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Energy Consumption Analysis of a Duty Cycle Wireless Sensor Network Model

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ABSTRACT A novel model is developed and investigated for a duty-cycle wireless sensor network (WSN), where each sensor node can stay at one of four statuses (i.e., full-active phase, two semi-active phases each with different functions, and a sleep phase) to make the system more efficient. The explicit result of the joint probability for data packet numbers and the sensor phases is achieved by suitably constructing a multi-dimensional Markov process. Furthermore, various energy consumption measures, such as energy consumption per unit time in each of the three energy consumptions and energy consumption switching from one phase to another, are obtained. System performance measures, such as the average delay time of a data packet in the system, throughput of the system, and the probability of the sensor node staying at any of the phases, are also achieved. The numerical analyses are provided to validate the model and the analytic results. The proposed model and the analysis method are expected to be applied to the design and analysis of WSN models with various phases of the sensor.

INDEX TERMS Energy consumption, performance analysis, system analysis and design, wireless sensor networks.

I. INTRODUCTION

Wireless Sensor Networks (WSNs) have become more and more attractive to researchers due to their great potential in civil and military applications [1]. They are composed of a large number of sensing nodes equipped with limited power and radio communication capabilities. Once deployed, the sensing nodes are used for monitoring, sensing, and forwarding event occurrences in the habitat of interest to the sink node for further processing. Due to their ability to support diverse applications and the availability of low-cost end devices (sensors), WSNs have attracted much interest in both academia and industry. The diversity of use is enhanced by technological advancement and subsequent explosion of inexpensive wireless communication, computation and sensing devices [2]. The application areas include seismic, acoustic, chemical, and physiological sensing that enables various applications such as battlefield surveillance, home security, habitat monitoring, forecast systems, smart agriculture, health monitoring, industrial systems, traffic control, and animal tracking, among many others. Based on the application scenario, the number of sensor nodes can range from a few to hundreds or even millions, and the density of nodes can range from a few to a few hundred in a region. These features of WSNs have made them advantages in modern network information systems, the complexity of which has generally reached a level that puts them beyond our ability to deploy, manage, and keep functioning correctly through traditional techniques [3]. A paradigm shift is required in confronting the complexity explosion problem to enable building robust and adaptive network information systems that are self-organizing and self-repairing. In this context, the study of complex systems seems promising for the future. WSNs, as complex systems, consist of spatially distributed autonomous nodes that provide infrastructure for several common applications [4] such as structural health monitoring [5], habitat monitoring [6], and intelligent transportation systems [7].

Energy resources are especially scarce in battery-powered sensor networks. To achieve a satisfactory length of network life, the problem of energy efficiency must be tackled on all levels of the wireless sensor network. Important principles for designing WSN protocols include executing in-network data processing, creating efficient adaptation algorithms, and providing quality of service (QoS) while maximizing net-

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work resource utilization. Hence, energy consumption is a crucial issue and optimizing power is highly important [8]. Two major techniques exist for maximizing the sensor network life span: the use of energy-efficient routing and the introduction of sleep/active modes for sensors [9]. Carle and Simplot-Ryl [10] present a good survey on energy-efficient area monitoring for sensor networks. The authors observe that the best method for conserving energy is to turn off as many sensors as possible, while still keeping the system functioning. A typical network configuration consists of sensors working unattended and transmitting their observation values to a processing or control center, the so-called sink node, which serves as a user interface [11]. If these sensors are managed by the sink node directly, control overhead, management delay, and management complexity may make such a network less responsive and less energy efficient. Due to the limited transmission range, data generated from sensors that are far away from the sink must be relayed through a number of intermediate clusters (i.e., multi-hop communications). Most applications for WSNs involve battery-powered nodes with limited energy. Their batteries may not be convenient to recharge or replace. When a node exhausts its energy, it can no longer sense or relay data. Thus, current research on sensor networks has mostly focused on protocols with energy-efficient mechanisms [12]–[15].

Over the years, several approaches and models have been proposed for energy evaluation and performance of WSNs [16], [17]. Chiasserini and Garetto [16] classify energy costs in relation to the state of the system and routing dynamics. Similar approaches have been used by Zhang and Li [17], [20], Jurdak *et al.* [18], and Odey and Li [19]. In addition, Jurdak *et al.* [18] present energy costs for listening, sensing, sleeping, and switching between the operation states. Studies in both [17] and [20], have extended the model used in [16]. Jones *et al.* [13] study wireless sensor networks that operate in low duty cycles, measured by the percentage of time a sensor is on or active. Musiani *et al.* [5] reiterate the significance of including energy costs for being in sleep and idle states. Agarwal *et al.* [21] present the development of a stochastic model to capture the expected energy consumption of a schedule-driven sensor node per cycle of operation by utilizing renewal theory, random stopping criteria, and Wald's inequality. Recently, Xu *et al.* [22] have used the average cycle analysis method and Newton solution of nonlinear equations to build a mathematical model, and the system throughput and energy consumption can be obtained. A new power minimization scheme, by which the average power consumption of individual nodes in the sensor network is reduced in accordance with the queue threshold during its period of active time, is investigated in [23]. Some properties of coverage over time as functions of individual sensor on/off schedules under a random duty-cycling assumption is studied in [24]. Muhammad *et al.* [25] consider the target-coverage and area-coverage applications of WSNs to analyze the usefulness of a proposed random scheduling algorithm based on the sub-region coverage algorithm. Ever [26] considers analytical modeling approaches of homogeneous clustered WSNs with a centrally located cluster head (CH) responsible for coordinating cluster communication. Unlike most of the existing studies, the performance and energy efficiency of the cluster head are considered together. Furthermore, potential failure of WSN nodes is considered and performance and availability models are integrated, as is the potential repair or replacement of sensor nodes. Various recent publications continue to make contributions to the WSN with energy considerations. Therefore, the investigation of energy-efficient WSNs together with uncertainty features becomes a crucial issue and shows promise for the future of WSN development. In WSNs, dynamic power management (DPM) is an effective tool in reducing system power consumption without significantly degrading performance. The basic idea is to shut down devices when not needed and wake them up when necessary [27]. It offers dynamic coil wiring configuration switching, using proprietary algorithms to optimize power electronics and efficiency at all speeds [28]. Whereas there are different approaches to a DPM strategy, they can be categorized in one of the following three approaches, dynamic operation nodes, dynamic scaling, and energy harvesting [29], [30]. In this paper, we continue the research on this investigation by concentrating on novel model development and identification of valuable system performance energy consumption; we expect our research to improve existing WSN development in both theory and application. Our proposed model targets on a specific power saving configuration problem among the approaches of above dynamic operation nodes. The paper [31] is one of the excellent survey paper in the routing protocol for duty-cycle WSNs. References [32] and [33] are related to this paper and also in our next research alone this subjects.

The major contributions of this paper includes: (1) A novel model for sleep/active is proposed for a wireless sensor network where each node can stay at one of four statuses, full-active phase, two different semi-active phases each with different functions, and a sleep phase to make the system more efficient; (2) The explicit result of the joint probability with a number of n ($n \geq 0$) data packets and the sensor in any of the four statuses is achieved by suitably constructing a multi-dimensional Markov process; (3) Various energy consumptions measures, such as energy consumption from one phase to another and energy consumption in each of the three phases, are obtained. System performance measures, such as the average delay time of a data packet, throughput of the system, and the probability when the sensor node is any of the phases, are achieved; (4) Numerical analyses are provided with clear explanations to validate the model and the analytic results.

The rest of this paper is structured as follows. Section II outlines a WSN model in this paper; Section III is devoted to analysis of this model by constructing a multi-dimensional Markov process and to an explicit solution of the model; Section IV concentrate on the derivation of various energy consumptions and system performance measures.

Section V provides the corresponding numerical analyses and validation.

II. DESCRIPTION OF THE MODEL

We considered a new WSN model in which each sensor node can alternatively stay in two major modes (i.e., active and sleep). The active mode consists of three phases, one of which is called the full-active phase (denoted by phase R); the second phase is called the semi-active N-phase (denoted by phase N) and the third phase is called the semi-active T-phase (denoted by phase T). The sleep mode consists of only one phase and, because of this, we alternatively say that the sensor is either in sleep mode S or in sleep phase S. The basic function and the relationships among these phases are as follows. In the full-active phase R, the sensor can generate data, receive data from other sensors, process data, and possibly change the phase to the semi-active phase N or sleep phase S. In the semi-active phase N, the sensor cannot generate data, or receive data from other sensors, but it mainly processes the data packets that are already waiting for transmission and can possibly change the phase to sleep phase S. In the semi-active phase T, the sensor can generate data, but it cannot receive data from other sensors, or process data packets; it can also change the phase to the full-active phase R. In the sleep phase S, the sensor may generate data, but it cannot receive data from other sensors, or process the data packets that arrive during the sleep phase; however, it can change the phase to full-active phase R. Figure 1 provides a description of the transition relationships between these phases and modes.

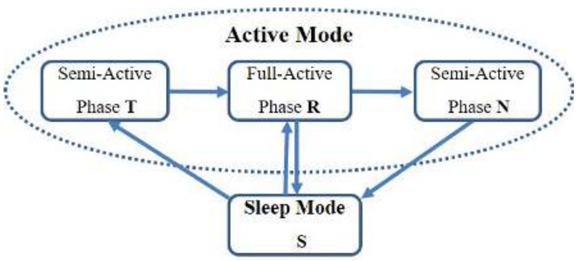

FIGURE 1. Transitions among the modes and phases.

Other assumptions and notations are given as below: When the sensor is in the full-active phase, we will need to consider two situations. One of which is that there are more than one data needing to be processed. In this situation, the sensor may process data packet with an exponential random time in a rate μ, may generate data packets in according with a Poisson process with rate λ, and accept packets coming from other sensors following a Poisson with rate $\lambda_{E,}$. In this situation, the sensor may also possibly change it's phase to semi-active phase N with an exponential distribution in rate α. Another situation in Phase R is that that there are no one data needing to be processed. This situation sometime is also called the sensor is in an idle status [11], [17], [20]. In this situation, the sensor may generate data packets in according with a Poisson process in rate λ, and accept packets coming from other sensors following a Poisson with rate $\lambda_{E,}$. The sensor may also change it's phase to sleep phase S with an exponential distribution with rate α_0. After the sensor changes it phase for phase R to the semi-active phase N, the sensor may only process the data packet with an exponential random time with a mean of $1/\mu$, will not generate data packets, will not receive packets coming from other sensors. After the sensor processed all data packet in this phase, the sensor will change its phase to the sleep phase S immediately. Whenever the sensor comes to its sleep mode or phase S, it will take a break before changing its phase to the full-active phase in its part of idle situation. This duration in the sleep phase S could be a bit longer and assumed to be an exponential distributed time with a rate β_0. To make the sensor more efficient, we may also assume that the sensor may detect the data and assume that it may detect data packets in according with a Poisson process with rate λ. Whenever the data are detected, the sensor may then change it's a semi-active phase T. The duration of a sensor in the semi-active phase T is distributed exponentially with a mean of $1/\beta$. The value of β is always much bigger than the value of β_0. This means that the sensor will take a longer time break whenever there is no data detected during the sleep mode. However, when a data is detected by the sensor, the sensor may need to wake up in processing the detected data. Before the sensor is waken up in semi-active phase T, the sensor may continuously detect the data packets in according with a Poisson process with rate λ. However, the sensor in phase T will not receive packets replayed from other sensors and will not process data packet. After the end of the phase T, the sensor will change its phase to the full-active phase R.

In general, power consumption models of the radio in embedded devices must take both transceiver and start-up power consumption into account, and there must be an accurate model of the amplifier. In general, the energy consumed per bit transmitted is given [1], [12] in terms of the energy per bit required by the electronic components of the transmitter (including the cost of startup energy), the electronic components of the receiver, the energy consumption of the transmitting amplifier to send one bit over one unit distance, and the path loss factor. In this paper, we consider the energy consumption in terms of number of packets transmitted, the sensor mode status, and the switches from one mode to another. With this description of the model, in next section, we will conduct the analysis of this model by constructing a two dimensional Markov process and to the explicit solution of the model.

III. DISTRIBUTION OF THE DATA PACKETS

In this section, we derive the formula of the joint steady-state probability of the node when there are i ($i \geq 0$) data packets (including the one being processed and the others that are waiting) and the sensor is in any one of the three phases. Here, we denote by:

- $P(R_i)$ as the joint steady-state probability of the node when there are i ($i \geq 0$) data packets in the referenced sensor node and the node is currently in the full-active phase R;
- $P(N_i)$ as the joint steady-state probability of the node when there are i ($i > 0$) data packets in the referenced sensor node and the node is currently in the semi-active phase N; and
- $P(T_i)$ as the joint steady-state probability of the node when there are i ($i > 0$) data packets in the referenced sensor node and the node is currently in the semi-active phase T.
- P(S) as the steady-state probability when the senor is in sleep phase. For the notation convenience, we will use a new notation $P(T_0)$ to represent P(S).
- Some other notations given by the given parameters:

$$\bar{\lambda} = \lambda + \lambda_E + \mu + \alpha; \quad r_{11} = \frac{\lambda}{\lambda + \beta};$$

$$r_{22} = \frac{1}{2\mu}\left(\bar{\lambda} - \sqrt{\bar{\lambda}^2 - 4\mu(\lambda + \lambda_E)}\right);$$

$$r_{12} = \frac{\beta r_{11}}{\bar{\lambda} - \mu(r_{11} + r_{22})}; \quad r_{13} = \frac{\alpha r_{12}}{\mu(1 - r_{11})(1 - r_{22})};$$

$$r_{23} = \frac{\alpha r_{22}}{\mu(1 - r_{22})}; \quad R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ 0 & r_{22} & r_{23} \\ 0 & 0 & 0 \end{bmatrix};$$

$$k_1 = \frac{r_{12}(1 + r_{23}) + (1 - r_{22})(1 + r_{13})}{(1 - r_{11})(1 - r_{22})}; \quad k_2 = \frac{1 + r_{23}}{(1 - r_{22})}.$$

Our major contribution in this Section is the following explicit result.

Theorem 1: Let

$$\pi_0 = (P(S), P(R_0)), \quad \text{and}$$
$$\pi_i = (P(T_i), P(R_i), P(N_i)), \quad (i \geq 1),$$

then for any $i \geq 1$, we have:

$$\pi_i = \pi_1 R^{i-1}, \tag{1}$$

and (π_0, π_1) is given by (2), as shown in the top of next page. in which e^τ is a unit row vector (0,0,0,0,1).

Proof: To obtain the result, we may need to introduce several stochastic processes as below.

- The first stochastic process is the sensor's phase status of the sensor node at time t, named as named I(t). The state space of this process consists of four value, R (means the node is in full-active phase R), N (means the node is in the semi-active phase N), T (means the node is in semi-active phase T) and S (means the node is in sleep phase S).
- The second stochastic process is the number of data packets in the sensor node when the sensor is in the full-active phase R at time t, named as called $R_I(t)$. The space of this process is from 0 to infinity.
- The third stochastic process is the number of data packets in the sensor node when the sensor is in the semi-active phase N at time t, named as called $N_I(t)$. The space of this process is from 1 to infinity.
- The forth stochastic process is the number of data packets in the sensor node when the sensor is in the semi-active phase T at time t, named as called $T_I(t)$. The space of this process is from 1 to infinity.

Based on the description of the sensor node proposed in the previous section and by noting a similar but different consideration as in our other papers [12], [34], it is fairly simple to show that the joint process $\{T_I(t), R_I(t), N_I(t)\}$ forms a multiple-dimensional Markov process with the transition rate diagram shown in Figure 2.

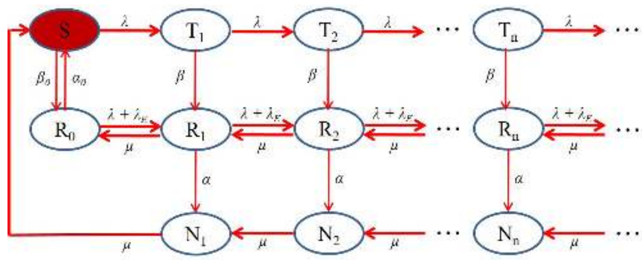

FIGURE 2. Transitions rate diagram.

In **Figure 2**, the circle notation with R_i inside means that the referenced sensor node is in full-active phase R and that there are i ($i \geq 0$) data packets in the referenced sensor node; the circle notation with N_i inside means that the referenced sensor node is in semi-active phase N and that there are i ($i > 0$) data packets in the referenced sensor node; the circle notation with T_i inside means that the referenced sensor node is in semi-active phase T and that there are i ($i > 0$) data packets in the referenced sensor node; The circle notation with S inside means that the referenced sensor is in sleep phase S. If we denote by

$$B_{00} = \begin{bmatrix} -(\lambda + \beta_0) & \beta_0 \\ \alpha_0 & -(\alpha_0 + \lambda + \lambda_E) \end{bmatrix};$$

$$B_{01} = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda + \lambda_E & 0 \end{bmatrix};$$

$$B_{10} = \begin{bmatrix} 0 & 0 \\ 0 & \mu \\ \mu & 0 \end{bmatrix}; \quad A_0 = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda + \lambda_E & 0 \\ 0 & 0 & 0 \end{bmatrix};$$

$$A_1 = \begin{bmatrix} -(\lambda + \beta) & \beta & 0 \\ 0 & -(\lambda + \lambda_E + \mu + \alpha) & \alpha \\ 0 & 0 & -\mu \end{bmatrix};$$

$$A_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \mu & 0 \\ 0 & 0 & \mu \end{bmatrix},$$

then the corresponding transition rate matrix, denote by Q, of the constructed multiple-dimensional Markov Process

$$e^{\tau}\cdot\begin{bmatrix} -(\lambda+\beta_0) & \beta_0 & \lambda & 0 & 1 \\ \alpha_0 & -(\alpha_0+\lambda+\lambda_E) & 0 & \lambda+\lambda_E & 1 \\ 0 & 0 & -(\lambda+\beta) & \beta+\mu\cdot r_{12} & k_1 \\ 0 & \mu & 0 & -(\bar{\lambda}+\mu\cdot r_{22}) & k_2 \\ \mu & 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \quad (2)$$

$\{T_I(t), R_I(t), N_I(t)\}$, can therefore be given by

$$Q=\begin{bmatrix} B_{00} & B_{01} & 0 & 0 & 0 & \cdots \\ B_{10} & A_1 & A_0 & 0 & 0 & \cdots \\ 0 & A_2 & A_1 & A_0 & 0 & \cdots \\ 0 & 0 & A_2 & A_1 & A_0 & \cdots \\ 0 & 0 & 0 & A_2 & A_1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}.$$

where

- B_{00} refers to no change in the number of data packets in the node (could be in sleep or active mode) when there are *no data packets*in the system on the transmission; Similarly, A_1 refers to no change in the number of data packets in the node (could be in semi-active phase T, active phase R, or semi-active phase N mode) when there are *data packets*in the system on the transmission;
- B_{01} refers to refers to an arrival of a data packet when there are *no data packet*s in the system on the transmission. Similarly, A_0 refers to an arrival of a data packet when there are data packets in the system on the transmission.
- B_{10} refers to refers to a departure of a data packet when there is only one*data packet*s in the system on the transmission. Similarly, A_2 refers to a departure of a data packet when there are data packets in the system on the transmission.

Now, by applying the matrix analytical methods in stochastic modeling, such as in [8, eq. (6.20)] or [35, eq. (1.1.12)], we can derive the following result:

$$\pi_i = \pi_1 R^{i-1}, \quad (3)$$

where $i \geq 1$ and matrix R is the minimal non-negative solution to the matrix-quadratic equations

$$R^2A_2 + RA_1 + A_0 = 0, \quad (4)$$

and π_0 and π_1 are the unique positive solution of the equations

$$[x_0, x_1]\cdot\begin{bmatrix} B_{00} & B_{01} \\ B_{10} & RA_2 + A_1 \end{bmatrix} = 0, \quad (5)$$

and

$$x_0e + x_1(I-R)^{-1}e = 1, \quad (6)$$

in which e is a corresponding dimensional column vector with all its components to be ones. It is worthy to point out that the minimal non-negative solution R to the matrix-quadratic equation (4) is a major concept in the theory of matrix-geometric analysis and can be used to characterize most of the features of the Markov chain. Non-negative means that all element of the matrix R is nonnegative and minimal means that $R \leq D$ for any other nonnegative solution D of the above matrix equation. Furthermore with more details in mathematics, the (i,j)-th element of matrix R is the expected number of visits to state $(n+1, j)$ before a return to lower phase-level given the process starts in state (n,i) which is independent with phase n. Two excellent references on this methodology are the books by Latouche and Ramaswami [8] and Neuts [35]. A more detailed probability explanation of the R matrix can be referred to the co-author's research paper [36]. In general, this matrix R is not known to have a closed form solution [37]. However, we will its explicit form here and this is indeed one of the major contributions of this paper.

Furthermore, by using [8, Th. 6.4.1] to proceed with solving equations (4), we finally obtain that the minimal non-negative solution of equation (4) is:

$$R=\begin{bmatrix} r_{11} & r_{12} & r_{13} \\ 0 & r_{22} & r_{23} \\ 0 & 0 & 0 \end{bmatrix}, \quad (7)$$

By submitting result from (7) into (5) and (6), we will know that π_0 and π_1 will be the unique positive solution of the equations

$$x_0e + x_1(I-R)^{-1}e = 1 \quad \text{and } (x_0, x_1)\cdot D = 0, \quad (8)$$

and Eq. (D), as shown in the top of next page.

By a direct mathematical argument on the linear equation (8), π_0 and π_1 can be explicitly derived as in (2) and then major theorem is verified together with (3). The verification is now completed.

Remark 1: It is straightforward to verify that if $\alpha\mu > 0$, we have

$$0 < r_{11} < 1 \quad \text{and } 0 < r_{22} < 1.$$

From the result in Theorem 1, we may explore an efficient ways to either consider the sensor energy consumption or sensor's performance. In next section, we will proceed the way to apply Theorem 1 to this direction.

IV. ENERGY CONSUMPTION AND PERFORMANCE

As long as the formula of the steady-state probability is derived as in Theorem 1, it is not difficult to find various energy consumption measures with unit as milliwatt (mW) of the sensor node. Here, some results are listed to demonstrate how to utilize this formula to obtain the sensor node's performance measures. We will use the following notations:

c_r: processing power consumption per data in phase R;

$$D = \begin{bmatrix} -(\lambda+\beta_0) & \beta_0 & \lambda & 0 & 0 \\ \alpha_0 & -(\alpha_0+\lambda+\lambda_E) & 0 & \lambda+\lambda_E & 0 \\ 0 & 0 & -(\lambda+\beta) & \beta+\mu\cdot r_{12} & \mu\cdot r_{13} \\ 0 & \mu & 0 & -(\bar{\lambda}+\mu\cdot r_{22}) & \alpha+\mu\cdot r_{23} \\ \mu & 0 & 0 & 0 & -\mu \end{bmatrix}.$$

c_n: processing power consumption per data in phase N;

c_t: processing power consumption per data in phase T (or S);

c_{rt}: power consumption when sensor switches from R to T;

c_{rn}: power consumption when sensor switches from R to N;

c_{tr}: power consumption when sensor switches from T to R;

c_{nt}: power consumption when sensor switches from N to T.

Theorem 2: For model introduced in Section II, denote by $e_1 = (1, 0, 0)^\tau, e_2 = (0, 1, 0)^\tau, e_3 = (0, 0, 1)^\tau$ are the transpose of vector (1,0,0), (0,1,0), and (0,0,1), respectively, we have

- If denote by E_R the average energy consumption per unit time when the sensor node is in full-active phase R, we will have

$$E_R = \pi_1(I-R)^{-2}e_2 \cdot c_r. \tag{9}$$

- If denote by E_N the average energy consumption per unit time when the sensor node is in semi-active phase N, then we will have

$$E_N == \pi_1(I-R)^{-2}e_3 \cdot c_n. \tag{10}$$

- If denote by E_T the average energy consumption per unit time when the sensor node is in semi-active phase T, then we will have

$$E_T == \pi_1(I-R)^{-2}e_1 \cdot c_t. \tag{11}$$

- If denote by E_{RT} the average energy consumed per unit time switching from full-active phase to semi-active phase T, then we will have

$$E_{RT} = \pi_0 e_2 \cdot \alpha c_{rt}. \tag{12}$$

- If denote by E_{RN} the average energy consumed per unit time switching from full-active phase to semi-active phase T, then we will have

$$E_{RN} = \pi_1(1-R)^{-1}e_2 \cdot \alpha c_{rn}. \tag{13}$$

- If denote by E_{TR} the average energy consumed per unit time switching from semi-active phase T to the full-active phase R, then we will have

$$E_{TR} = \pi_0 e_1 \beta c_{tr} + \pi_1(1-R)^{-1}e_1 \cdot \beta c_{tr}. \tag{14}$$

- If denote by E_{NT} the average energy consumed per unit time switching from semi-active phase T to the full-active phase R, then we will have

$$E_{NT} = \pi_1 e_3 \cdot \mu c_{nt}. \tag{15}$$

Proof: Since the verification of the first results are similar and the verification of the last four results are similar, we only briefly provide the versification of above first and fourth result as below.

Since the sensor will consume c_r mW of power for transmitting each data packet in phase $\boldsymbol{R}$, we know that the average energy consumption per unit time when the sensor node is in full-active phase **R**should be multiplication of c_r and the expected number of data packets in phase $\boldsymbol{R}$. Therefore, by noting the results in Theorem 1 and matrix operation, we will have

$$E_R = \sum_{i=1}^{\infty} iP(R_i)c_r = \sum_{i=1}^{\infty} i\pi_i e_2 c_r = \pi_1(I-R)^{-2}e_2 \cdot c_r.$$

Since the sensor will consume c_{rt} mW of power each time the sensor switches from the full-active phase **R** to the semi-active phase **N**, we know that average energy consumed per unit time switching from full-active phase to semi-active phase should be multiplication of c_{rt} and the expected switching number from the Phase R to phase T per unit time. Therefore, by noting the results in Theorem 1, we will have

$$E_{RT} = P(R_0)\alpha c_{rt} = \pi_0 e_2 \cdot \alpha c_{rt}.$$

The verification is now completed.

As for the sensor's performance measure, we have the following result:

Theorem 3: For model introduced in Section II, we have

- If denote by D the average delay of a data packet in the sensor node, we have

$$D = \frac{1}{(\lambda+\lambda_E)}\pi_1(I-R)^{-2}e. \tag{16}$$

- If denote by T the data packet throughput of a sensor node which is defined as the average number of the data packets transmitted from the sensor per unit time, then we have

$$T = \pi_1(I-R)^{-1}(e_2+e_3)\mu. \tag{17}$$

- If denote by P_R, P_N, P_T the probability that the sensor node is actually in full-active phase R (including in idle situation), in semi-active phase N and in semi-active phase T (including in sleep phase S), respectively, we have

$$P_T = \pi_0 e_1 + \pi_1(I-R)^{-1}e_1, \tag{18}$$
$$P_R = \pi_0 e_2 + \pi_1(I-R)^{-1}e_2, \tag{19}$$
$$P_N = \pi_1(I-R)^{-1}e_3. \tag{20}$$

Proof: Since the sensor's data generating rate is λ and the rate of the sensor's relay requests from other sensors is λ_E, by using Little's law [38], by using (3), we have

$$D = \frac{1}{\lambda+\lambda_E}\sum_{i=1}^{\infty} i\,[P(R_i)+P(N_i)+P(T_i)] = \frac{1}{(\lambda+\lambda_E)}\pi_1(I-R)^{-2}e.$$

By the definition of the throughput which is defined as the average number of the data packets transmitted from the sensor per unit time, then by considering that only in the Phase R and N the data packets can be transmitted, we will have

$$T = \sum_{i=1}^{\infty}[P(R_i)+P(N_i)]\,\mu = \pi_1(I-R)^{-1}(e_2+e_3)\mu.$$

Next, by using the result in Theorem 1, we can directly get that

$$P_T = \sum_{i=0}^{\infty} P(T_i) = \pi_0 e_1 + \sum_{i=1}^{\infty}\pi_i e_1 = \pi_0 e_1 + \pi_1(I-R)^{-1}e_1.$$

Similarly, we will have that

$$P_R = \sum_{i=0}^{\infty} P(R_i) = \pi_0 e_2 + \sum_{i=1}^{\infty}\pi_i e_2 = \pi_0 e_2 + \pi_1(I-R)^{-1}e_2,$$
$$P_N = \sum_{i=1}^{\infty} P(N_i) = \sum_{i=1}^{\infty}\pi_i e_3 = \pi_1(I-R)^{-1}e_3.$$

The verification is now completed.

Remark: Based on the explicit results presented above, it is apparent that the probability that the sensor is in sleep mode is not equal to $\beta_0/(\alpha_0+\beta_0)$, and that the probability that the sensor in active mode is not equal to $\alpha/(\alpha+\beta)$, since the sensor node has to relay all data packets in the node when the sensor's mode switches from phase R to phase N. This means that the active-sleep model we developed is not a standard stochastic renewal process.

V. NUMERICAL ANALYSIS

To verify the validity of the analytical expressions obtained in the previous section, we present the numerical results for a set of specific parameters in this section. The performance measures considered here are several energy consumptions, data package time delay, and the throughput. The value of the parameter set is given as follows:

The unit of energy parameters are milliwatt (mW) and the time parameters are given as millisecond. The performance of the proposed model are shown in Figures 2–6. As a comparison, in every figure, the performance measure is computed for three different α with value of 0.1, 0.05 and 0.04. That the value of α_0 is chosen to be bigger than α's value means that the sensor in phase R but no waiting data (or in idle status) will change the phase to sleep quicker than the rate to phase N when it is the phase R and there is at least one waiting data. That the value of β_0 is chosen to be smaller than β's value means that the sensor in phase T will change the phase to active phase R quicker than the rate to phase R when it is the sleep phase. We observe and compare the effects of various performance measures with respect to the sensor node's generating rate λ.

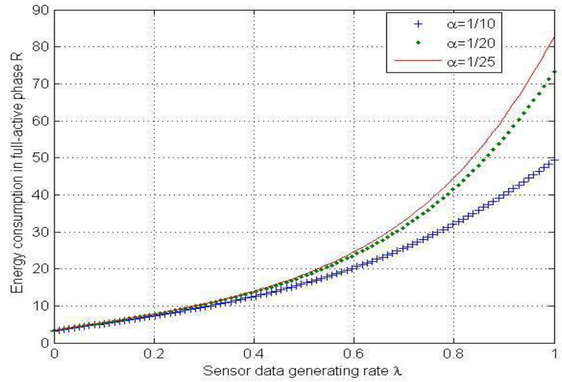

FIGURE 3. **The energy consumption in full-active phase.**

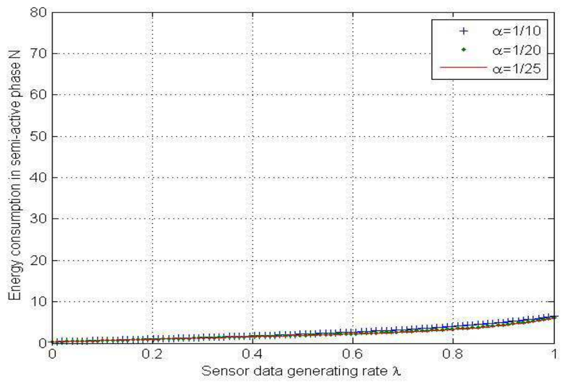

FIGURE 4. **Energy consumption in semi-active phase N.**

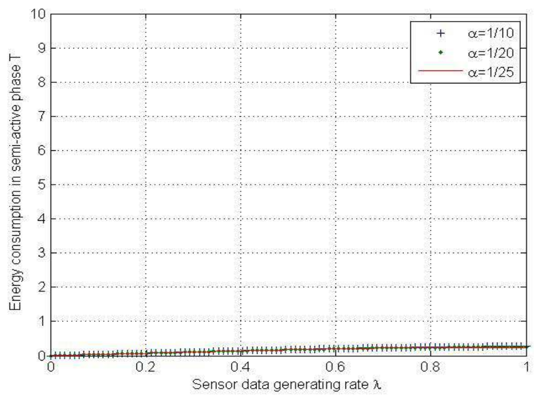

FIGURE 5. **The energy consumption in semi-active phase T.**

Figure 3 shows the energy consumption when the senor node is in the full-activity phase R. It is clear that, with the increase of the sensor's generating rate λ from 0 to 1, the average energy assumption increases quickly. This is

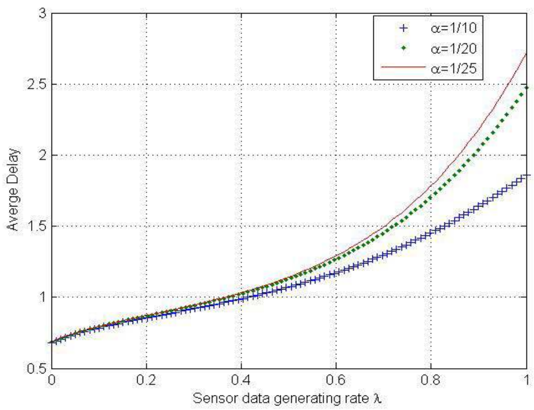

FIGURE 6. The average delay of a data in the system.

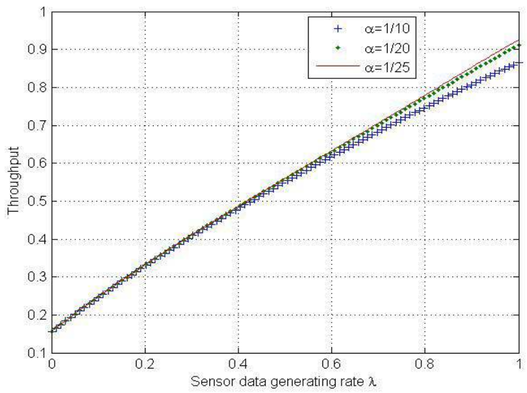

FIGURE 7. The throughput of the system.

reasonable since dealing with more number of data packages need spending more energy. In addition, as expected, we find that small value of α, the sensor with longer time on, corresponds to more energy assumption. We also investigated the relationship between the change of the sensor's generating rate λ and the average energy assumption in semi-active phase T or phase N.

The curves in **Figure 4** and **Figure 5** show that, for two semi-active phases, with the increase of the generating rate λ, these two kinds of energy consumption will increase slightly and there is not much affection for different value of α. This is as we expected since in these two phase, the sensor will not receive the incoming data. Therefore the results in Figure 4-5 verify that most energy consumption occurs at the full-active phase and processing more data packages does cause an increase in energy consumption. As mentioned during the discussion of the design of a WSN, the balance between energy consumption and the generation of packages is a critical consideration.

The average data delay is depicted in **Figure 6**. It is obvious that the data delay increases when either the sensor's generating rate λ increases or a greater number of relayed messages are being processed. The throughput of the sensor node is shown in **Figure 7**, and it depicts what we expected in that the throughput increases as the sensor's generating rate increases. As a brief remark, we would like to have a simple comparison between the model in this paper and the model in [17] from the viewpoint of numerical results. In fact, both papers consider the system energy consumption, throughputs and the average delay with respect to the sensor's data-packet generating rate under certain conditions. Even though the parameters used in these two papers are not the exactly same and the models is related but different, it is observed that the monotonic increasing property of these three system performance measures are the same. This observation is consistence with our expectation.

VI. CONCLUSION

In this paper, an explicit result for the joint probability of data packet numbers and the sensor phases is derived for a novel duty cycle wireless sensor network model with four phases. The energy consumption in switching from one phase to another and the cost incurred in each of the phases are derived. System performance measures such as delay, throughput and the probability in each phase are also provided. Numerical analyses are provided to validate the proposed model and the results obtained. The results show that the energy consumption for switching among phases does not depend significantly on the number of data packets. However, the energy consumption for transmitting the data packets depends on the rate at which data packets are generated, which means that processing high-density data requires the expenditure of more energy. The proposed model and analysis method are expected to be applied to the design and analysis of WSNs, taking into consideration the times spent in the various phases.

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