

Energy efficiency of an intracavity coupled, laser-driven linear accelerator pumped by an external laser

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(Received 26 January 2005; published 11 March 2005)

We calculate the optimum energy efficiency of a laser-driven linear accelerator by adopting a simple linear model. In the case of single bunch operation, the energy efficiency can be enhanced by incorporating the accelerator into a cavity that is pumped by an external laser. In the case of multiple bunch operation, the intracavity configuration is less advantageous because the strong wakefield generated by the electron beam is also recycled. Finally, the calculation indicates that the luminosity of a linear collider based on such a structure is comparably small if high efficiency is desired.

I. INTRODUCTION

Energy efficiency is a crucial parameter for operating a linear collider economically, and an acceleration scheme with high efficiency is required. In this paper, which is a continuation of earlier work [1], we first examine the case of single bunch operation and then the case of multiple bunch operation in a laser-driven linear accelerator. A simple linear model is adopted to calculate the concerned parameters and their optimization. In both cases we analyze the problem by introducing a realistic modeling of intracavity configuration, which is pumped by an external laser and is shown in Fig. 1. This is different from the earlier paper where a gain medium and an amplitude modulator are incorporated into the cavity while pumping it with external laser. (Schächter analyzed a similar configuration [2] but did not use a unitary matrix, as required for energy conservation, for the beam combiner [3].)

There are two considerations for removing the gain medium from the cavity with the accelerator. First, it was found in Ref. [1] that the external laser was necessary to establish the intracavity field at the accelerating phase, and, given this, the advantage of the gain medium within the cavity is not clear. A second consideration is the similarity to other applications where a function of some type is incorporated into a cavity. For example, it was found that externally pumping a cavity with a nonlinear crystal was important for second-harmonic generation [4], where the key to achieving high efficiency was adjusting the reflectivity of the beam combiner to accomplish impedance matching so that the incident power is coupled completely into the cavity. This is a case that is analogous to adjusting the coupling coefficient β in a standing wave accelerator cavity to take account of beam loading and will be expanded upon in the next section.

II. SINGLE BUNCH OPERATION

Following the analysis in Ref. [1], a single bunch produces wakefields in the accelerating mode and through Cherenkov radiation that are given by

$$G_k = q \frac{1}{4} \frac{\beta_g}{1 - \beta_g} \frac{cZ_C}{\lambda^2} \equiv qk, \quad (1)$$

$$G_h = q \frac{cZ_H}{\lambda^2} \equiv qh, \quad (2)$$

where Z_C is the characteristic impedance of the accelerating mode and Z_H is the impedance of Cherenkov radiation. The $1/\lambda^2$ dependences strongly limit the amount of charge that can be accelerated. The parameters from Lin [5] are used for the numerical examples in this paper. For them k and h can be calculated to be 2.0×10^{21} and 3.5×10^{22} V/(Cm), respectively.

The configuration is illustrated in Fig. 1 where a laser-driven linear accelerator is incorporated into a cavity that consists of three perfect mirrors and one beam combiner.

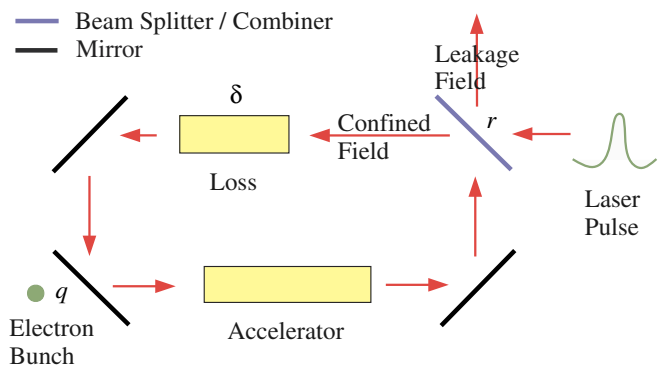


FIG. 1. (Color) The intracavity coupled, laser-driven linear accelerator pumped by the external laser.

Experimentally, if the accelerator were a photonic band gap fiber accelerator, two of the mirrors would be photonic couplers that couple to the accelerator laterally to avoid aligning the beam source and the laser source on the same axis. The purpose of the beam combiner is to regulate the relative strength of the transmitted external laser field and the reflected recycled fields so that these fields interfere appropriately. ‘‘Appropriately’’ here means minimizing the leakage field that leaves the cavity or maximizing the confined field that enters the cavity. In other words, the beam combiner serves to maximize the energy we can inject into the cavity by optimally matching the laser cavity impedance.

The energy efficiency of this particular structure is derived in the appendix and can be written as

$$\eta = \frac{4mkq}{E_{\text{pk}}^2 \sqrt{\pi}} \left\{ a(r, \delta) E_{\text{pk}} \sqrt{2\pi} \frac{\text{erf}(m/\sqrt{\delta})}{m} - b(r, \delta) 2kq - kq - hq \right\}, \quad (3)$$

$$a(r, \delta) \equiv \frac{(1 - \delta)\sqrt{1 - r^2}}{1 - (1 - \delta)r}, \quad b(r, \delta) \equiv \frac{(1 - \delta)r}{1 - (1 - \delta)r}, \quad (4)$$

where we assume a Gaussian pulse with peak value E_{pk} is used as the laser source. Parameter m is defined as the ratio of acceleration duration to laser pulse duration, q is the amount of charge for single bunch, r is the reflectivity of the beam combiner, δ is the modeled round-trip cavity loss, and erf is the error function. The brace of Eq. (3) is interpreted as the average loaded gradient for electron beam. The first term is the average acceleration gradient due to laser field, and the second term is the average deceleration gradient due to recycled wakefield. The third and fourth terms are the average retarding gradients due to wakefields in the accelerating mode and through Cherenkov radiation, respectively.

The a and b coefficients are functions of reflectivity and loss. These two parameters modify the strengths of the external laser field and the recycled wakefield, and it is intuitive to explain the values of a and b at certain values of reflectivity. For example, at $r = 0$, which is effectively equal to the situation with no cavity, there is no modification to the external laser field, but the recycled wakefield is eliminated (i.e., $a = 1 - \delta$ and $b = 0$). At $r = 1$, this is effectively equal to an impermeable cavity, and therefore the external laser field is rejected completely from the cavity and the recycled wakefield is of strong resonance [i.e., $a = 0$ and $b = (1 - \delta)/\delta$]. These features can be seen in Fig. 2.

In fact, the a coefficient can be derived intuitively. Let $T = 1 - \delta$ and then imagine the external laser pulse passes through the beam combiner [scaled by $(1 - r^2)^{0.5}$], suffers from the loss (scaled by T), and finally makes the round-

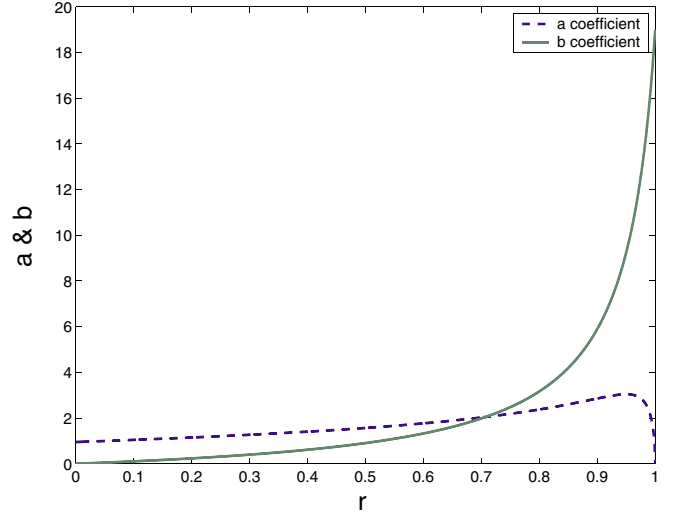


FIG. 2. (Color) Plots of the a and b coefficients for $\delta = 0.05$.

trip repeatedly [scaled by $1/(1 - Tr) = 1 + Tr + T^2r^2 + T^3r^3 \dots$]. The b coefficient can also be derived in the same fashion.

Now given parameters $k = 2.0 \times 10^{21}$ V/(Cm), $h = 3.5 \times 10^{22}$ V/(Cm), $\delta = 0.05$, $m = 3$, and $E_{\text{pk}} = 135$ MV/m, we can plot the energy efficiency as a function of charge and reflectivity shown in Fig. 3. The enhancement of energy efficiency by incorporating the accelerator into the cavity can be seen by comparing $r = 0$, i.e., effectively with no cavity, with $r > 0$, i.e., effectively with cavity. The values of r and q for the optimum case chosen in Fig. 3 can be calculated as $r_{\text{opt}} = 0.847$, $q_{\text{opt}} = 2.362$ fC by Eqs. (5) and (6):

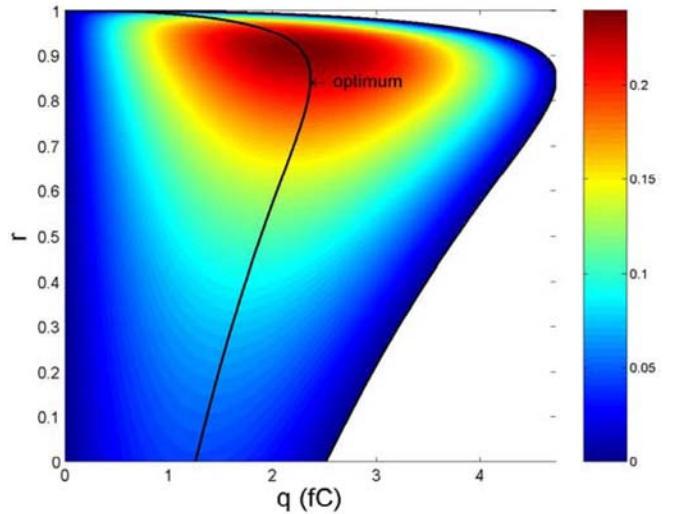


FIG. 3. (Color) Energy efficiency as a function of reflectivity and charge for $\delta = 0.05$. The blank part is where $\eta < 0$, and the hard black line gives the maximum efficiency for a given reflectivity. The optimum case is chosen as indicated in the figure.

$$r_{\text{opt}} = (1 - \delta) \frac{h - k}{h + k}, \quad (5)$$

$$q_{\text{opt}} = \frac{a(r_{\text{opt}}, \delta) E_{\text{pk}} \sqrt{2\pi} \frac{\text{erf}(m/\sqrt{8})}{m}}{2k[2b(r_{\text{opt}}, \delta) + 1] + 2h}. \quad (6)$$

The relevant fields for the optimum case are plotted as functions of time and are shown in Fig. 4. In Ref. [1], the estimated maximum unloaded gradient and average unloaded gradient sustainable by the photonic band gap fiber accelerator are about 320 and 160 MV/m, and therefore we chose $E_{\text{pk}} = 135$ MV/m in the beginning of calculation so that the maximum value of the black solid curve in Fig. 4 is below the breakdown threshold. By comparing the blue dashed and black solid curves in Fig. 4 we understand the intracavity configuration helps to build up the strength of the unloaded gradient and therefore increase the energy efficiency. In addition, by comparing the unloaded gradient in Fig. 4 with that in Fig. 7 of Ref. [1], we can see both of them share a similar pulse shape that is in general a superposition of a Gaussian pulse and a square pulse. As a consequence, it is intuitive that if the external laser field is of the same shape as the recycled wakefield, the efficiency can be further lifted because we would be able to have total destructive interference for the leakage field and total constructive interference for the confined field. In other words, we can completely inject the energy of the external laser into the cavity.

In this section we have shown that high efficiency can be achieved, but for single bunch operation the amount of charge that can be accelerated efficiently is small, on the

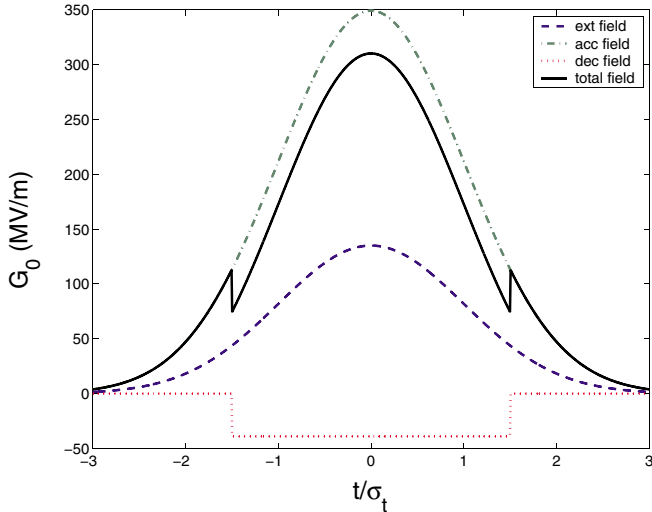


FIG. 4. (Color) Relevant fields for the optimum case. The blue dashed curve is the external laser field and the black solid curve is the unloaded gradient, which is the sum of acceleration gradient due to laser field and deceleration gradient due to wakefield. The time scale is normalized by the duration of the external laser pulse.

order of a femtocoulomb. Since luminosity is a basic requirement of any high-energy linear collider, to reach an interesting value we can (i) accelerate multiple bunches per laser pulse with high laser pulse repetition rate and/or (ii) reduce the collision spot size. The first consideration is covered in the next section.

III. MULTIPLE BUNCH OPERATION

The energy efficiency for multiple bunch operation is derived in the appendix and can be written as

$$\eta = \frac{4mkq}{E_{\text{pk}}^2 \sqrt{\pi}} \left\{ N \left[a(r, \delta) E_{\text{pk}} \sqrt{2\pi} \frac{\text{erf}(m/\sqrt{8})}{m} \right] - N^2 [b(r, \delta) 2kq] - N^2 [kq] - N[hq] \right\}, \quad (7)$$

where the a and b coefficients are again given by Eq. (4) and N is defined as the number of bunches.

Figure 5 is numerically generated for a given maximum average unloaded gradient sustainable by the photonic band gap fiber accelerator. The maximum energy efficiencies with and without the accelerator in a cavity are plotted along with the optimum reflectivity in the former case. Changing single bunch operation into multiple bunch operation can further enhance the energy efficiency. Qualitatively this result can be explained as follows. We can let $q_t = Nq$ and $h' = h/N$ then rewrite (7) as

$$\eta = \frac{4mkq_t}{E_{\text{pk}}^2 \sqrt{\pi}} \left\{ a(r, \delta) E_{\text{pk}} \sqrt{2\pi} \frac{\text{erf}(m/\sqrt{8})}{m} - b(r, \delta) 2kq_t - kq_t - h'q_t \right\}. \quad (8)$$

This equation is exactly the same as Eq. (3) except $q \rightarrow q_t$ and $h \rightarrow h'$, which implies changing single bunch op-

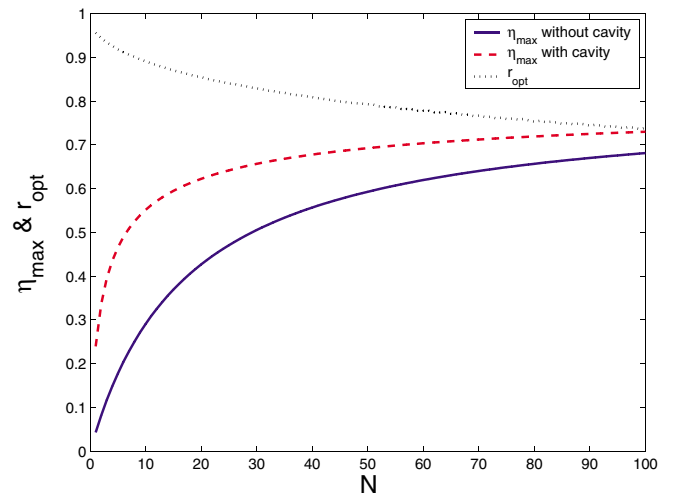


FIG. 5. (Color) The maximum efficiencies and optimum reflectivity versus number of bunches for $\delta = 0.05$. The average unloaded gradient is 160 MV/m.

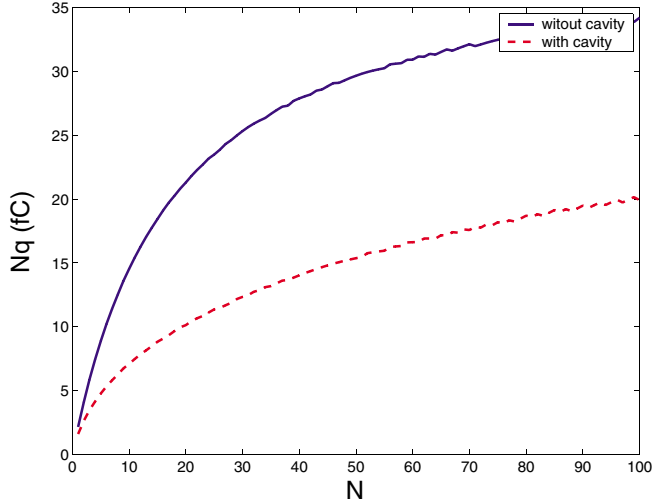


FIG. 6. (Color) The total charge versus number of bunches for $\delta = 0.05$. The average unloaded gradient is 160 MV/m.

eration into multiple bunches operation effectively reduces the value of Cherenkov impedance by a factor of N , and therefore increases the energy efficiency.

An interesting observation about Fig. 5 is that, as the number of bunches increases, the energy efficiencies with or without the cavity approach each other, which implies that the intracavity configuration is not as useful for multiple bunch operation. Physically this effect results because the recycled wakefield increases in strength with the number of bunches, and, therefore, recycling the fields is not as helpful.

The total amount of charge that can be accelerated is shown in Fig. 6 and is seen to saturate. Such saturation is expected because of differences in the bunch-to-bunch coherence of the contributions to the wakefield. Cherenkov radiation, which is broadband and characterized by h , effectively determines the single bunch current, and this component of the wakefield does not add coherently. In contrast the narrow band wakefield in the accelerating mode, characterized by k , adds coherently and becomes comparable to the Cherenkov radiation wake for a total charge of order h/k times the single bunch charge.

There would be a heavy beam loading and a resultant energy slew along the bunch train for a large number of bunches in addition to the observed charge saturation. It would not be possible to compensate this for the assumed high group velocity structure and Gaussian laser pulse.

IV. LUMINOSITY CONSIDERATION

The motivation for considering multiple bunches was increasing the beam current and power needed for luminosity. There are two regimes to consider. The first one is the regime where the bunch train length is longer than the depth of focus of the final focus, and in the second one it is

shorter. Now define the following two constants:

$$B_1 = N(q/e)^2 f, \quad B_2 = (Nq/e)^2 f, \quad (9)$$

where (q/e) is the number of electrons per bunch and f is the pulse repetition rate. These two quantities are proportional to the luminosity used in the two regimes described above. For a rf driven linear collider like the Next Linear Collider (NLC) [6] that works in the first regime, the constant $B_1 \sim 10^{24}$ Hz. Now from Fig. 6 with $N = 100$, $q \sim 0.35$ fC (without cavity) and assuming $f = 1$ GHz, constants B_1 and B_2 are about 5×10^{17} and 5×10^{19} Hz, respectively. The luminosity is much smaller than that for the NLC regardless of the regime, and luminosity must come from significant reduction in the collision spot size. The underlying cause is the $1/\lambda^2$ dependences in Eqs. (1) and (2).

V. SUMMARY AND CONCLUSION

We have analyzed the energy efficiency of a laser-driven linear accelerator by using a simple linear model. First, the energy efficiency of single bunch operation can be lifted significantly by incorporating the accelerator into a cavity that is pumped by an external laser. Then, the energy efficiency can be further enhanced with multiple bunch operation no matter whether the intracavity configuration is introduced or not. However, in both cases, the beam power is low and the luminosity must be achieved by reducing the collision spot size significantly as compared to a rf driven accelerator.

ACKNOWLEDGMENTS

This work is supported in part by U.S. Department of Energy Contracts No. DE-AC03-76SF00515 and No. DE-FG03-97ER41013. The authors have benefited from discussions with the members of ARDB group in SLAC.

APPENDIX: DERIVATION OF ENERGY EFFICIENCY—SINGLE BUNCH AND MULTIPLE BUNCH OPERATION WITH A CAVITY PUMPED BY AN EXTERNAL LASER

We start our analysis by considering multiple bunch operation as shown in Fig. 7 where we define N as the total number of bunches and n as the label of each bunch. The duration between each single bunch is given as

$$d\tau = \frac{l\lambda}{c}, \quad (A1)$$

where the bunches are spaced an integer number l of wavelengths apart. The laser pulse envelope slips by an amount

$$\Delta\tau = \frac{L}{c} \frac{1 - \beta_g}{\beta_g} \quad (A2)$$

relative to the beam as it passes through the accelerator. In

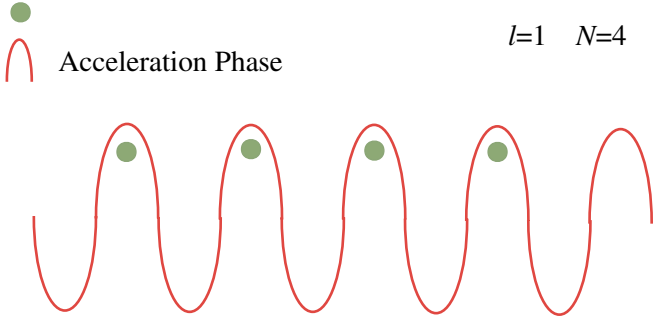


FIG. 7. (Color) The arrangement of multiple bunches operation. In this figure each single bunch is separated by one wavelength and the number of total bunches is taken as 4. The electron bunches are set to be on the peak of acceleration phase.

this expression L is the length of the linear accelerator and β_g is the group velocity. Then we can write down the kinetic energy gain for each bunch as

$$\Delta U_{\text{kin}}(n) = qL[\langle G_0(n) \rangle - kq - hq] - q \left[\sum_{i=1}^{n-1} \left(\frac{c\beta_g}{1-\beta_g} \right) (\Delta\tau - id\tau) \right] 2kq. \quad (\text{A3})$$

The average unloaded gradient for bunch n , $\langle G_0(n) \rangle$, depends on the temporal relation between the bunch and the laser envelope. The sum accounts for the fundamental mode wakefield produced by preceding bunches, and the factor of 2 comes from the fundamental theorem of beam loading [7]. Here we assume the validity of quasisingle bunch regime, i.e., that the following condition is true:

$$\Delta\tau \gg Nd\tau. \quad (\text{A4})$$

Then we can rewrite (A3) as

$$\Delta U_{\text{kin}}(n) \cong qL[\langle G_0(n) \rangle - kq - hq - 2kq(n-1)]. \quad (\text{A5})$$

The average unloaded gradient is calculated by using the following equation:

$$\langle G_0(n) \rangle = \frac{1}{\Delta\tau} \int_{-\Delta\tau/2}^{\Delta\tau/2} G_0(\tau) d\tau. \quad (\text{A6})$$

$G_0(\tau)$ is the unloaded gradient as a function of τ , which is a temporal coordinate with the origin located at the center of the pulse [1]. Notice that the integrand implies a symmetric setup with the bunch entering and leaving the structure at $\tau = -\Delta\tau/2$ and $\tau = \Delta\tau/2$, respectively.

We now consider the laser-driven linear accelerator incorporated into the cavity. There are two independent field sources: one is the external laser field, where by assuming a Gaussian pulse with peak value E_{pk} and duration σ_τ , it can be written as

$$F_{\text{laser}}(\tau) = E_{\text{pk}} \exp\left[-\frac{1}{2}\left(\frac{\tau}{\sigma_\tau}\right)^2\right]. \quad (\text{A7})$$

Another field we have to consider is the wakefield generated by the multiple bunches and emitted at the accelerator output. In quasisingle bunch regime this wakefield can be approximated as

$$F_{\text{wake}}(\tau) \cong 2NkqS(\tau), \quad (\text{A8})$$

where $S(\tau)$ is a rectangular function centered at the origin with full width equal to $\Delta\tau$ [1]. To find the steady state unloaded gradient we can write down a self-consistent equation:

$$G_0(\tau) = (1-\delta)\{\sqrt{1-r^2}[F_{\text{laser}}(\tau)] + r[G_0(\tau) - F_{\text{wake}}(\tau)]\}, \quad (\text{A9})$$

where δ is the round-trip cavity loss and r is the reflectivity of the beam combiner. This equation is derived by considering the steady state unloaded gradient at the accelerator input. The wakefield interferes destructively with the input field to produce the field at the accelerator output. This combined field is reflected by the beam combiner, and, in addition, it interferes with the transmitted external laser field. Then the whole field suffers the round-trip cavity loss to give the field at the accelerator input. Rearranging (A9) we have

$$G_0(\tau) = a(r, \delta)F_{\text{laser}}(\tau) - b(r, \delta)F_{\text{wake}}(\tau), \quad (\text{A10})$$

where a and b are the same as in Eq. (4). Now substituting (A10) into (A5) and making use of (A6)–(A8) we have

$$\Delta U_{\text{kin}}(n) = qL \left[a(r, \delta)E_{\text{pk}}\sqrt{2\pi} \frac{\text{erf}(m/\sqrt{8})}{m} - b(r, \delta)2Nkq - kq - hq - 2kq(n-1) \right], \quad (\text{A11})$$

where erf is the error function and m is defined as

$$m \equiv \Delta\tau/\sigma_\tau. \quad (\text{A12})$$

Finally, the energy efficiency is defined as the sum of all the kinetic energy gains divided by the input energy:

$$\eta = \frac{\Delta U_{\text{kin}}}{U_{\text{in}}} = \frac{\sum_{n=1}^N \Delta U_{\text{kin}}(n)}{\frac{\lambda^2}{Z_c} \int_{-\infty}^{\infty} F_{\text{laser}}^2(\tau) d\tau}, \quad (\text{A13})$$

where

$$\Delta U_{\text{kin}} = qL \left\{ N \left[a(r, \delta)E_{\text{pk}}\sqrt{2\pi} \frac{\text{erf}(m/\sqrt{8})}{m} \right] - N^2[b(r, \delta)2kq] - N^2[kq] - N[hq] \right\}, \quad (\text{A14})$$

$$U_{\text{in}} = \frac{\lambda^2 \sigma_\tau}{Z_c} \sqrt{\pi} E_{\text{pk}}^2. \quad (\text{A15})$$

Now substituting (A2), (A14), and (A15) into (A13) and using the definition of k in Eq. (1), we have

$$\eta = \frac{4mkq}{E_{pk}^2\sqrt{\pi}} \left\{ N \left[a(r, \delta) E_{pk} \sqrt{2\pi} \frac{\text{erf}(m/\sqrt{8})}{m} \right] - N^2[b(r, \delta)2kq] - N^2[kq] - N[hq] \right\}. \quad (\text{A16})$$

This is the same as Eq. (8) and is reduced to Eq. (3) if $N = 1$.

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