December 1968

Energy Deposition by Electron Beams and δ Rays

E. J. Kobetich
University of Nebraska-Lincoln

Robert Katz
University of Nebraska-Lincoln, rkatz2@unl.edu

Phys. Rev. 170, 391 - 396 (1968)

Energy Deposition by Electron Beams and δ Rays

E. J. Kobetich and Robert Katz
Behlen Laboratory of Physics, University of Nebraska, Lincoln, Nebraska

Received 18 December 1967

The product of two empirical relations, for the practical range and the transmission probability of normally incident electrons through plane sheets of matter, may be differentiated to yield a simple formulation of the energy deposition by electron beams, in agreement with more complex formulations and with experimental data. When combined with the δ-ray distribution formula, these results provide a theory of the spatial distribution of ionization energy about the path of a rapidly moving ion, which is basic to theories of radiation damage and detection.

Online at URL: http://link.aps.org/abstract/PR/v170/p391

DOI: 10.1103/PhysRev.170.391

Energy Deposition by Electron Beams and δ Rays*

E. J. Kobetich and Robert Katz
Behlen Laboratory of Physics, University of Nebraska, Lincoln, Nebraska
(Received 18 December 1967)

The product of two empirical relations, for the practical range and the transmission probability of normally incident electrons through plane sheets of matter, may be differentiated to yield a simple formulation of the energy deposition by electron beams, in agreement with more complex formulations and with experimental data. When combined with the δ-ray distribution formula, these results provide a theory of the spatial distribution of ionization energy about the path of a rapidly moving ion, which is basic to theories of radiation damage and detection.

I. INTRODUCTION

CALCULATIONS of the dissipation of the energy of normally incident beams of electrons in matter have been made by Spencer,[1] by Berger,[2] and others, while many experimental studies have been made for a variety of electron energies and materials. Nevertheless, simple and accurate formulations of the energy dissipation have not hitherto been achieved. For the present work a practical range-energy relationship in a form originally formulated by Weber,[3] and an expression for the transmission probability of electrons through slabs due to Rao,[4] have been combined to yield the dissipation of electron energy, in good agreement with calculations of Spencer and a wide variety of experimental data.

The spatial distribution of the energy loss of a rapidly moving ion has been calculated for radiobiological effects by Hutchinson[5] and by Butts and Katz,[6] and for studies of the width of heavy-ion tracks in electron-sensitive emulsion by Katz and Butts.[7] In these calculations rectilinear δ-ray paths and a power-law range-energy relation were assumed, while binding effects were neglected. To overcome these neglects, the dissipation of energy of normally incident electron beams was formulated and applied to a δ-ray distribution formula incorporating binding effects. The resulting calculation of the transverse distribution of ionization energy is basic to theories of radiation damage and detection.

II. PRACTICAL RANGE-ENERGY RELATION FOR ELECTRONS

The relation

$$r=Aw[1-B/(1+Cw)] \qquad (2.1)$$

given by Weber[3] for the practical range r in aluminum of monoenergetic electrons of energy w between 3 keV and 3 MeV has been extended to the energy interval 0.3 keV–20 MeV by small adjustments of the constants. These have the value $A=5.37\times10^{-4}$ g cm^{-2} keV^{-1}, $B=0.9815$, and $C=3.1230\times10^{-3}$ keV^{-1}. Of these, only the constant B lies outside the uncertainty assigned by Weber. In the limit of low energies, Eq. (2.1) reduces to

$$r=9.93w\ \mu\text{g cm}^{-2}\ \text{keV}^{-1} \qquad (2.2)$$

or about 1 Å/eV in water.

Experimental data reported by Katz and Penfold,[8] Young,[9] Kanter and Sternglass,[10] and Cosslett and Thomas[11] are plotted over Eq. (2.1) in Fig. 1, where the radius of a plotted point represents 10% uncertainty.

In subsequent sections of this paper, Eq. (2.1) is used to approximate practical range-energy relations for all materials.

III. TRANSMISSION OF NORMALLY INCIDENT ELECTRONS THROUGH FOILS

A simple formula for the transmission of monoenergetic (0.01–3 MeV) electrons normally incident on foils is given by Rao[4] as

$$\eta=[1+\exp(-gh)]/\{1+\exp[g(t/r-h)]\}, \qquad (3.1)$$

where η is the fraction of incident electrons of energy w and practical range r transmitted by an absorber of thickness t. In an absorber of atomic number Z and mass number A the constants g and h characteristic of the absorber are given by

$$g=9.2\text{Z}^{-0.2}+16\text{Z}^{-2.2} \qquad (3.2)$$

and

$$h=0.63\text{Z}/\text{A}+0.27.$$

For mixed materials and compounds, Z and A are replaced by their average values, weighted over the mass fractions.

* Supported by the U.S. Atomic Energy Commission and the National Science Foundation.

[1] L. V. Spencer, Natl. Bur. Std. (U.S.) Monograph **1** (1959).
[2] M. J. Berger, in *Methods in Computational Physics*, edited by B. Alder, S. Fernbach, and M. Rotenberg (Academic Press Inc., New York, 1963), Vol. 1, p. 135.
[3] K. H. Weber, Nucl. Instr. Methods **25**, 261 (1964).
[4] B. N. Subba Rao, Nucl. Instr. Methods **44**, 155 (1966).
[5] F. Hutchinson, in *Medical and Biological Aspects of the Energies of Space*, edited by Paul A. Campbell (Columbia University Press, New York, 1961), p. 349.
[6] J. J. Butts and R. Katz, Radiation Res. **30**, 855 (1967).
[7] R. Katz and J. J. Butts, Phys. Rev. **137**, B198 (1965).
[8] L. Katz and A. S. Penfold, Rev. Mod. Phys. **24**, 28 (1952).
[9] J. R. Young, Phys. Rev. **103**, 292 (1956).
[10] H. Kanter and E. J. Sternglass, Phys. Rev. **126**, 620 (1962).
[11] V. E. Cosslett and R. N. Thomas, Brit. J. Appl. Phys. **15**, 1283 (1964).

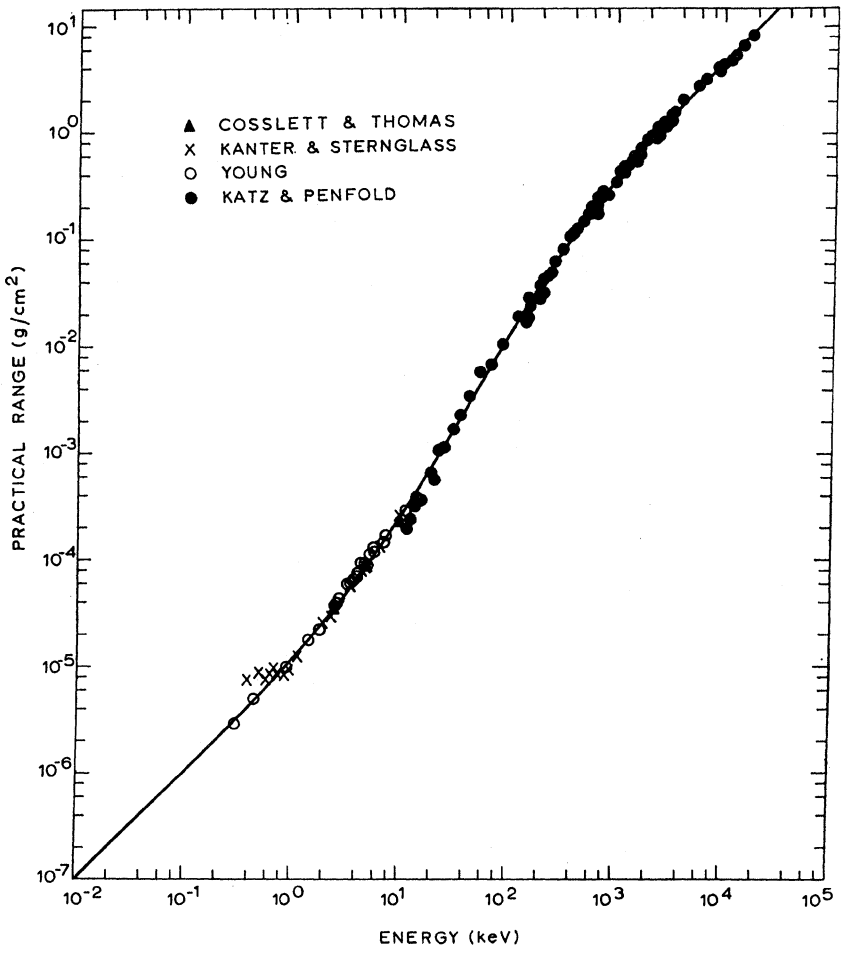

FIG. 1. Practical range versus energy for electrons in aluminum. Data from Refs. 8–11 are plotted over Eq. (2.1). The radius of each data point represents a 10% uncertainty.

IV. ENERGY DISSIPATION OF NORMALLY INCIDENT ELECTRONS

Electrons of range r which penetrate a foil of thickness t have residual energy W which can be found from Eq. (2.1) as the energy to go to the residual range $r-t$. The residual energy may be written in functional form as

$$W(r,t)=w(r-t). \tag{4.1}$$

The energy transmitted through a foil is then approximated by the product of η, the probability of transmission, and W, the residual energy. The energy E dissipated at depth t by a beam containing 1 electron per cm² may be represented as

$$E=d(\eta W)/dt. \tag{4.2}$$

Not all of the energy $d(\eta W)$, failing to penetrate a layer dt, is deposited there, for some of the energy is back scattered into earlier layers. In first approximation, however, there is a compensating energy deposition in dt from electrons back scattered from later layers.

The dissipation of the energy of normally incident beams of electrons has been calculated from Eq. (4.2) for a variety of materials and energies for comparison with the calculations of Spencer[1] and an assortment of experimental data, with good results. Energy dissipation as a function of depth in C, Al, Cu, Sn, and Pb for normally incident electrons of energies 0.025, 0.1, 1, and 10 MeV is displayed in Fig. 2, with the results of Spencer plotted for comparison. The agreement is best at low energies and at low Z, but is satisfactory for many purposes at all energies and all Z. Calculations from Eq. (4.2) are compared with experimental data in Figs. 3 and 4. The fractional energy dissipation of 1–8-keV electron beams measured by Kanter[12] for C is shown in Fig. 3(a), and for Al in Fig. 3(b).

Experimental data for low, intermediate, and high Z are shown in Figs. 4(a)–4(c). Energy dissipation data for 32-keV electrons in air from Grün,[13] for 57- and 104-keV electrons in Al from Huffman,[14] and for 1- and 2-MeV electrons in Al from Nakai,[15] are shown in Fig. 4(a). Data for Cu at 10 and 20 keV from Cosslett

[12] H. Kanter, Phys. Rev. **121**, 677 (1961).
[13] A. E. Grün, Z. Naturforsch. **12a**, 89 (1957).
[14] F. N. Huffman, J. S. Cheka, B. G. Saunders, R. H. Ritchie, and R. D. Birkhoff, Phys. Rev. **106**, 435 (1957).
[15] Y. Nakai, Japan J. Appl. Phys. **2**, 743 (1963).

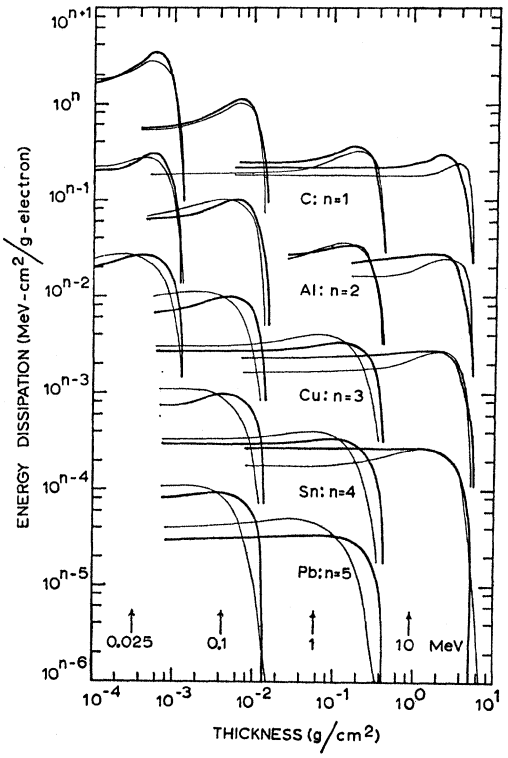

FIG. 2. Energy dissipation versus thickness for 0.025-, 0.1-, 1-, and 10-MeV electrons in C, Al, Cu, Sn, and Pb. Light lines are from Spencer's theory (see Ref. 1) while dark lines are computed from Eq. (4.2).

and Thomas[16] and at 1 and 2 MeV from Nakai[15] are shown in 4(b). Data for Au at 10 and 20 keV from Cosslett and Thomas[16] and for Pb at 1 and 2 MeV from Nakai[15] are shown in Fig. 4(c). In all cases, the experimental data are plotted over curves arising from the empirical formulation of Eq. (4.2). Again, agreement is best at low energy and for material of low Z.

For a large range of materials and energies, the agreement between the present empirical formulation and experimental data is of the order of magnitude of the experimental error, and is clearly adequate for the purpose intended; the calculation of the spatial distribution of ionization energy about the path of a rapidly moving ion, particularly in materials of low and intermediate Z.

In the absence of needed data, the present formulation provides a basis for extrapolation to the energy dissipation of electrons of lower energies.

V. SPATIAL DISTRIBUTION OF IONIZATION ENERGY

The effective charge ze, of an ion of atomic number Z, moving through matter at speed $v=\beta c$, has been given by Barkas[17] as

$$ze=Ze[1-\exp(-125\beta Z^{-2/3})]. \tag{5.1}$$

The number $dn/d\omega$ of δ rays per cm having energies between ω and $\omega+d\omega$ liberated from matter having N free electrons per cm³ by a passing ion of effective charge ze is given by Bradt and Peters[18] as

$$\frac{dn}{d\omega}=\frac{2\pi Nz^2e^4}{mv^2}\frac{1}{\omega^2}\left[1-\beta^2\frac{\omega}{\omega_m}+\frac{\pi\beta z}{137}\left(\frac{\omega}{\omega_m}\right)^{1/2}\left(1-\frac{\omega}{\omega_m}\right)\right], \tag{5.2}$$

where $\omega_m=2mc^2\beta^2\gamma^2$, and e and m are the electron charge and mass.

To treat the case of electrons initially bound to a parent atom with mean ionization potential I, the experiments of Rudd, Sauter, and Bailey[19] lead to the interpretation of ω as the total energy imparted to the ejected electron whose kinetic energy w is related to these quantities through the expression

$$\omega=w+I. \tag{5.3}$$

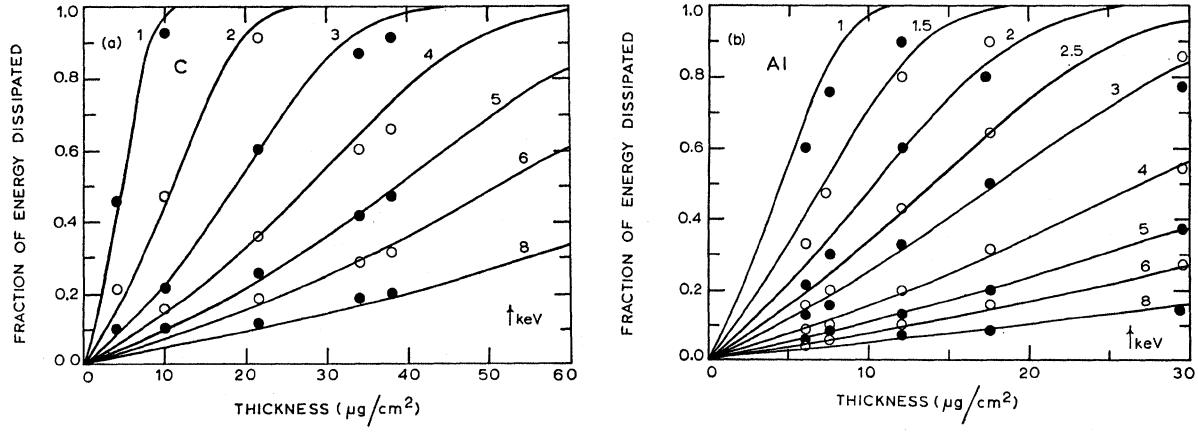

FIG. 3. Fractional energy dissipation of 1–8-keV electrons in (a) carbon and (b) aluminum. Data are from Kanter (see Ref. 12). Alternating solid and hollow circles are for clarity. The curves arise from Eq. (4.2).

[16] V. E. Coslett and R. N. Thomas, Brit. J. Appl. Phys. **16**, 779 (1965).
[17] W. H. Barkas, *Nuclear Research Emulsions* (Academic Press Inc., New York, 1963), Vol. 1, p. 371.
[18] H. L. Bradt and B. Peters, Phys. Rev. **74**, 1828 (1948).
[19] M. E. Rudd, C. A. Sauter, and C. L. Bailey, Phys. Rev. **151**, 20 (1966).

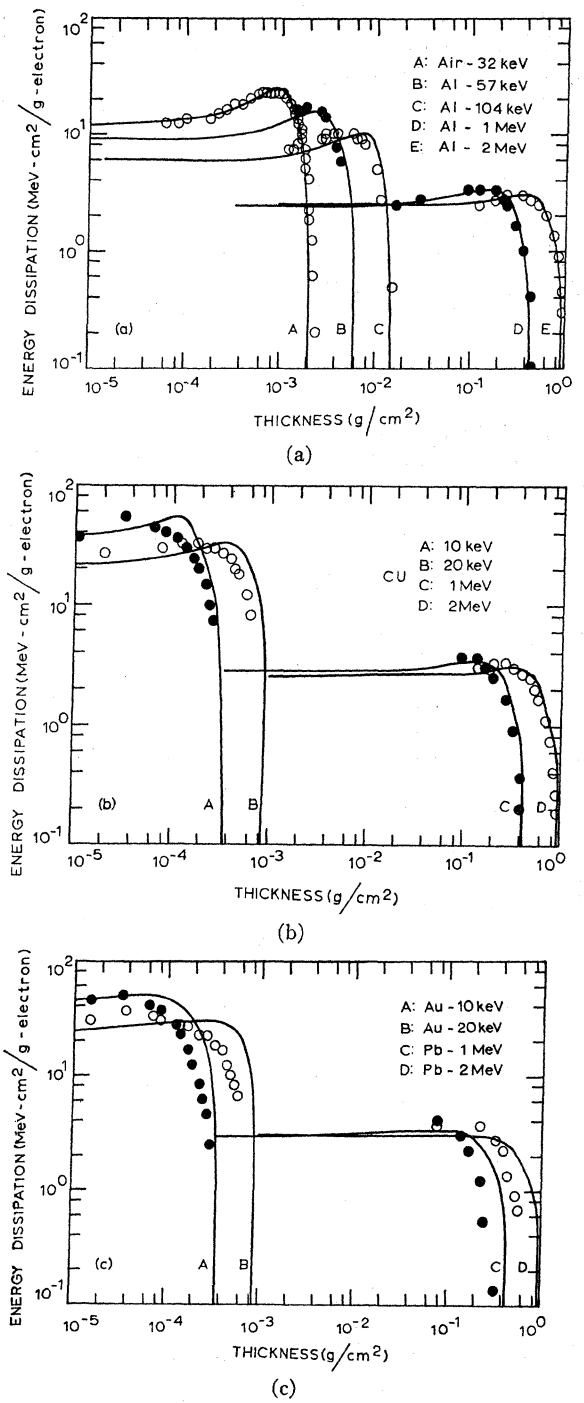

FIG. 4. Energy dissipation for electrons of various energies incident on (a) air and aluminum, (b) copper, and (c) gold and lead. In each case a curve based upon Eq. (4.2) is compared with experimental data.

Equation (5.2) must be summed for composite materials in which there are N_i electrons per cm³ having average binding energy I_i.

In many applications of the spatial distribution of ionization energy, interest is focused on the energy distribution relatively close to the ion's path. From classical kinematics it can be shown that electrons of energy ω are ejected at an angle θ to the path of the moving ion, given by

$$\cos^2\theta = \omega/\omega_m \tag{5.4}$$

for the collision between a free electron and the ion. For distances from the ion's path substantially less than the range of the δ ray of energy ω_m, it is therefore sufficient to consider that all δ rays are normally ejected; and that their energy dissipation in cylindrical shells, whose axis is the ion's path, may be found from knowledge of the energy dissipation of normally incident electrons onto sheets of matter.

If $\mathbf{E}$ is the energy flux carried by δ rays through a cylindrical surface of radius t whose axis is the ion's path, the energy density E deposited in a cylindrical shell of unit length and mean radius t is given by the expression

$$E = -(2\pi t)^{-1} d\mathbf{E}/dt. \tag{5.5}$$

To calculate the energy flux $\mathbf{E}$ we require appropriate values for the binding energy, obtained from energy loss studies as reported by Berger and Seltzer[20] and Hutchinson and Pollard,[21] and integrate the energy dissipation over the δ-ray distribution formula, summing over the atoms making up the medium.

We write

$$\mathbf{E} = \sum_{i=1}^{j} \int_{w_t}^{\omega_m - I_i} W(t, w)\eta(t, w)(dn_i/dw)\,dw. \tag{5.6}$$

Because of the analytic forms of each of the expressions making up the integrand, the computation may be simplified by carrying out the differentiation indicated in Eq. (5.5) before carrying out the integration of Eq. (5.6).

Numerical computations have been carried out for a variety of materials from $\beta = 0.01$ to $\beta = 0.99$. The reduction in the number of ejected δ rays due to the density effect at high ion speeds has been ignored, since this effect contributes a reduction in the stopping power of protons in Be of about 5% at the highest speed calculated and which drops off rapidly with decreasing speed.[22]

Results of the calculations are displayed in Figs. 5–7 for C, Cu, and Au. In order to present the results in a form independent of the atomic number of the bombarding ion, the contribution of the third term in the square brackets of Eq. (5.2) is separately presented as a correction to the first two terms, significant at large β and Z. Except for distances up to, say, 10^{-6} g/cm² (where the influence of binding is important) and at distances approximating the range of δ rays of maximum

[20] M. J. Berger and S. M. Seltzer, Natl. Acad. Sci.—Natl. Res. Council, Publ. **1133**, 205 (1964).

[21] F. Hutchinson and E. Pollard, in *Mechanisms in Radiobiology*, edited by M. Errera and A. Forssberg (Academic Press Inc., New York, 1961), Vol. 1, p. 1.

[22] W. H. Barkas and M. J. Berger, Natl. Acad. Sci. —Natl. Res. Council, Publ. **1133**, 103 (1964).

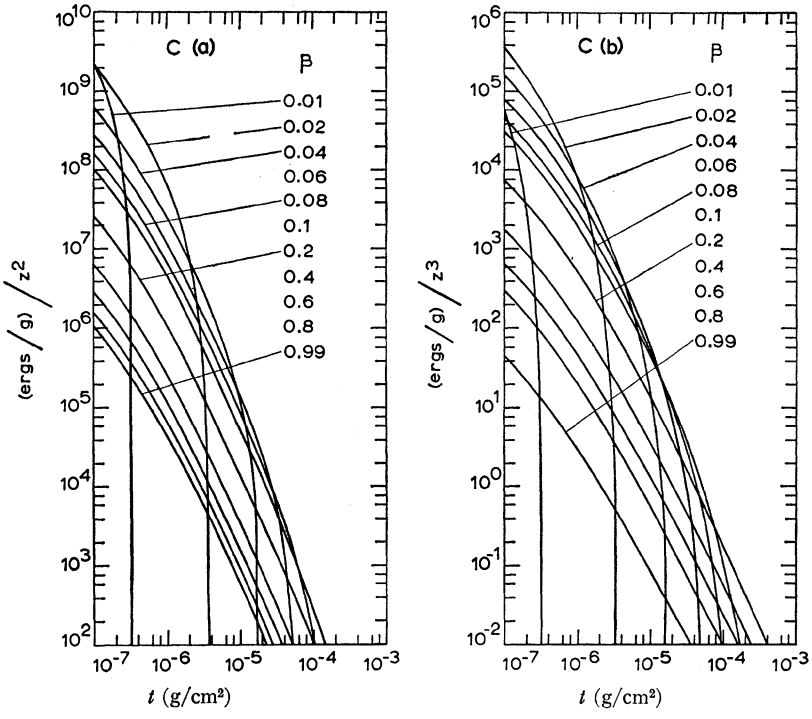

FIG. 5. Spatial distribution of ionization energy in carbon. To find the energy deposited at radius t, in erg/g, by an ion of atomic number Z moving at speed βc the value given in curve (b) must be multiplied by the effective charge number z [Eq. (5.1)], added to the value obtained from (a), and the sum multiplied by z^2.

energy, the results may be approximated at const β by the relation

$$E/z^2 \sim 1/t^2, \tag{5.7}$$

that is, the dose of deposited energy is directly proportional to the square of the effective charge of the ion and is inversely proportional to the square of the distance from the ion's path.

VI. APPLICATIONS

Calculations of the spatial distribution of ionization energy, as presented here, have been applied to the

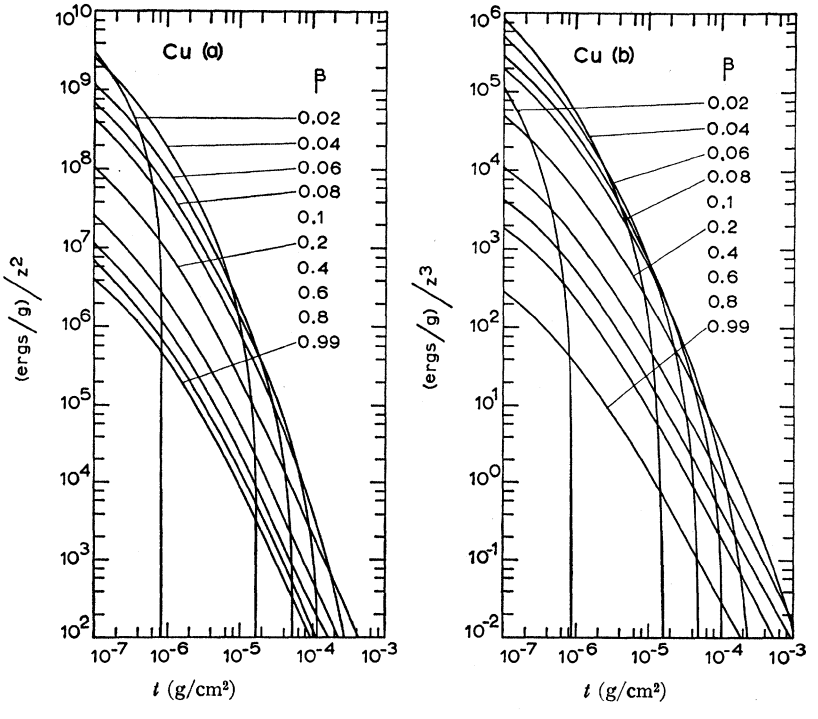

FIG. 6. Spatial distribution of ionization energy in copper. See caption for Fig. 5.

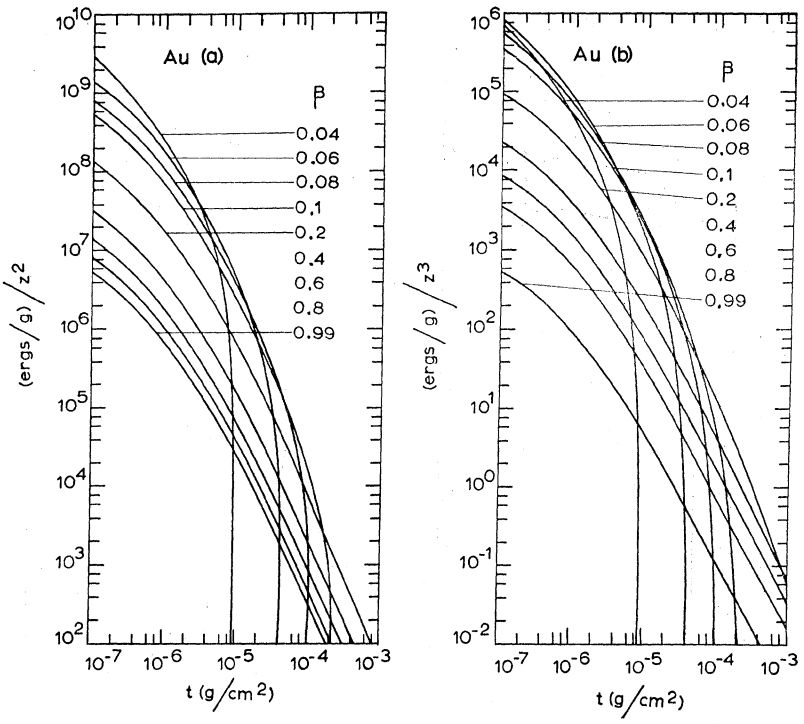

FIG. 7. Spatial distribution of ionization energy in gold. See caption for Fig. 5.

analysis of the response of NaI(Tl) to heavy ions[23] and to the analysis of the formation of etchable tracks in dielectrics.[24] They have been applied, with modification of the angular distribution of the ejected electrons, to the width of heavy-ion tracks in emulsion.[25] One immediate application of Eq. (5.7) is to the study of the width of the tracks of relativistic heavy ions in electron-sensitive emulsion. It may be assumed that the track width, whether characterized photometrically or by tracing track profiles, corresponds to profiles of constant energy deposition. At const β and E, $t \sim z$, so that the width of relativistic ion tracks should be proportional to the ion's charge. This is consistent with experimental findings of Kristiansson, Mathiesen, and Stenman.[26]

The energy dosage required for the several detection and damage processes thus far studied, ranges over many orders of magnitude, from 10^4 to 10^9 erg/g, as appropriate to the characteristic "sensitive volume" for the process. The sensitive volume ranges in size from a molecule to a photographic grain. Additionally, the response of the sensitive volume to radiation may be usefully characterized in terms of the cumulative Poisson distribution as a one-hit or a many-hit process.

The most sensitive material thus far studied is Ilford G-5 emulsion, which requires only 6×10^3 erg/cm³ for grain sensitization, while the least sensitive material is muscovite mica, which requires 3.5×10^9 erg/g for the production of etchable tracks. Both of these are many-hit processes.

It is somewhat surprising that two apparently unrelated phenomena, the response of scintillation crystals and the inactivation of biological molecules,[7] respond to identical analyses. Both are one-hit processes. For NaI(Tl) crystals the characteristic dose is 4×10^7 erg/cm³, close to the dose required to inactivate viruses, while enzymes require up to 10^9 erg/cm³.

[23] R. Katz and E. J. Kobetich, first following paper, Phys. Rev. **170**, 397 (1968).
[24] R. Katz and E. J. Kobetich, second following paper, Phys. Rev. **170**, 401 (1968).
[25] E. J. Kobetich and R. Katz, third following paper, Phys. Rev. **170**, 405 (1968).
[26] K. Kristiansson, O. Mathiesen, and A. Stenman, Arkiv Fysik **23**, 479 (1963).