Energy Detection of Unknown Signals in Fading and Diversity Reception

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***Abstract*—A comprehensive performance analysis of the energy detector over fading channels with single antenna reception or with antenna diversity reception is developed. For the no-diversity case and for the maximal ratio combining (MRC) diversity case, with either Nakagami-m or Rician fading, expressions for the probability of detection are derived by using the moment generating function (MGF) method and probability density function (PDF) method. The former, which avoids some difficulties of the latter, uses a contour integral representation of the Marcum-Q function. For the equal gain combining (EGC) diversity case, with Nakagami-m fading, expressions for the probability of detection are derived for the cases $L = 2, 3, 4$ and $L > 4$, where L is the number of diversity branches. For the selection combining (SC) diversity, with Nakagami-m fading, expressions for the probability of detection are derived for the cases $L = 2$ and $L > 2$. A discussion on the comparison between MGF and PDF methods is presented. We also derive several series truncation error bounds that allow series termination with a finite number of terms for a given figure of accuracy. These results help quantify and understand the achievable improvement in the energy detector's performance with diversity reception. Numerical and simulation results are also provided.**

***Index Terms*—Energy detection, MGF approach, cognitive radio, Rayleigh fading, Nakagami-*m* fading, Rician fading, maximal ratio combining, equal gain combining, selection combining, Marcum-Q integrals.**

I. Introduction

IN [1], the problem of energy detection of unknown signals over a noisy channel, which has a myriad of applications in traditional communications and in emerging cognitive radio networks and ultra-wideband (UWB) radio, has been addressed. An energy detector may help cognitive radios to determine whether or not a primary user signal is present. Cognitive radio, UWB and other applications have heightened the need for a more comprehensive analysis of the energy detector's performance in different wireless environments. Moreover, since diversity reception techniques can enhance the performance of the energy detector, the resulting performance improvement needs to be quantified.

The energy detector is a threshold device, whose output decision depends on the comparison of the incoming signal energy to the threshold. This decision problem is a binary hypothesis test with a chi-square (χ^2) distributed decision variable [1]. The main performance metrics are the probability of detection (P_d) and the probability of false alarm (P_f), which require averaging over the fading statistics when the energy detector is used for the detection of signals over a fading channel. Several papers have previously attacked this problem. For example, Kostylev [2] has derived the average P_d and the average P_f for Rayleigh, Rician and Nakagami-m fading channels. But he considers only the integer values of the Nakagami fading severity index m. Digham, Alouini and Simon [3] derive the P_d in Nakagami-m fading channel limiting to integer m and in Rician fading channel limiting to unity time bandwidth product ($u = 1$ in our notation). These prevailing results are neither completely general nor comprehensive enough to analyze the energy detection with and without diversity reception techniques.

For example, energy detection with maximal ratio combining (MRC), selection combining (SC), and switch-and-stay diversity is analyzed in [3] [4] and with SC and MRC in [5]. However, these results restricted to Rayleigh fading. This restriction may be perhaps due to lack of direct integral results of the Marcum-Q function [6], [7]. Our results presented in [8] partially fulfill this gap by deriving exact P_d and P_f for an equal gain combining (EGC) detector with i.i.d. Nakagami-m fading branches. Further, our work [9] proposes an energy detection performance analysis technique based on the moment generating function (MGF) while [10] considers SC diversity over Nakagami-m channels.

Previous performance analysis studies [1]–[5] primarily utilize the probability density function (PDF) based approach, i.e. the conditional detection probability is integrated over the PDF of the output signal-to-noise-ratio (SNR). All these works assume that the channel state information is available at the receiver.

In this paper, while using the very same conditions at the receiver, a new performance analysis approach – based on the contour integral representation of Marcum-Q function and MGF of the SNR – is presented. This approach along with

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the conventional PDF approach provides a flexible, general framework for analyzing the performance of the energy detector. The MGF approach, which avoids some difficulties of the PDF method, uses a contour integral representation of the Marcum-Q function [11]. Several fading models and diversity techniques are considered. For the no-diversity case and for the MRC diversity case, with either Nakagami-*m* or Rician fading, P_d expressions are derived by using the MGF and PDF approaches. For the EGC diversity case, with Nakagami-*m* fading, P_d expressions are derived for the cases $L = 2, 3, 4$ and $L > 4$, where L is the number of diversity branches. For the SC diversity case, with Nakagami-*m* fading, P_d expressions are derived for the cases $L = 2$ and $L > 2$. We also derive several truncation error bounds that allow series termination with a finite number of terms for a given figure of accuracy. These results help quantify and understand the achievable improvement in the energy detector's performance with diversity reception. Numerical and simulation results are also provided to complement theoretical formulations.

The rest of the paper is organized as follows. Section II describes the system model. For the no-diversity case, considered in section III derives P_d over Nakagami-m and Rician fading channels using the PDF and MGF approaches. Section IV treats the detector performance with MRC diversity over Nakagami-*m* and Rician fading. Section V presents the detector performance with EGC diversity for independent and identically distributed (i.i.d.) Nakagami-*m* diversity branches. The number of diversity branches $L = 2, 3$ and $L \geq 4$ cases are treated separately, and the PDF approach is followed. The results for P_d and P_f over SC with i.i.d. Nakagami-*m* fading branches are derived in Section VI using the PDF approach. The results are two fold. First, the dual SC performance is analyzed by deriving an exact equation for average P_d. Second, SC with an arbitrary number branches is considered, and the Nakagami parameter m is restricted to be an integer. The truncation error bounds are derived where applicable. Section VII discusses numerical and simulation results. Section VIII provides concluding remarks.

II. System Model and Detection over Fading Channel

For the sake of brevity, only a brief discussion of the system model is provided here. We refer the reader to [1], [2] for detailed derivations of these fundamental results. The received signal process $y(t)$, which contains an unknown deterministic signal and noise or noise only, may be modeled as [1]:

$$y(t) = \begin{cases} n(t) & : H_0 \\ hs(t) + n(t) & : H_1, \end{cases}$$

where h is the complex channel gain, $s(t)$ is the unknown transmit signal, $n(t)$ is an additive noise signal, and H_0 and H_1 refer to signal absence and signal presence, respectively. The energy detector operates by filtering, squaring and integrating the received signal $y(t)$. We suppress the details of these operations for brevity and note that the decision variable (Y) of the energy detector may be represented as

$$Y = c \int |y(t)|^2 \, dt,$$

where c is a constant. By using the sampling theorem representation for bandlimited signals, we can approximate the decision variable as a sum of squares of Gaussian random variables [1], [2], [4]. Then Y has a non-central chi-square distribution under H_1 and central chi-square distribution under H_0. Thus, the PDF of Y under H_0 and H_1 can be written as [4],

$$f_Y(y) = \begin{cases} \frac{1}{2^u \Gamma(u)} y^{u-1} e^{-\frac{y}{2}} & : H_0 \\ \frac{1}{2}\left(\frac{y}{2\gamma}\right)^{\frac{u-1}{2}} e^{-\frac{2\gamma+y}{2}} I_{u-1}(\sqrt{2\gamma y}) & : H_1 \end{cases}$$

where $\Gamma(.)$ is the gamma function and $I_n(.)$ is the n^{th} order modified Bessel function of the first kind. The parameter u depends on the time-bandwidth product. The signal-to-noise ratio (SNR) is defined by $\gamma = |h|^2 \frac{E_s}{N_0}$ where E_s is the signal energy and N_0 is the noise-power spectral density. Hence detection (P_d) and false alarm (P_f) probabilities conditional on the fading channel gain may be expressed as [4]

$$P_d = Q_u\left(\sqrt{2\gamma}, \sqrt{\lambda}\right), \tag{1}$$

$$P_f = \frac{\Gamma\left(u, \frac{\lambda}{2}\right)}{\Gamma(u)}, \tag{2}$$

where $Q_u(.,.)$ is the generalized Marcum-Q function and the incomplete gamma function $\Gamma(a, x) = \int_x^\infty t^{a-1} e^{-t} dt$, with $\Gamma(a, 0) = \Gamma(a)$ [3]. Note that in (2), the false alarm probability does not depend on SNR, fading or the diversity reception scheme.

III. Probability of Detection over Fading Channels – No-diversity case

A. Nakagami-m Fading - PDF approach

Here the instantaneous SNR is a Gamma random variable. The PDF of a Gamma $\mathcal{G}(\alpha, \beta)$ variable is given by

$$f(x) = \frac{1}{\Gamma(m)} \left(\frac{1}{\beta}\right)^{\alpha} x^{\alpha-1} \mathrm{e}^{-x/\beta}, \; x \geq 0, \tag{3}$$

where the shape parameter $\alpha > 0$ and the scale parameter $\beta > 0$. When the received signal amplitude follows Nakagami-*m* fading, the SNR γ is Gamma $\mathcal{G}(m, \overline{\gamma}/m)$ where $\overline{\gamma}$ is the average SNR and $m \geq \frac{1}{2}$ is the fading severity index.

The average detection probability over Nakagami-*m* fading $(\overline{P}_{d,Nak})$ can be evaluated by substituting the alternative series representation of Marcum-Q function [12, (4.63)] in (1) and integrating over $\mathcal{G}(m, \overline{\gamma}/m)$ as

$$\overline{P}_{d,Nak} = 1 - \frac{e^{-\frac{\lambda}{2}}}{\Gamma(m)} \left(\frac{m}{\overline{\gamma}}\right)^m \sum_{n=u}^{\infty} \left(\frac{\lambda}{2}\right)^{\frac{n}{2}} \times \int_0^\infty e^{-\left(1+\frac{m}{\overline{\gamma}}\right)\gamma} \gamma^{m-1-\frac{n}{2}} I_n(\sqrt{2\lambda\gamma}) \, d\gamma. \tag{4}$$

Using [13, (6.643-2)], [13, (9.220-2)] and appropriately selecting terms to satisfy the condition therein, $\overline{P}_{d,Nak}$ can be expressed as

$$\overline{P}_{d,Nak} = 1 - e^{-\frac{\lambda}{2}} \left(\frac{m}{\overline{\gamma}+m}\right)^m \sum_{n=u}^{\infty} \frac{1}{n!} \left(\frac{\lambda}{2}\right)^n \times {}_1F_1\left(m; \, n+1; \, \frac{\lambda\overline{\gamma}}{2(\overline{\gamma}+m)}\right). \tag{5}$$

The function ${}_1F_1(.,.,.)$ is a special case $(p=1, q=1)$ of generalized Hypergeometric function given in (6), [14, pp. 19]:

$$ {}_pF_q(a_1,...,a_p;\ b_1,...,b_q;\ x) = \sum_{n=0}^{\infty} \frac{(a_1)_n...(a_p)_n}{(b_1)_n...(b_p)_n} \frac{x^n}{n!}. \quad (6) $$

By expanding ${}_1F_1(.;\ .;\ .)$ in (5) using (6) and constructing the Hypergeometric function of two variables of the form given in (7) [14, pp. 25]:

$$ \Phi_2(\beta,\acute{\beta};\ \gamma;\ x,y) = \quad (7) $$
$$ \sum_{m,n=0}^{\infty} \frac{(\beta)_m(\acute{\beta})_n}{(\gamma)_{m+n}} \frac{x^m y^n}{m!n!}, \mid x \mid< \infty, \mid y \mid< \infty, $$

we can express $\overline{P}_{d,Nak}$ as

$$ \overline{P}_{d,Nak} = 1 - e^{-\frac{\lambda}{2}} \left(\frac{m}{\overline{\gamma}+m}\right)^m \left[\Phi_2\left(m,1;\ 1;\ \frac{\lambda\overline{\gamma}}{2(\overline{\gamma}+m)}, \frac{\lambda}{2}\right) - \sum_{n=0}^{u-1} \frac{1}{n!}\left(\frac{\lambda}{2}\right)^n {}_1F_1\left(m;\ n+1;\ \frac{\lambda\overline{\gamma}}{2(\overline{\gamma}+m)}\right)\right]. \quad (8) $$

The average probability of detection expressions (5) and (8) are more general than [3, (20)], which is restricted to integer values of m. Moreover, these expressions numerically coincide with [3, (20)] for integer values of m.

Although the series form of special function $\Phi_2(\beta,\acute{\beta};\ \gamma;\ x,y)$ can be implemented easily in common mathematical software such as Mathematica, series truncation is required. Thus, the error result in truncating the infinite series in (5) by N terms ($\mid E_{Nak} \mid$) is shown in (9). This bound is derived by using the monotonically decreasing property of ${}_1F_1\left(m;\ n+1;\ \frac{\lambda\overline{\gamma}}{2(\overline{\gamma}+m)}\right)$ over n for given values of m, $\overline{\gamma}$ and λ [15]:

$$ \mid E_{Nak} \mid< \left(\frac{m}{\overline{\gamma}+m}\right)^m {}_1F_1\left(m;\ N+1;\ \frac{\lambda\overline{\gamma}}{2(\overline{\gamma}+m)}\right) \times \left[1 - e^{-\frac{\lambda}{2}} \sum_{n=0}^{N} \frac{\left(\frac{\lambda}{2}\right)^n}{n!}\right]. \quad (9) $$

Using the bound in (9), the number of terms $(\tilde{N})$ required to compute $\overline{P}_{d,Nak}$ to a given figure of accuracy can be found (Table I).

B. Nakagami-m fading - MGF approach

Using the contour integral representation of generalized Marcum-Q function [11], (1) can be written as

$$ P_d = \frac{e^{-\frac{\lambda}{2}}}{2\pi j} \oint_{\Delta} \frac{e^{\left(\left(\frac{1}{z}-1\right)\gamma+\frac{\lambda}{2}z\right)}}{z^u(1-z)}\, dz, \quad (10) $$

where Δ is a circular contour of radius r that encloses origin and $0<r<1$. The MGF of γ is $M(s)=E(e^{-s\gamma})$ where $E(.)$ is the expected value. Thus, by taking the average of (10) over the distribution of γ, we find that the average detection probability $(\overline{P}_d)$ as

$$ \overline{P}_d = \frac{e^{-\frac{\lambda}{2}}}{2\pi j} \oint_{\Delta} M\left(1-\frac{1}{z}\right) \frac{e^{\frac{\lambda}{2}z}}{z^u(1-z)}\, dz. \quad (11) $$

This expression in (11) is fairly general and holds for any case where the MGF is available in suitable form. From residue calculus, we know that the integral in (11) depends on the residues at the poles of the integrand inside the contour Δ. The residues can be computed readily using mathematical software and are exact. We are going to employ this approach for Nakagami and Rician cases. For a Nakagami fading channel, the MGF of SNR can be written as

$$ M_{\gamma_{Nak}}(s) = \frac{1}{\left(1+\frac{\overline{\gamma}s}{m}\right)^m}, \quad m \geq \frac{1}{2}. \quad (12) $$

Hence by (11), the expression for the probability of detection over Nakagami fading is

$$ \overline{P}_{d,Nak} = \left(\frac{m}{m+\overline{\gamma}}\right)^m e^{-\frac{\lambda}{2}} \times \frac{1}{2\pi j} \oint_{\Delta} f(z)\, dz, \quad (13) $$

where $\theta_N = \frac{\overline{\gamma}}{m+\overline{\gamma}}$ and

$$ f(z) = \frac{e^{\frac{\lambda}{2}z}}{(z-\theta_N)^m z^{u-m}(1-z)}. \quad (14) $$

The contour integral in (13) is evaluated for an integer value of m. Suppose that $f(z)$ has a pole of order $k \geq 1$ at $z=z_0$. We need $|z_0|<1$, otherwise the pole will be outside the contour and need not be considered at all. The residue of the pole at $z=z_0$ of order $k \geq 1$ is given by

$$ \mathrm{Res}\,(f;\ z_0,k) = \frac{1}{(k-1)!} \frac{d^{k-1}}{dz^{k-1}} \left[f(z)(z-z_0)^k\right]\Bigg|_{z=z_0}. \quad (15) $$

Case I: $u>m$

In this case, the integrand (13) contains m and $(u-m)$ order poles at $z=\theta_N$ and $z=0$. From residue calculus, $\overline{P}_{d,Nak}$ can be derived as

$$ \overline{P}_{d,Nak} = \left(\frac{m}{m+\overline{\gamma}}\right)^m e^{-\frac{\lambda}{2}} \times \Big[\mathrm{Res}\,(f;\ \theta_N,m) + \mathrm{Res}\,(f;\ 0,u-m)\Big], \quad (16) $$

where $\mathrm{Res}\,(f;\ z_0,k)$ denotes the residue of the pole at $z=z_0$ of order $k \geq 1$ for function $f(z)$ - given in (15) above.

Case II: $u \leq m$

In this case, there is no pole at the origin, and only the pole at $z=\theta_N$ needs to be considered

$$ \overline{P}_{d,Nak} = \left(\frac{m}{m+\overline{\gamma}}\right)^m e^{-\frac{\lambda}{2}}\ \mathrm{Res}\,(f;\ \theta_N,m). \quad (17) $$

The evaluation of residues yields simpler expressions over Nakagami-*m* fading. However, these results are limited to integer values of m, while the results of the PDF approach are not. Further, this result is numerically equivalent to the (5), [3], [4] for integer values of m.

As a by product, the average probability of detection over Rayleigh fading channel $(\overline{P}_{d,Ray})$ can be obtained by substituting $m=1$ in (16) and (17) as given in (18) (top of the next page). The result in (18) is numerically equivalent to the expressions given in [3]–[5].

$$\overline{P}_{d,Ray}=\begin{cases} e^{-\frac{\lambda}{2(1+\overline{\gamma})}} & \text{for } u=1\\ \left(\frac{e^{-\frac{\lambda}{2}}}{1+\overline{\gamma}}\right)\left(\frac{e^{\frac{\lambda\theta_N}{2}}}{\theta_N^{u-1}(1-\theta_N)}+\frac{1}{(u-2)!}\frac{d^{u-2}}{dz^{u-2}}\left[\frac{e^{\frac{\lambda}{2}z}}{(z-\theta_N)(1-z)}\right]\right)\Bigg|_{z=0} & \text{for } u>1\end{cases} \tag{18}$$

C. Rician Fading Channel

For Rician fading, the MGF of the SNR is given by

$$M_{\gamma_{Ric}}(s)=\frac{1+K}{(1+K+s\overline{\gamma})}\exp\left(-\frac{sK\overline{\gamma}}{(1+K+s\overline{\gamma})}\right) \tag{19}$$

where K is the Rice factor. Hence, using (11), the average detection probability ($\overline{P}_{d,Ric}$) can be written as

$$\overline{P}_{d,Ric}=\frac{(1+K)\,\theta_R}{\overline{\gamma}}\exp\left(-\left(\frac{\lambda}{2}+K\theta_R\right)\right)\times\frac{1}{2\pi j}\oint_{\Delta}\frac{\exp\left(\frac{a}{z-\theta_R}+\frac{\lambda z}{2}\right)}{(z-\theta_R)\,z^{u-1}(1-z)}\,dz, \tag{20}$$

where $\theta_R=\frac{\overline{\gamma}}{\overline{\gamma}+K+1}$ and $a=K\theta_R(1-\theta_R)$. For the special case of $K=0$ (Rayleigh fading), (20) reduces to Rayleigh in (13). Applying Laurent series expansion for $\frac{\exp\left(\frac{a}{z-\theta_R}\right)}{(z-\theta_R)}$ when $K\neq 0$ and using the Residue theorem to integrate term by term, $\overline{P}_{d,Ric}$ for $u>1$ can be expressed as in (21) (top of the next page). When $u=1$, the pole at $z=0$ disappears. Hence the result can be obtained by setting the limit value of first derivative in (21) to 0. For $u=1$, the result given in (21) is numerically equivalent to the result given in [3]. However, it is difficult to derive the error result in truncating the infinite series in (21).

IV. Probability of Detection over Fading Channels – MRC Diversity Case

In the following study of energy detection with diversity reception, we assume the availability of channel state information (CSI) at the receiver. Although this assumption appears at odds with the notion of energy detection, the main aim of this assumption is to derive the gold standard of achievable performance. Other practical setups can then be compared against the gold standard. Thus, the following results clarify the fundamental performance limits of the energy detector with diversity reception.

Moreover, in cognitive radio applications, the CSI may be available to secondary users over a control channel or over a broadcast channel through an access point. Several such setups have recently been investigated [16], [17]. The more recent work [17] supports this setup where the CSI is assumed known to the secondary user access point. Further in [18], [19], soft combining of instantaneous SNR values of the secondary users is considered where CSI of individual secondary user is assumed available at a decision center which combines individual soft decisions coherently. References [20], [21] assume perfect CSI is available where the energy detector is employed in UWB systems.

A. Decision Variable Formulation

In MRC reception, the received signals $\{y_l(t)\}_{l=1}^{L}$ where L is the number of diversity branches, are weighted and combined to yield a new signal $y_{mr}(t)=\sum_{l=1}^{L}h_l^{*}y_l(t)$. The L branch MRC output under H_1 can thus be expressed as

$$y_{mr}(t)=gs(t)+n(t) \tag{22}$$

where $g=\sum_{l=1}^{L}|h_l|^2$ and $n(t)=\sum_{l=1}^{L}h_l^{*}n_l(t)$. Here $n_l(t)$ is the noise process in l^{th} indexed branch with $\mathcal{N}(0,N_0W)$ and h_l is the channel coefficient in l^{th} index branch. Hence $n(t)$ is a random process with $\mathcal{N}(0,\sum_{l=1}^{L}|h_l|^2N_0W)$. Thus, it is easy to show the effective SNR in this case is given by

$$\gamma_{mr}=g\frac{E_s}{N_0}. \tag{23}$$

B. Nakagami-m Fading Channel - PDF Approach

The conditional detection probability $P_{d,Nak,mr}$ is given in (1) with γ_{mr} replacing γ. The output SNR of the MRC receiver with i.i.d. Nakagami-*m* branches is $\mathcal{G}(Lm,\overline{\gamma}/m)$. By averaging (1) over the PDF of γ_{mr}, similar to (5), we find

$$\overline{P}_{d,Nak,mr}=1-e^{-\frac{\lambda}{2}}\left(\frac{m}{\overline{\gamma}+m}\right)^{Lm}\sum_{n=u}^{\infty}\frac{1}{n!}\left(\frac{\lambda}{2}\right)^n\times{}_1F_1\left(Lm;\;n+1;\;\frac{\lambda\overline{\gamma}}{2(\overline{\gamma}+m)}\right). \tag{24}$$

Following a similar procedure as in (8) and constructing $\Phi_2(\beta,\acute{\beta};\;\gamma;\;x,y)$ given in (7), $\overline{P}_{d,Nak,mr}$ is expressed as

$$\overline{P}_{d,Nak,mr}=1-e^{-\frac{\lambda}{2}}\left(\frac{m}{\overline{\gamma}+m}\right)^{Lm}\times\left[\Phi_2\left(Lm,1;\;1;\;\frac{\lambda\overline{\gamma}}{2(\overline{\gamma}+m)},\frac{\lambda}{2}\right)-\sum_{n=0}^{u-1}\frac{1}{n!}\left(\frac{\lambda}{2}\right)^n{}_1F_1\left(Lm;\;n+1;\;\frac{\lambda\overline{\gamma}}{2(\overline{\gamma}+m)}\right)\right]. \tag{25}$$

The computation of $\Phi_2(\beta,\acute{\beta};\;\gamma;\;x,y)$ requires the use of a software package such as Mathematica. The error result in truncating the infinite series in (24) by N terms ($|\;E_{Nak,mr}\;|$) is derived similar to (9) as

$$|\;E_{Nak,mr}\;|<\left(\frac{m}{\overline{\gamma}+m}\right)^{Lm}{}_1F_1\left(Lm;\;N+1;\;\frac{\lambda\overline{\gamma}}{2(\overline{\gamma}+m)}\right)\times\left[1-e^{-\frac{\lambda}{2}}\sum_{n=0}^{N}\frac{\left(\frac{\lambda}{2}\right)^n}{n!}\right]. \tag{26}$$

The bound is used to determine the number of terms ($\tilde{N}$) required to compute $\overline{P}_{d,Nak,mr}$ to a given figure accuracy.

$$\overline{P}_{d,Ric} = \frac{(1+K)\,\theta_R\, e^{-\left(\frac{\lambda}{2}+K\theta_R\right)}}{\overline{\gamma}} \sum_{n=1}^{\infty} \frac{a^{n-1}}{(n-1)!} \left(\frac{1}{(u-2)!} \frac{d^{u-2}}{dz^{u-2}} \left[\frac{e^{\frac{\lambda z}{2}}}{(1-z)(z-\theta_R)^n} \right] \Bigg|_{z=0} + \frac{1}{(n-1)!} \frac{d^{n-1}}{dz^{n-1}} \left[\frac{e^{\frac{\lambda z}{2}}}{z^{u-1}(1-z)} \right] \Bigg|_{z=\theta_R} \right) \tag{21}$$

C. Nakagami-m Fading Channel - MGF approach

The output SNR of L branch MRC combiner (γ_{mr}) is $\gamma_{mr} = \sum_{l=1}^{L} \gamma_l$ where γ_l is the l^{th} indexed branch SNR. Thus, for i.i.d. branch statistics, the MGF of output SNR ($M_{\gamma_{mr,Nak}}(s)$) is given by (27).

$$M_{\gamma_{mr,Nak}}(s) = \frac{1}{\left(1+\frac{\overline{\gamma}s}{m}\right)^{Lm}}, \quad m \geq \frac{1}{2} \tag{27}$$

The $\overline{P}_{d,mr,Nak}$ is

$$\overline{P}_{d,mr,Nak} = \left(\frac{m}{m+\overline{\gamma}}\right)^{Lm} e^{-\frac{\lambda}{2}} \frac{1}{2\pi j} \oint_{\Delta} f(z)\, dz, \tag{28}$$

where $f(z)$ is given by (14) with m replaced by Lm except in θ_N. Following a similar line of arguments as in subsection (III-B), $\overline{P}_{d,mr,Nak}$ can be expressed in closed form for integer values of Lm. For the cases $u > Lm$ and $u \leq Lm$, integral in (28) can be evaluated similar to (16) and (17), respectively. In computing the residues at θ_N and 0, m should be replaced by Lm in (16) and (17). Note that the results are limited to an integer of Lm and allow us to compute $\overline{P}_{d,mr,Nak}$ for certain non-integer values of m. For example, $\overline{P}_{d,mr,Nak}$ over a dual branch combiner can be computed for $\frac{1}{2}$ multiples of m values.

D. Rician Fading Channel - MGF Approach

Following a similar procedure as in subsection (IV-C) and by means of (19), the MGF of the output SNR of MRC receiver over i.i.d. Rician fading channel can easily be found. After substituting this MGF in (11), we arrive at an integral similar to (20). By following similar lines of arguments as in section (III-C), detection probability over a MRC combined Rician fading branches ($\overline{P}_{d,mr,Ric}$) can be derived as in (29) (top of the next page) for $u > L$ where $\theta_R = \frac{\overline{\gamma}}{\overline{\gamma}+K+1}$ and $a = K\theta_R L(1-\theta_R)$. When $u \leq L$, the pole at 0 disappears and thus the result can be obtained by setting the limit value of first derivative in (29) to 0. It is easy to verify that when $L = 1$, (29) reduces to (21).

V. Probability of Detection over Fading Channels – EGC Diversity Case

A. Received SNR

Note that MRC reception requires full channel knowledge (i.e., both channel amplitude and phase) for all diversity branches. However, EGC offers a somewhat reduced complexity alternative. In EGC reception, the received signals $\{y_l(t)\}_{l=1}^{L}$ where L is the number of diversity branches are weighted by phase only and combined to yield a new signal $y_{eg}(t) = \sum_{l=1}^{L} e^{-j\phi_l} y_l(t)$ where ϕ_l is the phase of the l^{th} channel gain. Then the L branch equal gain combiner output under H_1 can be expressed as

$$y_{eg} = g s(t) + n(t) \tag{30}$$

where $n(t) = \sum_{l=1}^{L} n_l(t) e^{-j\phi_l}$ and $g = \sum_{l=1}^{L} |h_l|$ [22, (6.32), pp. 285]. Here $n_l(t)$ is a random process with $\mathcal{N}(0, N_0 W)$ and h_l is the channel coefficient of l^{th} diversity branch. Hence $n(t)$ is a normal random process with $\mathcal{N}(0, LN_0W)$. Therefore Y defined under H_0 is a sum of square of $2u$ Gaussian random variables with $\mathcal{N}(0,1)$ and hence follows χ^2_{2u}. The output SNR of L branch EGC (γ_{eg}) is defined by

$$\gamma_{eg} = \left(\sum_{l=1}^{L} |h_l|\right)^2 \frac{E_s}{LN_0}.$$

See for example [22, (6.33), pp.285]. The PDF of γ_{eg} is required to calculate the average detection probability. Reference [23] derives the PDF of a sum of Nakagami-m variables, which we use next for our performance analysis.

B. Nakagami-m Fading Channel - PDF Approach

The detection probability when L diversity branches are used $\overline{P}_{d,eq,L}$ can be calculated by averaging (1) over PDF of γ_{eg} (≥ 0), i.e. $f_L(\gamma_{eg})$, $L = 1, 2, \ldots$ as

$$\begin{aligned}\overline{P}_{d,eq,L} &= \int_0^{\infty} Q_u(\sqrt{2\gamma_{eg}}, \sqrt{\lambda}) f_L(\gamma_{eg}) d\gamma_{eg} \\ &= 1 - e^{-\frac{\lambda}{2}} \sum_{n=u}^{\infty} \left(\frac{\lambda}{2}\right)^{\frac{n}{2}} \int_0^{\infty} \gamma^{-\frac{n}{2}} e^{-\gamma} I_n(\sqrt{2\lambda\gamma}) f_L(\gamma) d\gamma.\end{aligned} \tag{31}$$

C. Two i.i.d. branches ($L = 2$)

When the received signals in i.i.d. diversity branches are Nakagami-m, the PDF of the amplitude of the combined signal is given by [23, (4)]. Hence by following the same procedure for (3), $f_2(\gamma)$ can be derived as

$$\begin{aligned}f_2(\gamma) = &\frac{2\sqrt{\pi}\gamma^{2m-1} e^{-\frac{2m\gamma}{\overline{\gamma}}}}{2^{4m-1}} \frac{\Gamma(2m)}{\Gamma^2(m)\Gamma(2m+\frac{1}{2})} \left(\frac{2m}{\overline{\gamma}}\right)^{2m} \\ &\times {}_1F_1\left(2m;\, 2m+\frac{1}{2};\, \frac{m\gamma}{\overline{\gamma}}\right), \; \gamma \geq 0.\end{aligned} \tag{32}$$

By substituting (31) in (32) and doing some manipulations, $\overline{P}_{d,eg,2}$ can be expressed as

$$\begin{aligned}\overline{P}_{d,eg,2} = &1 - \rho_2(m) e^{-\frac{\lambda}{2}} \left(\frac{2}{\overline{\gamma}}\right)^{2m} \\ &\times \sum_{n=u}^{\infty} \left(\frac{\lambda}{2}\right)^{\frac{n}{2}} \int_0^{\infty} \gamma^{2m-\frac{n}{2}-1} e^{-\left(\frac{2m}{\overline{\gamma}}+1\right)\gamma} \\ &\times I_n(\sqrt{2\lambda\gamma})\, {}_1F_1\left(2m;\, 2m+\frac{1}{2};\, \frac{m\gamma}{\overline{\gamma}}\right) d\gamma\end{aligned} \tag{33}$$

$$\overline{P}_{d,mr,Ric} = e^{-\frac{\lambda}{2}} \left(\frac{(1+K)\,\theta_R\, e^{-K\theta_R}}{\overline{\gamma}}\right)^{L} \sum_{n=1}^{\infty} \frac{a^{n-1}}{(n-1)!} \left(\frac{1}{(u-L-1)!} \frac{d^{u-L-1}}{dz^{u-L-1}} \left[\frac{e^{\frac{\lambda z}{2}}}{(1-z)(z-\theta_R)^{L+n-1}} \right] \Bigg|_{z=0} + \frac{1}{(L+n-2)!} \frac{d^{L+n-2}}{dz^{L+n-2}} \left[\frac{e^{\frac{\lambda z}{2}}}{z^{u-L}(1-z)} \right] \Bigg|_{z=\theta_R} \right) \tag{29}$$

where $\rho_2(m)$ is defined by

$$\rho_2(m) = \frac{2\sqrt{\pi}\ \Gamma(2m) m^{2m}}{\Gamma^2(m)\Gamma(2m+\frac{1}{2})2^{4m-1}}. \tag{34}$$

Using [13, (6.643-2), pp.709], (6) and (33), $\overline{P}_{d,eg,2}$ can be evaluated as in (35)

$$\overline{P}_{d,eg,2} = 1 - \sqrt{\pi} e^{-\frac{\lambda}{2}} \sum_{n=u}^{\infty}\sum_{k=0}^{\infty} \left(\frac{\lambda}{2}\right)^n \left(\frac{m}{\overline{\gamma}+2m}\right)^{2m+k} \times \frac{\psi_2(m,n,k)}{2^{2m-2}\ k!}\ {}_1F_1\left(2m+k;\ n+1;\ \frac{\lambda\overline{\gamma}}{2(\overline{\gamma}+2m)}\right) \tag{35}$$

where

$$\psi_2(m,n,k) = \frac{\Gamma^2(2m+k)}{\Gamma^2(m)\Gamma(n+1)\Gamma(2m+k+\frac{1}{2})}. \tag{36}$$

In simplifying (35), well known relation of Pochhammer symbol to Gamma function i.e. $(a)_k = \frac{\Gamma(a+k)}{\Gamma(a)}$ is used. Replacing ${}_1F_1(.;\ .;\ .)$ using (6) and constructing two variable Hypergeometric function of the form $\Psi_1(\alpha,\beta;\ \gamma,\acute{\gamma};\ x,y)$ in (37) [14, pp. 26], $\overline{P}_{d,eg,2}$ can be expressed as in (38).

$$\Psi_1(\alpha,\beta;\ \gamma,\acute{\gamma};\ x,y) = \sum_{m,n=0}^{\infty} \frac{(\alpha)_{m+n}(\beta)_m}{(\gamma)_m(\acute{\gamma})_n} \frac{x^m}{m!}\frac{y^n}{n!},\ |\ x\ |< 1, |\ y\ |< \infty \tag{37}$$

$$\overline{P}_{d,eg,2} = 1 - \frac{e^{-\frac{\lambda}{2}}}{2^{2m-2}} \frac{\Gamma^2(2m)\,\Gamma\left(\frac{1}{2}\right)}{\Gamma^2\left(m\right)\Gamma\left(2m+\frac{1}{2}\right)} \left(\frac{m}{\overline{\gamma}+2m}\right)^{2m} \sum_{n=u}^{\infty} \frac{1}{n!}\left(\frac{\lambda}{2}\right)^n \times \Psi_1\left(2m, 2m;\ 2m+\frac{1}{2}, n+1;\ \frac{m}{\overline{\gamma}+2m}, \frac{\lambda\overline{\gamma}}{2(\overline{\gamma}+2m)}\right). \tag{38}$$

$\Psi_1(.,.;\ .,.;\ .,.)$ in (38) monotonically decreases as n increases for fixed values of m and $\overline{\gamma}$. Hence, the error result in truncating the infinite series in (38) by N terms ($|\ E_{eg,2}\ |$) can be bounded as in (39).

$$|\ E_{eg,2}\ | \ \leq\ \frac{e^{-\frac{\lambda}{2}}}{2^{2m-2}} \frac{\Gamma^2(2m)\,\Gamma\left(\frac{1}{2}\right)}{\Gamma^2\left(m\right)\Gamma\left(2m+\frac{1}{2}\right)} \times \left(\frac{m}{\overline{\gamma}+2m}\right)^{2m} \left(e^{\frac{\lambda}{2}} - \sum_{n=0}^{N}\frac{1}{n!}\left(\frac{\lambda}{2}\right)^n\right) \times \Psi_1\left(2m, 2m;\ 2m+\frac{1}{2}, N+1;\ \frac{m}{\overline{\gamma}+2m}, \frac{\lambda\overline{\gamma}}{2(\overline{\gamma}+2m)}\right). \tag{39}$$

Using the bound (39), the number of terms ($\tilde{N}$) required to compute $\overline{P}_{d,eg,2}$ to a given figure of accuracy can be found.

D. Three i.i.d. branches ($L = 3$)

When the received signal follows Nakagami-*m* distribution, the PDF of the amplitude of the combined signal is given by [23, (8)] and by following the similar procedure, $f_3(\gamma)$ can be expressed as in (40).

$$f_3(\gamma) = \frac{4\sqrt{\pi}\ \Gamma(2m)e^{-\frac{3m\gamma}{\overline{\gamma}}}}{\Gamma^3(m)2^{4m-1}} \sum_{n=0}^{\infty} \frac{\Gamma(2m+n)}{\Gamma(2m+n+\frac{1}{2})} \times \frac{\Gamma(4m+2n)\ \gamma^{3m+n-1}}{\Gamma(6m+2n)\,\Gamma(n+1)\ 2^n} \left(\frac{3m}{\overline{\gamma}}\right)^{3m+n} \times {}_2F_2\left(2m, 4m+2n;\ 3m+n+\frac{1}{2}, 3m+n;\ \frac{3m\gamma}{2\overline{\gamma}}\right), \gamma \geq 0. \tag{40}$$

Using the form of ${}_2F_2(.,.;\ .,.;\ .)$ in (6) ($q = 2, p = 2$) and by using (31) and (40), $\overline{P}_{d,eq,3}$ can be shown as in (41)

$$\overline{P}_{d,eq,3} = 1 - \rho_3(m) e^{-\frac{\lambda}{2}} \sum_{n=u}^{\infty}\sum_{p,k=0}^{\infty} \left(\frac{\lambda}{2}\right)^{\frac{n}{2}} \left(\frac{m}{2}\right)^{p+k} \left(\frac{3}{\overline{\gamma}}\right)^{3m+p+k} \times \frac{\Gamma(2m+p)\Gamma(4m+2p)(2m)_k(4m+2p)_k}{\Gamma(6m+2p)\Gamma(2m+p+\frac{1}{2})(3m+p)_k\left(3m+p+\frac{1}{2}\right)_k} \times \frac{1}{p!k!}\int_0^{\infty} \gamma^{3m+p+k-\frac{n}{2}-1}\ e^{-\left(\frac{3m}{\overline{\gamma}}+1\right)}\ I_n(\sqrt{2\lambda\gamma})d\gamma \tag{41}$$

where

$$\rho_3(m) = \frac{4\sqrt{\pi}\ \Gamma(2m) m^{3m}}{\Gamma^3(m)2^{4m-1}}. \tag{42}$$

Using [13, (6.643-2)] and [13, (9.220-2)], $\overline{P}_{d,eq,3}$ can be computed as in (43)

$$\overline{P}_{d,eq,3} = 1 - \sqrt{\pi} e^{-\frac{\lambda}{2}} \sum_{n=u}^{\infty}\sum_{p,k=0}^{\infty} \left(\frac{\lambda}{2}\right)^n \left(\frac{3m}{3m+\overline{\gamma}}\right)^{3m+p+k} \times \frac{\psi_3(m,n,p,k)}{2^{4m+p+k-3}\ n!p!k!}\ {}_1F_1\left(3m+p+k;\ n+1;\ \frac{\lambda\overline{\gamma}}{2(3m+\overline{\gamma})}\right) \tag{43}$$

where $\psi_3(m,n,p,k)$ is given in (44).

$$\psi_3(m,n,p,k) = \frac{\Gamma(2m+p)\Gamma(2m+k)\Gamma(3m+p)}{\Gamma^3(m)\Gamma(2m+p+\frac{1}{2})} \times \frac{\Gamma(3m+p+\frac{1}{2})\Gamma(4m+2p+k)}{\Gamma(3m+p+k+\frac{1}{2})\Gamma(6m+2p)} \tag{44}$$

E. Four or more i.i.d. branches ($L \geq 4$)

When the received signal follows Nakagami-*m* distribution, the PDF of the amplitude of the combined signal is given by [23, (9)] and $f_4(\gamma)$ can be derived as given in (45).

$$f_4(\gamma) = \frac{8\sqrt{\pi}\,\Gamma(2m)m^{4m}e^{-\frac{4m\gamma}{\overline{\gamma}}}}{\Gamma^4(m)2^{4m-1}} \tag{45}$$
$$\sum_{p,q=0}^{\infty} \frac{\Gamma(2m+p)\Gamma(2m+q)\Gamma(4m+2p+q)}{\Gamma(p+1)\Gamma(q+1)\Gamma(2m+p+\frac{1}{2})\Gamma(8m+2p+2q)}$$
$$\gamma^{(4m+p+q-1)}2^{(q-p)}m^{(p+q)}\left(\frac{4}{\overline{\gamma}}\right)^{(4m+p+q)}$$
$${}_2F_2\left(2m, 2(3m+p+q);\ 4m+p+q+\frac{1}{2}, 4m+p+q;\ \frac{2m\gamma}{\overline{\gamma}}\right)$$

Following a similar procedure, $\overline{P}_{d,eg,4}$ can be evaluated as given in (46)

$$\overline{P}_{d,eg,4} = 1 - \sqrt{\pi}\ e^{-\frac{\lambda}{2}} \sum_{n=u}^{\infty}\sum_{p,q,k=0}^{\infty}\left(\frac{\lambda}{2}\right)^n \tag{46}$$
$$\times\left(\frac{4m}{4m+\overline{\gamma}}\right)^{(4m+p+q+k)} \frac{\Psi_4(m,n,p,q,k)}{2^{(4m+p+k-q-4)}\ p!\ q!\ n!\ k!}$$
$$\times\ {}_1F_1\left(4m+p+q+k;\ n+1;\ \frac{\lambda\overline{\gamma}}{2(4m+\overline{\gamma})}\right)$$

where, $\psi_4(m,n,p,q,k)$ is defined in (47).

$$\psi_4(m,n,p,q,k) = \tag{47}$$
$$\frac{\Gamma(2m+p)\Gamma(2m+q)\Gamma(2m+k)\Gamma(4m+p+q)}{\Gamma^4(m)\Gamma(2m+p+\frac{1}{2})\Gamma(4m+p+q+k+\frac{1}{2})}$$
$$\times\frac{\Gamma(4m+2p+q)\Gamma(4m+p+q+\frac{1}{2})\Gamma(6m+2p+2q+k)}{\Gamma(6m+2p+2q)\Gamma(8m+2p+2q)}$$

When the received signal follows Nakagami-*m* distribution, the PDF of the amplitude of the combined signal is given by [23, (10)] for $L \geq 4$. By following the same line of arguments, $f_L(\gamma)$ can be evaluated as given in (48) (top of the next page). Hence, $\overline{P}_{d,eg,L}$ can be evaluated as given in (49) (top of the next page) and $\alpha_L = 1 + \frac{mL}{\overline{\gamma}}$.

Replacing k_1 by p, k_2 by q, p by k and $L = 4$ in (49), we can easily verify (46). Further simplification and deriving bounds for $\overline{P}_{d,eg,i}$ where $i \geq 3$ is a difficult task.

VI. Probability of Detection over Fading Channels – SC Diversity Case

The selection combiner picks the diversity branch with the maximum SNR. The PDF of output SNR of SC (γ_{sc}) can hence be obtained for i.i.d. branch statistics [24, (6)] as

$$f_{\gamma_{sc}}(y) = \frac{L}{\Gamma^L(m)}\left(\frac{m}{\overline{\gamma}}\right)^m y^{m-1}e^{-(\frac{my}{\overline{\gamma}})}\left[G\left(m, \frac{my}{\overline{\gamma}}\right)\right]^{L-1} \tag{50}$$

where $G(.,.)$ is the lower incomplete gamma function defined by the integral form $G(a,x) = \int_o^x t^{a-1}e^{-t}dt$. Thus, the average detection probability of L branch SC receiver ($\overline{P}_{d,sc,L}$) can be calculated averaging (1) over (50) as shown in (51) below.

$$\overline{P}_{d,sc,L} = \frac{L}{\Gamma^L(m)}\left(\frac{m}{\overline{\gamma}}\right)^m\int_0^{\infty} y^{m-1}e^{-(\frac{my}{\overline{\gamma}})}$$
$$\times\left[G\left(m,\frac{my}{\overline{\gamma}}\right)\right]^{L-1} Q_u(\sqrt{2y},\sqrt{\lambda})\,dy \tag{51}$$

By setting $L = 1$, it is easy to show that (51) reduces to (4), which is the no-diversity case for a Nakagami-*m* fading channel.

A. Dual Diversity Combiner ($L = 2$)

The special function $G(.,.)$ can be written as in (52) with the aid of [13, (8.351-2), pp. 899].

$$G(a,x) = \frac{x^a}{a}\,e^{-x}\ {}_1F_1(1;\ 1+a;\ x) \tag{52}$$

Substituting (52) in (51), $\overline{P}_{d,sc,2}$ (i.e. L=2) can be expressed as

$$\overline{P}_{d,sc,2} = 1 - \frac{2e^{-\frac{\lambda}{2}}}{m\ \Gamma^2(m)} \tag{53}$$
$$\times\sum_{n=u}^{\infty}\sum_{k=0}^{\infty}\left(\frac{m}{\overline{\gamma}}\right)^{k+2m}\left(\frac{\lambda}{2}\right)^{\frac{n}{2}}\frac{\Gamma(m+1)}{\Gamma(k+m+1)}$$
$$\times\int_0^{\infty} y^{(k+2m-\frac{n}{2}-1)}e^{-(\frac{2m}{\overline{\gamma}}+1)y}I_n(\sqrt{2\gamma y})dy.$$

Using [13, (6.643-2)] and [13, (9.220-2)], $\overline{P}_{d,sc,2}$ can be derived as

$$\overline{P}_{d,sc,2} = 1 - \frac{2e^{-\frac{\lambda}{2}}}{m\ \Gamma^2(m)}\sum_{n=u}^{\infty}\sum_{k=0}^{\infty}\left(\frac{\lambda}{2}\right)^n\left(\frac{m}{\overline{\gamma}\beta_2}\right)^{k+2m} \tag{54}$$
$$\frac{\Gamma(m+1)\Gamma(2m+k)}{\Gamma(m+k+1)n!}\ {}_1F_1\left(2m+k;\ n+1;\ \frac{\lambda}{2\beta_2}\right)$$

where $\beta_2 = 1 + \frac{2m}{\overline{\gamma}}$. Expanding ${}_1F_1(.;\ .;\ .)$ by using (6) and constructing the Hypergeometric function of two variables of the form given in (37), $\overline{P}_{d,sc,2}$ can also be expressed as

$$\overline{P}_{d,sc,2} = 1 - \frac{2e^{-\frac{\lambda}{2}}}{m}\frac{\Gamma(2m)}{\Gamma^2(m)}\left(\frac{m}{\overline{\gamma}+2m}\right)^{2m}\sum_{n=u}^{\infty}\frac{1}{n!}\left(\frac{\lambda}{2}\right)^n$$
$$\times\Psi_1\left(2m, 1;\ m+1, n+1;\ \frac{m}{\overline{\gamma}+2m}, \frac{\lambda\overline{\gamma}}{2(\overline{\gamma}+2m)}\right). \tag{55}$$

$\Psi_1\left(.,.;\ .,.;\ .,.\right)$ in (55) monotonically decreases as n increases for fixed values of m, λ and $\overline{\gamma}$. Hence, the error result in truncating the infinite series in (55) by N terms ($\mid E_{sc,2}\mid$) can be bounded as

$$\mid E_{sc,2}\mid \leq \frac{2e^{-\frac{\lambda}{2}}}{m}\frac{\Gamma(2m)}{\Gamma^2(m)} \tag{56}$$
$$\times\left(\frac{m}{\overline{\gamma}+2m}\right)^{2m}\left(e^{\frac{\lambda}{2}} - \sum_{n=0}^{N}\frac{1}{n!}\left(\frac{\lambda}{2}\right)^n\right)$$
$$\times\Psi_1\left(2m, 1;\ m+1, N+1;\ \frac{m}{\overline{\gamma}+2m}, \frac{\lambda\overline{\gamma}}{2(\overline{\gamma}+2m)}\right)$$

Using the bound given in (56), number of terms ($\tilde{N}$) required to compute $\overline{P}_{d,sc,2}$ to given figure accuracy can be found.

B. Integer m

By means of series form expansion of $G(.,.)$ [13, (8.352-6), pp. 900], the relation $\Gamma(m) = (m-1)!$ and the well known binomial expansion, $\left[G\left(m, \frac{my}{\overline{\gamma}}\right)\right]^{L-1}$ can be written as

$$\left[G\left(m,\frac{my}{\overline{\gamma}}\right)\right]^{L-1} = \Gamma^{L-1}(m) \tag{57}$$
$$\sum_{k=0}^{L-1}\binom{L-1}{k}(-1)^k\left[e^{-\frac{my}{\overline{\gamma}}}\sum_{i=0}^{m-1}\left(\frac{m}{\overline{\gamma}}\right)^i\frac{y^i}{i!}\right]^k.$$

$$\begin{aligned} f_L(\gamma) =& \frac{2^{L-1}\sqrt{\pi}\,\Gamma(2m) m^{Lm} e^{-\frac{mL\gamma}{\overline{\gamma}}}}{2^{4m-1}\Gamma^L(m)} \sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\cdots\sum_{k_{L-2}=0}^{\infty}\prod_{i=1}^{L-2}\left(\frac{\Gamma(2m+k_i)}{\Gamma(1+k_i)}\right)\frac{\Gamma(4m+2k_1+k_2)}{\Gamma(2m+k_1+\frac{1}{2})\Gamma(6m+2k_1+2k_2)} \\ &\times \frac{\Gamma(6m+2k_1+2k_2+k_3)\ \ldots\ \Gamma\left(2m(L-2)+2\sum_{i=1}^{L-3}k_i+k_{L-2}\right)}{\Gamma(8m+2k_1+2k_2+2k_3)\ \ldots\ \Gamma\left(2Lm+2\sum_{i=1}^{L-2}k_i\right)}\Gamma\left(2m(L-1)+2\sum_{i=1}^{L-2}k_i\right) \\ &\times \Gamma\left(2m(L-2)+2\sum_{i=1}^{L-3}k_i+k_{L-2}\right) m^{\left(\sum_{i=1}^{L-2}k_i\right)}\gamma^{\left(Lm+\sum_{i=1}^{L-2}k_i-1\right)}\left(\frac{L}{\overline{\gamma}}\right)^{\left(Lm+\sum_{i=1}^{L-2}k_i\right)} 2^{\left(\sum_{i=2}^{L-2}k_i-k_1\right)} \\ &\times {}_2F_2\left(2m,\ 2m(L-1)+2\sum_{i=1}^{L-2}k_i;\ \ Lm+\sum_{i=1}^{L-2}k_i+\frac{1}{2},\ Lm+\sum_{i=1}^{L-2}k_i;\ \ \frac{mL\gamma}{2\overline{\gamma}}\right),\ \ L\geq 4 \end{aligned} \tag{48}$$

$$\begin{aligned} \overline{P}_{d,eg,L} =& 1-\sqrt{\pi}e^{-\frac{\lambda}{2}}\sum_{n=u}^{\infty}\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\cdots\sum_{k_{L-2}}^{\infty}\sum_{p=0}^{\infty}\prod_{i=1}^{L-2}\left(\frac{\Gamma(2m+k_i)}{\Gamma(1+k_i)}\right)\frac{\Gamma(4m+2k_1+k_2)}{\Gamma(2m+k_1+\frac{1}{2})\Gamma(6m+2k_1+2k_2)} \\ &\times \frac{\Gamma(6m+2k_1+2k_2+k_3)\ \ldots\ \Gamma\left(2m(L-3)+2\sum_{i=1}^{L-4}k_i+k_{L-3}\right)}{\Gamma(8m+2k_1+2k_2+2k_3)\ \ldots\ \Gamma\left(2Lm+2\sum_{i=1}^{L-2}k_i\right)} \\ &\times \frac{\Gamma^2\left(2m(L-2)+2\sum_{i=1}^{L-3}k_i+k_{L-2}\right)\Gamma(2m+p)\Gamma\left(Lm+\sum_{i=1}^{L-2}k_i\right)\Gamma\left(Lm+\sum_{i=1}^{L-2}k_i+\frac{1}{2}\right)}{\Gamma(n+1)\Gamma^L(m)\Gamma\left(mL+\sum_{i=1}^{L-2}k_i+p+\frac{1}{2}\right)} \\ &\times \frac{\Gamma\left(2m(L-1)+2\sum_{i=1}^{L-2}k_i+p\right)\left(\frac{\lambda}{2}\right)^n\left(\frac{mL}{\alpha_L\overline{\gamma}}\right)^{\left(Lm+\sum_{i=1}^{L-2}k_i+p\right)}}{2^{\left(4m+k_1-\sum_{i=2}^{L-2}k_i-L+p\right)}\ p!}\ {}_1F_1\left(p+Lm+\sum_{i=1}^{L-2}k_i;\ \ n+1;\ \ \frac{\lambda}{2\alpha_L}\right),\ \ L\geq 4 \end{aligned} \tag{49}$$

Using [12, (4.63)] and (50) and applying multinomial expansion in (57), (51) can be written as

$$\begin{aligned} \overline{P}_{d,sc,L} =& 1-\frac{L\,e^{-\frac{\lambda}{2}}}{\Gamma(m)}\left(\frac{m}{\overline{\gamma}}\right)^m \\ &\times\sum_{n=u}^{\infty}\sum_{k=0}^{L-1}\binom{L-1}{k}(-1)^k\left(\frac{\lambda}{2}\right)^{\frac{n}{2}}\sum_{i=0}^{k(m-1)}\zeta_i\left(m,k,\overline{\gamma}\right) \\ &\times\int_0^{\infty}y^{\left(i+m-\frac{n}{2}-1\right)}e^{-\left(\frac{m(k+1)}{\overline{\gamma}}+1\right)y}\ I_n\left(\sqrt{2\lambda y}\right)dy \end{aligned} \tag{58}$$

where $\zeta_i\left(m,k,\overline{\gamma}\right)$ is the coefficient of multinomial expansion of $\left[\sum_{i=0}^{m-1}\left(\frac{m}{\overline{\gamma}}\right)^i\frac{y^i}{i!}\right]^k$. Hence, using [13, (6.643-2)] and with simplifications using [13, (9.220-2)], $\overline{P}_{d,sc,L}$ can be derived as in (59) where $\beta_L = 1+\frac{m(k+1)}{\overline{\gamma}}$.

$$\begin{aligned} \overline{P}_{d,sc,L} =& 1-Le^{-\frac{\lambda}{2}}\left(\frac{m}{\overline{\gamma}}\right)^m\sum_{n=u}^{\infty}\sum_{k=0}^{L-1}\binom{L-1}{k}\frac{(-1)^k}{n!}\left(\frac{\lambda}{2}\right)^n \\ &\times\sum_{i=0}^{k(m-1)}\frac{\zeta_i\left(m,k,\overline{\gamma}\right)(m)_i}{\beta_L^{(i+m)}}\,{}_1F_1\left(i+m;\ n+1;\ \frac{\lambda}{2\beta_L}\right) \end{aligned} \tag{59}$$

The multinomial expansion in (59) reduces to 1 for $m=1$ and to binomial expansion for $m=2$. Under the constraint of $m=1$, results in (54), (55) and (59) are numerically equivalent to [5, (24)] and [3, (30)] with the correction given in [4, pp. 22].

For fixed values of m, λ and $\overline{\gamma}$, ${}_1F_1\left(i+m;\ n+1;\ \frac{\lambda}{2\beta_L}\right)$ in (59) monotonically decreases as n increases [15]. Hence, the error result in truncating the infinite series in $\overline{P}_{d,sc,L}$ by N terms ($\mid E_{sc,L}\mid$) can be bounded as in (60).

$$\begin{aligned} \mid E_{sc,L}\mid \leq&\ L\,e^{-\frac{\lambda}{2}}\left(\frac{m}{\overline{\gamma}}\right)^m\left(e^{\frac{\lambda}{2}}-\sum_{n=0}^{N}\frac{1}{n!}\left(\frac{\lambda}{2}\right)^n\right) \\ &\times\sum_{k=0}^{L-1}\binom{L-1}{k}\sum_{i=0}^{k(m-1)}\frac{\zeta_i\left(m,k,\overline{\gamma}\right)(m)_i}{\beta_L^{(i+m)}} \\ &\times\ {}_1F_1\left(i+m;\ N+1;\ \frac{\lambda}{2\beta_L}\right) \end{aligned} \tag{60}$$

Using the bound given in (60), the number of terms $(\tilde{N})$ required to compute $\overline{P}_{d,sc,L}$ to a given figure of accuracy can be found.

VII. Results and Discussion

The proposed MGF method along with the PDF approach provide a general frame work for performance analysis of energy detector with diversity reception over Nakagami-m and Rician fading. The results can be used to determine the energy threshold value of the detector and the minimum number of samples required to meet a given false alarm rate. In evaluating the detection probabilities, the choice between MGF or PDF methods depends on the limitations imposed in derivations and the complexity. For example, for given u, m values, residues (MGF method) give simpler expressions

TABLE I
NUMBER OF TERMS REQUIRED TO OBTAIN A FIVE FIGURE ACCURACY ($\tilde{N}$)

$\mid E_{Nak} \mid$						
u	1	5	1	1	1	5
P_f	0.01	0.01	0.01	0.01	0.0001	0.0001
$SNR(dB)$	10	10	20	10	10	10
m	1	1	1	4	1	1
$\tilde{N}$	**14**	**27**	**13**	**12**	**23**	**36**
$\mid E_{Nak,mr} \mid$						
u	1	5	1	1	1	1
P_f	0.01	0.01	0.01	0.01	0.0001	0.01
$SNR(dB)$	10	10	20	10	10	10
L	2	2	2	2	2	4
m	1	1	1	4	1	1
$\tilde{N}$	**13**	**25**	**9**	**10**	**21**	**9**
$\mid E_{eg,2} \mid$						
u	1	5	1	1	1	5
P_f	0.01	0.01	0.01	0.01	0.0001	0.0001
$SNR(dB)$	10	10	20	10	10	10
m	1	1	1	4	1	1
$\tilde{N}$	**15**	**29**	**11**	**12**	**24**	**41**
$\mid E_{sc,2} \mid$						
u	1	5	1	1	1	5
P_f	0.01	0.01	0.01	0.01	0.0001	0.0001
$SNR(dB)$	10	10	20	10	10	10
m	1	1	1	4	1	1
$\tilde{N}$	**15**	**30**	**11**	**14**	**25**	**41**
$\mid E_{sc,L} \mid$						
u	1	5	1	1	1	1
P_f	0.01	0.01	0.01	0.01	0.0001	0.01
$SNR(dB)$	10	10	20	10	10	10
L	2	2	2	2	2	4
m	1	1	1	4	1	1
$\tilde{N}$	**16**	**28**	**14**	**16**	**24**	**18**

(16, 17) compared to that of the PDF method, which involves a double Hypergeometric function. But the former result is restricted to integer values of $m \geq \frac{1}{2}$ and latter handles any value of $m \geq \frac{1}{2}$. Mathematical software packages can readily implement both the MGF and PDF methods.

Table I is constructed to illustrate the number of terms required in evaluating the infinite series form expressions. These error bound expressions derived for no-diversity, SC, EGC and MRC cases can easily be implemented and computed with Mathematical software package Mathematica. The minimum number of terms required to obtained a five figure accuracy ($\tilde{N}$) is shown in the Table I.

To provide an insight to the performance of the detector, several complementary receiver operating characteristic (ROC) curves are provided [4]: P_m versus P_f where $P_m = 1 - P_d$. The Nakagami channel gains are generated as in [25]. EGC and MRC reception results are provided. We assume perfect channel estimates are available for diversity reception similar to [3], [5]. The detector binary decision is taken by comparing Y and respective λ values and average P_m and average P_f are calculated. Simulation results of average P_f are within the range specified for that particular set up. Simulation results exactly match the theoretical results.

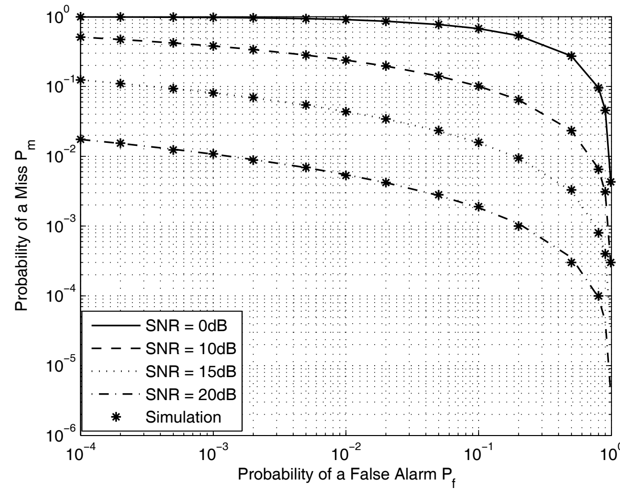

Fig. 1. Complementary ROC curves over Nakagami-m fading channel ($u = 1, m = 2$).

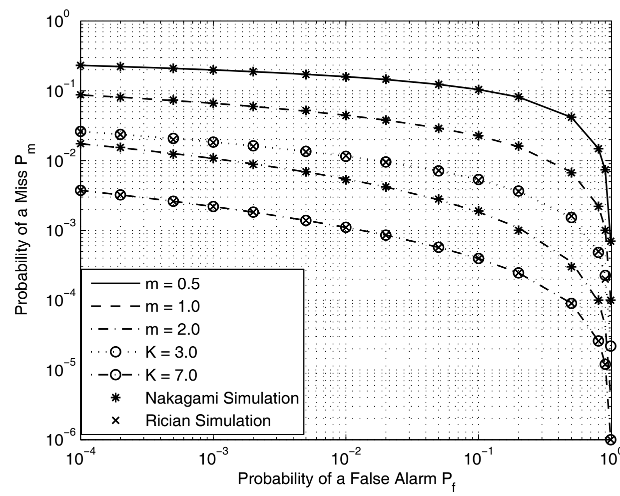

Fig. 2. Complementary ROC curves over Nakagami-m and Rician fading channels ($u = 1, SNR = 20\,dB$).

Fig. 1 shows the complementary ROC curves for non-diversity reception over a Nakagami channel ($m = 2$), parameterized over the average SNR ($\overline{\gamma}$). The probability of miss improves rapidly with increasing $\overline{\gamma}$; roughly a gain of one order of magnitude is achieved when $\overline{\gamma}$ increases from 15 dB to 20 dB. Fig. 2 plots complementary ROC performance curves for signals over Rayleigh, Nakagami-m and Rician fading channels. Rayleigh and Rician $K = 0$ curves coincide with the Nakagami $m = 1$ curve and therefore not shown. Roughly of about ten times performance improvement is observed for m, 1 to 2 and K, 3 to 7. Similar to Fig. 2, MRC, EGC and SC diversity receivers show better performance over higher m values and are therefore not shown here. However, the performance improvement for higher m and K values at lower SNR region is not significant. It is observed from Fig. 3 that at

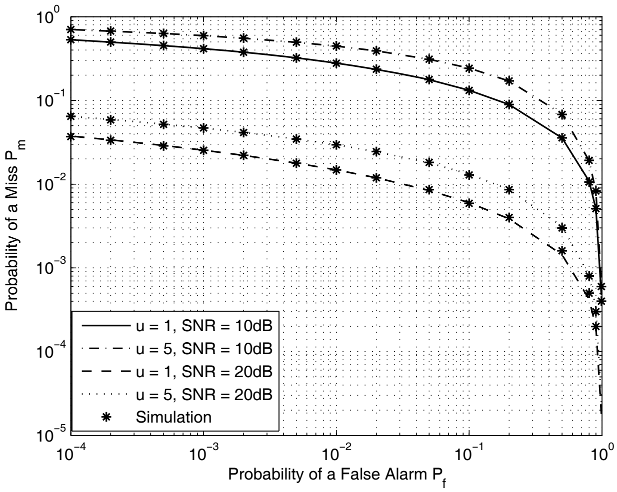

Fig. 3. Complementary ROC curves over Nakagami-*m* fading channel ($m = 1.5$).

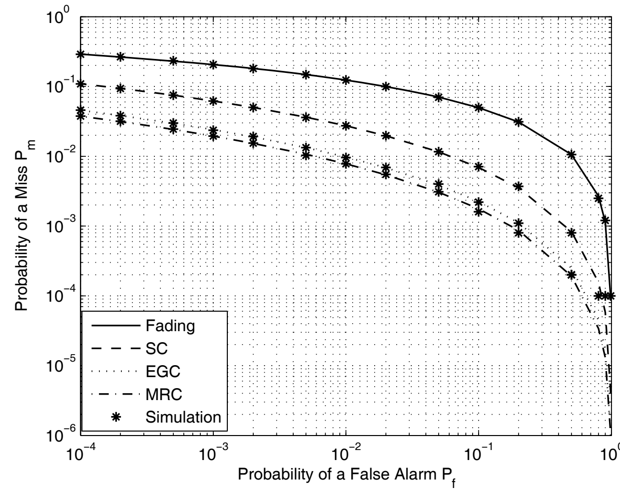

Fig. 5. Complementary ROC curves of dual branch diversity receivers over Nakagami-*m* fading channel ($u = 2, m = 2, SNR = 13\,dB$).

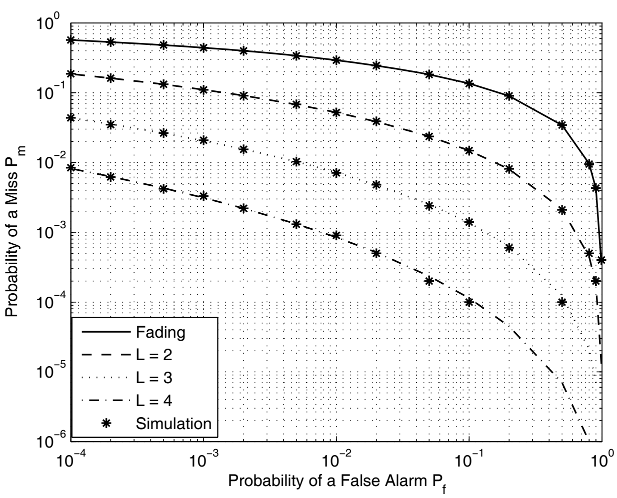

Fig. 4. Complementary ROC curves of MRC receiver over Rician fading channel ($u = 2, K = 3, SNR = 10\,dB$).

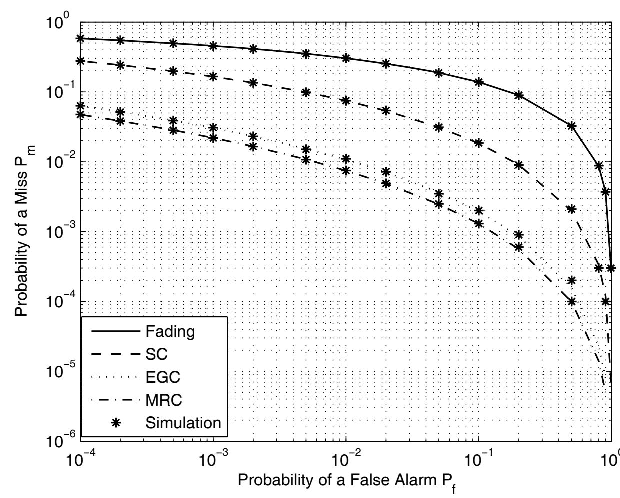

Fig. 6. Complementary ROC curves of triple branch diversity receivers over Nakagami-*m* fading channel ($u = 2, m = 2, SNR = 10\,dB$).

these values of $\overline{\gamma}$ (10 dB and 20 dB), the detector performance curves of $u = 1$ and $u = 5$ sketch closer and therefore, higher u values have no significant performance reduction in this $\overline{\gamma}$ region. Further, the Rician fading channel and the diversity combiners considered show similar variations over u.

How does diversity reception improve the performance of the energy detector? This question is investigated in Figs. 4-6. Fig. 4 shows the complementary ROC performance of the energy detector with MRC reception over Rician fading. The number of diversity branches varies for two to four. The dual and triple branch diversity detectors performance over all three combining schemes shown in Fig. 5 and Fig. 6 respectively (over a Nakagami-*m* fading channel). Observe that the slopes of the curves in Fig. 4 are steeper than those Fig. 5 and Fig. 6. The highest diversity gain is observed from no diversity fading case to the dual branch combiner in all the three combining schemes considered. The best performance of MRC is observed in Fig. 5, Fig. 6 and there is a gain of one order of magnitude improvement in P_m compared to the no-diversity case. It is interesting to note that EGC and MRC perform nearly identical. Note that Fig. 5 and Fig. 6 are for $\overline{\gamma}$ of 13 dB and 10 dB respectively, but the curves of the MRC of each plot are closer to each other. Hence, in this special case 3 dB SNR penalty is incurred by increasing the combiner branches from 2 to 3. When the no-diversity case is compared to the respective dual branch MRC energy detector, the SNR gain is higher than 3 dB. Therefore, diversity reception is a promising method of combating the inherent performance deterioration of the energy detector at moderately-low SNR region. However, the gain through diversity combining alone is not sufficient to operate the detector at low SNR values around, say, 0 dB, which are not uncommon in situations like shadowing environments. To use energy detection in such conditions, diversity reception and cooperative sensing schemes may be combined.

VIII. CONCLUSION

The performance of the energy detector with diversity reception has been studied. A new performance analysis method based on the contour integral representation of Marcum-Q function and MGF has been developed. This methods yields the energy detector's performance over Rician and Nakagami-*m* fading channels, whereas the conventional PDF method fails for Rician fading. As a by product of MGF approach, an alternative simple closed-form result over Rayleigh fading channel is also derived. Comprehensive performance results for energy detection with SC, MRC and EGC schemes have been derived. These results help quantify the performance gains for energy detection with diversity reception, which can help emerging applications such as cognitive radio and ultra wide-band radio.

REFERENCES

[1] H. Urkowitz, "Energy detection of unknown deterministic signals," *Proc. IEEE*, vol. 55, no. 4, pp. 523–531, 1967.

[2] V. Kostylev, "Energy detection of a signal with random amplitude," in *Proc. IEEE Int. Conf. Commun.*, vol. 3, Apr. 2002, pp. 1606–1610.

[3] F. Digham, M. Alouini, and M. Simon, "On the energy detection of unknown signals over fading channels," in *Proc. IEEE Int. Conf. Commun.*, vol. 5, 2003, pp. 3575–3579.

[4] ——, "On the energy detection of unknown signals over fading channels," *IEEE Trans. Commun.*, vol. 55, no. 1, pp. 21–24, Jan. 2007.

[5] A. Pandharipande and J.-P. Linnartz, "Performance analysis of primary user detection in a multiple antenna cognitive radio," in *Proc. IEEE Int. Conf. Commun.*, 2007, pp. 6482–6486.

[6] A. H. Nuttall, "Some integrals involving the Q function," Naval Underwater Systems Center (NUSC), Tech. Rep., Apr. 1972.

[7] —, "Some integrals involving the Q_M function," Naval Underwater Systems Center (NUSC), Tech. Rep., May 1974.

[8] S. P. Herath and N. Rajatheva, "Analysis of equal gain combining in energy detection for cognitive radio over Nakagami channels," in *Proc. IEEE Global Telecommun. Conf.*, Nov. 2008, pp. 1–5.

[9] S. P. Herath, N. Rajatheva, and C. Tellambura, "Unified approach for energy detection of unknown deterministic signal in cognitive radio over fading channels," in *Proc. IEEE Int. Conf. on Communications Workshops*, June 2009, pp. 1–5.

[10] ——, "On the energy detection of unknown deterministic signal over Nakagami channels with selection combining," in *Proc. IEEE Canadian Conf. Electrical Comput. Eng.*, May 2009, pp. 745–749.

[11] C. Tellambura, A. Annamalai, and V. Bhargava, "Closed form and infinite series solutions for the MGF of a dual-diversity selection combiner output in bivariate Nakagami fading," *IEEE Trans. Commun.*, vol. 51, no. 4, pp. 539–542, Apr. 2003.

[12] M. K. Simon and M.-S. Alouini, *Digital Communication over Fading Channels*, 2nd edition. Wiley, 2005.

[13] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 6th edition. Academic Press, Inc., 2000.

[14] H. M. Srivastava and P. W. Karlsson, *Multiple Gaussian Hypergeometric Series*. Ellis Horwood, 1985.

[15] C. Tan and N. Beaulieu, "Infinite series representations of the bivariate Rayleigh and Nakagami-m distributions," *IEEE Trans. Commun.*, vol. 45, no. 10, pp. 1159–1161, 1997.

[16] K. Cordeiro, C. Challapali, and D. Birru, "IEEE 802.22: an introduction to the first wireless standard based on cognitive radios," *J. Commun.*, vol. 1, no. 1, pp. 38-47, 2006.

[17] S. M. Almalfouh and G. L. Stuber, "Uplink resource allocation in cognitive radio networks with imperfect spectrum sensing," in *Proc. IEEE Veh. Technol. Conf. (VTC 2010-Fall)*, Sep. 2010, pp. 1–6.

[18] J. Ma, G. Zhao, and Y. Li, "Soft combination and detection for cooperative spectrum sensing in cognitive radio networks," *IEEE Trans. Wireless Commun.*, vol. 7, pp. 4502–4507, Nov. 2008.

[19] J. Ma and Y. Li, "Soft combination and detection for cooperative spectrum sensing in cognitive radio networks," in *Proc. IEEE Global Telecommun. Conf.*, Nov. 2007, pp. 3139–3143.

[20] S. Mekki, J. Danger, B. Miscopein, and J. Boutros, "Chi-squared distribution approximation for probabilistic energy equalizer implementation in impulse-radio UWB receiver," in *Proc. IEEE Singapore International Conf. Commun. Syst.*, Nov. 2008, pp. 1539–1544.

[21] S. Mekki, J. Danger, B. Miscopein, J. Schwoerer, and J. Boutros, "Probabilistic equalizer for ultra-wideband energy detection," in *Proc. IEEE Veh. Technol. Conf. (VTC) 2008-Spring*, May 2008, pp. 1108–1112.

[22] G. L. Stüber, *Principles of Mobile Communication*, 2nd edition. Kluwer Academic Publishers, 2001.

[23] P. Dharmawansa, N. Rajatheva, and K. Ahmed, "On the distribution of the sum of Nakagami-*m* random variables," *IEEE Trans. Commun.*, vol. 55, no. 7, pp. 1407–1416, 2007.

[24] R. Sannegowda and V. Aalo, "Performance of selection diversity systems in a Nakagami fading environment," in *Proc. IEEE Southeastcon Creative Technol. Transfer - A Global Affair*, Apr. 1994, pp. 190–195.

[25] N. C. Beaulieu and C. Cheng, "An efficient procedure for Nakagami-m fading simulation," in *Proc. IEEE Global Telecommun. Conf.*, Nov. 2001, pp. 3336–3342.

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