



# ENERGY EFFECTIVENESS OF SIMULTANEOUS HEAT AND MASS EXCHANGE DEVICES

G. Prakash Narayan<sup>a</sup>, Karan H. Mistry<sup>a</sup>, Mostafa H. Sharqawy<sup>a</sup>, Syed M. Zubair<sup>b</sup>, John H. Lienhard V<sup>a,†</sup>

<sup>a</sup>*Department of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, USA.*

<sup>b</sup>*Department of Mechanical Engineering, King Fahd University of Petroleum and Minerals, Dhahran, Saudi Arabia.*

## ABSTRACT

Simultaneous heat and mass exchange devices such as cooling towers, humidifiers and dehumidifiers are widely used in the power generation, desalination, air conditioning, and refrigeration industries. For design and rating of these components it is useful to define their performance by an effectiveness. In this paper, several different effectiveness definitions that have been used in literature are critically reviewed and an energy based effectiveness which can be applied to all types of heat and mass exchangers is defined. The validity and the limitations of the various effectiveness definitions are demonstrated by way of several examples including direct and indirect contact, parallel and counterflow heat and mass exchangers. The limiting case of a simple heat exchanger is also discussed. The importance of thermal balancing in minimizing entropy production and its implications for optimization and design of these devices is dealt with in detail. The application of the energy effectiveness to heat-exchanger-like  $\varepsilon$ -NTU correlations is also examined using a detailed numerical model.

**Keywords:** Exchangers, thermal balancing, cooling towers, humidifiers, dehumidifiers.

## 1. INTRODUCTION

A simultaneous heat and mass exchanger (HME) is a device that is used to transfer energy by both heat and mass transfer between two fluid streams at different temperatures and concentrations. Thermal contact between the fluid streams will occur through direct contact of the streams if mass is transferred between them or through indirect contact via a heat transfer surface if the mass transfer is associated with phase change in just one stream. Accordingly, they are classified as direct contact devices (e.g., cooling towers) and indirect contact devices (e.g., cooling coils). They can also be of storage type wherein two fluid streams alternatively flow through the same flow passages and intermittently exchange energy. The direct contact HME can be gas-liquid type such as a cooling tower, or liquid-vapor type such as a deaerator. Also depending on the flow configuration, they are classified as either counter or parallel flow exchangers.

The aforementioned devices are widely used in power generation, desalination, air conditioning, and refrigeration systems, and their performance is vital to the overall system performance. For example, the thermal performance of cooling towers is critical to the overall performance of steam or combined cycle power plants in which they are used (Mandi *et al.*, 2005; Tawney *et al.*, 2005). For analysis and optimization of cycles containing these components, defining an effectiveness to characterize their performance has considerable advantages (Mistry *et al.*, 2010; Narayan *et al.*, 2010b,c). It is therefore important to critically examine various definition of effectiveness that are used in the literature.

In HME literature, several definitions for effectiveness are in use, but they are specific to certain configurations and boundary conditions. The

effectiveness definitions that are used for cooling towers, humidifiers, and cooling coils are summarized in Table 1.

In cooling tower literature (Cheremisinoff and Cheremisinoff, 1981; Mandi *et al.*, 2005), an effectiveness is commonly defined based on the temperature change of one of either the air or the water streams, typically, the change in water temperature. This definition can also be written using terminology commonly used in cooling towers: (a) *Range*, the change in water temperature between the inlet and the outlet; and (b) *Approach*, the difference between the water exit temperature and the inlet air wet-bulb temperature:

$$\varepsilon_T = \frac{\text{Range}}{\text{Range} + \text{Approach}} \quad (1)$$

Nellis and Klein (2008) present a modified definition of effectiveness for a cooling coil (dehumidifier) based on humidity ratio and specific enthalpy of the moist air stream, and, they provide several examples of the use of these effectivenesses for cooling coil design in their textbook.

Jaber and Webb (1989) proposed a modified definition of effectiveness based on an analogy between counterflow heat exchangers and counterflow cooling towers. They defined the maximum possible heat transfer rate as the product of the minimum modified mass flow rate ( $\dot{m}_{\min}$ ) and the maximum air side enthalpy potential difference:

$$\dot{m}_w^{\text{mod}} = \frac{\dot{m}_w c_{p,w}}{f'} \quad (2)$$

<sup>†</sup> Corresponding author. Email: [lienhard@mit.edu](mailto:lienhard@mit.edu)

**Table 1** Various definitions of effectiveness for simultaneous heat and mass exchange components.

Reference	Effectiveness defined	Defined for
Cheremisinoff and Cheremisinoff (1981); Mandi <i>et al.</i> (2005)	$\varepsilon_T = \frac{\Delta T}{\Delta T^{ideal}}$	Cooling towers
Nellis and Klein (2008)	$\varepsilon_w = \frac{\Delta \omega}{\Delta \omega^{ideal}}$	Dehumidifier
Nellis and Klein (2008)	$\varepsilon_h = \frac{\Delta h_a}{\Delta h_a^{ideal}}$	Dehumidifier
Jaber and Webb (1989)	$\varepsilon_J = \frac{Q_{act}}{\dot{m}_{min}^{mod} \cdot \Delta h_a^{ideal}}$	Cooling towers

where

$$f' = \frac{dh_{a,sat}}{dT_w} \quad (3)$$

$$\dot{m}_{min}^{mod} = \min(\dot{m}_w^{mod}, \dot{m}_{da}) \quad (4)$$

where  $f'$  is the average slope of the saturated moist air enthalpy with water temperature. It should be noted that they neglect the effect of evaporation on  $\dot{m}_w$  and assume that the saturation enthalpy curve for moist air is linear with temperature. This definition is examined in more detail in a succeeding section (Sec. 2.2).

The above literature review suggests a need to critically examine the existing effectiveness definitions and to identify one which is robustly applicable to a large class of HME devices. Thus, this paper seeks to answer the following questions: Can a single definition of effectiveness be given which will apply to all types of HME? Can this definition be used for developing reliable  $\varepsilon$ -NTU models? For what situations can one apply the existing definitions of effectiveness? What is the significance of the heat capacity rate ratio to HME devices and how can it be defined without any approximations about fluid properties? What is the analogy of a balanced heat exchanger (which minimizes entropy generation (Narayan *et al.*, 2010a)) in the case of a heat and mass exchanger? How does thermal balancing affect the effectiveness definitions?

## 2. HEAT AND MASS EXCHANGER TERMINOLOGY

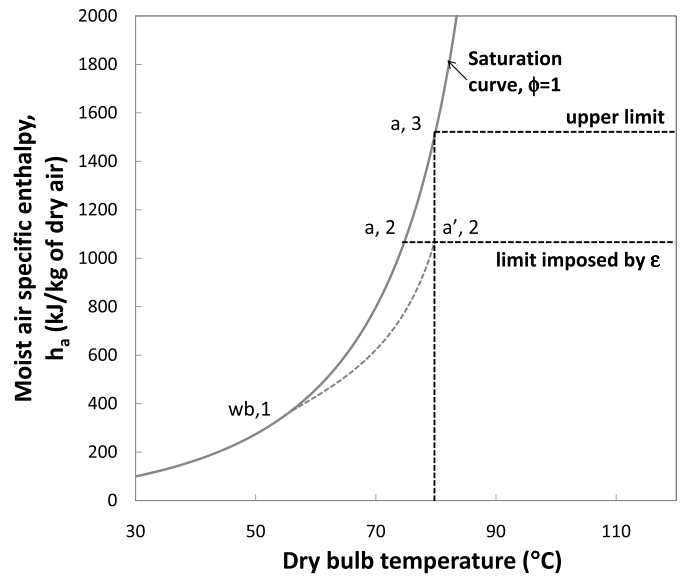
In this section, new terminology for HME devices is defined. This includes an energy-based effectiveness and a modified heat capacity rate ratio.

### 2.1. Energy effectiveness

An energy based effectiveness, analogous to the effectiveness defined for heat exchangers, is given in Eq. (5):

$$\varepsilon = \frac{\Delta \dot{H}}{\Delta \dot{H}_{max}} \quad (5)$$

This definition is based on the maximum change in total enthalpy rate that can be achieved in an adiabatic heat and mass exchanger. It is defined as ratio of change in total enthalpy rate to maximum possible change in total enthalpy rate. The maximum possible change in total enthalpy rate can be of either the cold or the hot stream, depending on the heat capacity rate of the two streams. The stream with the minimum heat capacity rate dictates the thermodynamic maximum that can be attained. To elucidate this concept, consider the example of a counterflow cooling tower.



**Fig. 1** Psychrometric chart illustrating an example of the Second Law limits on a counterflow cooling tower operation.

Figure 1 illustrates the Second Law limitations imposed on such a device. Here, 'wb,1' is the wet-bulb point of the air at the inlet to the humidifier and 'a,2' is the exit air state. The air is assumed to be saturated at the inlet and hence,  $T_{wb,1} = T_{a,1}$ . The saturation line connecting the point 'wb,1' to 'a,2' represents one possible process path for the humidification process.

The maximum dry bulb temperature that can be achieved by the saturated air at the exit of the humidifier is the water inlet temperature (indicated by point 'a,3'). From Fig. 1, it is seen that the maximum enthalpy change possible ( $\Delta \dot{H}_{max}$ ) for saturated air entering the humidifier occurs if the air can be brought to saturation at the water inlet temperature. The required energy is drawn from the water stream, which may or may not have the capacity rate ( $\dot{m}_w c_{p,w}$ ) necessary to supply that amount of energy within the limits imposed by the air and water inlet temperatures. If the water stream lacks sufficient capacity, the maximum change in total enthalpy rate ( $\Delta \dot{H}_{max}$ ) will be that which cools the water to the air inlet temperature. In this case the outlet air will be cooler than the water inlet temperature, and it may or may not be saturated. That is, for any given case, a particular range of exit relative humidities is possible (corresponding to points from 'a,2' to 'a',2' shown in Fig. 1). Hence, another parameter is required to fix the exit state of the air apart from effectiveness. In this analysis, the exit relative humidity is treated as a free design variable which can be controlled by adjusting the dimensions of, say, the packing of a cooling tower.

It is important to note that the energy effectiveness concept applies to all types of HME devices, not simply counterflow cooling towers.

### 2.2. Heat capacity rate ratio

In the limit of infinite heat transfer area for a simple heat exchanger, the entropy generation rate in the exchanger is entirely due to what is known as thermal imbalance or remanent irreversibility. This thermal imbalance is associated with conditions at which the heat capacity rate ratio is not equal to unity (Bejan, 1996). In other words, a heat exchanger is said to be thermally 'balanced' at a heat capacity rate ratio of one. This concept of thermodynamic balancing, even though very well known for heat exchangers, was only recently extended to HME devices (Narayan *et al.*, 2010a). It is important to establish a reliable definition for the heat capacity rate ratio for an HME in order to understand its influence on selecting the appropriate definition of effectiveness.

In some cases, the heat capacity rate of one of the streams of a si-

multaneous heat and mass exchangers cannot be defined readily. For example, in a counterflow cooling tower (where hot water is losing heat and mass to humid air), it is not possible to define the change in enthalpy of the moist air stream as the product of specific heat capacity at constant pressure and change in temperature because the humidity ratio also affects moist air enthalpy,  $h_a$ :

$$h_a = f\{T, p, \omega\} \quad (6)$$

If the effect of pressure variation on enthalpy is neglected,

$$dh_a = c'_{p,a} \cdot dT + \left( \frac{\partial h_a}{\partial \omega} \right)_{p,T} \cdot d\omega \quad (7)$$

where

$$c'_{p,a} \equiv \left( \frac{\partial h_a}{\partial T} \right)_{p,\omega} \quad (8)$$

Hence, the heat capacity rate ratio cannot be calculated as it is for heat exchangers.

Braun *et al.* (1989) found a way around this problem by defining an effective heat capacity of the moist air stream as slope of the saturated specific enthalpy line evaluated at water temperatures:

$$c_{p,a} = \frac{dh_{a,sat}}{dT_w} \quad (9)$$

which is identical to the enthalpy correction factor  $f'$ , Eq. (3), defined by Jaber and Webb (1989). This approximation has reasonable accuracy (under certain operating conditions) for cooling tower cases. However, in certain cases where the humidity levels are relatively high (e.g., in a direct contact counterflow air humidifier or an indirect contact dehumidifier), a sizable error is induced.

Therefore, we introduce a modified heat capacity ratio based on the total enthalpy rate change which is accurate in all ranges of humidity and temperature levels. This is defined using an analogy to heat exchangers. For heat exchangers,

$$HCR_{HE} = \frac{\dot{m}_c c_{p,c}}{\dot{m}_h c_{p,h}} \quad (10)$$

This can be rewritten as

$$HCR_{HE} = \left( \frac{\Delta \dot{H}_{max,c}}{\Delta \dot{H}_{max,h}} \right) \quad (11)$$

since the maximum temperature difference in  $\Delta \dot{H}_{max,c}$  and  $\Delta \dot{H}_{max,h}$  is the same (i.e.,  $\Delta \dot{H}_{max,k} = \dot{m}_k c_{p,k} \cdot (T_{h,i} - T_{c,i})$ , where  $k = c$  or  $h$ ). Similarly, for an HME device, we define

$$HCR = \left( \frac{\Delta \dot{H}_{max,c}}{\Delta \dot{H}_{max,h}} \right) \quad (12)$$

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### 2.3. Non-dimensional entropy generation

For heat exchangers, it is well known that entropy generation can usefully be non-dimensionalized as  $\frac{\dot{S}_{gen}}{(\dot{m}c_p)_{min}}$  (Bejan, 1996). Similarly, we define a non-dimensional term for heat and mass exchangers. The example of a cooling tower is used to describe this term below:

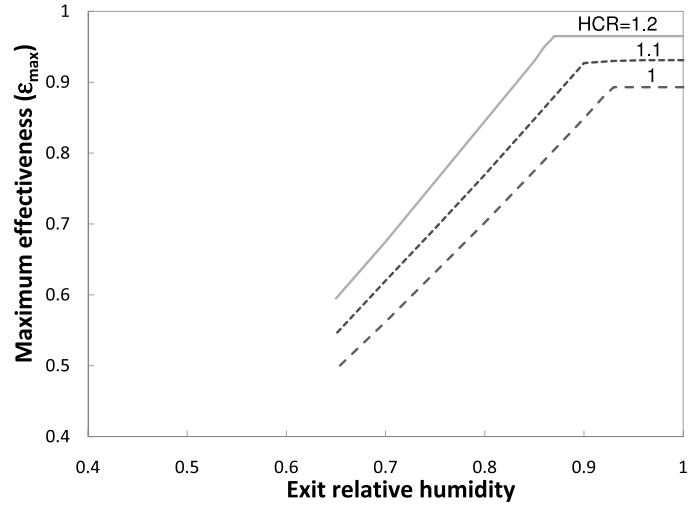
Case I,  $\Delta \dot{H}_{max,w} < \Delta \dot{H}_{max,a}$ :

$$\sigma = \frac{\dot{S}_{gen}}{(\dot{m}c_p)_{min}} = \frac{\dot{S}_{gen}}{(\dot{m}_{w,i}c_{pw})} \quad (13a)$$

Case II,  $\Delta \dot{H}_{max,w} > \Delta \dot{H}_{max,a}$ :

$$\sigma = \frac{\dot{S}_{gen}}{(\dot{m}c_p)_{min}} = \frac{\dot{S}_{gen}}{(\dot{m}_{w,i}c_{pw}HCR)} \quad (13b)$$

The typical difference in water mass flow rate from inlet to outlet in a cooling tower is 5%. Hence, the inlet water mass flow can also be used as a good approximation to the average water mass flow for the cooling tower cases presented in this paper.



**Fig. 2** Maximum value of effectiveness versus exit relative humidity for a cooling tower:  $T_{w,i} = 55^\circ\text{C}$ ;  $T_{a,i} = 34^\circ\text{C}$ ;  $\phi_i = 100$ ;  $p = 100$  kPa.

### 2.4. Limiting value of energy effectiveness

The effectiveness of an HME defined by Eq. (1) varies from zero to a maximum value that might be less than one. The maximum value depends significantly on the heat capacity ratio (HCR) defined above and is constrained by the Second Law and by transport processes.

An example of the variation of the maximum effectiveness is shown in Fig. 2. The curves are plotted at various values of HCR versus exit relative humidity. Each curve consists of two linear segments: one, the horizontal segment at high relative humidities ( $\approx 0.9$ ) and; two, the sloped straight line at lower relative humidities. The first (horizontal) segment consists of data points that are constrained by the Second Law ( $\dot{S}_{gen} = 0$ ); in this segment the outlet air temperature does not reach the water inlet temperature ( $T_{a,o} < T_{w,i}$ ). The second segment consists of points that are constrained by the temperature cross, that is the outlet air temperature reaches the inlet water temperature ( $T_{a,o} = T_{w,i}$ ) but entropy generation is non-zero and positive ( $\dot{S}_{gen} > 0$ ). This second segment (the line with a positive slope) is linear because enthalpy varies almost linearly with relative humidity in the range of temperature changes considered here.

To incorporate the two constraints in the effectiveness definition, the maximum value of enthalpy change can be redefined as follows:

$$\Delta \dot{H}_{max}^{mod} = \varepsilon_{max} \cdot \min(\Delta \dot{H}_{max,w}, \Delta \dot{H}_{max,a}) \quad (14)$$

and Eq. (5) would be modified to:

$$\varepsilon^{mod} = \frac{\Delta \dot{H}}{\Delta \dot{H}_{max}^{mod}} \quad (15)$$

Equation (15) ensures that the effectiveness varies from 0 to 1 (as for heat exchangers). However, this modification to the definition of effectiveness will make it very cumbersome to evaluate effectiveness a priori. Hence, the definition given in Eq. (5) is recommended and will be used in this paper.

## 3. EQUATIONS AND MODELING DETAILS

This section discusses the conservation equations for various configurations of HME devices and the corresponding effectiveness definitions. Additionally, the fluid property packages and models used to solve the defined equations are described.

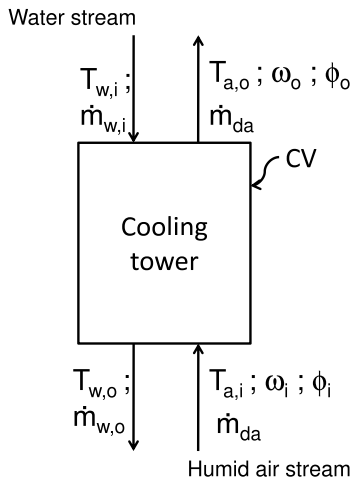


Fig. 3 Control volume for counterflow cooling tower.

### 3.1. Direct contact heat and mass exchangers

**3.1.1. Counterflow** Consider a counterflow cooling tower (Fig. 3) in which one fluid stream is pure water and the other stream is a mixture of air and water vapor. Since the dry air that enters the device in the humid air stream all leaves in the humid air stream, the mass flow rate of dry air is constant:

$$\dot{m}_{da} = \dot{m}_{da,i} = \dot{m}_{da,o} \quad (16)$$

A mass balance on the water in the cooling tower gives the mass flow rate of the water leaving the humidifier in the water stream:

$$\dot{m}_{w,o} = \dot{m}_{w,i} - \dot{m}_{da}(\omega_{a,o} - \omega_{a,i}) \quad (17)$$

Based on Eq. (5), the energy effectiveness,  $\varepsilon$ , may be written in terms of mass flow rates, temperatures, and humidity ratios. However, in order to determine the maximum possible change in enthalpy rate, it must be known whether the air stream or the water stream is the hot stream.

When the water enters hotter than the air, the ideal condition that the water stream can attain is that the temperature at the exit equals the inlet air wet-bulb temperature. This corresponds to the enthalpy driving force (which is nothing but the enthalpy potential difference between the two streams driving the heat and mass transfer) becoming zero at the water exit (Kloppers and Kröger, 2005a). The ideal condition that the moist air stream can reach is saturation at the inlet water temperature. As explained earlier this is a limit imposed by the rate processes ( $T_{a,o} \leq T_{w,i}$ ). When the air enters hotter than the water stream, the ideal conditions that can be attained by the air and the water is different to the case with hot water entering the HME. These again correspond to the driving enthalpy difference becoming zero for the respective streams.

Based on the above discussion, the effectiveness definition of a counterflow direct contact HME device with hot water entering is written as follows:

Case I,  $\Delta\dot{H}_{\max,w} < \Delta\dot{H}_{\max,a}$ :

$$\varepsilon = \frac{\dot{m}_{w,i}h_{w,i} - \dot{m}_{w,o}h_{w,o}}{\dot{m}_{w,i}h_{w,i} - \dot{m}_{w,o}h_{w,o}^{ideal}} \quad (18)$$

Case II,  $\Delta\dot{H}_{\max,w} > \Delta\dot{H}_{\max,a}$ :

$$\varepsilon = \frac{\dot{m}_{da}(h_{a,o} - h_{a,i})}{\dot{m}_{da}(h_{a,o}^{ideal} - h_{a,i})} \quad (19)$$

Note that the First Law for the cooling tower gives,

$$0 = \underbrace{\dot{m}_{da}(h_{a,i} - h_{a,o})}_{\Delta\dot{H}_a} + \underbrace{\dot{m}_{w,i}h_{w,i} - \dot{m}_{w,o}h_{w,o}}_{\Delta\dot{H}_w} \quad (20)$$

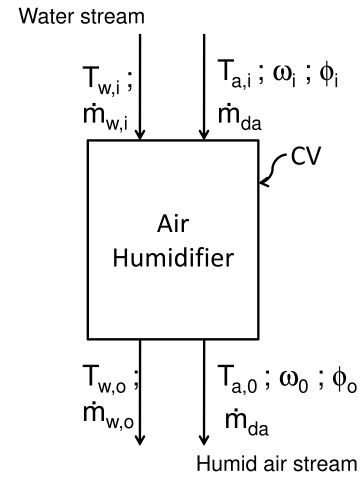


Fig. 4 Control volume for parallel flow cooling tower.

where  $\Delta\dot{H}_w$  is the change in total enthalpy rate for the feed water stream and  $\Delta\dot{H}_a$  is the change in total enthalpy rate of the moist air stream. One can similarly write down the effectiveness definition when hot air enters the tower.

**3.1.2. Parallel flow** Now let us consider a parallel flow direct contact air humidifier (Fig. 4) in which one fluid stream is pure water and the other stream is a mixture of air and water vapor.

The energy balance, mass balance, and effectiveness equations are identical to the counterflow case. However, for finite heat capacities, the cold stream can never simultaneously gain mass and reach the temperature of the hot stream at the inlet (unlike in the counterflow HME). The ideal condition at the exit corresponds to the driving enthalpy force reaching zero at the exit. Depending on the heat capacity rate ratio the enthalpy driving force is controlled by either the water or the air streams. At a HCR < 1, the cold stream is the minimum heat capacity stream and the enthalpy potential that drives the energy transfer is the difference of the maximum possible enthalpy of the cold stream (at the corresponding hot stream temperature) and the enthalpy of the cold stream at its own temperature. For example, in a cooling tower this enthalpy difference (for HCR < 1) corresponds to the difference in moist air saturated enthalpy at the water temperature and the actual moist air enthalpy at the corresponding location. Similarly, when the minimum heat capacity stream is the hot stream, the driving enthalpy difference is that between the enthalpy of the hot stream and the minimum possible enthalpy of the hot stream (at the corresponding cold stream temperature). Moreover, the ideal condition is also constrained by rate processes which cannot simultaneously transfer both mass and heat from the cold to the hotter stream.

### 3.2. Indirect contact heat and mass exchangers

**3.2.1. Counterflow** Now consider a counterflow dehumidifier (Fig. 5) in which one fluid stream is pure water and the other stream is a mixture of air and water vapor. The air-vapor mixture is transferring heat to the water stream. In this process, some of the water vapor in the mixture condenses out and forms a separate condensate stream. Since all the dry air in the air stream and the water in the other fluid stream that enters the dehumidifier also leaves the device, the mass flow rate of dry air and mass flow rate of the water is constant.

$$\dot{m}_{da} = \dot{m}_{da,i} = \dot{m}_{da,o} \quad (21)$$

$$\dot{m}_{w,o} = \dot{m}_{w,i} \quad (22)$$

The mass flow rate of the condensed water can be calculated using a

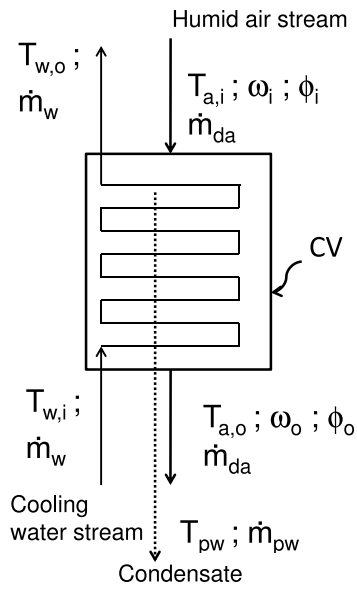


Fig. 5 Control volume for counterflow dehumidifier.

simple mass balance:

$$\dot{m}_{pw} = \dot{m}_{da} (\omega_{a,o} - \omega_{a,i}) \quad (23)$$

To calculate maximum total enthalpy rate change possible, the inlet temperatures and mass flow rates must be known. As explained before, the ideal condition corresponds to the enthalpy driving force becoming zero at the water exit or the air exit. The ideal condition that the air can reach at the exit is saturation at the inlet temperature of water. The water can at best reach the dry bulb temperature of the air at inlet. Again, this corresponds to the enthalpy driving force reaching zero at the air inlet end.

Based on the above discussion, the effectiveness definition of a counterflow indirect contact HME device is as follows:

Case I,  $\Delta \dot{H}_{max,w} < \Delta \dot{H}_{max,a}$ :

$$\varepsilon = \frac{h_{w,i} - h_{w,o}}{h_{w,i}^{ideal} - h_{w,o}} \quad (24)$$

Case II,  $\Delta \dot{H}_{max,w} > \Delta \dot{H}_{max,a}$ :

$$\varepsilon = \frac{\dot{m}_{da}(h_{a,o} - h_{a,i}) + \dot{m}_{pw}h_{pw}}{\dot{m}_{da}(h_{a,o} - h_{a,i}^{ideal}) + \dot{m}_{pw}h_{pw}} \quad (25)$$

Note that the First Law for the dehumidifier can be expressed as,

$$0 = \underbrace{\dot{m}_{da}(h_{a,i} - h_{a,o}) - \dot{m}_{pw}h_{pw}}_{\Delta \dot{H}_a} + \underbrace{\dot{m}_w(h_{w,i} - h_{w,o})}_{\Delta \dot{H}_w} \quad (26)$$

where  $\Delta \dot{H}_w$  is the change in total enthalpy rate for the feed water stream and  $\Delta \dot{H}_a$  is the change in total enthalpy rate of the moist air stream.

**3.2.2. Parallel flow** In the case of parallel flow dehumidifier (Fig. 6), the energy and mass balance equations are the same as for counterflow. The energy effectiveness depends on the maximum possible enthalpy change in an adiabatic counterflow type HME. Depending on the heat capacity rate ratio, the enthalpy potential is either of the water or of the air streams; in an ideal scenario, it reaches zero at the exit.

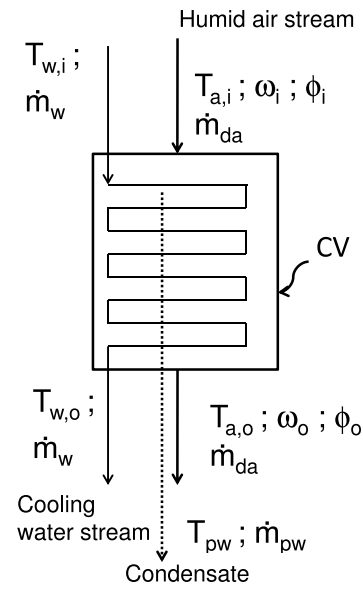


Fig. 6 Control volume for parallel flow dehumidifier.

### 3.3. Modeling approximations

In order to illustrate the usefulness of the concepts discussed in previous sections, a thermodynamic analysis of various examples has been carried out. In performing the analysis, the following approximations have been made:

- The processes involved operate at steady-state conditions.
- There is no heat loss from the components to the surroundings.
- Kinetic and potential energy terms are neglected in the energy balance.

### 3.4. Solution technique

The solution of the governing equations was carried out using **Engineering Equation Solver (EES)** (Klein, 2009) which uses accurate equations of state to model the properties of moist air and water. EES evaluates water properties using the IAPWS (International Association for Properties of Water and Steam) 1995 Formulation (Wagner and Pruss, 2002). Dry air properties are evaluated using the ideal gas formulations presented by Lemmon *et al.* (2000). Moist air properties are evaluated assuming an ideal mixture of air and steam using the formulations presented by Hyland and Wexler (1983b). Moist air properties from EES are in close agreement with the data presented in ASHRAE Fundamentals (Wessel, 2001) and pure water properties are equivalent to those found in NIST's property package, REFPROP (Lemmon *et al.*, Version 8.0 (2007)).

EES is a numerical solver, and it uses an iterative procedure to solve the equations. The convergence of the numerical solution is checked by using the following two variables: (1) 'Relative equation residual' — the difference between left-hand and right-hand sides of an equation divided by the magnitude of the left-hand side of the equation; and (2) 'Change in variables' — the change in the value of the variables within an iteration. The calculations converge if the relative equation residuals is lesser than  $10^{-6}$  or if change in variables is less than  $10^{-9}$ . These are standard values used to check convergence in EES. There are several publications which have previously used them for thermodynamic analysis (Qureshi and Zubair, 2006; Zmeureanu and Wu, 2007).

## 4. LIMITING CASE: HEAT EXCHANGERS

In this section, it is shown that the energy effectiveness of an HME in the limiting case of no mass transfer in a counterflow cooling tower is

equivalent to that of a counterflow heat exchanger. When mass is not transferred, the humidity ratio of the moist air stream remains unchanged from inlet to outlet. Therefore,

$$\omega_i = \omega_o \quad (27)$$

Since the humidity ratio of the air stream remains constant, the effectivenesses reduce to the following

Case I, water is the minimum heat capacity stream ( $HCR > 1$ )

$$\begin{aligned} \varepsilon &= \frac{h_{w,i} - h_{w,o}}{h_{w,i} - h_{w,o}^{ideal}} \\ &= \frac{c_{p,w} (T_{w,i} - T_{w,o})}{c_{p,w} (T_{w,i} - T_{w,o}^{ideal})} = \frac{c_{p,w} (T_{w,i} - T_{w,o})}{c_{p,w} (T_{w,i} - T_{a,i})} \end{aligned} \quad (28)$$

Case II, moist air is the minimum heat capacity stream ( $HCR < 1$ )

$$\varepsilon = \frac{h_{a,i} - h_{a,o}}{h_{a,i} - h_{a,o}^{ideal}} = \frac{c_{p,a} (T_{a,i} - T_{a,o})}{c_{p,a} (T_{a,i} - T_{w,i})} \quad (29)$$

Finally, the maximum value of the above two equations gives the energy effectiveness expression

$$\begin{aligned} \varepsilon &= \frac{\dot{m}_w c_{p,w} (T_{w,i} - T_{w,o})}{\dot{C}_{\min} (T_{w,i} - T_{a,i})} \\ &= \frac{\dot{m}_{da} c_{p,a} (T_{a,i} - T_{a,o})}{\dot{C}_{\min} (T_{a,i} - T_{w,i})} \end{aligned} \quad (30)$$

which is the usual definition for the effectiveness of a two stream heat exchanger (Lienhard IV and Lienhard V, 2008).

## 5. RESULTS OF THE CONTROL VOLUME MODEL

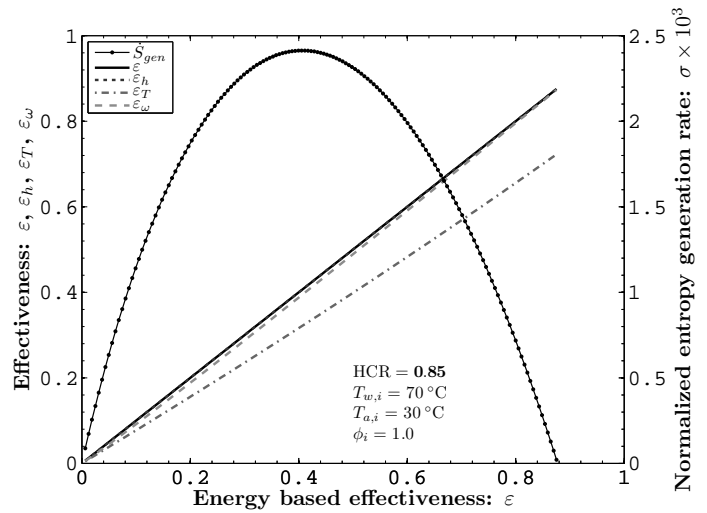
In this section, the differences and similarities between the various definitions of effectiveness and their relationship to the energy effectiveness are examined. The importance of the heat capacity rate ratio and how it affects the aforementioned relationships is also discussed together with the concept of thermal balancing of HME devices. The entropy generation as calculated from the results of an unconstrained First Law analysis can appear to be negative for certain unphysical cases which must be excluded by proper choice of effectiveness and boundary conditions.

All graphs in this section plot the various definitions of effectiveness versus the energy based effectiveness. Additionally, the non-dimensional entropy generation rate is also plotted versus the energy based effectiveness.

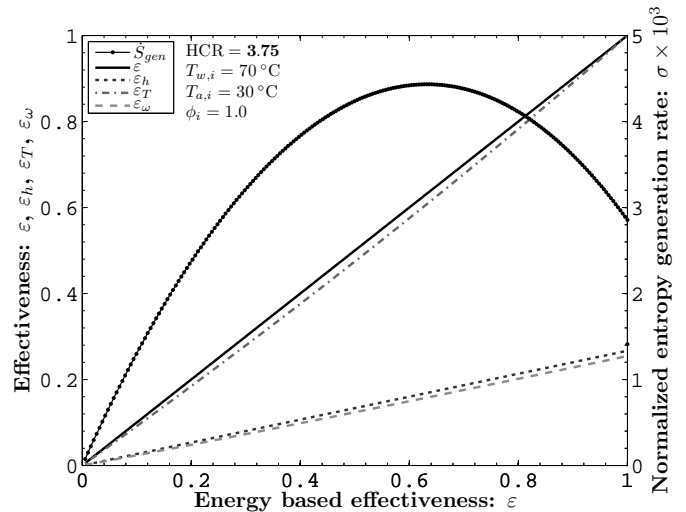
### 5.1. Direct contact heat and mass exchangers

**5.1.1. Counterflow cooling towers** The comparison of various definition of effectiveness for a counterflow cooling tower is shown in Fig. 7. In this example, the device operates at atmospheric pressure with the inlet water temperature at  $70^\circ\text{C}$ , and the moist air entering saturated at  $30^\circ\text{C}$  and exiting saturated. In this case, the device is operating at a heat capacity rate ratio of less than one which means that moist air is the minimum heat capacity rate stream. For the given case the maximum values of  $\varepsilon$ ,  $\varepsilon_h$  and  $\varepsilon_w$  are about 0.9 and that of  $\varepsilon_T$  is about 0.7. The entropy generation rate becomes negative when these values are exceeded. Moreover, the values of  $\varepsilon$ ,  $\varepsilon_h$  and  $\varepsilon_w$  correlate very well with each other, but  $\varepsilon_T$  differs. This is because  $\varepsilon_T$  is a water-temperature-based definition, and, in the current case the minimum heat capacity stream is moist air.

Figure 8 shows the comparison for a situation in which  $HCR > 1$ , which means that the minimum heat capacity stream is the water. In this case, the values of  $\varepsilon$  and  $\varepsilon_T$  correlate very well with each other but  $\varepsilon_h$  and  $\varepsilon_w$  differ significantly from  $\varepsilon$ . This indicates that in situations where the  $HCR > 1$ , the temperature based effectiveness is a good approximation for the energy effectiveness, provided the inlet air is saturated. We will discuss the correlation for unsaturated air entering the inlet in Sec. 5.1.2.



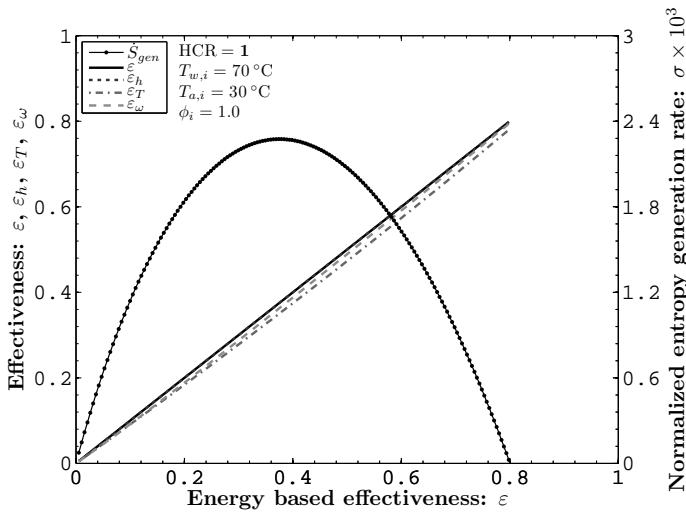
**Fig. 7** Comparison of different effectiveness definitions in a counterflow cooling tower when moist air is the minimum heat capacity stream ( $HCR < 1$ ).



**Fig. 8** Comparison of different effectiveness definitions in a counterflow cooling tower when water is the minimum heat capacity stream ( $HCR > 1$ ).

Figure 9 illustrates the comparison for a situation in which  $HCR = 1$ . This condition is called the thermally balanced condition, at which remanent irreversibility is minimum (Narayan *et al.*, 2010a). Since the temperature profiles and the humidity levels are balanced, all the various definitions of effectiveness are very similar. This further demonstrates the concept of thermal balancing. In simple heat exchangers, this concept is well known and corresponds to the balancing of the temperature profiles (Bejan, 1996; Narayan *et al.*, 2010a).

Another interesting observation that can be made from the figures in this section is that the maximum value of the energy effectiveness is lower at the balanced condition and increases as  $HCR$  moves further away from a balanced condition (see Table 2). This is because irreversibility is lower at  $HCR = 1$ , compared to the unbalanced case, and the entropy production reaches zero at a lower value of effectiveness (See Figs. 7, 8 and 9). Hence, a lower value of maximum effectiveness exists for the balanced condition. At values of  $HCR$  sufficiently away from one (like the case were  $HCR = 3.75$  in the table) the energy effectiveness has a maximum value of one.



**Fig. 9** Comparison of different effectiveness definitions in a counterflow cooling tower at thermally balanced condition (HCR = 1).

**Table 2** Examples of maximum effectiveness for a counterflow cooling tower with following boundary condition:  $T_{w,i} = 70\text{ }^\circ\text{C}$ ;  $p_{w,i} = 1\text{ atm}$ ,  $T_{a,i} = 30\text{ }^\circ\text{C}$ ;  $p_{a,i} = 1\text{ atm}$ ,  $\phi_i = 1.0$ .

Thermal balance	Maximum value of effectiveness			
	$\epsilon$	$\epsilon_w$	$\epsilon_h$	$\epsilon_T$
unbalanced, HCR = 0.85	0.875	0.873	0.874	0.723
unbalanced, HCR = 3.75	1	0.258	0.270	1
balanced, HCR = 1	0.794	0.789	0.794	0.778

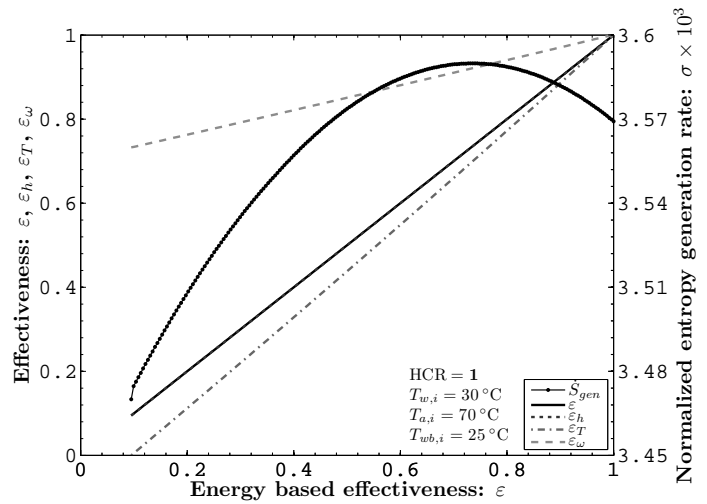
**5.1.2. Counterflow, unsaturated air at inlet** The case where the inlet air in unsaturated is discussed below. Table 3 illustrates the effect of this condition on the maximum values of effectivenesses. The comparisons are very similar to the saturated air condition discussed above.

**5.1.3. Counterflow humidifier with air entering hotter than water** The implications of having hotter air entering a direct contact HME on the ideal conditions that be achieved by the fluid streams were discussed in the previous section. Figure 10 illustrates the comparison of the effectivenesses for this boundary condition in a thermally balanced situation. It is observed that the values of  $\epsilon$ ,  $\epsilon_h$  and  $\epsilon_T$  correlate reasonably well with each other but  $\epsilon_w$  differs. This is because  $\epsilon_w$  is a humidity-based definition and the humidity change alone does not capture the change in total energy very well in hot air cases.

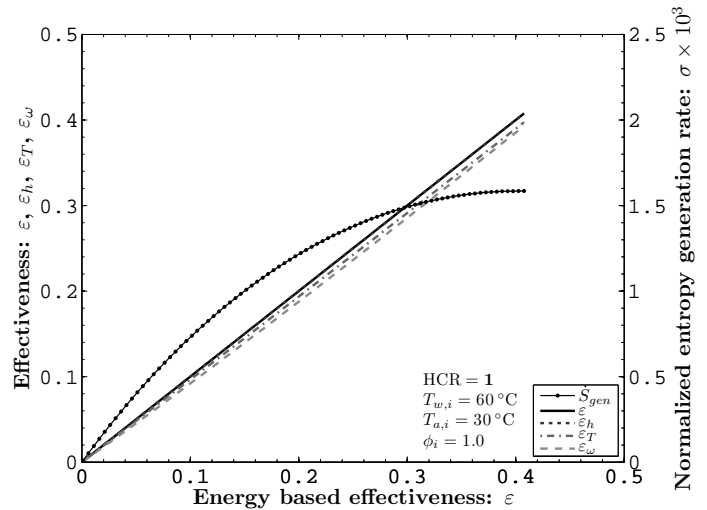
**5.1.4. Parallel flow cooling tower** Figure 11 shows a plot of the various definitions of effectiveness versus the energy based effectiveness for a balanced (HCR = 1) parallel flow, direct contact HME. For this particular configuration both streams are at atmospheric pressure, the inlet water temperature is 60 °C, and the moist air enters saturated at 30 °C and exits saturated. Several important observations can be made. First, as

**Table 3** Examples of maximum effectiveness for a counterflow cooling tower with following boundary condition:  $T_{w,i} = 70\text{ }^\circ\text{C}$ ;  $p_{w,i} = 1\text{ atm}$ ,  $T_{a,i} = 30\text{ }^\circ\text{C}$ ;  $p_{a,i} = 1\text{ atm}$ ,  $\phi_i = 0.5$ .

Thermal balance	Maximum value of effectiveness			
	$\epsilon$	$\epsilon_w$	$\epsilon_h$	$\epsilon_T$
unbalanced, HCR = 0.85	0.780	0.776	0.780	0.761
unbalanced, HCR = 4	1	0.242	0.250	1
balanced, HCR = 1	0.776	0.772	0.776	0.756



**Fig. 10** Comparison of different effectiveness definitions at thermally balanced condition (HCR = 1) for counterflow humidifier with air entering hotter than water.



**Fig. 11** Effectiveness of a parallel flow, direct contact heat and mass exchanger with HCR = 1.

with a parallel flow HE, it is not possible to achieve 100% effectiveness. Instead,  $\epsilon \leq \epsilon_{\max}$ , where  $\epsilon_{\max}$  is based on the pinch point, or the minimum temperature difference between the two fluid streams, at the outlet of the component. The overall entropy generation does not approach zero at  $\epsilon_{\max}$ . Second,  $\epsilon \equiv \epsilon_h$  since the mass flow rate of dry air does not change and therefore, cancels out in the definition of energy based effectiveness when the air stream has the minimum capacity. Finally, both  $\epsilon_w$  and  $\epsilon_T$  are approximately the same as  $\epsilon$  since the component considered here is balanced.

The effectiveness values for a balanced parallel flow, direct contact HME with unsaturated inlet moist air and the operating conditions of the previous case are plotted in Fig. 12. Once again, the actual effectiveness range is  $0 \leq \epsilon \leq \epsilon_{\max}$  and  $\epsilon \equiv \epsilon_h$  since the dry air mass flow rate cancels out. Changing the inlet air condition from saturated to unsaturated results in significant differences between the various definitions of effectiveness. The temperature-based effectiveness,  $\epsilon_T$ , is substantially different since it is based on the water stream and the air stream has the minimum capacity. The humidity-based effectiveness,  $\epsilon_w$ , is more accurate than  $\epsilon_T$  since it is still based on the air stream, but at low effectiveness,  $\epsilon_w$  deviates significantly.

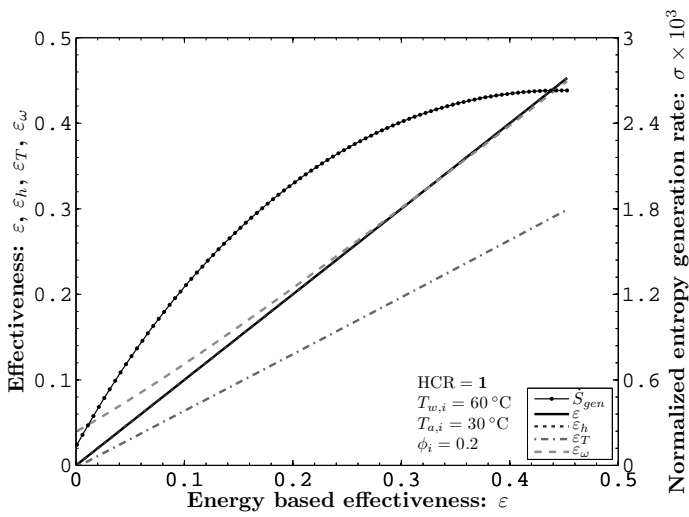


Fig. 12 Effectiveness of a parallel flow, direct contact heat and mass exchanger with HCR = 1.

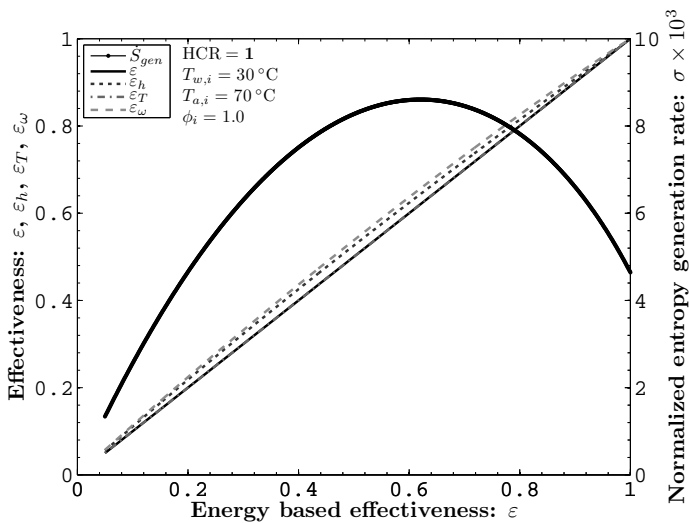


Fig. 13 Comparison of different effectiveness definitions at thermally balanced condition (HCR = 1) for counterflow air dehumidifier.

## 5.2. Indirect contact heat and mass exchangers

Indirect contact or surface type HME are different from direct contact type HME in that the fluid streams are not mixed. Hence, it is important to investigate the comparison between the different effectiveness definitions. Also, it is interesting to investigate the influence of thermal balancing in these devices.

**5.2.1. Counterflow** First consider the counterflow type using the example of an air dehumidifier. Figure 13 illustrates that at the thermally balanced condition, the air dehumidifier can be defined by any of the four values of effectivenesses (which is similar to the direct contact counterpart). Also, when  $HCR < 1$  and  $HCR > 1$ , all trends are similar to the direct contact type.

It was previously observed that for the direct contact HME the maximum value of effectiveness is lower at the balanced condition and increases as HCR moves further away from a balanced condition (Table 2). Table 4 shows that for counterflow indirect contact HME the maximum value of effectiveness is close to one for cases when  $HCR = 1$  and  $HCR > 1$ . This is a significant observation and shows that the performance of these exchangers reaches a true optimum at  $HCR = 1$  and

**Table 4** Examples of maximum effectiveness for a counterflow air dehumidifier with following boundary condition:  $T_{w,i} = 30\text{ }^\circ\text{C}$ ;  $p_{w,i} = 1\text{ atm}$ ,  $T_{a,i} = 70\text{ }^\circ\text{C}$ ;  $p_{a,i} = 1\text{ atm}$ ,  $\phi_i = 1.0$ .

Thermal balance	Maximum effectiveness			
	$\epsilon$	$\epsilon_w$	$\epsilon_h$	$\epsilon_T$
unbalanced, HCR = 0.25	1	0.270	0.263	1
unbalanced, HCR = 2	1	1	1	1
balanced, HCR = 1	1	1	1	1

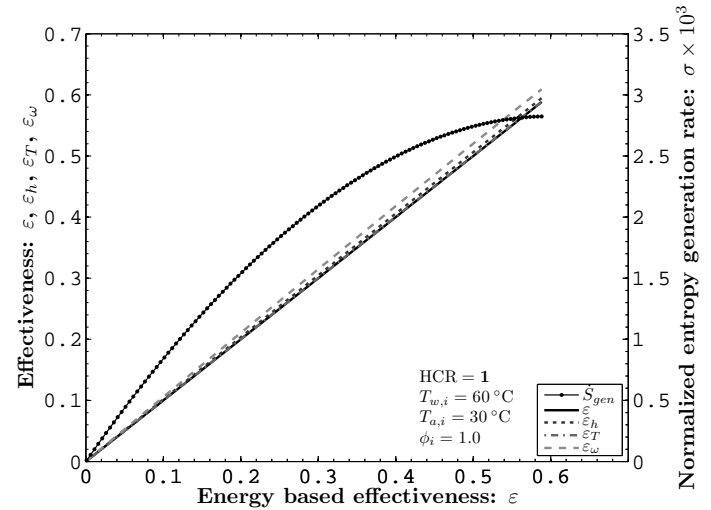


Fig. 14 Effectiveness of a parallel flow, non-direct contact heat and mass exchanger with HCR = 1.

can be effectively used in engineering applications to optimize the design as illustrated by Narayan *et al.* (2009).

**5.2.2. Parallel flow** Figure 14 shows a plot of the various definitions of effectiveness for a balanced ( $HCR = 1$ ) parallel flow, non-direct contact HME. Both streams are at atmospheric pressure, the inlet water temperature is  $60\text{ }^\circ\text{C}$ , and the moist air enters saturated at  $30\text{ }^\circ\text{C}$  and exits saturated. Once again, since the exchanger is operating at  $HCR = 1$ , all of the definitions of effectiveness are approximately equal. The specific enthalpy based effectiveness,  $\epsilon_h$ , is very close to the energy based effectiveness,  $\epsilon$ , since the mass flow rate of the product in this stream is very small compared to mass flow rate of air which means the effect of condensation is very small. The only difference between  $\epsilon$  and  $\epsilon_h$  is the effect of condensation. Finally, as with the parallel flow, direct contact exchangers, the effectiveness range is again limited to  $\epsilon \leq \epsilon_{\max}$  due to the flow configuration.

The effectiveness values for a balanced parallel flow, non-direct contact HME with unsaturated inlet moist air and the same operating conditions as stated in the previous paragraph is plotted in Fig. 15. As with the saturated case,  $\epsilon_h$  is very close to the energy based effectiveness,  $\epsilon$ , since the mass flow rate of the product in this stream is very small compared to mass flow rate of air. Additionally, the effectiveness is limited to  $\epsilon \leq \epsilon_{\max}$  rather than  $\epsilon \leq 1$ . Unlike the saturated case, however,  $\epsilon_T$  and  $\epsilon_w$  significantly over- and under-predict the effectiveness.

**5.2.3. Counterflow with phase change in one of the streams** Figure 16 shows a plot of the various definitions of effectiveness for a balanced counterflow, indirect contact HME in which one of the streams changes phase. Note that the entropy generation is not non-dimensionalized because the heat capacity of both streams varies significantly along the length of the exchanger and Eqs. (13a) and (13b) are not readily evaluated. For this particular configuration, the water stream enters as saturated



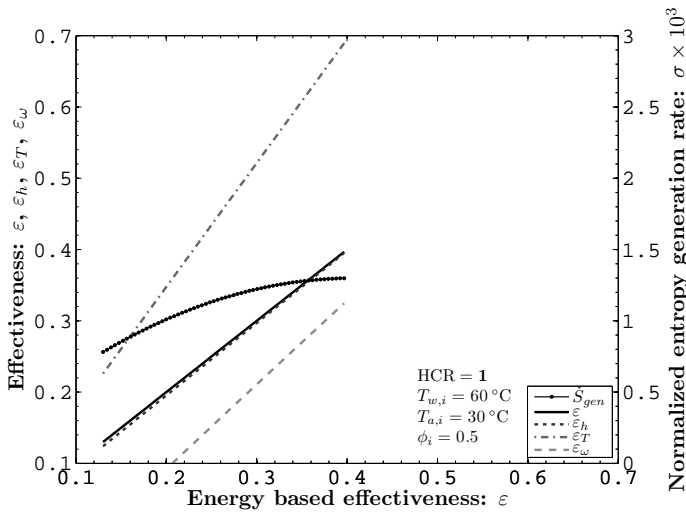


Fig. 15 Effectiveness of a parallel flow, non-direct contact heat and mass exchanger with HCR = 1.

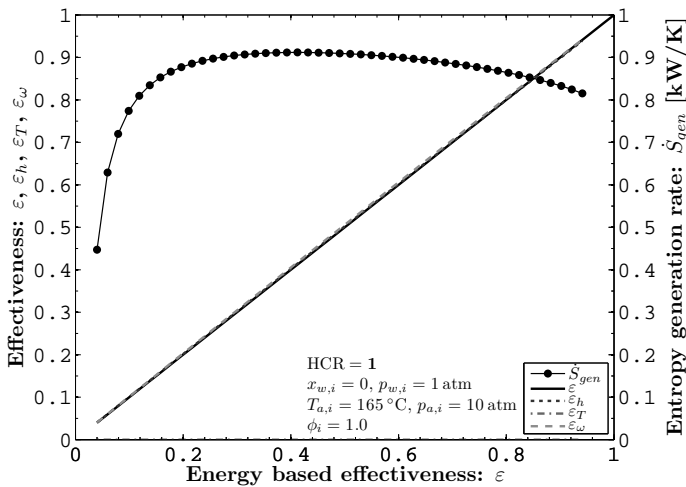


Fig. 16 Effectiveness of a heat and mass exchanger with phase change in one of the streams with HCR = 1.

liquid at 1 atm while the moist air stream enters saturated at  $165^\circ\text{C}$  and 10 atm and exits in the saturated state. As the effectiveness increases, the water stream quality increases toward 1. When the water stream is undergoing phase change, the temperature based effectiveness,  $\epsilon_T$  always has a null value since there is no temperature change in the water stream. This definition of effectiveness is inappropriate to use in this class of exchangers. The other three definitions all yield similar values.

### 6. $\epsilon$ -NTU MODEL FOR HEAT AND MASS TRANSFER EXCHANGERS

As shown in Table 1, the literature contains many definitions for the effectiveness of simultaneous heat and mass exchange devices. All of these definitions are based on the well-known concept that the effectiveness is the ratio of the actual heat transfer to the maximum possible heat transfer. This maximum heat transfer is limited by the Second Law of Thermodynamics. However, some approximations are imposed on the  $\epsilon$ -NTU models in which these effectivenesses are used. These approximations are application and operating condition specific. In general heat exchangers, it is easy to relate the effectiveness to the heat capacity rate ratio and the number of transfer units. This can be mathematically proven if constant

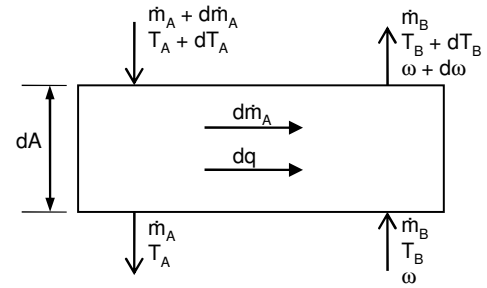


Fig. 17 A differential control volume of a direct contact counterflow heat and mass exchanger.

specific heats and overall heat transfer coefficient are assumed.

However, in heat and mass exchangers, the capacity rate ratio is not constant as it depends strongly on the amount of mass transferred from one stream to the other (see Sec. 2.2). Also, the mass transfer coefficient is difficult to calculate, and there is usually no closed form correlation that accurately estimates the overall mass transfer coefficient between streams that exchange both heat and mass. Therefore, the mass transfer coefficient is usually related to the heat transfer coefficient by a Lewis factor based on Chilton-Colburn analogy, and experimental measurements are normally carried out to determine the effective transfer characteristics.

Analytical expressions for the effectiveness of a heat and mass exchanger as a function of the heat capacities and transfer characteristics are generally not available unless substantial approximations are made. A robust and easy way to solve the governing equations of the simultaneous heat and mass exchanger is by numerical simulation.

Hence, in this section, the governing equations of a heat and mass exchanger are solved numerically and the energy effectiveness is calculated from the outlet conditions. Knowing the inlet conditions of the hot and cold streams and the transfer characteristics (i.e., the heat and mass transfer coefficients) the governing differential equations can be solved to find the exit conditions. From the given inlet conditions and the solved outlet conditions, it is possible to calculate the energy effectiveness and heat capacity rate ratio. This value is then compared with an approximate analytical expression given in the literature (Jaber and Webb, 1989).

A differential control volume in a direct contact counterflow heat and mass exchanger is shown in Fig. 17. The assumptions that are used to derive the modeling equations are as follows:

- Negligible heat transfer between the HME walls and the external environment.
- Constant heat and mass transfer coefficients.
- The Lewis factor that relates the heat and mass transfer coefficients is not constant.
- Uniform cross-sectional area.
- The total pressure is constant along the HME and equal to 1 bar.
- The solubility of stream B (e.g., air) in stream A (e.g., water) is neglected, so that only one fluid is transferred to the other fluid.

Steady-state heat and mass balances are performed on the incremental volume shown in Fig. 17 results in the following differential equations:

#### Mass Balance

$$d\dot{m}_A = \dot{m}_B d\omega = K(\omega_{sat,A} - \omega) dA \quad (31)$$

#### Energy balance on stream A

$$\dot{m}_A dh_A + h_A d\dot{m}_A = [U(T_A - T_B) + Kh_A^v(\omega_{sat,A} - \omega)] dA \quad (32)$$

### Energy balance on stream B

$$\dot{m}_B dh_B = [U(T_A - T_B) + Kh_A^v(\omega_{sat,A} - \omega)] dA \quad (33)$$

where  $\omega$  is the mass concentration of stream B vapor in the mixture and  $\omega_{sat}$  is the saturated concentration at the interface temperature of both fluid streams. For water-air system, this is the humidity ratio which is the mass fraction of water vapor in moist air per unit mass of dry air. The enthalpy of stream A vapor,  $h_A^v$  in cold fluid, represents the enthalpy or energy transfer associated with mass transfer.  $U$  and  $K$  are the heat and mass transfer coefficients respectively which can be related by Lewis factor ( $Le$ ) using Chilton-Colburn analogy as follows:

$$Le = \frac{U}{Kc_{p,B}} \quad (34)$$

where  $c_{p,B}$  is the specific heat of stream B including the vapor transferred from stream A per dry mass of stream B. The Lewis factor is assumed to be unity in the literature for the air-water system. However, a general expression for Lewis factor is given by Kloppers and Kröger (2005b) as

$$Le = 0.865^{0.667} \left[ \frac{\left( \frac{0.622 + \omega_{sat,A}}{0.622 + \omega_{sat,B}} \right) - 1}{\ln \left( \frac{0.622 + \omega_{sat,A}}{0.622 + \omega_{sat,B}} \right)} \right] \quad (35)$$

The mass flow rate ratio,  $m_r$ , is defined as the ratio of the inlet mass flow rate of stream A to the mass flow rate stream B:

$$m_r = \frac{\dot{m}_{A,i}}{\dot{m}_B} \quad (36)$$

and the Merkel number is given as:

$$Me = \frac{KA}{\dot{m}_{A,i}} \quad (37)$$

Using Eqs. (34), (36), and (37) into Eqs. (31), (32), and (33) together with enthalpy of the cold stream gives the following differential equations (Sharqawy *et al.*, 2010):

$$\frac{d\omega}{dz} = Me \cdot m_r \cdot (\omega_{sat,A} - \omega) \quad (38)$$

$$\frac{dh_A}{dz} = \left[ \frac{1}{m_r - (\omega_0 - \omega)} \right] \cdot \left[ \frac{dh_B}{dz} - h_A \frac{d\omega}{dz} \right] \quad (39)$$

$$\frac{dh_B}{dz} = Me \cdot m_r \cdot [Le(h_{B,sat,A} - h_B) + (1 - Le)(\omega_{sat,A} - \omega)h_A^v] \quad (40)$$

For a given mass transfer characteristic or Merkel number ( $Me$ ), mass flow rate ratio ( $m_r$ ) and inlet conditions ( $T_{A,i}$ ,  $T_{B,i}$ ,  $\omega_i$ ), Eqs. (38)–(40) can be solved numerically to find the outlet conditions for both streams ( $T_{A,o}$ ,  $T_{B,o}$ ,  $\omega_o$ ).

It is very difficult mathematically to find an analytical solution to Eqs. (38)–(40) without some approximations. Jaber and Webb (1989) developed the equations for the  $\varepsilon$ -NTU method for counterflow cooling towers by using the following additional approximations:

- The Lewis factor is unity ( $Le = 1$ ).
- The water evaporated into the air stream is neglected ( $d\omega = 0$ ).
- The saturated enthalpy of moist air is linearly proportional to the temperature.

These approximations simplify Eqs. (38)–(40) into

$$\frac{dh_A}{dz} = \frac{1}{m_r} \cdot \frac{dh_B}{dz} \quad (41)$$

$$\frac{dh_B}{dz} = Me \cdot m_r \cdot (h_{B,sat,A} - h_B) \quad (42)$$

Equations (41) and (42) alone are known as the Merkel model for cooling towers (Merkel, 1925) which use the first two approximations mentioned above. However, by using the third approximation, Jaber and Webb (1989) were able to solve analytically for the effectiveness by analogy to a counterflow heat exchanger. The effectiveness for a cooling tower (defined by Jaber and Webb (1989)) modeled with Eqs. (41) and (42) is given by

$$\varepsilon_h^{\text{mod}} = \frac{1 - \exp[-NTU_J \cdot (1 - HCR_J)]}{1 - HCR_J \exp[-NTU_J \cdot (1 - HCR_J)]} \quad (43)$$

where  $NTU_J$  is the number of transfer units given by

$$NTU_J = Me \cdot m_r \quad \text{if } \dot{m}_B > \dot{f}' \quad (44a)$$

$$NTU_J = Me \cdot \dot{f}' / c_{p,A} \quad \text{if } \dot{m}_B < \dot{f}' \quad (44b)$$

where  $\dot{f}'$  is the average slope of the saturated enthalpy-temperature relation. This treats the slope as an effective specific heat for the moist air stream. In addition, a heat capacity rate ratio,  $HCR_J$ , defined by Jaber and Webb (1989) is as follows.

$$HCR_J = \min \left\{ \frac{\dot{m}_A c_{p,A}}{\dot{m}_B \dot{f}'}, \frac{\dot{m}_B \dot{f}'}{\dot{m}_A c_{p,A}} \right\} \quad (44c)$$

To compare the  $\varepsilon$ -NTU model of Jaber and Webb (1989) given by Eq. (43) with the full numerical solution of the cooling tower model represented by Eqs. (38)–(40), the effectiveness is calculated using Eq. (43) and the energy based effectiveness given by Eq. (5) is evaluated by making use of the outlet conditions from the numerical solution. This is performed at different values of Merkel number and mass flow rate ratios. The comparison is given in Fig. 18 hereunder. In Fig. 18a, the mass flow rate ratio is 0.5 which makes the heat capacity of water the minimum (lower than the heat capacity of moist air), whereas, in Fig. 18b the mass flow rate ratio is 2, which makes the heat capacity of air the minimum. Therefore, the effectiveness in Fig. 18a is for the water stream while in Fig. 18b it is for the air stream. It is clearly seen that there is a large error in the effectiveness if the water stream has the minimum heat capacity. The same results are obtained if the air temperature is higher than the water temperature. In that case, the air will be dehumidified and some water vapor will be condensed from the air stream as shown in Fig. 19.

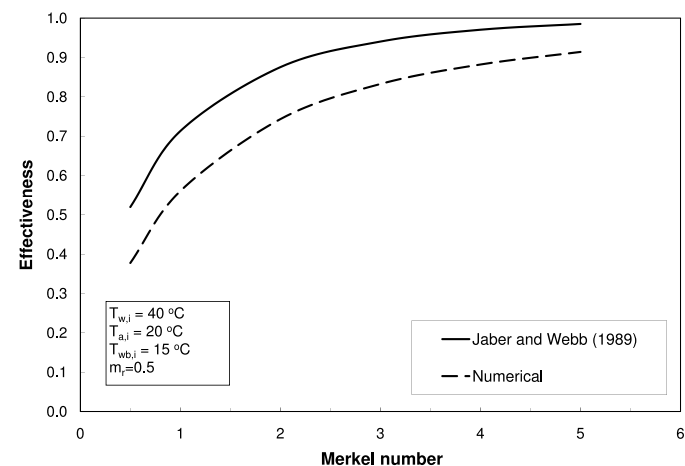
A better correlated numerical result for the energy effectiveness can be obtained if the  $\varepsilon_h^{\text{mod}}$  is replaced by  $\varepsilon$ , and HCR and NTU are replaced as follows:

$$HCR = \min \left\{ \frac{\Delta \dot{H}_{\max,B}}{\Delta \dot{H}_{\max,A}}, \frac{\Delta \dot{H}_{\max,A}}{\Delta \dot{H}_{\max,B}} \right\} \quad (45a)$$

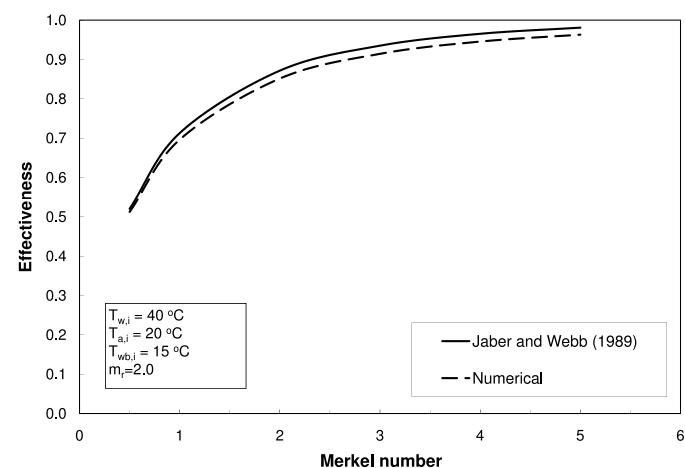
$$NTU = Me \quad \text{if } \Delta \dot{H}_{\max,A} > \Delta \dot{H}_{\max,B} \quad (45b)$$

$$NTU = Me \cdot m_r \quad \text{if } \Delta \dot{H}_{\max,B} > \Delta \dot{H}_{\max,A} \quad (45c)$$

Using the above values of the HCR and NTU defined in Eq. (45) into Eq. (43) of the counter flow heat exchanger, the resulting effectiveness has a lower deviation from the numerical results than by using the HCR and NTU values defined by Jaber and Webb [Eq. (44)] as shown in Fig. 20. In Fig. 20, the deviation of Jaber and Webb effectiveness from the numerical solution increases as the water temperature increases because the amount of water evaporation increases, which is basically neglected in Jaber and Webb model. The deviation reaches about  $-85\%$  from the numerical solution (the third group of data in Fig. 20). However, by using NTU and HCR defined in Eq. (45), the maximum deviation from the numerical solution is  $20\%$ . When the air is hotter than the water, both methods show almost the same deviation from the numerical solution (the fourth group of data in Fig. 20). However, using NTU and HCR as defined in Eq. (45) still gives a lower deviation than the Jaber and



(a)



(b)

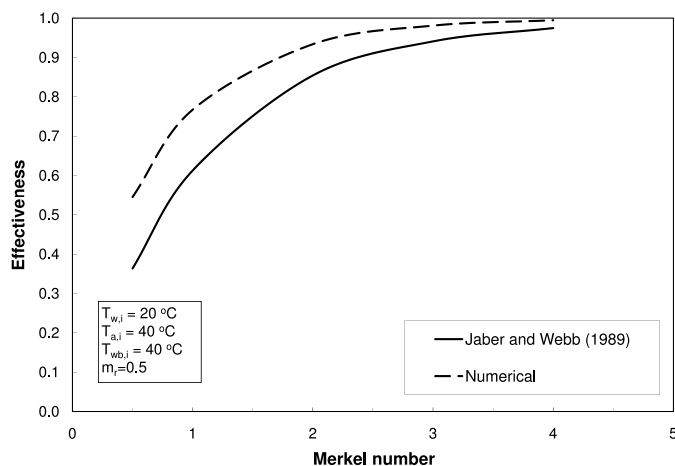
**Fig. 18** Comparison of effectiveness calculated by numerical and analytical methods for a direct contact counter flow humidifying HME for following cases: (a) water is the minimum heat capacity stream; (b) air is the minimum heat capacity stream.

Webb method. Therefore, it may be concluded that neglecting the water evaporation and assuming a linear relationship between the enthalpy of saturated air and temperature leads to a large deviation from the accurate numerical solution of the heat and mass exchanger. However, modifying the definition of the heat capacity ratio and number of transfer units, as in Eq. (45), reduces this deviation and gives more accurate results for the effectiveness.

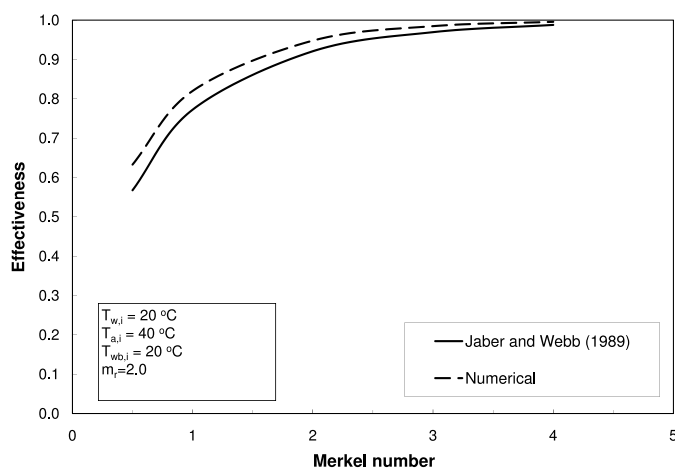
## 7. CONCLUSIONS

In this paper, the following significant conclusions have been reached:

1. A simple definition for energy effectiveness, which can be applied to all types of HMEs, has been developed. It is based on the total energy change of each fluid stream participating in the transfer processes.
2. A reliable definition for the heat capacity rate ratio, without any simplifying assumptions on the fluid properties, has been developed.
3. Temperature, humidity and enthalpy based effectivenesses are each applicable in some cases, depending on the value of HCR and  $\phi_i$ .



(a)



(b)

**Fig. 19** Comparison of effectiveness calculated by numerical and analytical methods for a direct contact counter flow dehumidifying HME for following cases: (a) water is the minimum heat capacity stream; (b) air is the minimum heat capacity stream.

The range of applicability is noted in Table 5.

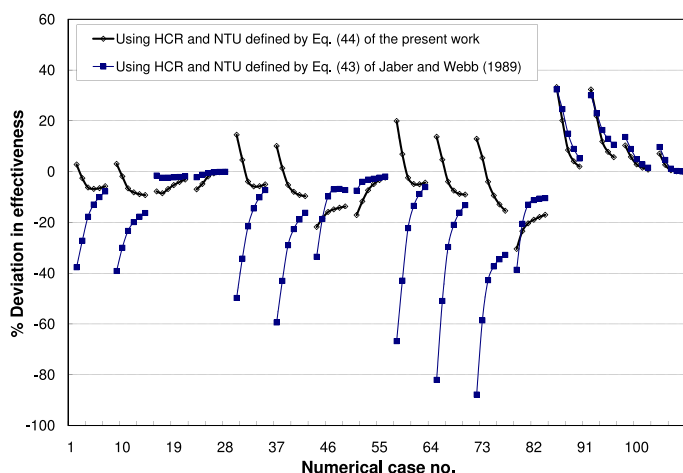
4. Using the comparison of the different effectivenesses at various values of HCR and looking at the non-dimensional entropy generation, the concept of thermal balancing of heat and mass exchange devices was demonstrated.
5. There is a maximum value of effectiveness for certain configurations ( $0 < \epsilon < \epsilon_{max}$ ).
6. Finally, it was demonstrated that it is possible to use the energy effectiveness in developing reliable  $\epsilon$ -NTU models for cooling towers. The  $\epsilon$ -NTU models developed in this paper have better accuracy than those already existing in literature.

## ACKNOWLEDGMENTS

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**Table 5** Applicable range of various definitions of effectiveness of simultaneous heat and mass exchange devices.

Reference	Effectiveness defined	Range of applicability
Cheremisinoff and Cheremisinoff (1981); Mandi <i>et al.</i> (2005)	$\varepsilon_T = \frac{\Delta T}{\Delta T^{ideal}}$	Cooling towers and humidifiers when $HCR \geq 1$
Nellis and Klein (2008)	$\varepsilon_w = \frac{\Delta \omega}{\Delta \omega^{ideal}}$	Humidifier, dehumidifiers and cooling coils when $HCR \leq 1$ ; but not applicable for parallel flow when $\phi_i < 1$
Nellis and Klein (2008)	$\varepsilon_h = \frac{\Delta h_a}{\Delta h_a^{ideal}}$	Humidifier, dehumidifiers and cooling coils when $HCR \leq 1$
Jaber and Webb (1989)	$\varepsilon_J = \frac{Q_{act}}{\dot{m}_{min}^{mod} \cdot \Delta h_a^{ideal}}$	All heat and mass exchangers (but needs evaluation of slope of moist air enthalpy line)
Present work	$\varepsilon = \frac{Q_{act}}{Q_{max}}$	All heat and mass exchangers; all situations



**Fig. 20** Deviation of energy effectiveness using the NTU, HCR of Jaber and Webb (1989) and using the modified NTU and HCR from present work in the general  $\varepsilon$ -NTU expression (43).

**NOMENCLATURE**

**Acronyms**

- HE Heat Exchanger
- HME Heat and Mass Exchanger

**Symbols**

- $\dot{C}$  heat capacity rate (W/K)
- $c_p$  specific heat capacity at constant pressure (J/kg-K)
- $c'_{p,a}$  rate of change of moist air enthalpy with temperature at constant humidity and pressure (J/kg-K)
- $f'$  slope of moist air saturated enthalpy with temperature (J/kg-K)
- $\dot{H}$  total enthalpy rate (W)
- $h$  specific enthalpy (J/kg)

- HCR heat capacity rate ratio (-)
- $\Delta \dot{H}_{max}$  maximum possible change in total enthalpy rate (W)
- K mass transfer coefficient (kg/m<sup>2</sup>-s)
- Le Lewis factor (-)
- $\dot{m}$  mass flow rate (kg/s)
- $m_r$  mass flow rate ratio (-)
- Me Merkel number (-)
- NTU number of transfer units (-)
- $p$  absolute pressure (Pa)
- $\dot{Q}$  heat transfer rate (W)
- $\dot{S}_{gen}$  entropy generation rate (W/K)
- $T$  temperature (°C)
- $T_{wb}$  wet bulb temperature (°C)
- U overall heat transfer coefficient (W/m<sup>2</sup>·K)
- z dimensionless coordinate along length of exchanger (-)

**Greek**

- $\Delta$  difference or change
- $\varepsilon$  energy based effectiveness (-)
- $\varepsilon_h$  enthalpy based effectiveness (-)
- $\varepsilon_T$  temperature based effectiveness (-)
- $\varepsilon_w$  humidity based effectiveness (-)
- $\phi$  relative humidity (-)
- $\omega$  absolute humidity (kg water vapor per kg of dry air)

**Subscripts**

- A fluid stream in HME
- a humid air
- act actual
- B fluid stream in HME
- c cold stream
- da dry air
- h hot stream
- i inlet
- J defined by Jaber and Webb (1989)
- max maximum
- min minimum

<i>o</i>	outlet
<i>sat</i>	saturated
<i>v</i>	vapor
<i>w</i>	water

#### Superscripts

<i>ideal</i>	ideal condition
<i>mod</i>	modified

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