of the σ-particle, this becomes

$$V(\sigma)=\frac{m_\sigma^2}{4\sigma_0^2}\sigma^4\left(\ln\frac{\sigma}{\sigma_0}-\tfrac{1}{4}\right). \tag{49}$$

Scale invariance also dictates the form of the interaction with the chiral meson fields

$$\mathcal{L}=\mathcal{L}_\sigma+\mathcal{L}_U$$

$$\mathcal{L}_U=\frac{F^2}{4}\left\{\left(\frac{\sigma}{\sigma_0}\right)^2 \operatorname{Tr}(D_\mu U D^\mu U^+) +\left(\frac{\sigma}{\sigma_0}\right)^3 \operatorname{Tr}(\chi U^+ + U\chi^+)\right\}. \tag{50}$$

The trace of the energy momentum tensor of this model contains two contributions:

$$\theta_\mu^\mu=-\frac{m_\sigma^2}{4\sigma_0^2}\sigma^4-\frac{F^2}{4}\left(\frac{\sigma}{\sigma_0}\right)^3 \operatorname{Tr}(\chi U^+ + U\chi^+). \tag{51}$$

The first term stems from the logarithmic scale of the potential which reproduces the scale breaking generated by the conformal anomaly, while the second originates in the quark mass matrix. The relation (51) shows that the position of the minimum is determined by the one-particle matrix element of θ_μ^μ; in the chiral limit we have

$$\langle 0|\theta_\mu^\mu|\sigma\rangle=-m_\sigma^2\sigma_0. \tag{52}$$

The dilaton model is not renormalizable and the above effective Lagrangian is meant to be used only at tree level. If we restrict ourselves to matrix elements involving momenta which are small compared to m_σ, then the σ-field can be integrated out explicitly. As we are working at tree level, this is achieved by solving the classical equation of motion

$$\Box\sigma-\tfrac{1}{6}R\sigma+V'(\sigma)=\frac{F^2\sigma}{2\sigma_0^2}\left\{\operatorname{Tr}(D_\mu U D^\mu U^+) +\tfrac{3}{2}\frac{\sigma}{\sigma_0}\operatorname{Tr}(\chi U^+ + U\chi^+)\right\}. \tag{53}$$

At small momenta, the term $\Box\sigma$ is small compared to the mass term $m_\sigma^2\sigma$ contained in $V'(\sigma)$ and can be dropped. What remains is an algebraic equation for the field σ which is readily solved,

$$\sigma=\sigma_0+\frac{F^2}{2m_\sigma^2\sigma_0}\left\{\operatorname{Tr}(D_\mu U D^\mu U^+) +\tfrac{3}{2}\operatorname{Tr}(\chi U^+ + U\chi^+)+\frac{R\sigma_0^2}{3F^2}\right\}. \tag{54}$$

Accordingly, the effective Lagrangian reduces to

$$\mathcal{L}=\frac{F^2}{4}\operatorname{Tr}(D_\mu U D^\mu U^+ +\chi U^+ + U\chi^+) +\frac{F^2}{8\sigma_0^2}\left\{\operatorname{Tr}(D_\mu U D^\mu U^+) +\tfrac{3}{2}\operatorname{Tr}(\chi U^+ + U\chi^+)+\frac{R\sigma_0^2}{3F^2}\right\}^2. \tag{55}$$

Comparison with the general effective Lagrangian of (8) and (10) shows that the model corresponds to

$$L_1=\frac{F^4}{8\sigma_0^2 m_\sigma^2};\quad L_4=\frac{3F^4}{8\sigma_0^2 m_\sigma^2};\quad L_6=\frac{9F^4}{32\sigma_0^2 m_\sigma^2};\quad L_{11}=\frac{F^2}{12m_\sigma^2};\quad L_{13}=\frac{F^2}{8m_\sigma^2}, \tag{56}$$

all other couplings being equal to zero. In particular, the dilaton model thus leads to a theoretical prediction for the new couplings L_{11} and L_{13}, relating their value to the mass of the σ-particle. The prediction represents a scalar analogue of the tensor meson dominance formula for L_{12} given above. In fact, relations of this sort are by no means a special feature of the dilaton model. Similar estimates relating the values of all the "old" couplings $L_1,\ldots,L_{10}$ to the properties of low lying resonances are given in [18]. In this perspective, the scope of the dilaton model is rather narrow as it exclusively accounts for the exchange of a scalar flavor neutral particle and ignores the contribution generated by other particles of low mass. In the case of L_1, e.g., the leading contribution turns out to arise from vector meson exchange, while in the case of L_4, L_6, singlet and octet scalars generate comparable contributions.

VIII Scalar meson dominance

In view of these limitations of the dilaton model, we now examine the above predictions for the new couplings in a model independent manner. We wish to show that these predictions immediately follow if one assumes that the dispersion relations obeyed by the scalar form factor $\theta_0(q^2)$ and by the two-point-function $\langle 0|T\theta_{\mu\nu}\bar{q}q|0\rangle$ are saturated by the contributions from an intermediate state of mass m_σ. For simplicity, we work in the chiral limit where the scalar form factor reduces to $q^2(3\theta_1-\theta_2)/2$, such that the dispersion relation can be written in the once-subtracted form

$$\theta_0(q^2)=\frac{q^2}{\pi}\int_0^\infty\frac{ds}{s}\frac{\operatorname{Im}\theta_0(s)}{s-q^2-i\varepsilon}. \tag{57}$$

Furthermore, in the chiral limit, the slope of θ_0 at $q^2=0$ is equal to one. The imaginary part therefore obeys the low energy theorem

$$\frac{1}{\pi}\int_0^\infty\frac{ds}{s^2}\operatorname{Im}\theta_0(s)=1. \tag{58}$$

Now, we invoke the scalar meson dominance hypothesis and assume that the integrals in (57) and (58) are saturated by the contributions from the region around $s \cong m_\sigma^2$. This leads to the approximate representation

$$\theta_0(q^2) = q^2 m_\sigma^2/(m_\sigma^2 - q^2) = q^2 + q^4/m_\sigma^2 + \dots \tag{59}$$

Comparing this with the chiral representation and ignoring the loop contributions generated by two pion intermediate states, we obtain

$$3L_{11} + L_{12} = F^2/4m_\sigma^2 . \tag{60}$$

In view of (37), the meson dominance formula for L_{11} therefore reads

$$L_{11} = \frac{F^2}{12 m_\sigma^2} + \frac{F^2}{6 m_{f_2}^2}, \tag{61}$$

a result which indeed reduces to the prediction (56) of the dilaton model if the tensor contribution is dropped.

Finally, consider the two point function

$$\begin{aligned} &\mathrm{i}\int \mathrm{d}^4x \mathrm{e}^{\mathrm{i}qx} \langle 0 | T\theta_{\mu\nu}(x)\bar{q}q|0\rangle \\ &= g_{\mu\nu}\phi_0(q^2) + (g_{\mu\nu}q^2 - q_\mu q_\nu)\phi_1(q^2). \end{aligned} \tag{62}$$

On account of energy-momentum conservation, the imaginary part of $\phi_0(q^2)$ vanishes, such that this term is a polynominal and can be removed with a suitable redefinition of the time-ordered product. In fact, the Fourier integral is ambiguous, because the integrand is singular at $x=0$. The structure of the singularity is controlled by the operator product expansion. We again consider the chiral limit where chirality conservation implies that the coefficient of the unit operator vanishes, such that the leading singularity stems from the operator $\bar{q}q$,

$$\theta_{\mu\nu}(x)\bar{q}q = c_{\mu\nu}(x)\bar{q}q + \dots . \tag{63}$$

Since $\theta_{\mu\nu}$ carries canonical dimension, the leading contribution in the short distance expansion of the coefficient $c_{\mu\nu}$ is the same as for free quarks,

$$\begin{aligned} c_{\mu\nu}(x) = &(\partial_\mu\partial_\nu - g_{\mu\nu}\Box) \\ &\times \frac{1}{4\pi^2 x^2}\{1 + 0(1/\log x)\}. \end{aligned} \tag{64}$$

The Fourier integral occuring in (62) is therefore only logarithmically divergent - the two point function is unique up to a constant. In particular, the transverse part $\phi_1(q^2)$ is free of ambiguities; it tends to zero for $q \to \infty$ and therefore satisfies an unsubtracted dispersion relation,

$$\phi_1(q^2) = \frac{1}{\pi}\int_0^\infty \frac{\mathrm{d}s\,\mathrm{Im}\,\phi_1(s)}{s - q^2 - \mathrm{i}\varepsilon}. \tag{65}$$

Furthermore, since the behaviour at $q \to \infty$ is determined by the leading short distance singularity, the imaginary part obeys the sum rule

$$\frac{1}{\pi}\int_0^\infty \mathrm{d}s\,\mathrm{Im}\,\phi_1(s) = -\langle 0|\bar{q}q|0\rangle . \tag{66}$$

Assuming that the integrals (65) and (66) are dominated by the contributions from the region $s \cong m_\sigma^2$, we obtain

$$\phi_1(q^2) = -\langle 0|\bar{q}q|0\rangle/(m_\sigma^2 - q^2). \tag{67}$$

In particular, scalar meson dominance predicts that the value of ϕ_1 at $q^2=0$ is approximately given by $-\langle 0|\bar{q}q|0\rangle/m_\sigma^2$. This is to be compared with the systematic expansion in powers of the momentum provided by chiral perturbation theory. The effective Lagrangian given in Sect. II allows us to calculate the two-point function we are considering here up to and including terms of order q^2. In this framework, the leading term is a contact contribution to ϕ_0, which - as discussed above - is a matter of convention. The function $\phi_1(q^2)$ starts showing up at order q^2 where two graphs contribute: a tree graph involving the coupling L_{13} and a one loop graph associated with two pion intermediate states. Ignoring the latter, the effective Lagrangian predicts

$$\phi_1(0) = -\frac{8L_{13}}{F^2}\langle 0|\bar{q}q|0\rangle . \tag{68}$$

The scalar meson dominance formula (67) therefore implies the estimate

$$L_{13} = \frac{F^2}{8 m_\sigma^2}. \tag{69}$$

More generally, the above analysis shows that the value of L_{13} can be expressed in terms of the function $\mathrm{Im}\,\phi_1(s)$ which is sensitive only to flavor neutral states of spin zero - this is why, in this case, the meson dominance formula agrees with the prediction of the dilaton model.

Note that the two point functions involving the trace of the energy momentum tensor is given by $4\phi_0 + 3q^2\phi_1$ and therefore in general tends to a constant as $q^2 \to \infty$. There is however one particular definition of the time-ordered product for which this constant vanishes, viz.

$$\phi_0 = -\tfrac{3}{4}\langle 0|\bar{q}q|0\rangle . \tag{70}$$

With this choice, the energy momentum tensor is not transverse, but the trace is well-behaved at high energies and obeys the sum rule of [17],

$$\mathrm{i}\int \mathrm{d}x \langle 0|T\theta_\mu^\mu(x)\bar{q}q|0\rangle = -3\langle 0|\bar{q}q|0\rangle . \tag{71}$$

Let us now confront the meson dominance formula for the couplings L_{11} and L_{13} with the results of the dispersive analysis given in Sect. VI. There are two problems with these formulas. First, the structure of the $I=J=0$ channel is rather complex, particularly around $K\bar{K}$ threshold - it is not clear whether it makes sense to replace this structure by a single narrow peak and, if so, it is not clear what mass to choose. Second, the two pion continuum generates substantial contributions even at low energies, because the $I=J=0$ phase shift rapidly grows with energy, reaching values of order 40^0 already at $\sqrt{s}=500$ MeV: the scalar resonance which is supposed to dominate the dispersion integrals, sits on top of a sizeable background. This problem manifests itself in the fact that the values of the coupling constants L_{11}^r, L_{13}^r depend on the scale μ at which the one loop contributions are renormalized. Numerically, using $m_\sigma=\mu=1$ GeV, the meson dominance formulae (61) and (69) predict $L_{11}=1.6\times 10^{-3}$ and $L_{13}=1.1\times 10^{-3}$, to be compared with the dispersive values $L_{11}=1.4\times 10^{-3}$ and $L_{13}=0.9\times 10^{-3}$. This shows that the result of the dispersive analysis is reasonable, both in the size and in magnitude. It is clear, however, that the comparison is sensitive to the values taken for m_σ and μ, a notorious problem with the scalar channel where there are several candidates to play the role of the σ-particle and where, as witnessed by the sensitivity of the couplings to the value of μ, the $\pi\pi$ continuum generates an important background. The problem does not occur with vector or tensor meson dominance where the mass of the resonance is unmistakable and where the background is neglibible.

IX Summary and conclusion

1. We have extended the effective Lagrangian of chiral perturbation theory to incorporate matrix elements of the energy momentum tensor. At one loop order, the extension requires three new coupling constants.
2. As an application of this machinery we have evaluated the energy expansion of the form factors $\langle\pi|\theta_{\mu\nu}|\pi\rangle$, $\langle K|\theta_{\mu\nu}|K\rangle$ and $\langle\eta|\theta_{\mu\nu}|\eta\rangle$ to order E^4. The calculation illustrates the physical significance of the three new couplings: one of the three is related to the slope of the scalar form factor $\langle\pi|\theta_\mu^\mu|\pi\rangle$, the second to the slope of the spin-two component and the third determines the flavor asymmetries generated by the quark masses.
3. Two of the three couplings are determined by comparing the chiral representation with a dispersive analysis of the scalar form factors $\langle\pi|\theta_\mu^\mu|\pi\rangle$ and $\langle K|\theta_\mu^\mu|K\rangle$. The third is estimated on the basis of the hypothesis that the slope of the spin two form factors is dominated by the contribution from f_2-exchange.
4. The results found for the new coupling constants are compared with the predictions of the dilaton model and the significance of this model is discussed in the more general context of scalar meson dominance. The analysis shows that these model predictions are rather soft, because the scalar channel is not well represented by a single narrow resonance. On a semi-quantitative level, the model expectations do match our dispersive results.

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