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UNITED STATES DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
Washington, D. C.
H. H. Bennett, Chief

ENERGY-BALANCE IN STREAM-FLOWS CARRYING SUSPENDED LOAD

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PASADENA
PUBLICATION NO. 13

Reprinted from the Transactions of the American
Geophysical Union, Nineteenth Annual Meeting,
April 27-30,1938, Washington,D.C., pp.501-505.

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Problem.--The mechanism by which a flowing fluid transports solid material in suspension has received a great deal of thought. Many avenues of approach have been explored in attempts to gain more insight into the phenomenon. One of these which has been employed with the object of establishing the basic criteria of transportation in suspension has been the study of the energy-balance of the flow. The Transactions of the American Geophysical Union contain important discussions that fall in this category. In 1933, W. W. Rubey (Trans. 1933, p. 497) presented a paper on "Equilibrium-conditions in debris-laden streams" to the Section of Hydrology. This article proposes a general expression of stream-equilibrium in the form of an energy-equation. In this equation the loss in potential-energy of the flowing mixture plus the decrease in kinetic energy of flow is equated to the energy consumed in friction plus the energy consumed in supporting the debris. One of the significant conclusions reached by Rubey on the basis of this analysis, together with pertinent field-data, is that the energy consumed in supporting the debris is normally a very small fraction of the total energy in the flow.

In 1935 another paper in this category was presented to the Section of Hydrology by Howard L. Cook (Trans. 1935, p. 456) entitled "Outline of the energetics of stream-transportation of solids." In this presentation Cook outlines six energy-demands on the flow and concludes that "until the various energy-demands have been fairly well investigated there is little possibility of using the energy-equations to predict the behavior of artificial channels, or to apply this line of reasoning to investigations in the morphology of streams."

Purpose of present article.--Both of these papers emphasize the necessity of examining in detail the mechanism of debris-transportation before any useful conclusions in this field can be drawn from the application of the general energy-equation. The purpose of this article, therefore, is to present some further considerations involved in the transportation of that portion of debris which is carried in suspension. The conclusions reached are general in character and are put forward in the form of working hypotheses, with the hope that they may serve as the basis of additional experimental study of the phenomenon.

Viewpoint.--The phrase "energy-balance" is a very loose one and can be interpreted in several ways. The general usage in hydraulics is to apply it to the mechanical energy only. Transformations from mechanical energy to heat are considered "losses" instead of merely manifestations of another form of energy. All such equations simply define special cases of the general principle of conservation of energy, and as such are very useful. Indeed, since energy is conserved it is perfectly proper to set up equations to account for the disposition of any particular subdivision of it that may be of interest. As has been indicated, one of the normal approaches to the study of the energy-considerations in the transportation of suspended load is to attempt to set up the energy-balance of the flow as a whole. However, it is also helpful to investigate simply the energy-exchange between the fluid and the suspended particles. This latter viewpoint leads to the following hypothesis concerning the capacity of a given stream to transport material of a specified kind.

Statement of hypothesis.--Briefly stated, this hypothesis begins with the postulate that a particle of sediment in suspension in a flowing stream of fluid simultaneously adds and subtracts energy from the stream through two independent processes. It then reasons that if the amount of energy added is less than the amount subtracted, the net result is that the particle is a burden on the energy of the stream and that, therefore, there must be a definite limit to the number of such particles that can be kept in suspension in a unit-volume of the fluid. On the other hand, if the amount of energy added by the particle is greater than the amount subtracted, then the energy of the stream is increased by its presence. Under the latter conditions it would appear that there would be no limit to the number of such particles that could be kept in suspension and transported by the unit-volume of the fluid. In other words, if the suspended sediment is a drain on the energy of the flow then there will be a definite load-carrying capacity of the stream for that particular size of material, whereas, if the sediment adds to the net energy of the flow then the only limit to the concentration of such sediment that can be transported will be that imposed by the transition from fluid to plastic flow.

Mechanism of energy-contribution from suspended load.--Now consider the mechanisms through which the suspended particles and the fluid exchange energy. The first concerns the energy added to the fluid by the particle. If, over a given length of channel, the concentration of sediment of a given particle-size remains constant, then statistically it can be considered that each particle moves parallel to the channel bed. Examine one such particle suspended in a stream flowing at a velocity v in a channel of slope s as shown in Figure 1. In time t the fluid, and therefore the particle, moves downstream a distance $l = vt$. The vertical component of this movement is $l \sin \theta = vt \sin\theta \cong vts$ (since $\sin \theta \cong \tan \theta$). If ρ_f is the density of the fluid, ρ_p is the density of the particle, and V its volume, then the apparent weight of the immersed particle is $(\rho_p - \rho_f)gV$. Thus the energy given up by the particle to the stream in time t in excess of the energy that would have been given by the displaced fluid is

$$\Delta e = vts\ (\rho_p - \rho_f)gV \qquad (1)$$

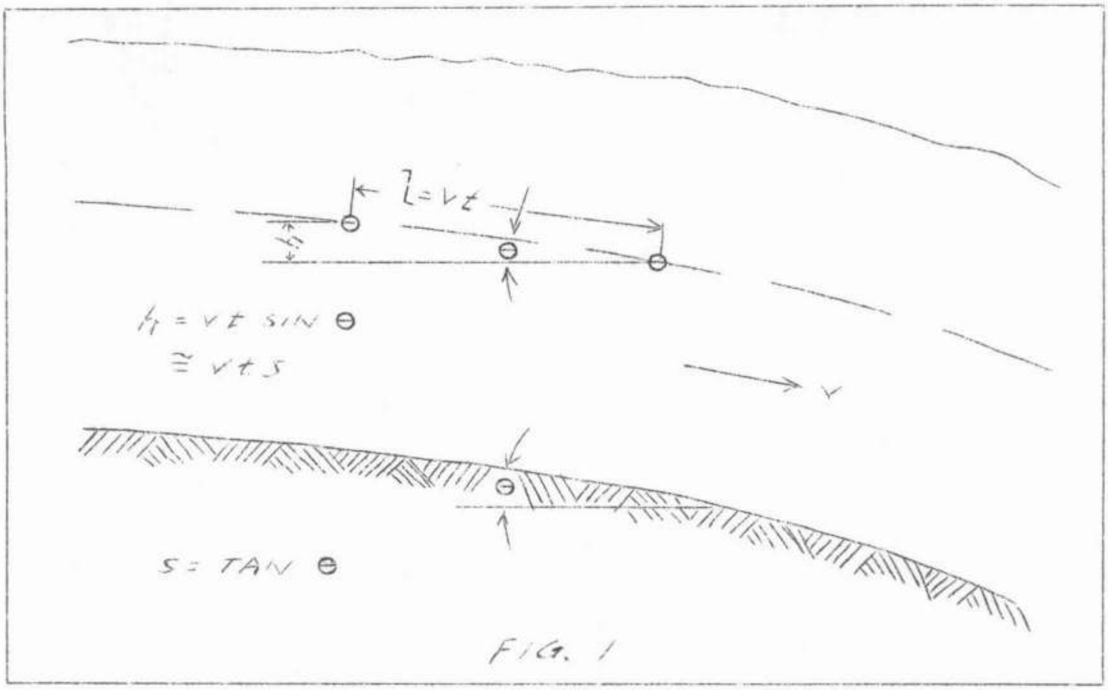

FIG. 1

If there are n such articles per unit-volume of the mixture then their total weight will be $n \rho_p gV$. This may be called the concentration c. The apparent weight, however, will be less, being equal to $c(\rho_p - \rho_f)/\rho_p$. The energy-contribution of such a suspension, per unit-time, per unit-volume of mixture will be

$$E = vsc\ (\rho_p - \rho_f)/\rho_p \tag{2}$$

Mechanism of energy-utilization by suspended load.--The second mechanism is that by which the particle subtracts energy from the fluid. Consider a particle falling freely in a body of fluid which is at rest. It will quickly attain an equilibrium velocity v_p which represents the force-balance between the skin-friction and the apparent weight of the particle. The particle is obviously dissipating to the fluid in time t an amount of energy

$$\Delta e_1 = v_p t \ (\rho_p - \rho_f) \ gV \qquad (3)$$

If the fluid contains a concentration c of such particles, then per unit-time and per unit-volume of the mixture, these particles dissipate energy at the rate

$$E_1 = v_p c \ (\rho_p - \rho_f) \ / \rho_p \qquad (4)$$

However, if the particles are suspended in a moving stream of fluid, and therefore remain statistically in the same location relative to the fluid, then, in order to produce the forces on the particles needed to keep them in suspension, the fluid must supply the mechanical energy dissipated in heat in the skin-friction and the rate of supply must be E_1.

Energy-balance due to presence of suspended load.--The net unit-energy contributed to the stream by the suspension is therefore

$$E - E_1 = (vs - v_p) \ c \ (\rho_p - \rho_f) \ / \rho_p \qquad (5)$$

According to our hypothesis, if this value is positive then there is no limit to the concentration of such material that can be transported in suspension, but if it is negative then apparently there must be a definite limit to the concentration that can be supported.

Alternate formulation of hypothesis.--An examination of equation (5) shows that the energy will always be positive as long as vs is greater than v_p. This leads to the following alternate method of stating the original hypothesis: The concentration of sediment of a given size which can be transported by a given

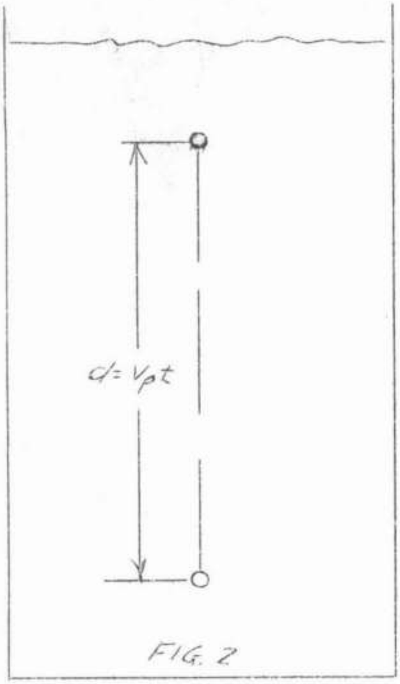

FIG. 2

stream is unlimited, except by the fluidity, if the vertical component of the stream-velocity is greater than the velocity of free fall of the same sediment at the same concentration in the same fluid when the fluid is at rest.

This alternate wording of the hypothesis makes allowance for the experimental fact that the fall velocity of particles in a fluid at rest actually varies somewhat with the particle-concentration, decreasing as the concentration increases. This effect is commonly referred to as "hindered settling." A physical explanation of this variation in fall velocity can be obtained if the fall of the particle in the stationary fluid be examined more closely. It is generally assumed that in relative motion between a fluid and a solid surface, if the fluid wets the surface, a layer adheres to the solid and moves with it without slip. The difference in velocities of the surface and the fluid causes shear in the fluid layers. This shear is high near the surface, and decreases in intensity with the distance from the surface. Actually, therefore, no energy is dissipated in heat on the surface itself, but such dissipation takes place all through the fluid. Therefore, in the case of falling particle the energy is not actually dissipated on its surface. Instead, the particle drags a layer of fluid down with it. This in turn drags more fluid but now not without slippage. Thus the particle pulls down with it a large amount of fluid compared to its own volume, but at an average velocity lower than that of its own fall. This movement of fluid downward in the immediate vicinity of the particle necessitates the flow of an equal amount upward, and thus a circulation is set up in the fluid. This velocity-energy is soon dissipated as heat through the viscous friction in the fluid. However, in case other similar particles are falling in the fluid at the same time, their fall will be slowed up by this upward current, or in effect, part of the fall-energy of the first particle is used to help support the second one. Obviously, the greater the concentration of particles the more of this velocity-energy can be used for support before it is transformed into heat, and thus the lower is the observed fall-velocity. It is evident, therefore, that in equations (3), (4), and (5) v_f should represent the fall-velocity at the same concentration as that existing in the flowing stream.

Factors determining the normal load-capacity of a flow.--If the vertical component of the stream-velocity is less than the fall-velocity of the particle, the question naturally arises as to what factors determine the concentration of this sediment that can be carried by the stream, that is, the maximum load-carrying capacity for material of this particular particle-size. In an attempt to indicate the possible direction for further study on this problem, another simple hypothesis is proposed as follows: A given stream is fully loaded with particles of a given size when the vertical-velocity components of the turbulent motion have been reduced in magnitude until they are equal to the free fall-velocity of particles of this size and concentration in fluid at rest. The reasoning back of this statement is very simple. The velocity of fall of the particle in the container in Figure 2 will be the same with respect to the container irrespective of whether this container is at rest or is moving at a constant velocity in a straight line in any given direction. This is a direct consequence of Newton's laws of motion if the fluid in the container is at rest with respect to the container. Now, if under a special set of conditions the particle was observed to have no velocity of fall with respect to the container, it would be concluded that there must be an upward fluid velocity in the container with respect to the particle of a magnitude equal to the normal velocity of fall of the particle, since the conditions for force-equilibrium require a relative velocity between the fluid and the particle equal to this free fall-velocity. Lower upward velocities would reduce the settling rate of the particle, but eventually it would reach the bottom of the container and once there would remain there. Thus, if particles in a stream flowing in a steady uniform state remain statistically in suspension, it must mean that vertical fluid velocities exist in the stream with magnitudes equal to or greater than the fall-velocity of the particles. If only lower vertical velocities existed the particles would soon settle out and stay out of suspension. Higher velocities are permissible, however, as vertical components of velocities can exist only in a balanced state, that is, on the average there must be the same quantity of fluid flowing downward as upward through a given horizontal cross section. In the normal stream-flow these vertical currents are the vertical components of the turbulent motion. Thus the turbulence of the fluid is responsible for the suspension of the particles in the flow. However, each little particle acts as an energy-dissipator with a net effect as indicated by equation (5). This dissipation must reduce the magnitude of the vertical velocity of the turbulent fluctuations, and when the velocity is reduced to the fall-velocity of particles there would be no apparent way of supporting any higher concentration in suspension. Indeed, it is very probable that this criterion does not specify a sufficiently high residual vertical-velocity component in the turbulent fluctuations of the fully loaded flow as it is likely that the concentration of the given sized particles would start to decrease before the vertical-velocity components were reduced to the particle fall-velocities.

The above picture naturally represents an over-simplication of the situation, since, even at one given location in the flow, the vertical-velocity components of the turbulent motion vary greatly in magnitude about their statistical mean value. Also, it is well known that these turbulent velocity-fluctuations are not constant in average magnitude throughout the cross section of the flow, but vary roughly with the rate of shear, being greatest near the sides and bottom, and decreasing with the distance away from these boundaries. The interrelation between this velocity-distribution and the distribution of sediment-concentration has been pointed out by von Karman, O'Brien (Trans. 1933, p. 487) and others. (In fact, the criterion of capacity here proposed is contained implicitly in their results, although it was not pointed out directly.) This inter-relationship, of course, makes the present problem more complicated since the total carrying capacity of a stream would be the sum of the capacities of the different zones, and due to the damping effect of the energy-dissipation discussed above, the relative distribution of sediment-concentration would be expected to vary with the total amount of load carried. However, it is much beyond the scope of this article to attempt to calculate any actual carrying capacities.

Effect of other material in suspension on load-carrying capacity of stream for a given particle-size.--One or two secondary conclusions may be of interest in passing. For example, on the basis of the above hypothesis the load-carrying capacity of a given stream for a material of given size would not be independent of the loads of material of other size being carried. According to the energy-balance, all material whose fall-velocities were smaller than vs would increase the capacity of the stream to carry the larger particles, while all material with fall-velocities larger than vs would reduce the capacity of the stream to carry any of the particle-sizes coming within this latter class.

Transportation of large fragments.--The behavior of large fragments in flows carrying high concentrations of sediments is also of interest. It is well known that the fall-velocity of a particle increases with increasing density-difference but decreases as the effective viscosity of the fluid increases. The effective density-difference between a large fragment of solid material and the fluid would be reduced materially by the presence of the fine sediment in suspension, since such sediment acts to increase the apparent density of the fluid with respect to the fragment. At the same time, the presence of the sediment also acts to increase somewhat the effective viscosity of the fluid. Therefore, the fall-velocity of the fragment may be greatly decreased and thus it may be carried in suspension in this case, whereas in clear fluid flowing at the same velocity it might settle out immediately. Thus the fact that large rocks are transported in flood-times by flows carrying high silt-contents may not indicate the high velocities of flow that preliminary considerations might first suggest.

Effect of energy utilized in suspension upon energy-balance of stream as whole.--In conclusion, it may be worth while to examine briefly the role that the energy utilized in suspension of the debris-load plays on the energy-balance as a whole. O'Brien, in the paper previously referred to, pointed out some of the important aspects of the role, but it is felt that they are not sufficiently recognized by all. Neglecting velocity-changes, the entire amount of energy supplied to the flow must come from the decrease of potential-energy of the flowing mixtures. In accounting for the utilization of this energy-supply it is very important to differentiate between transitional and end processes in order to avoid having the same quantity of energy appear more than once in the general energy-equations. For example, the concept of the frictional resistance to flow at the channel-boundaries implies a dissipation of mechanical energy into heat at these surfaces. Actually, little if any such dissipation takes place. Some transformation of mechanical energy to heat occurs in the laminar boundary-layer, but most of the energy represented by the product of the forces at the boundaries and the mean velocity of flow is transmitted through it to the main body of the fluid, where it appears as "turbulence." However, the energy of turbulence is still mechanical energy and thus turbulence represents only a state through which the original potential-energy passes in its transformation into heat. The final mechanism of this transformation must be viscous shear in all cases, as this is the only process available through which an incompressible fluid can convert mechanical energy into heat. In a clear flow the energy in the turbulence must be dissipated through the viscous shear in the eddies. The addition of suspended load supplies a secondary method producing viscous shear within the turbulence units. It does not necessarily increase the amount of energy dissipated, but may simply change the distribution or the mechanism of the dissipation. Thus it is evident that in an energy-balance which attempts to account for the dissipation of the potential-energy, the energy in the turbulent motions actually does not appear at all, being accounted for by the energy dissipated in viscous shear either in the eddies within the turbulence units or adjacent to the particles of suspended load. Likewise, a term for bed-friction would be redundant, as would all similar terms that represent simply stages through which the mechanical energy passes before it is finally transferred into heat. If it is desired to trace the entire path of the energy, then it must be done through a series of equations, each of which represents a stage in the transformation-process.

Application.--It must be emphasized that the problems of transportation of suspended load are too complex ever to permit of being fitted in their entirety into the simple hypotheses presented here. However, it is felt that the general viewpoint on which they are based must be taken into account in any rational analysis of the problem of suspended load. It is hoped that in the near future more and better experimental data will be made available with which to test these and other allied hypotheses and that finally it will become possible to predict quantitatively the behavior of debris-laden flows of any fluid, either liquid or gas, carrying in suspension in its movement any material of different density than itself.