Energy Efficiency in Wireless Networks via Fractional Programming Theory

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Contents

Abstract

This monograph presents a unified framework for energy efficiency maximization in wireless networks via fractional programming theory.

The definition of energy efficiency is introduced, with reference to single-user and multi-user wireless networks, and it is observed how the problem of resource allocation for energy efficiency optimization is naturally cast as a fractional program. An extensive review of the state-of-the-art in energy efficiency optimization by fractional programming is provided, with reference to centralized and distributed resource allocation schemes.

A solid background on fractional programming theory is provided. The key-notion of generalized concavity is presented and its strong connection with fractional functions described. A taxonomy of fractional problems is introduced, and for each class of fractional problem, general solution algorithms are described, discussing their complexity and convergence properties.

The described theoretical and algorithmic framework is applied to solve energy efficiency maximization problems in practical wireless networks. A general system and signal model is developed which encompasses many relevant special cases, such as one-hop and two-hop heterogeneous networks, multi-cell networks, small-cell networks, device-to-device systems, cognitive radio systems, and hardware-impaired networks, wherein multiple-antennas and multiple subcarriers are (possibly) employed. Energy-efficient resource allocation algorithms are developed, considering both centralized, cooperative schemes, as well as distributed approaches for self-organizing networks.

Finally, some remarks on future lines of research are given, stating some open problems that remain to be studied. It is shown how the described framework is general enough to be extended in these directions, proving useful in tackling future challenges that may arise in the design of energy-efficient future wireless networks.

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List of Acronyms

AF: amplify-and-forward

BC: broadcast channel

BER: bit error rate

BR: best-response

BRD: best-response dynamics

CCFP: concave-convex fractional problem

CLFP: concave-linear fractional problem

CoMP: coordinated multi-point

CSI: channel state information

D2D: device-to-device

DPC: Dirty paper coding

EVD: eigenvalue decomposition

GEE: global energy efficiency

GNE: generalized Nash equilibrium

IBC: interference broadcast channel

IC: interference channel

IMAC: interference multiple-access-channel

KKT: Karush-Kuhn-Tucker

LFP: linear fractional problem

LHS: left-hand-side

LMMSE: linear minimum mean squared error

MAC: multiple-access-channel

MC: multi-carrier

MIMO: multiple-input multiple-output

MMFP: max-min fractional problem

NE: Nash equilibrium

OFDMA: orthogonal frequency division multiple-access

P2P: point-to-point

PC: pseudo-concave

PL: pseudo-linear

PoRP: product-of-ratios problem

PPP: Poisson point process

QC: quasi-concave

QL: quasi-linear

QoS: quality-of-service

RHS: right-hand-side

RMT: random matrix theory

SC: single-carrier

SG: stochastic geometry

SIC: successive interference cancellation

SINR: signal-to-interference-plus-noise-ratio

SISO: single-input single-output

SNR: signal-to-noise-ratio

SoRP: sum-of-ratios problem

SPC: strictly pseudo-concave

SQC: strictly quasi-concave

SVD: singular value decomposition

WMEE: weighted minimum energy efficiency

WPEE: weighted product energy efficiency

WSEE: weighted sum energy efficiency

ZF: zero-forcing

Notation

Boldface upper-case and lower-case letters denote matrices and vectors, respectively.

$\|\boldsymbol{x}\|$, $\boldsymbol{x}^T$, $\boldsymbol{x}^H$ denote Euclidean norm, transpose, and conjugate transpose of the n-dimensional column vector $\boldsymbol{x} = \{x_i\}_{i=1}^n$. $\mathbf{0}_n$ and $\mathbf{1}_n$ denote an all zero and an all one n-dimensional vector, respectively. Component-wise vector ordering is used, i.e. $\boldsymbol{x} \succeq \boldsymbol{y}$ means $x_i \geq y_i$, for all $i = 1, \ldots, N$.

$\mathrm{tr}(\boldsymbol{X})$, $\boldsymbol{X}^T$, $\boldsymbol{X}^H$, $|\boldsymbol{X}|$, $\boldsymbol{X}^{-1}$, $\boldsymbol{X}^+$, $\|\boldsymbol{X}\|$ denote trace, transpose, conjugate transpose, determinant, inverse, pseudo-inverse, and Frobenius norm of the matrix $\boldsymbol{X}$. $\boldsymbol{I}_n$, $\boldsymbol{O}_{m,n}$, and $\mathrm{diag}(\boldsymbol{x})$ denote the identity matrix of order n, an all zero $m \times n$ matrix, and a diagonal matrix with $\boldsymbol{x}$ on the diagonal, respectively. Löwner matrix order is used, i.e. $\boldsymbol{X} \succeq \boldsymbol{Y}$ means $\boldsymbol{X} - \boldsymbol{Y}$ is positive semidefinite. $\otimes$ denotes the Kronecker matrix product.

When applied to a set $\mathcal{S}$, the symbol $|\mathcal{S}|$ denotes the cardinality of $\mathcal{S}$.

$\mathbb{E}$, $\mathbb{R}$, and $\mathbb{C}$ denote statistical expectation, the field of real numbers, and the field of complex numbers. $\mathbb{R}_+$ and $\mathbb{R}_{++}$ denote the set of non-negative real numbers and the set of positive real numbers, respectively. We say that a function $f(p)$ is $o(p)$ if $\lim\limits_{p \to +\infty} \dfrac{f(p)}{p} = 0$.

1

Introduction

Over the last years, energy-aware optimization of wireless communication networks has become a very popular research topic [194, 74, 104], due to at least three main reasons:

- The exponential increase of connected devices that wireless communications have been experiencing, poses serious sustainable growth concerns. The number of devices connected to the internet is larger than the size of the world population. By 2020, there might be more than 50 Billion devices or more than 6 devices per person connected to the internet. The IP traffic forecast for 2014 is more than 60 Exabytes (1000^6 Bytes) per month. Most connections will be wireless because most devices are small and mobile. In 5G networks, the goal is set to 1000 times higher data rates compared to present systems, but it is clear that trying to achieve this goal by a proportional increase of the transmit power would soon lead to an unmanageable energy demand. The 1000x datarate increase should be achieved at a similar energy consumption as today's networks, which calls for a 1000x increase of the energy efficiency.

- The rapid expansion of wireless networks causes also environmental concerns. Even nowadays, information and communication technology (ICT) infrastructures consume more than 3% of the world-wide energy, out of which about 60% is consumed by base stations, which causes about 2% of the world-wide CO_2 emissions. Given the rate at which wireless connected devices are increasing, the situation will eventually escalate, if no countermeasure is taken. This is acknowledged as key-issue by GSMA (green manifesto), which demands a reduction of CO_2 emissions per connection by 40% until 2020, and by NGMN, which also declares energy saving as a top priority. In addition to CO_2 emissions, electromagnetic pollution is also another concern to be dealt with.
- Besides sustainable growth and ecological concerns, economical reasons drive the development of novel energy-efficient ICT, too. Reducing energy consumptions allows network operators to save on electricity bills and maintenance costs, which makes energy efficiency a very popular topic not only in the academic world but also among industries. As an example, many leading telecommunication companies have formed the GreenTouch consortium (http://www.greentouch.org), whose goal is an increase of energy efficiency by a factor 1000 compared to 2010.

The numbers for the breakdown of the conventional cellular system base station power consumption differ between vendors, base station types, and location. More than 50% are consumed by the power amplifier including feeder, about 10% by the digital and analog signal processing, and about 20% by the cooling system. In general, energy efficiency can be improved on all technology layers including the network deployment strategies: hardware components and RF front-ends, physical layer with link adaptation, adaptive multiple access and resource allocation, and adaptive network management.

This monograph focuses on energy-efficient wireless network design including resource allocation, scheduling, precoding, relaying, and decoding. Starting from simple point-to-point (P2P) systems and then gradually moving towards more complex interference networks, the energy efficiency is defined and its properties characterized. It is shown how the energy efficiency is naturally defined by fractional functions, thus establishing that a key-role in the modeling, analysis, and optimization of energy efficiency is played by *fractional programming*, a branch of optimization theory specifically concerned with the properties and optimization of fractional functions. The monograph introduces fractional programming theory, illustrating how it can be used to formulate and handle energy efficiency optimization problems. More in detail, the monograph is divided into three main parts.

The first part provides a solid description of fractional programming theory and generalized concavity. The classes of quasi-concave and pseudo-concave functions are characterized, detailing their properties and highlighting the key role they play in fractional programming theory. The connection between fractional programming and generalized concavity is described in detail, and a taxonomy of fractional problems is introduced. We consider the classes of fractional problems that have proved most useful in the energy-efficient design of wireless networks: linear fractional problem (LFP), concave-convex fractional problem (CCFP), max-min fractional problem (MMFP), sum-of-ratios problem (SoRP), and product-of-ratios problem (PoRP).

After illustrating the theoretical aspects of fractional programming, the second part of the monograph focuses on the algorithmic side of fractional programming. For each class of introduced fractional problems, general solution algorithms are presented, which are able to solve any generic instance of the corresponding problem class. The analysis provides both intuitive insights into the algorithms and a mathematically rigorous treatment, including complexity/performance trade offs and sub-optimal algorithm development.

Finally, the third part of the monograph deals with practical applications, in which we show how the developed framework is used to optimize the energy efficiency of practical wireless networks. In particular, we show how fractional programming is extremely useful for the optimization of the energy efficiency of multiuser, multi-hop, multiple antenna, multi-carrier (MC), and multi-cell networks, which will all be key technologies in future wireless networks such as 5G. Both cooperative schemes to be centrally implemented and decentralized algorithms which allow for self-organizing networks are discussed, clearly pointing out advantages and disadvantages of the two approaches. In the decentralized setting, it is shown how fractional programming is used together with game theory to analyze the existence and uniqueness of stable equilibria of the resource allocation problem, and to devise distributed resource allocation algorithms.

We should mention that as far as applications to wireless systems are concerned, the focus will be on networks with deterministic topology, which are more well-established at this point in time, as opposed to stochastic topologies which have started to emerge recently and in which the position of the nodes and the interference levels are treated as random processes. However, we stress that fractional programming is not restricted only to deterministic topologies, but can be applied to stochastic networks, too. However, in this case, additional mathematical tools like random matrix theory (RMT) [165, 47] and stochastic geometry (SG) [11, 175] are required to statistically characterize the network interference levels. More details on this issue will be reported in Chapter 6.

1.1 Energy efficiency of a point-to-point system

A general definition of the efficiency with which a system uses a given resource, is the benefit-cost ratio. The ratio between the goods produced by using a given resource and the corresponding incurred cost naturally measures the income per unit cost. This general definition of efficiency applies to all fields of science, from physics to economics, and wireless communication is no exception.

As far as wireless communications are concerned, the cost is represented by the amount of energy consumed to operate the system. To elaborate, let us consider a P2P link, and as a first introductory example assume a single-input single-output (SISO), single-carrier (SC) system. The transmit power is p and the signal-to-noise ratio (SNR) at the receiver is γ. If the transmission takes T seconds to complete, the consumed energy will be

$$\mathrm{E}(p) = T(\mu p + P_c) \text{ [Joule]} , \tag{1.1}$$

wherein $\mu = 1/\eta$, with η the efficiency of the transmit power amplifier, while P_c includes the power dissipated in all other circuit blocks of the transmitter and receiver to operate the terminals. We should remark that the two underlying assumptions in (1.1) are that the transmit amplifier operates in its linear region, and that the circuit power P_c is a fixed power cost which depends neither on the transmit power p, nor on the communication rate R. Both assumptions are typically met in wireless communication systems, which are operated so as to ensure the amplifiers operate in the linear region of their transfer function, and in which the hardware-dissipated power is a constant power offset [8]. Indeed, the energy model in (1.1) has become a canonical choice in wireless network design [105, 84, 60] and for this reason in this monograph we will focus on the model in (1.1). Nevertheless a more general energy consumption model can be written as

$$\mathrm{E}(p,R) = T\left(\mu p + \sum_{n=2}^{N} c_n p^n + P_c(p,R)\right) \text{ [Joule]}\,, \tag{1.2}$$

wherein $\mu p + \sum_{n=2}^{N} c_n p^n$ is the transfer function of the amplifier, also including non-linear terms, whereas $P_c(p,R)$ models the functional relationship between the hardware-dissipated power and the transmit power and communication rate. We remark that here R is the fixed rate at which communication is taking place, and is not a function of the transmit power. This more general model can be useful in specific scenarios. Some examples are:

- Very high peak-to-average power ratio of the transmit signals, or very narrow linear region of the transmit amplifier, which make it difficult to guarantee the absence of higher-order distortions.
- Resource allocation schemes in which the modulation order is one optimization variable, which would make P_c dependent on R through the allocation of the modulation scheme. Some examples in this sense can be found in [119, 46].
- In multiple-antenna systems, if the transmit radio-frequency chains can be adaptively switched-off and on, it would be possible to reduce the power consumption by switching off the in-

active transmit chains. This would cause the circuit power P_c to depend on the rank of the transmit covariance matrix, and so on the transmit power. However, this not a typical operation condition in communication networks, in which instead the inactive transmit chains are usually left in idle mode, thereby leaving the per-chain hardware power consumption unaltered.

As a final remark on the choice of the energy consumption model, we stress that the theory of fractional optimization to be described in the sequel, applies to the more general model in (1.2), too. However, when (1.2) is used, we might have to deal with computational complexity issues. As it will become clear after Chapter 3, energy efficiency optimization problems can be globally solved with limited complexity, only if the function $E(p, R)$ is convex. If this requirement does not hold, then the complexity required for resource allocation is in general exponential.

As for the income resulting from the use of a given amount of energy, it can be measured by many different metrics, and the choice depends on the particular system to design, but also on subjective considerations regarding the particular physical quantity which needs to be optimized [181, 78]. For example, if the goal is to have a system with a large coverage, then the income is well-represented by the system coverage radius. Instead, if the goal is to serve as many users as possible, than the number of serviced users should be taken as system benefit. Many similar examples can be made, and in all cases we would be left with a ratio to optimize, which can be managed by fractional programming. However, in this monograph we will focus our attention on the communication aspect of a wireless network, measuring the income of the system as the amount of data that can be reliably transmitted to the receiver in the time interval T. Even restricting our focus to this scenario, more than one performance function can be used to measure the system income, each one with advantages and disadvantages. A first approach is to use the system capacity, defining the income in the interval T as $TW \log_2(1 + \gamma)$, with W the communication bandwidth. However, this metric does not account for the actual communication bit error rate (BER). For this reason, another proposed function is $TR(1 - e^{-\gamma})$, with R the communication rate and $(1 - e^{-\gamma})$ an approximation of the BER. If the channel h is rapidly varying, these two functions can be replaced by their ergodic counterpart, obtained by averaging with respect to the channel. Instead, in scenarios with a slowly-varying channel h, a meaningful performance function is $TR(1 - p_{out}(\gamma))$, with p_{out} the communication outage probability. A common feature of all presented metrics, is that they depend on the SNR γ. Then, we can include all of these special cases by considering a generic function $Tf(\gamma)$, with f the so-called efficiency function, to be specified according to the particular considered system.

Finally, we can give the following definition.

Definition 1.1. SISO SC Energy Efficiency
The energy efficiency of a P2P SISO SC system is

$$\mathrm{EE} = \frac{Tf(\gamma)}{T(\mu p + P_c)} = \frac{f(\gamma)}{\mu p + P_c}[\text{bit/Joule}]\ . \tag{1.3}$$

The energy efficiency (1.3) is measured in bit/Joule, thereby measuring the amount of data that can be reliably transmitted per Joule of consumed energy. Fig. 1.1 shows the shape of (1.3) when the efficiency function is chosen as the achievable rate.

It is seen that the energy efficiency is not monotone in p, but instead is a unimodal function that increases up to a point, and then decreases to zero. This is the key feature which allows us to save energy by energy-efficient resource allocation. Indeed, unlike traditional performance measures such as achievable rate, BER, mean square error, which are typically optimized by using all of the available power, the transmit power level that maximizes the energy efficiency is in general lower than the maximum feasible power. For example, in Fig. 1.1 the energy efficiency is maximized for $p = 0.48\,\mathrm{W}$ and the transmitter will use exactly this power level even if it is lower than the maximum feasible power. This optimum power level represents the best trade-off between achieving fast and reliable communication and saving energy. These consideration also hold for the other mentioned efficiency functions, which result in similar shapes of the energy efficiency.

However, it is clear that not all functions $f(\gamma)$ yield a similar energy efficiency as in Fig.1.1. More in general, what are the properties that

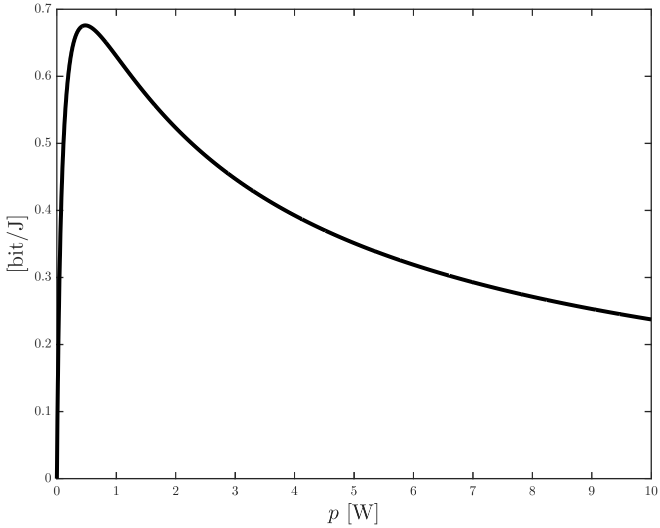

Figure 1.1: Energy efficiency with $f(\gamma(p)) = \log_2(1+p)$ [bit/s], $\mu = 1$, $P_c = 0.1$ [W]. The optimum power level is $p = 0.48$ [W].

$f(\gamma)$ should enjoy to obtain a physically meaningful energy efficiency? We have already implicitly introduced the first property, when we expressed the energy efficiency in bit/Joule. Indeed, for this to be true, we should require the following.

Property 1. *The efficiency function $f(\gamma)$ is measured in [bit/s].*

We have also intuitively already introduced the second property. For energy efficiency optimization to result in energy savings, we require that the energy efficiency must decrease for growing p, approaching 0 as $p \to \infty$. Moreover, clearly the efficiency should be zero if we do not transmit at all. This translates into the following requirements for f.

Property 2. *$f(\gamma)$ must be such that $f(0) = 0$ and $f(\gamma(p)) = o(p)$.*

The third and final property derives by the consideration that a meaningful f must reflect the fact that the benefit obtained from the system increases with the SNR γ.

Property 3. *$f(\gamma)$ must be increasing with γ.*

Equipped with this framework, we can move on and extend our definition to slightly more complex scenarios. We always consider a P2P system, but now we assume the optimization variable is not a scalar power but rather an N-dimensional matrix $\boldsymbol{P}$. For example, this could be the case of a multiple-input multiple-output (MIMO) system with $\boldsymbol{P}$ being the transmit covariance matrix. Considering the capacity as efficiency function, denoting by σ^2 the noise power at the receiver, and by $\boldsymbol{H}$ the MIMO channel matrix, the following definition should be clear.

Definition 1.2. MIMO Energy Efficiency
The energy efficiency of a MIMO P2P system is

$$\mathrm{EE} = \frac{W \log_2 \left| \boldsymbol{I}_N + \frac{1}{\sigma^2} \boldsymbol{H}\boldsymbol{P}\boldsymbol{H}^H \right|}{\mu \mathrm{tr}(\boldsymbol{P}) + P_c} [\mathrm{bit/Joule}] \, . \tag{1.4}$$

If instead, we have a MC P2P system with N orthogonal subcarriers, the matrix $\boldsymbol{P}$ reduces to its diagonal vector $\boldsymbol{p} = \{p_n\}_{n=1}^N$. Then, denoting by γ_n the SNR on subcarrier n, Definition 1.2 simplifies as follows

Definition 1.3. MC Energy Efficiency
The energy efficiency of a MC P2P system is

$$\mathrm{EE} = \frac{W \sum_{n=1}^N \log_2(1+\gamma_n)}{\mu \sum_{n=1}^N p_n + P_c} [\mathrm{bit/Joule}] \, . \tag{1.5}$$

Typically, an energy efficiency optimization problem is formulated as the maximization of the energy efficiency with respect to $\boldsymbol{P} \in \mathcal{S}$ where the set $\mathcal{S}$ of feasible power allocations models all constraints including power constraints $\mathrm{tr}(\boldsymbol{P}) \leq P_{max}$, quality of service (QoS) constraints $\log_2 \left| \boldsymbol{I}_N + 1/\sigma^2 \boldsymbol{H}\boldsymbol{P}\boldsymbol{H}^H \right| \geq R_{min}$, or other constraints.

Before closing this section, we should mention that another popular approach to energy efficiency does not consider the fractional approach, but rather takes a difference-based approach. More specifically, denoting by $R(p)$ the system achievable data-rate, another possible definition of energy efficiency is the function [143]

$$u(p) = R(p) - \mu(p + P_c) \; [\mathrm{bits/s}] \, , \tag{1.6}$$

with μ a constant term with dimensions bit/s/W. This approach is typically used in conjunction with pricing techniques and is not directly related to ratios. However, in the following we will not consider the difference-based approach, focusing instead on the fractional definition of energy efficiency. The motivation for this choice is based on both physical and mathematical arguments:

- As already mentioned, an efficiency is intrinsically a ratio, and in particular the ratio between the *output of the system* over the *consumed goods by the system*. This definition is well-established in communication theory, too. For example, the spectral efficiency of a communication system is defined as the number of transmitted bits per unit of bandwidth. Replacing spectrum with energy yields the definition of energy efficiency as the number of transmitted bits per unit of energy.

- In the difference-based approach, the energy efficiency is defined as $R(p) - \mu p$. This quantity is measured in bit/s and represents the difference between the rate minus a term which measures the cost in bit/s associated to the use of the transmit power. Thus, this definition of energy efficiency lacks the strong physical interpretation of *output of the system* over *consumed goods by the system*, being instead more related to the net gain in bit/s resulting from the use of the system, rather than to the efficiency with which the available Joules of energy are used to transmit information bits.

- Finally, from a mathematical point of view, the difference-based approach is closely connected to the fractional-based approach. More in detail, the maximization of the ratio $f(p)/g(p)$ is mathematically equivalent to the maximization of the difference $f(p) - \mu g(p)$ for a specific choice of the parameter μ. In order to elaborate further on this point, we need to introduce a second, more mathematical, approach to energy efficiency, based on multi-objective optimization. This is done in the coming section.

1.1.1 Bi-objective energy-efficient optimization

Leveraging the theory of multi-objective optimization, we can mathematically capture the trade-off between achieving a reliable communication and saving power, by formulating a bi-objective optimization problem in which the objectives to maximize are the benefit of the system $f(\gamma(p))$, and the negative of the power consumption, namely,

$$\max_{p\in\mathcal{P}} \{f(\gamma(p)); -(p+P_c)\} \; , \tag{1.7}$$

with $\mathcal{P}$ the set of admissible transmit powers, which can account for constraints such as a maximum feasible power constraint as well as QoS constraints. Employing the scalarization technique [20, 28], (1.7) can be converted into the scalar problem

$$\max_{p\in\mathcal{P}} w_1 f(\gamma(p)) - w_2(p+P_c) \; , \tag{1.8}$$

wherein the scalar objective is given by the linear combination of the two objectives in (1.7) and the non-negative weights w_1 and w_2 weigh the priority given to the two contrasting objectives. For any choice of the weights, the solution of (1.8) is on the Pareto-boundary of (1.7), and we see that the two extreme points of the boundary are obtained when either $w_2 = 0$, and (1.8) reduces to the maximization of $f(\gamma(p))$ (e.g. rate maximization), or when $w_1 = 0$, and (1.8) reduces to power minimization.

Using this approach, the critical question is how to choose the weights in order to obtain a meaningful point on the Pareto-boundary. The answer is obtained by considering the maximization of the ratio $f(\gamma(p))/(p+P_c)$. Indeed, in Chapter 3 we will show that maximizing the ratio $f(\gamma(p))/(p+P_c)$ also yields a point on the Pareto-boundary of (1.7), which corresponds to a specific choice of the weights w_1 and w_2. Such a Pareto-optimal point corresponds to the maximum benefit-cost ratio of the system, which, in light of the physical consideration described in the previous section, is the most meaningful operating point as far as energy efficiency maximization is concerned.

1.1.2 Energy-spectral efficiency trade-off

Some more insight on the operational meaning of the energy efficiency in (1.3) can be gained considering an additive white Gaussian noise (AWGN) channel with band B, noise spectral density N_0, and channel power gain h^2. In this context, taking the channel capacity as the measure of the system benefit, the spectral efficiency and energy efficiency can be written as

$$\mathrm{EE} = \frac{B\log_2\left(1+\frac{ph^2}{N_0 B}\right)}{\mu p + P_c} = \frac{B\eta_B}{\mu p + P_c} \tag{1.9}$$

$$\eta_B = \log_2\left(1+\frac{ph^2}{N_0 B}\right) . \tag{1.10}$$

The energy efficiency in (1.9) can be expressed as a function of the energy per bit E_b, by observing that $p = \eta_B E_b$, which yields

$$\mathrm{EE} = \frac{B\log_2\left(1+h^2\eta_B\frac{E_b}{N_0}\right)}{\mu E_b \eta_B + \frac{P_c}{B}} , \tag{1.11}$$

which shows the connection between the energy efficiency and the energy contrast E_b/N_0. We see that in AWGN channels, the energy contrast plays a similar role as the SNR γ in (1.3). Indeed, in AWGN channels, the energy contrast is proportional to the SNR $\frac{h^2 p}{N_0 B}$ through the spectral efficiency η_B [173].

Elaborating again on equations (1.9) and (1.10) we can gain more insight on the fundamental trade-off between energy efficiency and spectral efficiency. From (1.10) we obtain the transmit power as

$$p = (2^{\eta_B}-1)\frac{N_0 B}{h^2} , \tag{1.12}$$

which plugged into (1.9) yields the energy efficiency as a function of the spectral efficeincy

$$\mathrm{EE} = \frac{\eta_B}{\mu(2^{\eta_B}-1)\frac{N_0}{h^2} + \frac{P_c}{B}} . \tag{1.13}$$

The functional relation in (1.13) is reported in Fig. 1.2, showing that the energy efficiency is unimodal with respect to η_B.

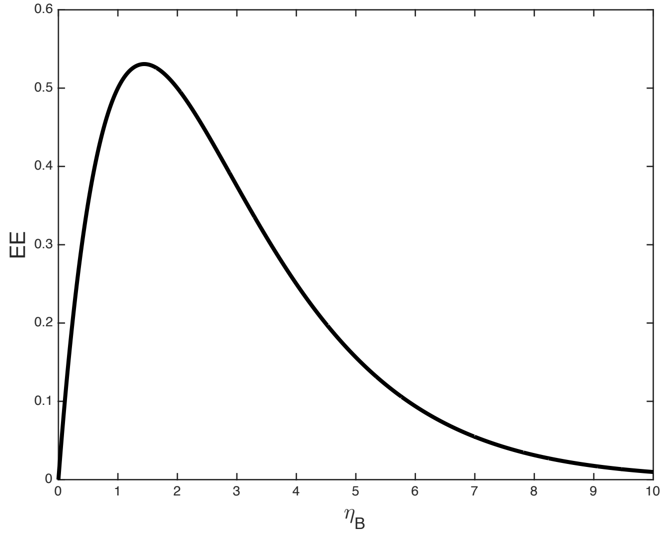

Figure 1.2: EE vs. η_B, for $\mu = 1$,$N_0/h^2 = P_c/B = 1$ [W/Hz].

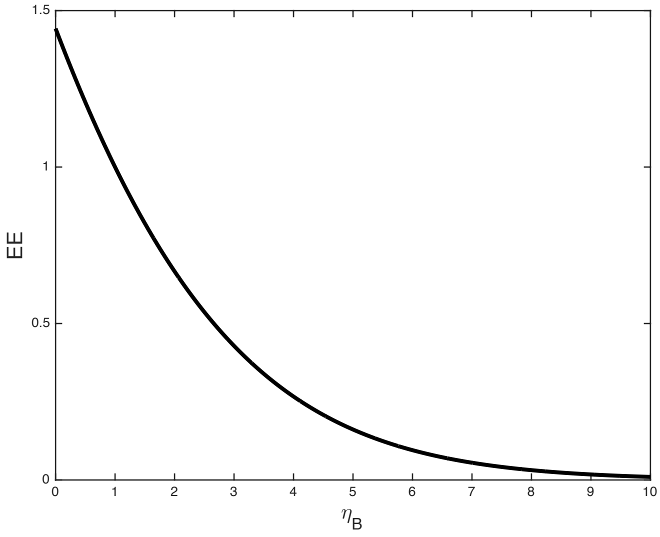

Figure 1.3: EE vs. η_B, for $\mu = 1$, $N_0/h^2 = 1$, $P_c/B = 0$ [W/Hz].

Instead, if $P_c = 0$, the energy efficiency becomes strictly decreasing in η_B, as shown in Fig. 1.3. It is interesting to observe that a similar behavior is observed for the energy efficiency versus the transmit power, which is unimodal if $P_c > 0$, as shown in Fig. 1.1, but which becomes strictly decreasing if $P_c = 0$. Before closing this section, we remark that by similar elaborations on (1.9) and (1.10), it is possible to analyze other energy-related trade-offs, such as the energy-delay and bandwidth-power trade-offs [42, 78].

1.2 Energy efficiency of a communication network

After learning how to define the energy efficiency of a P2P system, a natural question is how to extend such definition to a network composed of multiple nodes. It is intuitively clear that we should somehow combine the energy efficiencies of the single links, but what is the best way to do so? As we will see, there is no objective answer. Several different choices are possible, each with advantages and disadvantages, and the best choice depends on our subjective goals or on the particular constraints of the network to optimize.

In this section we introduce the most common performance metrics that have been proposed, describing merits and drawbacks of each approach. To elaborate, let us denote by L the number of links in the networks, and by γ_ℓ and P_ℓ the signal to interference plus noise ratio (SINR) and power consumption experienced by the ℓ-th link. Accordingly, we can define the energy efficiency of the ℓ-th link as $\mathrm{EE}_\ell = f(\gamma_\ell)/P_\ell$, with $f(\cdot)$ any meaningful efficiency function.

1.2.1 Global energy efficiency

A first approach is to stick to the physical meaning of energy efficiency as benefit-cost ratio, where now the benefit and cost should be related to the whole network, rather than to a single link. Following this approach leads to the following definition.

Definition 1.4. GEE
The global energy efficiency (GEE) of a network is defined as

$$\mathrm{GEE} = \frac{\sum_{\ell=1}^{L} f(\gamma_\ell)}{\sum_{\ell=1}^{L} \mu_\ell p_\ell + P_{c,\ell}} \,. \tag{1.14}$$

The GEE is the ratio between the total amount of data that can be reliably transmitted per unit of time and the total amount of consumed power, thus representing the efficiency with which the network resources are being used to produce the necessary goods. Otherwise stated, the GEE is exactly the total benefit produced by the network, divided by the total incurred cost.

However, while having a clear and strong physical interpretation, the GEE does not allow to tune the individual energy efficiencies of the different links. This is a drawback in heterogeneous networks where terminals with different features and specifications coexist and possibly have different energy-efficient requirements. As an example, consider a multi-cell system in which regular base stations are deployed together with small access points operated by renewable sources of energy. Clearly, the energy-harvesting nodes require a higher energy efficiency than regular terminals which are plugged to the electrical network.

A second observation regarding GEE concerns fairness issues, in terms of an unbalanced distribution of the energy efficiency among the different communication links. More formally, we define fairness according to the Max-Min fairness concept[1] [123], considering unfair those resource allocations which result in some links experiencing very low energy efficiency with respect to some other links. This might be the case when maximizing the GEE, which is a sum-based performance metric and therefore tends to favor the links with better propagation channels. Moreover, the GEE does not explicitly depend on the energy efficiencies of the different links. Therefore, maximizing the GEE makes it difficult to adjust the individual energy efficiencies according to specific needs.

For these reasons, we also describe a second approach to network energy efficiency, based on multi-objective optimization, which allows more flexibility in the choice of the network operating point.

[1] A resource allocation is said to be max-min fair if it is not possible to increase the energy efficiency EE_i of link i, without decreasing the energy efficiency EE_j of link j which is smaller than or equal to EE_i.

1.2.2 Multi-objective network energy efficiency

An alternative approach to defining the network energy efficiency is based on a multi-objective formulation, in which the objectives to optimize are the L energy efficiencies of the network $\{\mathrm{EE}_\ell\}_{\ell=1}^L$. This leads to considering the network energy-efficient region as the set of all feasible vectors $\{\mathrm{EE}_\ell\}_{\ell=1}^L$, and to defining the network energy efficiency as a performance measure whose maximization yields a point on the Pareto boundary of the energy-efficient region. This can be accomplished by defining the network energy efficiency as

$$\phi(\mathrm{EE}_1, \ldots, \mathrm{EE}_L) \; , \tag{1.15}$$

wherein $\phi : \mathbb{R}^L \to \mathbb{R}$ is an increasing function of each argument. The choice of the particular function ϕ to employ depends on the particular point on the Pareto boundary that needs to be achieved. In the following we list the most widely used choices, describing advantages and disadvantages of each approach.

Weighted sum of the energy efficiencies

If the function ϕ is set to the weighted sum of the energy efficiencies, (1.15) becomes the weighted arithmetic mean of the energy efficiencies.

Definition 1.5. WSEE
The weighted sum energy efficiency (WSEE) of a network is defined as

$$\mathrm{WSEE} = \sum_{\ell=1}^{L} w_\ell \frac{f(\gamma_\ell)}{\mu_\ell p_\ell + P_{c,\ell}} \; . \tag{1.16}$$

This choice results in the point where the hyperplane $y = \sum_{\ell=1}^{L} w_\ell \mathrm{EE}_\ell$ is tangent to the Pareto boundary. In the multi-objective jargon such a point is called utilitarian point. Similarly to the GEE, the WSEE is a sum-based metric and therefore still tends to favor links with better propagation channels. However, unlike the GEE, the WSEE allows one to counteract this effect because it explicitly depend on the individual energy efficiencies, which can be assigned different priorities through the choice of the weights.

Weighted product of the energy efficiencies

If the function ϕ is set to the exponentially weighted product of the energy efficiencies, (1.15) becomes the weighted geometric mean of the energy efficiencies.

Definition 1.6. WPEE
The weighted product energy efficiency (WPEE) of a network is defined as

$$\mathrm{WPEE} = \prod_{\ell=1}^{L} \left(\frac{f(\gamma_\ell)}{\mu_\ell p_\ell + P_{c,\ell}} \right)^{w_\ell} . \tag{1.17}$$

This choice yields the point where the hyperbola $y = \prod_{\ell=1}^{L} \mathrm{EE}_\ell^{w_\ell}$ is tangent to the Pareto boundary. Unlike previous performance measures, WPEE allows a more balanced resource allocation, thanks to its product-based definition. In particular, it is known that maximizing the product of given utility functions yields the point on the Pareto-boundary corresponding to the Nash Bargaining solution [138]. Moreover, just as WSEE, also WPEE enables the possibility to assign priorities to the individual energy efficiencies through the choice of the weights. However, even though WPEE maximization ensures that no link will experience a near-zero energy efficiency, it can not guarantee to achieve the max-min fair allocation.

Weighted minimum of the energy efficiencies

In order to obtain the max-min fair resource allocation, we set the function ϕ in (1.15) to the weighted minimum of the energy efficiencies. This leads to the following definition.

Definition 1.7. WMEE
The weighted minimum energy efficiency (WMEE) of a network is defined as

$$\mathrm{WMEE} = \min_{\ell \in \{1,\dots,L\}} \left(w_\ell \frac{f(\gamma_\ell)}{\mu_\ell p_\ell + P_{c,\ell}} \right) . \tag{1.18}$$

This choice results in the point where the hyperplane through the origin and in the direction $[w_1, \dots, w_L]$ intersects the Pareto boundary.

In the multi-objective jargon such a point is called egalitarian point, and it has the property to make $w_\ell \mathrm{EE}_\ell$ the same for all $\ell = 1, \ldots, L$. Hence, if the weights are all equal, maximizing the WMEE results in all of the energy efficiencies to be equal.

Fig. 1.4 summarizes the global-performance/fairness trade-off for the mentioned metrics and shows an example of energy-efficient Pareto region with the points on the boundary achieved by the different metrics. It should be remarked that by varying the weights, it is possible to describe the whole Pareto region by considering the maximization of any metric among WSEE, WPEE, and WMEE.

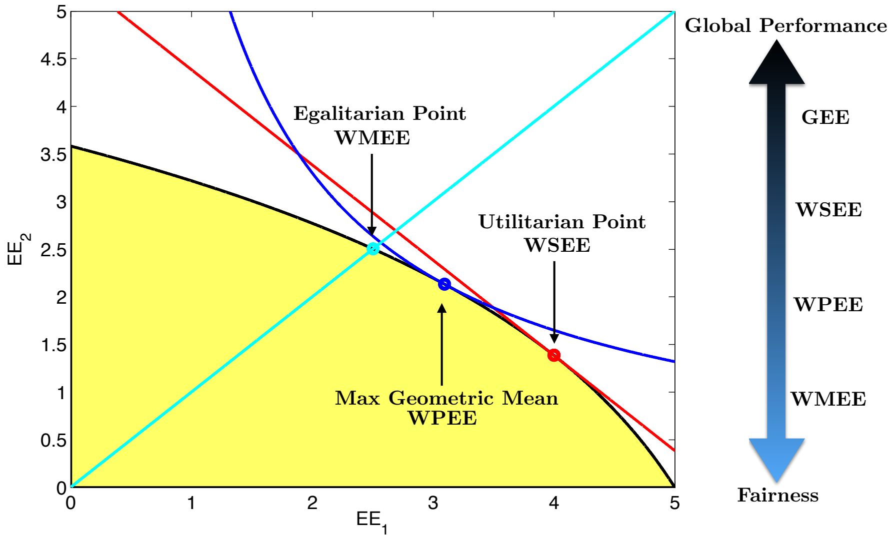

Figure 1.4: Different operating point for multi-objective network energy efficiency optimization. In all performance measures, weights are assumed all equal.

However, none of the functions that fall within the described multi-objective framework can be interpreted as the network benefit-cost ratio, as the GEE. Indeed, although the GEE will be in general inside the Pareto region, not depending directly on the links' energy efficiencies, it still retains the strongest physical meaning among the discussed performance functions.

1.3 Fractional programming for energy efficiency

Fractional programming theory is the branch of optimization theory that is concerned with the properties and optimization of fractional functions [153]. From previous sections it should appear clear the strong link between fractional programming and energy efficiency optimization. Be it a single link or a network, and regardless of the particular performance measure that is adopted, the energy efficiency of a communication system is always expressed through fractional functions and the energy-efficient resource allocation problem is naturally cast as a fractional program. Thus, fractional programming represents a fundamental tool in the energy-efficient modeling and design of wireless networks, and of communication systems in general.

Fractional programming studies problems of the form

$$\max_{\boldsymbol{x}} \frac{f(\boldsymbol{x})}{g(\boldsymbol{x})} \tag{1.19a}$$

$$\text{s.t. } \boldsymbol{x} \in \mathcal{X} \tag{1.19b}$$

with $f : \mathcal{C} \subseteq \mathbb{R}^n \to \mathbb{R}$, $g : \mathcal{C} \subseteq \mathbb{R}^n \to \mathbb{R}_+$ and $\mathcal{X} \subseteq \mathcal{C} \subseteq \mathbb{R}^n$. The challenge in tackling Problem (1.19) is that the objective is generally not concave, even making very restrictive assumptions on f and g. For example, even assuming f and g are both affine functions, in general (1.19a) is not concave. An immediate consequence of this fact is that the strong results of convex optimization theory in general do not apply for fractional problems, and Karush Kuhn Tucker (KKT) optimality conditions are only necessary for a generic fractional problem. However, under specific assumptions on f and g, the objective (1.19a), although not concave, falls into the class of generalized concave functions. In this case, it is possible to develop computationally-efficient methods and algorithms to find the global solution of (1.19). This will be described in detail in Chapters 2 and 3.

In the field of resource allocation for energy efficiency, while (1.19a) represents the energy-efficient metric to optimize, the set $\mathcal{X}$ models the constraints typically enforced in wireless communication systems. The most common constraint is the maximum feasible power constraint, related to the fact that the power which can be used for transmission is upper bounded by the power amplifier characteristics. Other common constraints are QoS constraints, which ensure that the result of the energy-efficient resource allocation fulfills specific service requirements to the network subscribers. Many different kinds of QoS constraints can be formulated in connection to energy-efficient problems, with two of the most common being minimum rate constraints [12] and maximum delay constraints [119, 51].

In the former case, the resource vector $\boldsymbol{x}$ must be such that each subscriber of the network enjoys at least a minimum achievable rate, or such that the global sum-rate is above a given threshold. Enforcing minimum rate constraints might degrade the resulting energy efficiency, if the target rates are large, because in this case it is necessary to increase the transmit power beyond the unconstrained maximizer of the energy efficiency, in order to ensure that each subscriber achieves his target achievable rate.

In the latter case, the resource vector $\boldsymbol{x}$ must be such that each subscriber experiences a transmission and/or queuing delay below a threshold. Employing a lower transmit power might result in decoding errors at the receiver, and hence in retransmission requests. Therefore, a maximum delay constraint requires each user's SINR to be above a certain threshold and this may again require the use of a transmit power beyond the unconstrained maximizer of the energy efficiency.

In both cases, and in general regardless of the particular constraints which define the set $\mathcal{X}$, the fractional programming framework to be described can always be applied and is always guaranteed to yield the global solution. The critical issue is about the required complexity. As it will be formally shown in Chapters 2 and 3, in order to solve (1.19) with polynomial complexity, $\mathcal{X}$ needs to be convex, while f and g need to be concave and convex, respectively. However, in Chapter 4 a technique will be introduced, to obtain a point fulfilling the KKT conditions of (1.19) with polynomial complexity, also when one of these assumptions is not met.

1.4 State of the art and future directions

Over the last years, fractional programming has become a well-established mathematical tool to solve energy-efficient resource allocation problems in wireless networks.

Paper [141] represents, to the best of authors' knowledge, the first application of fractional programming theory to the optimization of wireless networks. In particular, a set of parallel non-interfering channels is considered, and, using Dinkelbach's algorithm, the power allocation profile for energy efficiency maximization is derived. Since then, fractional programming has been largely used in the field of energy-efficient optimization of wireless systems, and some fractional programming methods and applications are outlined in [84].

Papers [137, 174, 112, 193, 142, 109] employ fractional programming to optimize the energy efficiency of cognitive radio systems. [137] considers a sensing scenario in which secondary (unlicensed) users are allowed to use licensed frequency bands that are temporarily not used by the primary (licensed) users. In [174] an underlay approach is used in which secondary users are allowed to use licensed subcarriers provided this does not cause too much interference to the primary users. Underlay approaches are also used in [112], where a MIMO broadcast channel is considered, in [193], which includes QoS requirements in the optimization process, in [142] which provides energy-efficient resource allocation algorithms for ad-hoc cognitive networks, and in [109], where fractional programming together with the rate-splitting technique is employed in cognitive MIMO systems.

In [45, 41, 29, 52, 72, 131, 169, 177] fractional programming is used for energy-efficient resource allocation in wireless MIMO systems. In [45] a single-user system with time-varying channel is considered. In [41] fractional programming is used to minimize the energy consumption per transmitted bit, while in [29] energy and outage probability optimization are considered. The energy-spectral efficiency trade-off is analyzed via fractional programming in [131]. In [169] optimal, energy-efficient precoding design in a single-user MIMO system is performed. In [177] a full-duplex multi-user system is considered in which uplink and downlink communications take place on the same frequency band.

Instead [178, 72] focus on MIMO broadcast channels. Also, applications of fractional programming in systems with a large number of antennas are considered in [128, 80, 40, 25], thus showing that fractional optimization proves useful in massive and large MIMO systems, too.

Fractional programming in MC and orthogonal frequency division multiple access (OFDMA) systems is employed in [176, 73, 97, 68, 179, 107]. In [176, 107, 179] energy-efficient resource allocation is performed subject to QoS constraints. In [97] energy per bit minimization is performed by fractional programming, while in [68] fractional programming is applied for energy efficiency optimization subject to proportional fairness constraints.

Fractional programming has been applied to relay-assisted systems, too, in [43, 160, 98, 185, 186, 71, 184]. In [43] an OFMDA network with multiple relays is considered, while in [160] the energy efficiency of a two-way relay system is discussed. In [98] the trade-off between spectral and energy efficiency is analyzed for relay-systems, while in [185, 186] a relay-assisted multi-stream MIMO system is considered, for different channel state information (CSI) assumptions. A MIMO system is also considered in [71], where low-complexity algorithms are provided for energy efficiency optimization with perfect CSI. In [184], a multi-user relay-assisted MIMO system is considered and fractional programming is employed to perform joint transceiver and relay allocation.

The papers [127, 69, 171, 55, 70] consider the issue of energy-efficient resource allocation in a coordinated multi-point (CoMP) system in which a cluster of base stations coordinate their resource allocations via backhaul connection. Fractional programming is employed to optimize different energy-efficient performance metrics, including the global energy efficiency of the cluster [127, 171], the sum of the individual energy efficiencies [69, 171], the product of the individual energy efficiencies [171], and the minimum of the energy efficiencies [55]. In [70], fractional programming is used to come up with beamforming allocation in single-stream MIMO systems.

Fractional programming has also been used to optimize the energy efficiency of systems with secure communications. In [39, 126] secure OFDMA networks are considered, while in [191] secure communication in multiple-antenna systems is studied.

The above works mainly consider centralized resource allocation approaches. However, a key-feature of future cellular systems is anticipated to be self-organization, given the sheer amount of nodes to manage. In particular, the study of energy-efficient resource allocation problems in wireless networks has started considering power control algorithms for competitive scenarios in single-cell and multi-cell networks [63, 148], where a game-theoretic approach is taken. The results in [63, 148] paved the way to many following contributions, which also employed game theory to devise competitive resource allocation algorithms. In [120, 121, 32], the algorithms provided in [63, 148] are extended considering also transceiver design, whereas [118] and [119] consider competitive power control in MC systems and in presence of maximum communication delay constraints. In [101] a hierarchical power control algorithm is proposed, using a Stackelberg game formulation. The results indicate that the equilibrium point is more efficient that in [148]. A further improvement can be obtained using a repeated-game formulation, as done in [162]. In [16], energy-efficient precoding matrix allocation in single-user MIMO systems is studied. In [13] and [31] competitive power control algorithms have been proposed for ultra-wide-band systems and for networks subject to frequency-selective fading, respectively. In [33] competitive resource allocation for multi-user wireless systems is performed, while [35] considers competitive resource allocation in cognitive systems. In [34] the impact of widely linear signaling on competitive, energy-efficient resource allocation algorithms is analyzed for multi-user wireless networks, while [30, 182] consider competitive power and subcarrier allocation in MC and multiple-antenna wireless networks. Fractional programming in competitive resource allocation problems has been used also in [122] for OFDMA networks, in [183, 187, 12, 190] for relay-assisted systems, and in [89, 189] for multiple-antenna systems.

The common denominator to all the mentioned contributions in the area of competitive resource allocation, is that they use game theory to analyze the competition among the network nodes. As it will be explained in more detail in Chapter 5, fractional programming proves very useful in this context, too, because it allows one to analyze the generalized concavity properties of the individual utility functions, thereby leading to the optimal resource allocation for each node and to determining the existence of equilibria points.

It is thus seen that fractional programming is a general theory that has been widely applied in recent years to the design of resource allocation protocols for wireless networks. It is clearly anticipated that fractional programming will continue to be a valuable tool also for the design and optimization of future wireless networks such as 5G cellular networks. In particular, the following technologies appear among the most promising candidates to meet the requirements of future cellular networks [3, 27, 83].

- *Densely deployed small cells* [79, 4, 82], which try to cope with the sheer number of connected nodes by an extensive use of small-cells and relay stations to be placed in critical areas that would be otherwise difficult to serve. From an energy-efficient point-of-view, network densification is particularly attractive because it reduces the distances between nodes, thus leading to higher data-rates at lower transmit powers. However, deploying more infrastructure nodes results in increased inter-cell interference and leads to an heterogeneous network where nodes with different features and specifications should co-exist.

- *Device-to-device communications* [197], which aim at increasing a cellular system resource reuse factor and reducing the infrastructure load by allowing neighboring mobiles wishing to communicate with each other, to establish a direct communication link, bypassing the network infrastructure. device-to-device (D2D) communications in cellular networks can be either operator-controlled [103], or opportunistically activated by the devices, thus underlaying the regular cellular system [139]. In the former case, the system resources are centrally optimized by the network operator, which dictates what resource blocks can be used for D2D communications. In the latter, neighboring devices can autonomously decide to directly communicate, reusing the same resource blocks used by the cellular system, provided the interference caused to the cellular users remains under a predetermined threshold.

- *Massive MIMO systems* [147, 99], which deal with the large number of connected nodes by drastically increasing the amount of deployed antennas. Owing to the law of large numbers, using many antennas has the potential to average out multi-user interference, provided the so-called *favorable propagation condition* holds [116]. However, deploying a number of antennas much larger than the mobiles to serve, has some drawbacks, too, among which we mention pilot contamination effects that complicate channel estimation, and more significant hardware impairments due to the fact that low-cost hardware must be used in practical systems, given the large amount of circuitry required to feed hundreds of antennas.

The debate as to what candidate technology is the most suited to the implementation of 5G networks is still ongoing in the wireless community, and it is common opinion that the final selected technology will not be based on a single approach, but rather on a combination of different approaches. However, even if it is not clear yet, what 5G networks will look like, it is clear that they will have to be energy-efficient in order to meet the exponentially increasing rate demands, while at the same time guaranteeing a sustainable growth and affordable costs. This makes energy efficiency one enabler of 5G networks, and fractional programming an invaluable mathematical tool for the energy-efficient analysis and optimization of future wireless networks.

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