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# Energy Control of Multi-Switch Power Supplies; An Application to the Three-Phase Buck type Rectifier with Input Filter

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*Abstract* – The first part of the paper presents a systematic method for the modeling of power electronic networks with multiple switches using the classical Lagrangian framework. The main advantage of this method is its structured and general character compared to other methods. The power of working with Lagrangian dynamics, especially when very large networks have to be analyzed, is that Kirchhoff's voltage law is given in advance, while the interconnection structure is based on Kirchhoff's current laws only. The physical nonlinear structure of these systems is revealed and can be used for feedback controller design and stability analysis. The second part concentrates on the recently proposed passivity-based controller design technique based on the Lagrangian structure. Besides its physical significance, this approach appears to be an interesting alternative when dealing with the problem of input filter influences in power electronic systems. With this technique artificial damping is injected such that the impedances of the in- and output filters can be matched. In this way power is not reflected and resonance problems, especially during the start-up and transient conditions, are minimized. A unity power factor passivity-based control algorithm for the three-phase Buck type rectifier with an input filter is proposed that does not require current sensors but only measurements of the capacitor voltages.

## I INTRODUCTION

A continuous flow of papers is presenting the theoretical and practical approaches for modeling and control of power converters. A variety of methods are proposed for obtaining a description of the dynamics of these electrical switching networks. The analysis is mainly based on small-signal models following from state space averaging techniques . In recent developments power converters are considered from a physical (energy and interconnection) point of view. It is first shown in [8, 9] that this class of systems actually correspond to systems derivable from the classical Euler-Lagrange (or Hamiltonian [2]) dynamic considerations. The main advantage of this approach is that it reveals the often overlooked physical (nonlinear) properties of power electronic networks. The physical properties can be used for the design of feedback controllers that do not involve the cancellation of nonlinearities. None of these methods, however, offer a straight forward and systematic method to obtain dynamic models in a general structured way such that the method is applicable to all kinds of switch-mode power converters or inverters. In [11] a method is proposed to obtain the dynamic models from the Euler-Lagrange (EL) dynamic considerations for single-switch DC-to-DC converters, and is extended to networks containing coupled-magnetics [12]. In the first part of this paper we show that this procedure can also be used for large power electronic networks with multiple switches. The power of working with Lagrangian dynamics is that Kirchhoff's voltage law is given in advance, while the interconnection structure is based on Kirchhoff's current laws only. A six-switch three-phase Buck rectifier with LC input filter is used to illustrate the method.

Apart from its practical relevance, the three-phase Buck rectifier is an interesting case study because (due to the input filter) it is a non-minimum phase system and the phase shift (also caused by the input filter) between input currents and voltages varies with the load and with the magnitude of the input voltages. In a practical situation these magnitudes are often uncertain parameters. In the second part of the paper we design a controller based on the EL properties derived in the modeling part. This passivity-based controller (PBC) preserves the EL structure of the system but modifies the energy and dissipation structure. It is shown that the dissipation characteristics can be modified in such a way that the impedance of the input and output filters can both be matched dynamically. In this way power is not reflected and resonance problems, especially during the start-up and transient conditions, are minimized. Simulations are performed to evaluate the proposed controller when the system is subject to changes in the reference.

## II LAGRANGIAN MODELING

The main part of the modeling procedure consists of establishing a suitable set of *EL parameters*, which fully describes the (idealized) dynamics of an electrical circuit $\Sigma$. The system is then expressed by means of a five-tuple as [11, 12]:

$$\Sigma = \{\mathcal{T}(q,\dot{q}), \mathcal{V}(q), \mathcal{D}(\dot{q},u), \mathcal{F}_q(u), \mathcal{A}(u)\}, \qquad (1)$$

where $q(t)$ is the electric *charge* and $\dot{q}(t)$ represents the vector of flowing *current*. For sake of brevity we omit the time dependency in the remaining of the document. The vectors $q$ and $\dot{q}$ represent the generalized coordinates describing the circuit and are assumed to have $n$ components related to the $n_L$ inductors and $n_C$ capacitors, i.e. $q = [q_L^T, q_C^T]^T$ with $q_L^T = [q_{L_1}, \ldots, q_{L_{n_L}}]$, and $q_C^T = [q_{C_1}, \ldots, q_{C_{n_C}}]^T$, $n = n_L + n_C$. We denote by $\mathcal{T}(q,\dot{q})$ the total *magnetic co-energy*, and by $\mathcal{V}(q)$, the total *electric field energy* of the circuit, which for linear inductors and capacitors are

defined as

$$T(\dot{q}_L) = \tfrac{1}{2}\dot{q}_L^T L \dot{q}_L, \quad \mathcal{V}(q_C) = \tfrac{1}{2}q_C^T C^{-1} q_C, \tag{2}$$

where $L$ is a $n_L \times n_L$ positive definite matrix containing the inductor values, and $C$ is a $n_C \times n_C$ positive definite matrix containing the capacitor values, respectively. If the network contains one or more switches, we denote the switch position(s) by $u = [u_1, \ldots, u_m]$, with $u_i \in U := \{0,1\}, i = 1, \ldots, m$, i.e., ON or OFF, or in other words $u$ is in the discrete set $U^m$. Depending on the application, re-definition of the switching function may also result in for example $u_i \in U := \{-1,0,1\}$. The remaining EL properties are defined as follows: The function $\mathcal{D}(\dot{q}, u)$ is the *Rayleigh dissipation co-function* of the system. This term is necessary to include the resistive elements, representing the load resistance(s) and the losses in the dynamic elements, the sources and the switches. By $\mathcal{A}(u)$ we denote the *constraint force matrix*, resulting from Kirchhoff's current laws not defined by the Rayleigh dissipation function, $\mathcal{A}^T(u)\dot{q} = 0$, and by $\mathcal{F}_q(u)$ we denote the *generalized forcing functions*, related to the external voltage sources (supply voltage, semiconductor junction voltage drops, back electromotive force of electrical machines) associated with each charge coordinate. The latter parameters are then simply obtained by following the first five steps as given in full details in [11, 12]. The last step is defining the *Lagrangian* of the circuit as $\mathcal{L}(q, \dot{q}) = T(q, \dot{q}) - \mathcal{V}(q)$ and plugging the information of (1) in the constraint equations given by

$$\frac{d}{dt}\left[\frac{\partial \mathcal{L}(q,\dot{q})}{\partial \dot{q}}\right] - \frac{\partial \mathcal{L}(q,\dot{q})}{\partial q} = -\frac{\partial \mathcal{D}(\dot{q},u)}{\partial \dot{q}} - \mathcal{A}(u)\lambda + \mathcal{F}_q(u)$$

$$\mathcal{A}^T(u)\dot{q} = 0, \quad \lambda \in \mathbb{R}^k, \tag{3}$$

After solving for the Lagrange multipliers $\lambda$, a state space description is obtained by choosing the currents corresponding with the inductive elements, and by selecting either the charge or the voltage corresponding with the capacitive elements, as state variables. Note that the upper part in (3) simply represents Kirchhoff's voltage law, while the constraint represents the current laws.

In three- or multi-phase power electronic networks it is often assumed that the source voltages satisfy certain constraints. For a $g$-phase network with $n$ dynamic elements these constraints will be of the form $\mathcal{F}_{i_1}^q + \ldots + \mathcal{F}_{i_g}^q = 0$, where $i_j \in \hat{n}$, $\hat{n} := \{1, \ldots, n\}, j = 1, \ldots, g, g \leq n$ and $i_j \neq i_k$ if $j \neq k$. For a symmetrical $g$-phase network this will often result in constraints on the current coordinates of the form $\dot{q}_{i_1} + \ldots + \dot{q}_{i_g} = 0$. From a system theoretic point of view this type of constraints implies that the system is non-minimal in the present description. In general there are many ways to deal with this type of algebraic dependence. In the field of electrical machines and power electronic networks, a very often used and convenient method is to shift the system into an orthogonal fixed $(\alpha, \beta)$ or rotating $(d, q)$ reference frame, see e.g. [6]. Later on we shall see that this transformation is actually necessary for the application of passivity-based controller design.

## III APPLICATION TO THE BUCK RECTIFIER

To illustrate the proposed modeling procedure we will consider the Buck rectifier with LC input filter depicted in Fig. 1. We assume that the rectifier operates in continuous conduction mode (CCM), and that the source voltages are balanced, i.e., $e_1 + e_2 + e_3 = 0$. We also assume that there is no neutral line. The circuit consists in its main part of three subcircuits: an LC input filter, $L_i = L_k$, $C_i = C_k$, $k = 1, 2, 3$, which is used to attenuate the effect of the pulsating input currents, a bridge of three legs with two switches each, and an LC output filter, $L_o$ and $C_o$. The upper switches are denoted by $s_{ak}$ and the lower switches by $s_{bk}$, $k = 1, 2, 3$. Let the switching functions $s_{jk}$ be defined as

$$s_{jk}(t) = \begin{cases} 1, & s_{jk} \text{ closed} \\ 0, & s_{jk} \text{ open} \end{cases}, \quad j = a, b, \; k = 1, 2, 3. \tag{4}$$

Since the converter is supplied by a voltage source, the input capacitor voltages must not be shortened and at any time a path should be provided for the output inductor current. These restrictions can be expressed as $s_{j1} + s_{j2} + s_{j3} = 1$. For convenience we define $u_k = s_{ak} - s_{bk}$, $u_k \in \{-1, 0, 1\}$, so that the constraints on the switching functions become $u_1 + u_2 + u_3 = 0$. Following the procedure, we define as the configuration variables

$$q = [q_{L_k}^T, q_{L_o}, q_{C_k}^T, q_{C_o}]^T, \tag{5}$$

with $q_{L_k}^T = [q_{L_1}, q_{L_2}, q_{L_3}]$ and $q_{C_k}^T = [q_{C_1}, q_{C_2}, q_{C_3}]$. The magnetic co-energy and the electric of the system are readily found as

$$T(\dot{q}_{L_k}, \dot{q}_{L_o}) = \tfrac{1}{2}\sum_{k=1}^{3} L_k \dot{q}_{L_k}^2 + \frac{1}{2}L_o \dot{q}_{L_o}^2$$

$$\mathcal{V}(q_{C_k}, q_{C_o}) = \tfrac{1}{2}\sum_{k=1}^{3} \frac{q_{C_k}^2}{C_k} + \frac{1}{2}\frac{q_{C_o}^2}{C_o}, \tag{6}$$

The Rayleigh dissipation function and the generalized forces acting on the system are

$$\mathcal{D}(\dot{q}_{L_k}, \dot{q}_{L_o}, \dot{q}_{C_o}) = \tfrac{1}{2}R_o(\dot{q}_{L_o} - \dot{q}_{C_o})^2 + \tfrac{1}{2}\sum_{k=1}^{3} R_k \dot{q}_{L_k}^2,$$

$$\mathcal{F}_{L_1}^q = e_1; \; \mathcal{F}_{L_2}^q = e_2; \; \mathcal{F}_{L_3}^q = e_3; \tag{7}$$

$$\mathcal{F}_{L_o}^q = \mathcal{F}_{C_1}^q = \ldots = \mathcal{F}_{C_o}^q = 0,$$

where $R_k = R_i, k = 1, 2, 3$ represents the internal resistance of the source and the switching losses (not shown in the figure). The constraints[1] that follow from Kirchhoff's current law are expressed as

$$\mathcal{A}^T(u) = \begin{bmatrix} 1 & 0 & 0 & -u_1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -u_2 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & -u_3 & 0 & 0 & -1 & 0 \end{bmatrix}. \tag{8}$$

---

[1]Note that rank$\{\mathcal{A}\} = 3$ everywhere, and that by adding the second and the last row of $\mathcal{A}^T$ to the first one results in the constraint $\sum \dot{q}_{L_k} - \sum \dot{q}_{C_k} = 0$, which agrees with the assumption that there is no neutral line.
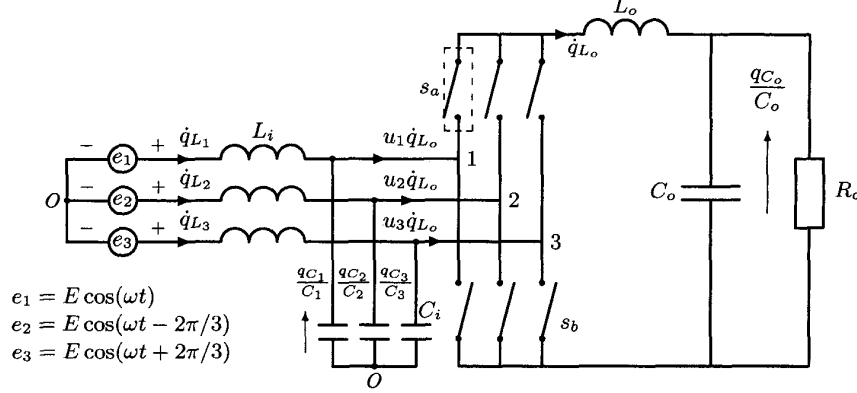
Fig. 1: Three-phase (current source) Buck rectifier with LC input filter

$e_1 = E\cos(\omega t)$
$e_2 = E\cos(\omega t - 2\pi/3)$
$e_3 = E\cos(\omega t + 2\pi/3)$

In this case $\lambda \in \mathbb{R}^3$. We now have all the information that characterizes the dynamic behavior of the circuit. Plugging (6), (7) and (8) into (3) yields the following EL representation of the circuit:

$$
\begin{aligned}
L_i \ddot{q}_{L_k} &= e_k - R_i \dot{q}_{L_k} + \lambda_k \\
L_o \ddot{q}_{L_o} &= -R_o(\dot{q}_{L_o} - \dot{q}_{C_o}) - \sum_{k=1}^{3} u_k \lambda_k \\
C_i^{-1} q_{C_k} &= -\lambda_k \\
C_o^{-1} q_{C_o} &= R_o(\dot{q}_{L_o} - \dot{q}_{C_o}) \\
0 &= \mathcal{A}^T(u)\dot{q},
\end{aligned}
\tag{9}
$$

for $k = 1, 2, 3$. Then after a change of coordinates

$$
x = \left[ \dot{q}_{L_k}^T, \dot{q}_{L_o}, \tfrac{1}{C_i} q_{C_k}^T, \tfrac{1}{C_o} q_{C_o} \right]^T,
\tag{10}
$$

denoting the inductor currents and capacitor voltages, respectively, one obtains the following matrix representation

$$
\mathcal{M}\dot{x} + (\mathcal{J} + \mathcal{J}'(u))x + \mathcal{R}x = \mathcal{E},
\tag{11}
$$

where $\mathcal{M}$ is a positive definite diagonal matrix containing the inductor and capacitor values, $\mathcal{R}$ is the dissipation matrix containing the resistive elements, $\mathcal{J} + \mathcal{J}'(u)$ is the interconnection structure, and $\mathcal{E}$ is the vector representing the voltage sources. The property $\mathcal{J} + \mathcal{J}'(u) = -(\mathcal{J} + \mathcal{J}'(u))^T$ (skew-symmetry) is a typical feature of Lagrangian systems and forms a necessary and sufficient condition for application of the passivity-based approach [8, 10]. Although not shown in detail, (11) is exactly the same model as obtained in e.g. [1]. In the new coordinates (10) the (co-)energy and the dissipation (co-)energy of the system is now given by

$$
\mathcal{H}(x) = \tfrac{1}{2}x^T \mathcal{M}x \quad \text{and} \quad \mathcal{D}(x) = \tfrac{1}{2}x^T \mathcal{R}x,
\tag{12}
$$

respectively.

*Remarks:*

*(i)* As shown in [9], the switched EL equations are closely related to the average PWM (pulse-width modulation) models. For the Buck rectifier given by the equations (11) this means that the state variable $x$ is replaced by the average state $\bar{x}$, representing the average inductor currents and capacitor voltages, and the discrete control vector $u$ is replaced by its duty ratio function vector $\mu$.

*(ii)* The source voltages satisfy a constraint of the form $e_1 + e_2 + e_3 = 0$. In that case (11) is actually a non-minimal description. In contrast to [1] we have that $x \in \mathbb{R}^6$ instead of $x \in \mathbb{R}^8$. A minimal description can be found by using the properties $x_3 = -(x_1 + x_2)$, $x_7 = -(x_5 + x_6)$ and $u_3 = -(u_1 + u_2)$. It is easily checked that if we eliminate the constraints in this way the interconnection structure of the Buck rectifier in the minimal state space representation does not satisfy the skew-symmetric property anymore. This is due the fact that such transformation is not energy (or power) preserving. To overcome this problem, the system has to be rewritten in a orthogonal reference frame by performing $\alpha\beta$- or $dq$-transformation.

Under the assumption that all three-phase variables are balanced, we have after performing some straight forward calculations, e.g. [1], for the *average* dynamics of the Buck rectifier in the $dq$-reference frame with the coordinates, $z = \bar{x}_{dq}$,

$$
z = \left[ \dot{q}_{L_i}^d, \dot{q}_{L_i}^q, \dot{q}_{L_o}, \tfrac{1}{C_i} q_{C_i}^d, \tfrac{1}{C_i} q_{C_i}^q, \tfrac{1}{C_o} q_{C_o} \right]^T
\tag{13}
$$

we have that

$$
\begin{aligned}
L_i \dot{z}_1 &= e_d - R_i z_1 + L_i \omega z_2 - z_4 \\
L_i \dot{z}_2 &= e_q - R_i z_2 - L_i \omega z_1 - z_5 \\
L_o \dot{z}_3 &= \mu_d z_4 + \mu_q z_5 - z_6 \\
C_i \dot{z}_4 &= z_1 + C_i \omega z_5 - \mu_d z_3 \\
C_i \dot{z}_5 &= z_2 - C_i \omega z_4 - \mu_q z_3 \\
C_o \dot{z}_6 &= z_3 - R_o^{-1} z_6,
\end{aligned}
\tag{14}
$$

or in matrix form:

$$
\mathcal{M}_{dq}\dot{z} + (\mathcal{J}_{dq} + \mathcal{J}'_{dq}(\mu))z + \mathcal{R}_{dq}z = \mathcal{E}_{dq},
\tag{15}
$$

It is then easily checked that for the PWM model with states $\bar{x}$ we have

$$
\begin{aligned}
\tfrac{1}{2}\bar{x}^T \mathcal{M}\bar{x} &\equiv \tfrac{1}{2}z^T \mathcal{M}_{dq}z \\
\tfrac{1}{2}\bar{x}^T \mathcal{R}\bar{x} &\equiv \tfrac{1}{2}z^T \mathcal{R}_{dq}z
\end{aligned}
\tag{16}
$$

which shows that the transformation is indeed energy preserving. In following sections we shall use the above properties for the control design.

## IV CONTROLLER DESIGN

The basic idea behind passivity-based controller design is to modify the energy function and add damping by modification of the Rayleigh dissipation function. Full details of this design technique can be found in e.g. [8, 9]. Since we have given a general procedure to build an Euler-Lagrange model for (multi-switch) electrical networks, we can also generally apply the passivity based control design technique. However, one issue that remains is the choice of the state variable to be stabilized to a certain value, in order to, possibly indirectly, regulate our output toward a desired equilibrium value. For the Buck circuit with input filter it can be shown by bringing the system in normal form (see e.g., [4, 12]) or by linearization (see e.g., [1, 3]) that the zero-dynamics of all states with respect to the control inputs are unstable, except for the input current. This means that we are dealing with a non-minimum phase system and thus we can not control the input capacitor voltage, the output current, and output capacitor voltage directly, when using the passivity-based approach [8, 9]. The only feasible states that remain are the input currents.

### A  The Single-Switch Buck Cell

Before we design a controller for the three-phase Buck rectifier let us first show the idea of impedance matching by studying a simple circuit such as the single-switch Buck cell depicted in Fig. 2. The (average) EL dynamics are expressed as

$$
\Sigma_B = \{\tfrac{1}{2}L\dot{q}_L^2, \tfrac{1}{2C}q_C^2, \tfrac{1}{2}R(\dot{q}_L - \dot{q}_C)^2, \mu E, 0\}
\tag{17}
$$

with $\mu$ the duty ratio of the switch $0 < \mu < 1$. In [9] the following control law is proposed

$$
\mu = \frac{1}{E}\left[V_d - R^i\left(\dot{q}_L - \frac{V_d}{R}\right)\right],
\tag{18}
$$

with $V_d$ a desired value for the output voltage and $R^i$ a design parameter.   After a change of coordinates $z_1 = \dot{q}_L$ and $z_2 = C^{-1}q_C$ the closed-loop system satisfies

$$
\begin{aligned}
L\dot{z}_1 &= \frac{R + R_s^i}{R}V_d - R_s^i z_1 - z_2 \\
C\dot{z}_2 &= z_1 - \frac{z_2}{R}
\end{aligned}
\tag{19}
$$

Note that the dissipation matrix of the closed-loop is now defined as $\mathcal{R}_{cl} = \mathcal{R} + \mathcal{R}^i = \mathrm{diag}\{R_s^i, R^{-1}\}$, where $\mathcal{R} = \mathrm{diag}\{0, R^{-1}\}$ is
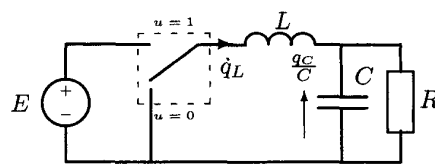


Fig. 2: Single PWM switch Buck cell

the open-loop dissipation matrix, while $\mathcal{R}^i = \mathrm{diag}\{R_s^i, 0\}$ is the injected dissipation. Equations (19) have a nice network-theoretic interpretation as shown in Fig. 3. It is seen that $R_s^i$ actually acts as a virtual non-dissipating *series damping* resistance. The ideal transformer is necessary to compensate for the voltage drop over $R_s^i$.
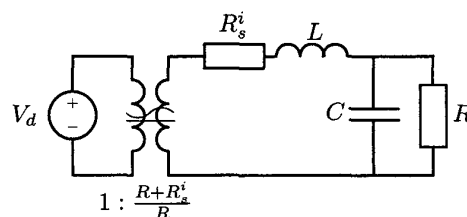


Fig. 3: Closed-loop interpretation of the PBC controlled Buck cell

The differential equation describing the input-output relation from $V_d$ to $z_2$ can be derived from (19) as

$$
\ddot{z}_2 + \left(\frac{R_s^i}{L} + \frac{1}{CR}\right)\dot{z}_2 + \frac{R + R_s^i}{LCR}z_2 = \frac{R + R_s^i}{LCR}V_d
\tag{20}
$$

From classical control theory we know that in order to have a perfect match (zero overshoot), $R_s^i$ has to be chosen such that

$$
\frac{R_s^i}{L} + \frac{1}{CR} = 2\sqrt{\frac{R + R_s^i}{LCR}}
\tag{21}
$$

and thus that

$$
R_s^i = \begin{cases} \frac{L + 2R\sqrt{LC}}{CR}, & \text{for } R > Z_c \\ \frac{L - 2R\sqrt{LC}}{CR}, & \text{for } R \le Z_c, \; Z_c = \tfrac{1}{2}\sqrt{\frac{L}{C}} \end{cases}
\tag{22}
$$

We could also modify the original EL parameters of the Buck cell to inject *parallel damping*. This is done by setting the injected damping matrix

$$
\mathcal{R}^i = \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{R_p^i} \end{bmatrix} \text{ such that } \mathcal{R}_{cl} = \begin{bmatrix} 0 & 0 \\ 0 & \frac{R + R_p^i}{R_p^i R} \end{bmatrix}
$$

with, for example, the control law

$$
\mu = \frac{V_d}{E} - \frac{L\dot{q}_C}{ECR_p^i}, \; R \ne Z_c, \; R_p^i = \frac{Z_c R}{R - Z_c}.
\tag{23}
$$

The resulting input-output relation from $V_d$ to $z_2$ is now given by

$$
\ddot{z}_2 + \frac{R + R_p^i}{CR_p^i R}\dot{z}_2 + \frac{1}{LC}z_2 = \frac{1}{LC}V_d
\tag{24}
$$

The closed-loop dynamics (24) imply that there is a resistor $R_p^i$ connected in parallel with the output capacitor. Note that for both the controllers the source voltage is eliminated from the closed-loop dynamics, which is an inherent property of the passivity-based approach. Let us now return to our more complicated three-phase rectifier example.

## B Stabilizing Controller for the Three-Phase Buck Rectifier

We are now ready to provide a controller for the three-phase rectifier that stabilizes the output capacitor voltage as well as the input capacitor voltages by *dynamically* matching the filter impedances. As a result the in- and output inductor currents are stabilized as well. As briefly discussed at the start of the previous section, due to the non-minimum phase nature of the rectifier the only feasible regulation is achieved through regulation of the input currents, in the $dq$ reference frame: $I_d^d$ and $I_q^d$. Following [8], we start by modifying the energy of the rectifier (15) to arrive at a desired one that preserves the structure of the original energy, i.e., $\mathcal{H}^d = \frac{1}{2}(z - z^d)^T \mathcal{M}_{dq}(z - z^d)$, where $z^d$ denote the desired auxiliary states of the controller. The minimum of this energy will then be located at the desired equilibrium point, which as a function of the desired output capacitor voltage, $V_o^d$, is given by $z_*^d = [I_d^d, I_q^d, I_o^d, V_d^d, V_q^d, V_o^d]^T$, with

$$I_d^d = \frac{(V_o^d)^2}{R_o E}\sqrt{\frac{2}{3}}; \quad I_q^d = 0; \quad I_o^d = \frac{V_o^d}{R_o};$$

$$V_d^d = e_d = E\sqrt{\frac{3}{2}}; \quad V_q^d = -\omega L_i I_d^d.$$

The internal resistances, $R_i$, of the source and the switches are for ease of calculations set to zero. We choose to inject parallel damping on the capacitors, $C_i$ and $C_o$. This is done by forcing the closed-loop dissipated energy to be of the form

$$\mathcal{R}_{dq}^{cl} = \text{diag}\left\{0, 0, 0, \frac{1}{R_i^i}, \frac{1}{R_i^i}, \frac{R_o R_o^i}{R_o + R_o^i}\right\},$$

which is accomplished through the following *implicit* definition of the control law:

$$\mathcal{M}_{dq}\dot{z}^d + (\mathcal{J}_{dq} + \mathcal{J}_{dq}'(\mu))z^d + \mathcal{R}_{dq}z^d - \mathcal{R}_{dq}^i(z - z^d) = \mathcal{E}_{dq},$$

where $\mathcal{R}_{dq}^i$ is the required damping modification given by

$$\mathcal{R}_{dq}^i = \text{diag}\left\{0, 0, 0, \frac{1}{R_i^i}, \frac{1}{R_i^i}, \frac{1}{R_o^i}\right\},$$

Hence, we find an *explicit* definition of our controller by choosing as the control laws

$$\mu_d = \frac{1}{z_3^d}\left[I_d^d + \omega C_i V_q^d + \frac{z_4 - V_d^d}{R_i^i}\right] \quad (25)$$

$$\mu_q = \frac{1}{z_3^d}\left[-\omega C_i V_d^d + \frac{z_5 - V_q^d}{R_i^i}\right], \quad z_3^d > 0 \quad (26)$$

TABLE I
DESIGN PARAMETERS

| Source Voltage | $E = 100V$, $\omega = 2\pi 50$rad/s |
|---|---|
| Input Filter | $R_i = 0\Omega$, $L_i = 3$mH, $C_i = 3\mu$F |
| Switching Frequency | $f_s = 25$kHz (PWM) |
| Output Filter | $L_o = 1.6$mH, $C_o = 1$mF |
| Load Resistance | $R_o = 10\Omega$ |

and let the functional relations $z_3^d$, $z_6^d$ be the solutions of

$$L_o \dot{z}_3^d = \mu_d V_d^d + \mu_q V_q^d - z_6^d, \quad z_3^d(0) = I_o^d$$

$$C_o \dot{z}_6^d = z_3 - \frac{R_o + R_o^i}{R_o R_o^i}z_6^d - \frac{z_6}{R_o^i}, \quad z_6^d(0) = V_o^d \quad (27)$$

The above developments lead to a partial state feedback PWM scheme that does *not* involve the use of current sensors but only measurements of the capacitor voltages. Using Lyapunov theory and LaSalle's invariance principles, one can easily proof that the proposed controller indeed stabilizes the closed-loop dynamics of the system, e.g. [10].

*Remarks:*

*(i)* If we would like to inject series damping both the input inductor current and voltages have to be measured, resulting in the use of extra (expensive) sensors. Besides the use of extra sensors, the closed-loop response will also be slower because the input time constants are increased.

*(ii)* In order to account for parametric uncertainty the above control scheme can also be extended to an adaptive version, as presented in e.g. [8, 12].

*(iii)* Because the controller is based on the average dynamics of the rectifier the actual measured states need to be filtered. In this way extra phase shift is injected into the loop which can cause unaccounted stability problems. This is a drawback of working with control algorithms based on average dynamics. However, in our case the simulations in the following section show promising results with realistic switching frequencies.

## V  SIMULATION RESULTS

Computer simulation with SIMULINK was performed in order to validate the closed-loop dynamics of the controlled Buck rectifier. We now use (11) with the discrete values for the switches. Table I shows the parameters used for the analysis. The control parameter for output matching is set to

$$Z_o^c = \frac{1}{2}\sqrt{\frac{L_o}{C_o}} \Rightarrow R_o^i = \frac{RZ_o^c}{R - Z_o^c} = 1.88\Omega,$$

while $R_i^i$ is set to $R_i^i = Z_i^c = 15.8\Omega$. First-order low pass filters are used to suppress the ripples in the actual input capacitor voltages, the cut-off frequency is set to $0.5f_s$.
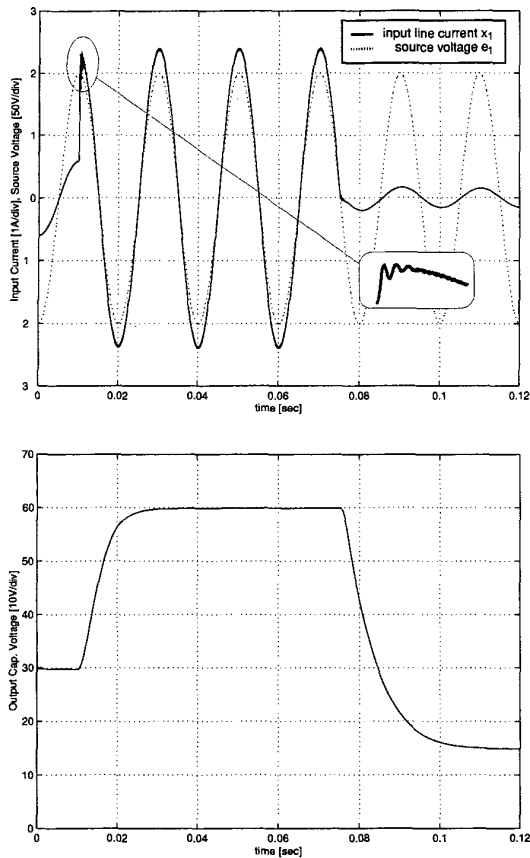
Fig. 4: Closed-loop response to step changes in the reference signal $V_o^d$; input current $x_1 = \dot{q}_{L_1}$ (top), output voltage $x_8 = C_o^{-1} q_{C_o}$ (bottom).

The response of the system for step changes at $t = 0.01$ and $t = 0.075$ in the desired output capacitor voltage is depicted in Fig. 4. As can be seen from the bottom figure, the controller achieves the desired indirect stabilization of the output voltage around the desired equilibrium value without any oscillations or overshoot. The input current shows a small oscillation but no overshoot. This oscillation is due to a small impedance mismatch caused by the additional low-pass filters. Note in the top figure that the power factor reaches unity almost instantly, i.e., the input current $x_1 = \dot{q}_{L_1}$ (solid line) is exactly in phase with the source voltage $e_1$ (dashed line).

## VI Conclusion

In the first part of this paper the systematic method to build dynamical models for (power) electronic networks as proposed in [11, 12] is extended to a broad class of electrical networks containing multiple switches. The power of working with Lagrangian dynamics, especially when very large networks have to be analyzed, is that Kirchhoff's voltage law is given in advance, while the interconnection structure is based on Kirchhoff's current laws

only. Orthogonal transformations to obtain minimal state space descriptions are in many cases necessary to preserve the EL structure. In the second part we have given some circuit-theoretic interpretations of the PBC design method. For the Buck rectifier it appears that, the dissipation structure can be modified such that impedance of the input and output filters can be matched by injecting virtual damping resistors. The resulting PBC strategy does not require current sensors but only measurements of the capacitor voltages. The simulations show a good agreement with the developed theory. Although not shown here, several other tests have shown that the closed-loop system is very robust against voltage source perturbations.

Further research is recommended towards the application of impedance matching by way of a passivity-based controller to other power electronic circuits. Involvement of more non-ideal physical effects, like saturation and (non-ideal) diodes has to be studied in the Lagrangian framework. The influences of the measurement filters has to be studied. An adaptive scheme should be added to account for parametric uncertainty. Future plans include the design of an experimental setup to test the performance of the passivity-based controlled Buck rectifier in a real-time environment. It is also of interest to compare the performance of the PBC Buck rectifier with other existing control strategies.

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