

Energy-Dependent Production Functions and the Optimization Model “PRISE” of Price-Induced Sectoral Evolution

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Abstract

In an attempt to integrate thermodynamics with economics, production functions are proposed that depend on capital, labor, energy and technological parameters associated with the energy conversion efficiency of the capital stock. Based on these production functions, which resolve most of the unexplained Solow residual of conventional economic growth theory, we develop the optimization model PRISE of PPrice-Induced Sectoral Evolution. The model is designed to analyze potential changes of inputs, outputs and profits in differently energy- and labor-intensive sectors of an economy in response to changing factor prices. The model has been tested by comparing its predictions with the German sectoral economic evolution, 1968-1989.

Key words: energy conversion, production functions, factor prices, economic evolution

“Anything as important in industrial life as power deserves more attention than it has yet received by economists... a theory of production that really explains how wealth is produced must analyze the contribution of the element energy” (Tryon, 1927). “The decisive mistake of traditional economics... is the neglect of energy as factor of production” (Binswanger and Ledergerber, 1974).

1. Introduction

In conventional economic theory the production factor energy is either neglected altogether or attributed only marginal importance. The argument is that energy’s share in total factor cost is small compared to the cost shares of labor and capital. However, the recessions after the oil price crises in 1973/74 and 1979/81 have raised the question of how a production factor of monetarily minor importance can have such large economic impacts.

The conventional view of the low economic importance of energy dates back to the first stages in the development of a neoclassical economic

theory. Initially, the focus was not on the generation of wealth, but on its distribution and the efficiency of markets. Consequently, the early thinkers in economics started with a model of pure exchange of goods, without considering their production. On the basis of a set of assumptions about rational consumer behavior, it is shown that through the exchange of goods in markets an equilibrium results in which all consumers maximize their utility in the sense that it is not possible to improve the situation of a single consumer without worsening the situation of at least one other (Pareto optimum). This benefit of (perfect) markets is generally considered as the foundation of free-market economics. It shows why markets, where “greedy“ individuals meet, work at all. But later, when the model was extended to include production, the problem of the physical generation of wealth was coupled inseparably to the problem of the distribution of wealth, as a consequence of the model structure: Since the neoclassical equilibrium is characterized by a (profit-maximizing) optimum in the interior – and not on the boundary – of the region in factor space

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accessible to the production system according to the state of technology, factor productivities had to equal factor prices. In the resulting production model the weights with which the production factors contribute to the physical generation of wealth, i.e. the elasticities of production have to be equal to the factor cost shares. These cost shares, in the industrialized countries, are typically 0.7 (labor), 0.25 (capital) and 0.05 (energy).

Consequently, according to the neoclassical model, the elasticities of production of the factors, which – roughly speaking – measure the percentage of output growth if a factor input increases by one percent, would have to have these values: labor 0.7, capital 0.25 and energy 0.05. With these input weights a decrease of energy utilization of up to 7%, as observed during the first oil crisis between 1973 and 1975, could explain a decrease of value added of only $0.05 \times 7\% = 0.35\%$. The actually observed decreases of economic output, however, were roughly ten times larger.

Furthermore, a substantial part of observed long-term economic growth cannot be explained by the growth of the input factors, if these are weighted by their cost shares. Large residuals remain. They are associated with a time-dependent multiplier in the aggregate production function and interpreted as the effects of “technological progress” (Solow, 1957). In most cases, up to 70% of industrial output growth has to be attributed to the unexplained residual “technological progress”.¹ Therefore, this residual usually plays a more important role than the explanatory factors, and this, according to Gahlen (1972), makes the neoclassical theory of production tautological. Solow, after noting “...it is true that the notion of time-shifts in the [production] function is a confession of ignorance rather than a claim of knowledge” (Solow 1960), comments: “This ... has led to a criticism of the neoclassical model: it is a theory of growth that leaves the main factor in economic growth unexplained” (Solow 1994). Within the “New Growth Theory” (Romer 1986, Lucas 1988), a variety of approaches to explain technical progress and growth has been put forward. Howard Pack’s (1994) conclusion is: “But have the recent theoretical insights succeeded in providing a better guide to explaining the actual

growth experience than the neoclassical model? This is doubtful”.²

As has been shown recently, most of the residuals of the neoclassical growth theory can be removed by appropriately taking into account the production factor energy.³ Thereby the previously unexplained technological progress reveals its two principal elements. The first one is the activation of the increasingly automated capital stock by energy; and, of course, the people who handle capital have to be qualified appropriately. The second one consists of improvements in the organizational and energetic efficiency of the capital stock. The short-term impact of the first element is much bigger than that of the second element, but the reverse may be true for the long-term impact, if efficiency improvements fundamentally change the course of economic evolution (Kümmel et al. 2002). The efficiency improvements are identified by shifts of the corresponding technology parameters in the production functions, whereas energy’s high productive power in increasingly automated production processes is revealed by its high elasticity of production. Energy’s elasticity, in industrial sectors of the economy, is typically of the order 0.5, i.e. as large as those of capital and labor together. In service sectors it still exceeds energy’s low cost share significantly (Lindenberger 2000). Both in industrial and service sectors, labor’s elasticity is far below its cost share. Only in the case of capital, elasticities of production and cost shares turn out to be roughly in equilibrium, as neoclassical theory presumes.⁴

In order to analyze the non-equilibrium process toward increasing automation, i.e. the substitution of high-cost human labor by energy-driven and increasingly information-processing capital, we propose the optimization model PRISE of PRice-Induced Sectoral Evolution. The model is designed to analyze potential changes of inputs, outputs and profits in differently energy- and labor-intensive sectors of an economy in response to changing factor prices. Section 2 briefly summarizes the required energy-dependent production theory for industrial and service

¹ Solow attributed 87% of (US non-farm 1909-1949 per capita) growth to the residual. For a more recent review, see (Boskin and Lau, 1992).

² While emphasizing that it is the responsibility of the engineers to specify the production function, Dorfman, Samuelson and Solow (1958), remark “...there seems to have been a misunderstanding somewhere... nearly all the production functions that have actually been derived are the work of economists rather than of engineers.”

³ See, e.g., Ayres (2001), Beaudreau (1998), Hall et al. (2001), Kümmel et al. (1985, 2000, 2002), Lindenberger (2000).

⁴ The production systems are operating in *boundary* cost minima in factor space, where the boundaries, at a given point in time, are established by the state of technology in information processing and automation and prevent the system from sliding at once into the absolute cost minimum of nearly vanishing labor input.

production. In section 3 the optimization model is presented and tested empirically by comparing its predictions with the German sectoral economic evolution between 1968 and 1989. Section 4 presents some conclusions. We hope that our paper contributes to the attempts of incorporating the laws of thermodynamics into economic theory (Hall et al. 2001).

2. Energy-Dependent Production Functions⁵

In deriving energy-dependent production functions our starting point is the observation that in industrial economies the *capital* stock consists of all energy conversion devices and the facilities necessary for their operation and protection. Its fundamental components are heat-engines and transistors, activated by *energy* and handled by *labor*.⁶ They provide every citizen of the industrially developed countries with services that are energetically equivalent to those of ten to thirty hard laboring people. These numbers would more than triple if one included energy for room and process heat.⁷

Economic output Q of value added is created by the cooperation of the production factors capital K , labor L , and energy E . Materials are the passive partners of the production process, which do not contribute actively to the generation of value added. However, if materials become scarce in spite of recycling, growth will of course be constrained. Limits to growth resulting from resource or environmental constraints may be incorporated into the model outlined below by recycling and pollution functions as proposed by Kümmel (1980). Capital K is measured in inflation-corrected monetary units, and so is the output Q , whereas appropriate measures for L are man-hours worked per year, and for E Petajoules (PJ) per year.⁸ Strictly speaking, the production factor E is really exergy. Since, however, the principal fossil and nuclear energy carriers used in industrial economies

are practically all exergy, we stay with the concept of energy for empirical reasons. We normalize all variables to their values Q_0, K_0, L_0, E_0 in a base year. For the quantitative analysis of growth, we employ production functions $q=q[k(t),l(t),e(t);t]$ that describe the evolution of the normalized output $q=Q/Q_0$ as the normalized inputs of capital, $k=K/K_0$, labor, $l=L/L_0$, and energy, $e=E/E_0$ change in time t ; we allow for an explicit time-dependence of q in order to model structural changes, i.e. improvements of organizational and energy conversion efficiencies due to human creativity. We derive production functions from the following growth equation that relates the (infinitesimal) relative change of the normalized output, dq/q , to the relative changes of the normalized inputs, $dk/k, dl/l, de/e$, and creativity's action:

$$\frac{dq}{q} = \alpha \frac{dk}{k} + \beta \frac{dl}{l} + \gamma \frac{de}{e} + C \quad (1)$$

The coefficients α, β , and γ are the above-mentioned elasticities of production of capital, labor, and energy.⁹ Deviating from the neoclassical approach, we drop the assumption that the elasticities are equal to the corresponding factor cost shares. Rather, we determine the elasticities of production by technological and empirical analysis. As long as the influence of human creativity is negligible, technical causality of work performance and information processing in production by capital, labor and energy uniquely determines the output q , and k, l , and e represent, by definition, all factors of production. As a consequence, returns to scale are constant, i.e., $\alpha+\beta+\gamma=1$. A non-zero C , on the other hand, represents influences like human ideas, inventions and value decisions, which, in principle, cannot be measured in physical terms. We model this dichotomy as follows. In a first step, we set C to zero and derive production functions from purely physical/technological considerations in k - l - e -space. In a second step, the effects of a non-zero C will be associated with time-changes of technological parameters to be introduced on the way of deriving the production function (i.e., parameters indicating organizational and energy conversion efficiencies). The requirement that the second-order mixed derivatives of α, β , and γ with respect to the physical factors of production have to be equal results in a set of three differential equations for α, β , and γ (which correspond to the Maxwell relations in thermodynamics): $k(\partial\beta/\partial k)=l(\partial\alpha/\partial l)$, $k(\partial\gamma/\partial k)=e(\partial\alpha/\partial e)$ and $l(\partial\gamma/\partial l)=e(\partial\beta/\partial e)$. Due to the constant returns to scale in k - l - e -space, one of the three elasticities can

⁵ This section is based on Kümmel et al. (1985, 2000) and Lindenberger (2000).

⁶ Other mechanical, electro-mechanical or fluidic conversion devices are, of course, essential, too.

⁷ In 1995 primary energy consumption per capita and day was 133 kWh in Germany and 270 kWh in the USA. Numerically, this corresponds to more than 40 and 90 energy slaves per capita in Germany and the USA, respectively, each one consuming about 3 kWh per day.

⁸ E and L are obtained from the national energy and labor statistics and K and Q from the national accounts. Ideally, one would like to measure K by the amount of work performance and information processing that capital is capable of delivering when being totally activated by energy and labor. Likewise the output Q might be measured by the work performance and information processing necessary for its generation. The detailed, quantitative technological definitions of K and Q are given in Kümmel (1980) and Kümmel et al. (2000). However, information on these quantities is not available. Therefore, we assume proportionality between them and the constant currency data.

⁹ Eq. (1) results from the total differential of the production function. The elasticities of production are $\alpha(k,l,e) \equiv (k/q)(\partial q/\partial k)$, $\beta(k,l,e) \equiv (l/q)(\partial q/\partial l)$, $\gamma(k,l,e) \equiv (e/q)(\partial q/\partial e)$, and the term due to the creativity-induced explicit time-dependence of the production function is $(t/q)(\partial q/\partial t)(dt/t)$.

be eliminated. If one eliminates γ , the differential equation for α is $k(\partial\alpha/\partial k)+l(\partial\alpha/\partial l)+e(\partial\alpha/\partial e)=0$, the equation for β has an identical structure, and the coupling equation reads $l(\partial\alpha/\partial l)=k(\partial\beta/\partial k)$. The most general solutions of the first two equations are $\alpha=f(l/k, e/k)$ and $\beta=g(l/k, e/k)$ with arbitrary differentiable functions f and g . The boundary conditions that determine the solutions of this system of partial differential equations unequivocally would require the knowledge of α on a surface and of β on a curve in k, l, e space. Since it is practically impossible to obtain such knowledge, one has to choose approximate or asymptotic technological boundary conditions (Kümmel 1980).

2.1. Industry

The simplest non-constant solutions of the above differential equations with technologically meaningful boundary conditions for industrial production are $\alpha=a_0(1+e)/k$, $\beta=a_0(c_0l/e-l/k)$, and $\gamma=1-\alpha-\beta$ with technology parameters a_0 and c_0 . Here, a_0 gives the weight with which the ratios of labor to capital and energy to capital contribute to the productive power of capital, and c_0 indicates the energy demand $e_t=c_0k_t$ of the fully utilized capital stock k_t that would be required in order to generate the industrial output totally automated, i.e. with virtually no labor: β goes to zero as e and k approach e_t and k_t . If one inserts these elasticities of production into eq. (1) and integrates, with $C=0$, one obtains the (first) LINEX production function:

$$q_{LI}(k, l, e) = q_0 e \exp \left\{ a_0 \left(2 - \frac{1+e}{k} \right) + a_0 c_0 \left(\frac{1}{e} - 1 \right) \right\} \quad (2)$$

which depends linearly on energy and exponentially on quotients of capital, labor and energy. The integration constant q_0 is the third technology parameter indicating changes in the monetary valuation of the original basket of goods and services making up the output unit Q_0 . Creativity-induced innovations and structural change make a_0 , c_0 , and q_0 time-dependent. To give an example, investments improving the energy conversion efficiency of the capital stock, as experienced substantially in response to the oil-price explosions in the 1970s, lead to a decrease of the parameter c_0 , as analyzed econometrically for the USA, Japan and Germany (Hall et al. 2001, Kümmel et al. 2000, 2002, Lindenberg 2000).

It is important to note that α , β , and γ must be non-negative in order to make sense economically. For instance, the requirement of a non-vanishing β implies that one cannot feed more energy into the machines and energy conversion devices of the capital stock than they can receive according to their technical design, when operating at full capacity. The requirement of non-negative α , β ,

and γ imposes restrictions on the admissible factor quotients in α , β , and eq. (2).

The simplest, i.e. constant α , β , and γ , yield the well-known Cobb Douglas production function $q_{CDE} = q_0 k^{\alpha_0} l^{\beta_0} e^{1-\alpha_0-\beta_0}$. Ex post, this function works also reasonably well (Kümmel et al. 1985, Lindenberg 2000). However, since it allows the (asymptotically) complete substitution of energy by capital, which is not consistent with thermodynamics, it should be avoided when calculating economic scenarios of the future.

2.2. Services

In the following we derive a production function suited to model the evolution of service production. We note that, empirically, in the medium term most progress of automation by computer-based information processing is expected in the traditional service industries of trade, banking, insurance, and public administration (Thome, 1997). Therefore, it is plausible to take into account the possibility that in service production, human routine labor can be substituted by energy-driven and increasingly information processing capital, to some extent, i.e., up to a state of maximum automation.

We model the potential approach toward the state of maximum automation in service production by employing the law of diminishing returns: We assume that the approach toward the state of maximum automation in service production is associated with decreasing returns due to (additional) energy utilization. The simplest corresponding ansatz for energy's elasticity of production is: $\gamma=a_0(c_m-e/k)$, where $c_m=e_m/k_m$ measures the energy demand of the maximum automated capital stock, and a_0 indicates the organizational efficiency of capital, as above. This γ fulfills the differential equation coupling γ and α , using $\alpha=a_0(1+e)/k$ and $\beta=1-\alpha-\gamma$. Inserting these elasticities of production into eq. (1) and integrating, with $C=0$, yields the service production function:

$$q_{DI}(k, l, e) = q_0 l \left(\frac{e}{1} \right)^{a_0 c_m} \exp \left\{ a_0 \left(2 - \frac{1+e}{k} \right) \right\} \quad (3)$$

where, again, non-vanishing effects of human creativity make the technology parameters c_m , a_0 , and q_0 time dependent.

3. The Optimization Model PRise

Based on the sketched energy-dependent production functions for industrial and service sectors of an economy, we propose the optimization model PRise of PRice-Induced Sectoral Evolution. PRise combines the new technological models with standard behavioral assumptions, i.e. utility and profit maximization.

3.1. Model algebra

Due to the evolutionary character of the production functions that incorporate potential progress of automation and the resulting energy-dependence of production, PRISE is a model of time step-wise optimization. At each time step, the sectoral profits are maximized:

$$\max_{p_j, k_j, l_j, e_j} \{p_j q_j(k_j, l_j, e_j) - [w_{k,j} k_j + w_{l,j} l_j + w_{e,j} e_j]\} \quad (4)$$

Optimization variables are the sectoral (index j) factor inputs k , l , e , and the output prices p . In eq. (4), the production function $q_j(k_j, l_j, e_j)$ represents eq. (2) or (3), if j denotes industrial or service production, respectively. The exogenous factor prices, which may be sector-specific, are denoted w .¹⁰

The optimization is performed subject to a set of technological and demand-sided constraints (5)-(17). In each sector and at each time-step Δt of optimization limits to technical progress are modeled by (exogenous) simple bounds on the changes of the sectoral factor-inputs, i.e.,

$$\Delta k_j(t - \Delta t, t) \leq \Delta k_{j, \max}(t - \Delta t, t) \quad (5)$$

$$\Delta l_j(t - \Delta t, t) \leq \Delta l_{j, \max}(t - \Delta t, t) \quad (6)$$

$$\Delta e_j(t - \Delta t, t) \leq \Delta e_{j, \max}(t - \Delta t, t) \quad (7)$$

Since we are interested in the sectoral outputs in real terms (without inflation effects), the optimization is performed subject to an overall time-constant price level,

$$\bar{P}(t) = \bar{P}(t - \Delta t) \quad (8)$$

where

$$\bar{P}(t) = \frac{\sum_j p_j(t) q_j(t)}{\sum_j q_j(t)} \quad (9)$$

The sectoral capital stocks take into account the rates of capital depreciation δ_j and the shares v_j of the investments that serve for emission mitigation at constant energy services (and do not expand that part of the capital stock that produces the standard basket of goods and services defined by the national accounts):

$$k_j(t) = (1 - \delta_j(t - \Delta t))k_j(t - \Delta t) + (1 - v_j(t))I_j(t) \quad (10)$$

The sum of the sectoral investments $I_j(t)$ is constrained by the available (non-consumed) share μ of output produced in the preceding period,

$$\sum_j I_j(t) \leq \mu(t - \Delta t) \sum_j p_j(t - \Delta t) q_j(t - \Delta t). \quad (11)$$

Since investment decisions aim at certain degrees of capacity utilization, the optimization is performed subject to (exogenously) fixed load factors $\eta_j^*(t)$, where $\eta_j(k, l, e, t)$ are homogeneous functions of degree zero characterizing the factor-dependence of the degree of capacity utilization:¹¹

$$\eta_j(k_j, l_j, e_j; t) = \eta_j^*(t) \quad (12)$$

For the sectorally produced goods and services, $q_j(t)$, there is market clearing through the demands $d_j^c(t)$ of the representative consumers c ,

$$q_j(t) = \sum_c d_j^c(t) \quad (13)$$

On the demand side we follow a common approach: The simplest possible demand functions d_j^c result from maximizing a Cobb Douglas-like consumer utility function, observing the consumer's budget restriction, leading to

$$d_j^c = \frac{a_j^c J_c}{p_j} \quad (14)$$

where the a_j^c characterize the consumers' preferences, and J_c are the consumers' incomes. Closing the circular flow of income in a simple manner, the owners of the production factors labor, energy, and capital are chosen as representative consumers, whose incomes read:

$$J_L(t) = \sum_j w_{l,j}(t) l_j(t) \quad (15)$$

$$J_E(t) = \sum_j w_{e,j}(t) e_j(t) \quad (16)$$

$$J_K(t) = \sum_j p_j(t) q_j(t) - J_L(t) - J_E(t) = \sum_j w_{k,j}(t) k_j(t) + \sum_j g_j(t) \quad (17)$$

where the sectoral profits g_j are part of the capital income J_K .

Inserting (14) into (13), multiplying by the output prices p_j , and summing over the production sectors (j), while observing $\sum_j a_j^c = 1$, yields eq. (17). This means that one of the above equations is redundant: market clearing for $N-1$ of the N sectoral markets implies that the N th market is

¹⁰ In the test of PRISE below, the (unweighted) sum of the sectoral profits will be maximized. Alternatively, using the methods of vector optimization, the trade-offs between those profits may be analyzed.

¹¹ The capacity utilization η is an intensive quantity, $\eta(\phi k, \phi l, \phi e) = \eta(k, l, e)$. The employed functional form is $\eta(k, l, e) = (q(k, l, e)/k)(l/k)^\lambda (e/k)^\varepsilon$, where λ and ε are technological parameters to be determined empirically (Lindenberger 2000).

cleared as well, i.e. in PRISE the law of Walras is valid for the produced goods and services.

3.2. Empirical Test of PRISE

In a first model application, the model PRISE was tested by the attempt to reproduce the observed sectoral growth of the German economy 1968-89 by time step-wise optimization. For this purpose, two industrial and two service sectors were considered which together make up about 90 percent of the German gross domestic product (GDP): i) basic materials and producer goods, ii) capital and consumer goods, iii) market services, and iv) non-market services. The model test required considerable effort to collect and construct the appropriate time series data on inputs, outputs, prices and other parameters from various sources.

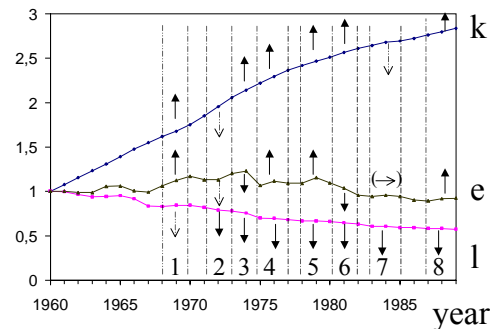
The technological parameters of the sectoral production functions and capacity utilization functions are exogenous to the PRISE-model and were estimated econometrically. The required data (time series of sectoral gross value added, capital stocks, labor and energy inputs, and degrees of capacity utilization) were collected from the German national accounts, labor statistics, energy balances and the German Council of Economic Experts. The PRISE-model is driven by the exogenous historical factor prices. The time series of the user price of capital was constructed according to Jorgenson (1963) as the sum of depreciation, interest, and taxes minus subsidies (Jorgenson, 1963). The price of labor includes wages and social security contributions. Like the capital price, it was assumed to be sector-independent. Since energy prices differ significantly between industrial and non-industrial users, they were constructed separately for manufacturing and service industries. Energy carriers were aggregated according to energy content. Prices were taken from the German Federal Ministry of Economics. The sectoral factor input changes (eqs. (5)-(7)) were allowed to be positive or negative; for the purpose of the model test they were bounded by the magnitudes of the historical factor changes. More details on data construction and model parametrization are given in (Lindenberger 2000).

Based on the historical factor prices, the changes of the sectoral inputs capital, labor, and energy (and thus, via the production functions, also the outputs) were computed by optimization for a number of selected time-steps including the recessions after the oil-price shocks of 1973/74 and 1979/81.

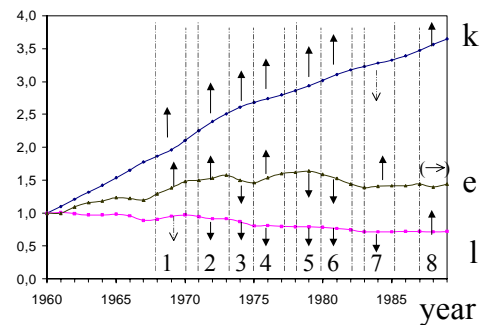
It turns out that the model reproduces the historical evolution of the sectoral factor inputs satisfactorily (*Figure 1*): if the computed input-changes have the same direction as the historical ones, this is indicated by filled arrowheads, otherwise by unfilled ones. (The few horizontal

arrows indicate that the historical factor changes were negligibly small, thus leaving no room for optimization.) The model computes factor changes in accordance with the empirically observed directions for capital in 91% (29/32), for labor in 84% (26/31), and for energy in 97% (29/30) of the cases. Quantitatively, in case of 'correctly' computed directions, for the ratios of computed and empirical factor changes values higher than 0.9 are achieved for capital in 59% (17/29), for labor in 96% (25/26), and for energy in 66% (19/29) of the cases.

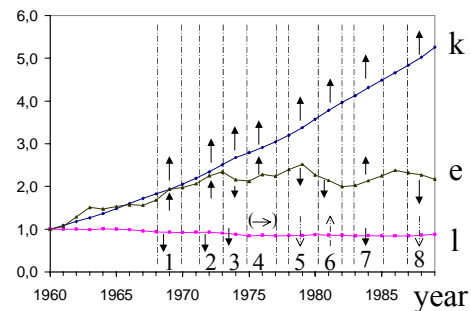
1. Basic materials and producer goods



2. Capital and consumer goods



3. Market services



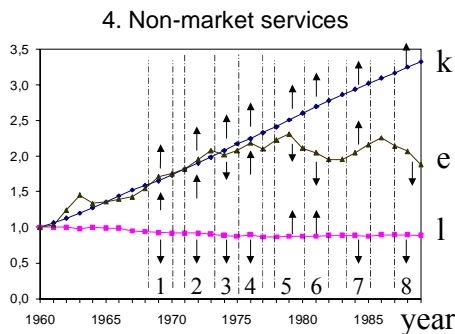


Figure 1: The lines represent the historical evolution of the factor inputs in industrial and service producing sectors of the German economy 1960-1989 (multiples of the respective quantities in 1960). The arrows show the direction of the computed factor input changes by time step-wise optimization with the PRISE-model. Only in case of the unfilled arrowheads the computed directions do not coincide with the empirical ones.

4. Conclusion

Although the laws of thermodynamics state that no production process can be driven without energy conversion, conventional production theory neglects the input factor energy altogether or assigns it only marginal importance. As a consequence, when one tries to reproduce observed economic growth, large unexplained residuals remain. If, on the other hand, the production factor energy is taken into account properly, the activation of the increasingly automated capital stock can be modeled, and the residuals mostly disappear.

Based on econometrically estimated energy-dependent production functions for industrial and service sectors of an economy, we proposed the model PRISE of PRice-Induced Sectoral Evolution. Given the historical factor prices, the model reproduces the historical sectoral growth of the German economy 1968-1989 satisfactorily by time step-wise optimization. Therefore, the model might be a helpful tool to estimate possible future economic developments under varying assumptions on factor prices.

Acknowledgements

Helpful comments and suggestions by four anonymous referees are gratefully acknowledged.

References

Ayres, R. U., 2001, "The minimum complexity of endogenous growth models: the role of physical resource flows", *Energy – The International J.* 26, 817-838.

Beaudreau, B. C., 1998, *Energy and organization: growth and distribution re-examined*, Westwood (CT), Greenwood Press.

Binswanger, H. C., Ledergerber, E., 1974, "Bremsung des Energiezuwachses als Mittel der Wachstumskontrolle", *Wirtschaftspolitik in der Umweltkrise*, J. Wolff (ed), dva, Stuttgart.

Boskin, M. J., Lau, L. J., 1992, "Capital, Technology and Economic Growth, in: *Technology and the Wealth of Nations*, hrsg. von Rosenberg R. et. al.", Stanford University Press, California.

Dorfman, R., Samuelson, P., Solow, R. 1958, *Linear Programming and Economic Analysis*, McGraw-Hill, New York.

Gahlen, B., 1972, "Der Informationsgehalt der neoklassischen Wachstumstheorie für die Wirtschaftspolitik", Tübingen.

Hall, C., Lindenberger, D., Kümmel, R., Kroeger, T., Eichhorn, W., 2001, "The Need to Reintegrate the Natural Sciences with Economics", *BioScience* 51 (8), 663-673.

Jorgenson, D. W., 1963, "Capital Theory and Investment Behavior", *American Economic Review* 53, 247-259.

Kümmel, R., 1980, "Growth Dynamics of the Energy Dependent Economy, *Mathematical Systems in Economics*", Vol. 54, Hain, Königstein/Ts. and Oelgeschlager, Gunn & Hain, Cambridge Mass.

Kümmel, R., Strassl, W., Gossner, A.; Eichhorn, W., 1985, "Technical Progress and Energy Dependent Production Functions", *J. of Economics* 45, 285-311.

Kümmel, R., Lindenberger, D., Eichhorn, W., 2000, "The Productive Power of Energy and Economic Evolution", *Indian J. of Applied Economics* 8 (Special Issue in Honor of P.A. Samuelson), 231-262.

Kümmel, R., Henn, J., Lindenberger, D., 2002, "Capital, Labor, Energy and Creativity: Modelling Innovation Diffusion, Structural Change and Economic Dynamics", (in press) <http://theorie.physik.uniuerzburg.de/TP1/kuemme1/profile.html>.

Lindenberger, D., 2000, "Wachstumsdynamik industrieller Volkswirtschaften – Energieabhängige Produktionsfunktionen und ein faktorpreisgesteuertes Optimierungsmodell, *Metropolis*", Marburg.

Lindenberger, D., Eichhorn, W., Kümmel, R. 2001, "Energie, Innovation und Wirtschaftswachstum", *Z. für Energiewirtschaft*, 25, 273-282.

Lukas, R. E., 1988, "On the Mechanics of Economic Development", *J. of Monetary Economics* 22, 3-42.

Pack, H., 1994, "Endogenous Growth Theory: Intellectual Appeal and Empirical Shortcomings", *J. of Economic Perspectives*. 8, 55-72.

Romer, P. M., 1986, "Increasing Returns and Long Run Growth", *J. of Political Economy* 94, 1002-37.

Solow, R. M., 1957, "Technical Change and the Aggregate Production Function", *The Review of Economics and Statistics* 39, 312-320.

Solow, R. M., 1960, "Investment and Technical Progress", *Mathematical Methods in the Social Sciences*, K. J. Arrow, S. Karlin, and P. Suppes ed., Stanford, 89-104.

Solow, R. M., 1994, "Perspectives on Growth Theory", *J. of Economic Perspectives* 8, 45-54.

Thome, R., 1997, "Arbeit ohne Zukunft? Organisatorische Konsequenz der wirtschaftlichen Informationsverarbeitung", Vahlen, München.

Tryon, F. G., 1927, "An index of consumption of fuels and water power", *J. of the American Statistical Association* 22, 271-282.