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Energy-Efficient Clustering Mechanism of Routing Protocol for Heterogeneous Wireless Sensor Network Based on Bamboo Forest Growth Optimizer

Qing Feng¹, Shu-Chuan Chu^{1,*}, Jeng-Shyang Pan^{1,2}, Jie Wu³ and Tien-Szu Pan⁴

- ¹ College of Computer Science and Engineering, Shandong University of Science and Technology, Qingdao 266590, China; 202082060009@sdust.edu.cn (Q.F.); jspan@cc.kuas.edu.tw (J.-S.P.)
- ² Department of Information Management, Chaoyang University of Technology, Taichung 41349, Taiwan
- ³ School of Electrical and Information Engineering, Zhengzhou University of Light Industry, Zhengzhou 450002, China; wujie@zzuli.edu.cn
 - ⁴ Department of Electronic Engineering, National Kaohsiung University of Science and Technology, Kaohsiung 82445, Taiwan; tpan@nkust.edu.tw
 - * Correspondence: scchu0803@sdust.edu.cn

Abstract: In wireless sensor networks (WSN), most sensor nodes are powered by batteries with limited power, meaning the quality of the network may deteriorate at any time. Therefore, to reduce the energy consumption of sensor nodes and extend the lifetime of the network, this study proposes a novel energy-efficient clustering mechanism of a routing protocol. First, a novel metaheuristic algorithm is proposed, based on differential equations of bamboo growth and the Gaussian mixture model, called the bamboo growth optimizer (BFGO). Second, based on the BFGO algorithm, a clustering mechanism of a routing protocol (BFGO-C) is proposed, in which the encoding method and fitness function are redesigned. It can maximize the energy efficiency and minimize the transmission distance. In addition, heterogeneous nodes are added to the WSN to distinguish tasks among nodes and extend the lifetime of the network. Finally, this paper compares the proposed BFGO-C with three classic clustering protocols. The results show that the protocol based on the BFGO-C can be successfully applied to the clustering routing protocol and can effectively reduce energy consumption and enhance network performance.

Keywords: wireless sensor networks; energy-efficient clustering mechanism; bamboo forest growth optimizer

1. Introduction

Guided by the development trend of the "sensor city", wireless sensor networks (WSN) have been widely used in the field of information, from the field of military and national defense to the fields of medical care, industry and agriculture, urban management, environmental monitoring [1], and smart homes [2] that are closely related to people. The WSN is an ad hoc network composed of randomly distributed sensor nodes [3]. The nodes perceive the environment to enable data collection, processing and transmission. However, the sensor nodes in WSN use limited energy (such as batteries), and in some complex working environments, it is difficult to supply power in time, which will lead to the unreliable lifetime and quality of the network. In addition, for scenarios with high real-time requirements, such as disaster monitoring, military supervision, medical inspection, etc., it is even more necessary to consider how to balance the energy consumption of nodes [4]. Therefore, how to balance the energy consumption and extend the lifetime of network is the research focus in WSN.

The selection of cluster head (CH) nodes in routing protocols is a key to efficient communication in WSN. The CH nodes undertake the tasks of information collection,



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Copyright: © 2022 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/). data fusion and data transmission in the cluster, and their energy consumption is faster than other nodes. By designing an effective clustering protocol, each sensor node can use the limited energy more reasonably. The clustering routing protocol is divided into four processes: cluster head election, cluster formation, data fusion, and data transmission. It divides the entire network into multiple clusters. In each cluster, select one node as the cluster head (CH) node and the other nodes as the cluster member (CM) nodes. The CM nodes communicate with the CH node in each cluster and forward the data to the CH node. The CH nodes integrate the data and send it to the sink node, and then the sink node transmits the data to the network for communication management between users [5], as shown in Figure 1. In addition, the performance of routing clustering protocols in homogeneous networks is not good in heterogeneous networks, so the research based on clustering routing protocols is one of the research hotspots in heterogeneous networks at present.

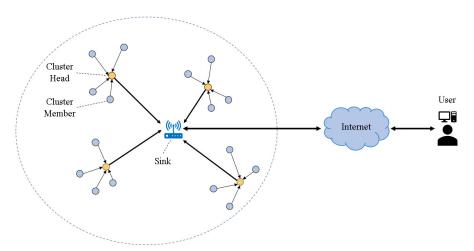


Figure 1. Clustered routing structure of WSN.

Using the metaheuristic algorithm to design the cluster routing protocol has always been a popular direction of industry research [6]. The metaheuristic algorithm is a powerful tool to solve complex optimization problems [7]. It can obtain the best approximate solution for more complex NP-hard problems in polynomial time [8]. As the research on bionics becomes more and more mature, metaheuristic algorithms are proposed one after another; for example, the particle swarm optimization algorithm (PSO) [9,10], genetic algorithm (GA) [11,12], bat algorithm (BA) [13], seagull optimization algorithm (SOA) [14], and the grey wolf optimizer (GWO) [15], etc. The formulas for individual movements of many metaheuristic algorithms are based on operations such as addition, subtraction, multiplication, and division. The mathematical model of the algorithm is not closely related to the essence of things, and there is no specific scientific theory support. In order to come up with a metaheuristic algorithm with good performance and a close connection between the mathematical model and the essence of things, we searched for formula derivation about the growth principle of bamboo forest in biology. Based on the differential model of the bamboo growth and the Gaussian mixture models [16], this study proposes a new metaheuristic optimization algorithm named the bamboo forest growth optimizer (BFGO) and demonstrates the effectiveness of the algorithm's optimization ability is proved on the CEC test sets and engineering optimization problems.

In addition, for the problem of energy consumption in WSN, based on the BFGO algorithm, this paper proposes an energy-efficient clustering mechanism of protocol (BFGO-C) for two-level heterogeneous WSN is proposed. The following are the characteristics of this study:

 A small number of heterogeneous nodes in WSN can usually be used to improve network life and stability, and different types of nodes have different initial energy and consumption rates. In the two-level heterogeneous WSN studied, only energy heterogeneity is considered, and the sensor nodes are divided into advanced nodes and normal nodes;

- The encoding method of the BFGO algorithm is redesigned in this paper. Each individual in the algorithm represents a set of cluster heads;
- The fitness function is improved for BFGO-C, which first considers the relationship between remaining energy and node energy, and also considers the separation of inter-cluster and the compactness of intra-cluster. The purpose of both is to reduce energy consumption and shorten the transmission distance of node communication;
- In experimental, using the four indicators of the lifetime of network, the lifetime of network until the first node dies, remaining energy, and data transmission volume to analysis, and it uses the entropy weight method [17] to conduct a comprehensive analysis.

The remaining structure of the paper is as follows. Section 2 reviews related work. Section 3 presents the BFGO algorithm. Section 4 analyzes the proposed network clustering mechanism of routing protocol (BFGO-C). Section 5 tests the performance of the BFGO algorithm. Section 6 simulates and analyzes the protocol in heterogeneous WSN. In Section 7, the conclusions are given.

2. Related Work

As the key to the clustering mechanism of routing protocol in WSN, many clustering techniques are applied to the clustering mechanism of WSN. The clustering schemes are divided into three categories: hierarchical clustering algorithm, heuristic clustering algorithm, and grid-based clustering algorithm. The related clustering protocol for WSN is shown in Figure 2.

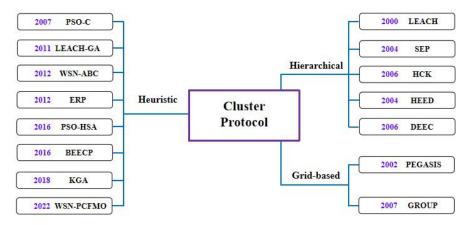


Figure 2. Related clustering protocols for WSN.

2.1. Hierarchical Clustering Algorithm

Low energy adaptive clustering architecture (LEACH) [18], as the earliest network clustering routing protocol, randomly selects CH nodes in the cycle process and distributes the energy load evenly, effectively reducing network power consumption. Still, due to the randomness of cluster head selection, low-energy nodes can easily be selected as CH nodes and die, shortening the life of the network. Later, there were many improved versions of the LEACH protocol, such as mobile-LEACH [19], LEACH-balanced, LEACH-C [20], and other protocols. The stable election protocol (SEP) [21] is also improved based on the LEACH protocol, giving more energy to advanced nodes, effectively using heterogeneous networks. The cluster head selection of a hybrid, energy-efficient, distributed clustering approach (HEED) [22] focuses on the remaining energy and the energy consumed in the cluster, which improves energy utilization. Nodes in intelligent hierarchical cluster-based routing (HCR) [23] are self-organized into clusters, also improved using an agent-based architecture, resulting in energy-efficient hierarchical clusters. The distributed energy-

efficient clustering algorithm (DEEC) [24] provides a different algorithm for estimating network lifetime and retains the distributed nature of the HEED protocol. As the most popular clustering algorithm, the LEACH protocol has been improved and applied by many subsequent algorithms.

2.2. Heuristic Clustering Algorithm

The heuristic clustering algorithm is a combination of a clustering algorithm and an intelligent algorithm based on biological principles. Due to the complexity of the problem and the high computational costs, people pay more attention to adding heuristic optimization algorithms to clustering algorithms to achieve more efficient results. PSOclustering (PSO-C) is the earliest research that applies intelligent optimization algorithms to cluster routing protocols. It selects CH nodes by minimizing the distance between CHS and other nodes in the cluster [25]. In 2011, the adaptive probabilistic prediction clustering protocol, called LEACH-GA, was introduced, effectively extending the lifetime of the LEACH protocol [26]. In 2012, an artificial bee colony algorithm was used to cluster nodes in WSN, and energy was used as an indicator to design the fitness function [27]. In 2012, an evolutionary algorithm is used in routing clustering protocol (ERP), and a fitness function based on separation degree and cohesion degree is proposed [28]. In 2016, the PSO-HSA was improved based on PSO-C, and CH nodes were selected by combining the harmony search algorithm and PSO algorithm, which balanced the local constraints of the two algorithms and extended the life of the network [29]. In 2016, the biogeography optimization-based energy-efficient clustering protocol (BEECP) used the binary version of the biogeography optimization algorithm for clustering and optimized it with discrete coding, making the routing protocol more efficient to extend the life cycle [30]. In 2018, a protocol merged the GA algorithm with the K-means clustering algorithm (KGA) for cluster head selection in network [31]. In 2022, parallel fish migration optimization with compact technology based on the memory principle (PCMFO-Memory) proposed the idea of memory reduction, saving the best set of CH nodes in each round and applying it to the next round, which improved the efficiency of the clustering algorithm [32].

2.3. Grid-Based Clustering Method

Grid-based clustering method includes power-efficient gathering algorithm (PEGA-SIS) [33] and GROUP [34] protocols. Unlike the multi-cluster structure of the LEACH protocol, PEGASIS uses signal strength to measure the distance between nodes. It assumes that every node can communicate with the sink node. It is twice as fast as the LEACH protocol to improve the lifetime of the network. The cluster network established by the GROUP protocol is dynamically random. The grid seed is selected first, then the grid seeds along the radius identify the CH nodes in the grid. Its selection operation is based on the remaining energy in the node.

3. Proposed Bamboo Forest Growth Optimization Algorithm

3.1. Inspiration from the Growth Principle of Bamboo Forest

Bamboo is an herbaceous plant that grows explosively to the height of a tree. This rapid growth occurs during its shoot stage. As the "bamboo law" says, bamboo grows only 3 cm in four years and then grows at a rate of 30 cm per day from the fifth year onwards, reaching 15 m in just six weeks [35]. The bamboo extends its roots hundreds of square meters in the soil, and a short period of rapid growth occurs during the bamboo shoot. Therefore, the growth of bamboo forest can be divided into two stages: (a) the underground expansion of the bamboo whip; (b) the shoot growth of the bamboo.

In addition, a bamboo forest is composed of multiple bamboo whips, and the bamboos belonging to one bamboo whip are a group. The bamboo whip undergoes cell division and differentiation by absorbing nutrients from the soil to store energy. Some shoots emanating from the bamboo whip become vigorous bamboo shoots that burst from the ground, while others grow laterally and develop into new bamboo whips.

The two stages of bamboo forest growth can respectively correspond to the global exploration and local exploitation when the metaheuristic algorithm searches for solutions. So combined with the differential equation of bamboo forest growth, a bamboo forest growth optimizer (BFGO) algorithm can be constructed.

3.2. Mathematical Model

3.2.1. Underground Extension of the Bamboo Whip

Based on the characteristics of the bamboo whip, the idea of grouping is added to the algorithm, and individuals are dynamically grouped in the optimization process of the algorithm. Dynamic grouping is the dynamic scheduling of uniform grouping and disrupted grouping, and the fitness of individuals is used as the criterion for grouping. Among them, 'fitness' refers to the performance of an individual to survive in the population. The description of the idea of dynamic grouping is shown in Figure 3.

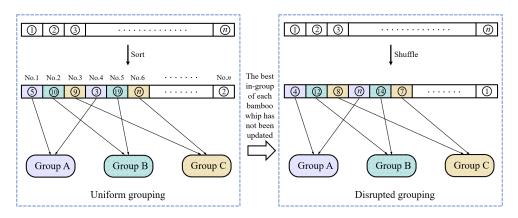


Figure 3. The idea of dynamic grouping.

The direction of the underground bamboo whip expansion depends on three factors: the directives for the group cognitive items, bamboo whip memory, and bamboo forest center, which means that the global optimum, the intra-group optimum, and the location of the central solution simultaneously affect the solution search direction. The formula for the direction of expansion is shown in Equations (1)-(3).

$$\cos\alpha = \frac{\vec{X}_t \cdot \vec{X}_G}{|\vec{X}_t| \times |\vec{X}_G|} \tag{1}$$

$$\cos\beta = \frac{\vec{X}_t \cdot \vec{X_{P(k)}}}{|\vec{X}_t| \times |\vec{X_{P(k)}}|}$$
(2)

$$\cos\gamma = \frac{\vec{X}_t \cdot \vec{C(k)}}{|\vec{X}_t| \times |\vec{C(k)}|},\tag{3}$$

where \vec{X}_t is the position of the current solution, and \vec{X}_G is the position of the globally optimal individual. $\vec{X}_{P(k)}$ and $\vec{C}(k)$ are the intra-group optimal solution and the central solution on the *k*-th bamboo whip, respectively. α , β and γ represent the extension direction of the current individual on \vec{X}_G , $\vec{X}_{P(k)}$ and $\vec{C}(k)$, respectively.

The formula for updating the solution at this stage is shown in Equation (4).

$$X_{t+1} = \begin{cases} X_G + Q \times (c_1 \times X_G - X_t) \times \cos \alpha, & if \ R = P_1(t) \\ X_{P(k)} + Q \times (c_1 \times X_{P(k)} - X_t) \times \cos \beta, & if \ R = P_2(t) \\ C(k) + Q \times (c_1 \times C(k) - X_t) \times \cos \gamma, & if \ R = P_3(t) \end{cases}$$
(4)

$$Q = 2 - \frac{t}{T},\tag{5}$$

where Q is a decreasing parameter, which decreases from 2 to 1 with the iteration of the algorithm, and can influence the development process of the algorithm to a certain extent; c_1 is a random number between 0 and 2. t represents the current iteration number, and T is the maximum iteration number; where R represents the probability of the moving direction of the individual and always takes the maximum probability of the three directions. $P_1(t)$, $P_2(t)$, and $P_3(t)$ are the probabilities of the moving trend of the solution simulated by the Gaussian mixture model, as shown in Figure 4. The dotted line parts represent the values of the three orientation probabilities, respectively, and the solid line parts represent the value of R.

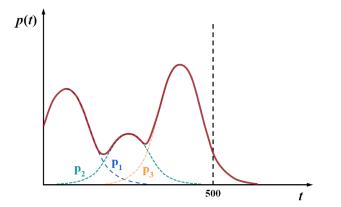


Figure 4. Gaussian mixture models for individual trend probabilities.

The Gaussian mixture model moves individuals to a globally optimal solution in the early iteration process. As the number of iterations increases, the algorithm is more likely to fall into local optimization. To avoid the algorithm getting stuck in local optima, increase the probability of individuals tending towards a central solution. In this way, the distribution of solutions in the iterative process is more diverse, and the capacity of the algorithm to find the optimal solution is enhanced.

3.2.2. Shoot Growth of the Bamboo

Combined with the stochastic process of the growth model proposed by Sloboda [36], different growth environments and random factors lead to different cumulative growth of each bamboo at time t. The cumulative growth of the shooting stage is shown in Equation (6).

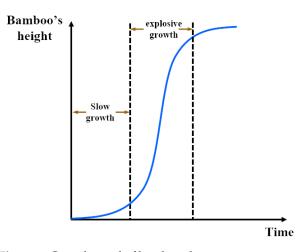
$$q(t) = X_G \times e^d \times e^{\frac{v}{\psi \times t^{\psi}}}.$$
(6)

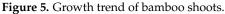
The shape of the bamboo population incremental growth model is shown in Figure 5, including two stages: slow growth and explosive growth [37], where X_G represents the maximum height of bamboo under a particular growth environment, *b* and ψ are the shape parameters of the model.

Given the bamboo accumulation at a specific time t, the change over time can be calculated, as shown in Equation (7).

$$\Delta H = \frac{q(t) - q(t-1)}{X_G - C(k) + 1},$$
(7)

where $\triangle H$ represents the change in the cumulative amount of two iterations per unit distance, the denominator represents the distance from the optimal individual of the population to the center position, and q(t) represents the total cumulative amount of bamboo growth within the *t*-th generation.





The renewal of individuals in this phase is shown in Equations (8) and (9).

$$X_{temp} = \begin{cases} X_t + X_D \times \triangle H, rand < 0.5\\ X_t - X_D \times \triangle H, else \end{cases}$$
(8)

$$X_D = 1 - \left| \frac{X_t - C(k)}{X_G - C(k) + 1'} \right|$$
(9)

where X_D represents the ratio of the distance from the current individual to the optimal individual and the distance from the current individual to the central individual. Increases with the number of iterations, and the cumulative amount shows a trend of rapid growth in the early stage and slow growth in the late stage or even unchanged. Therefore, these two parameters will affect the breadth of algorithm exploration. In the stage of rapid accumulation growth, the algorithm is explored more widely, while in the stage of slow growth, the algorithm gradually reaches the convergence state.

In Equation (8), '+' represents being away from the current individual, '-' represents being close to the current individual. The smaller the value of X_D , the smaller the difference between the current individual and the optimal individual, and then search near the current individual. On the contrary, it is far away from the current individual to find a better solution.

4. Proposed Energy-Efficient Clustering Mechanism of Routing Protocol Based on BFGO Algorithm (BFGO-C)

This study presents an energy-efficient clustering mechanism protocol based on the BFGO algorithm. The goal of this work is to rationalize node distribution and minimize the energy consumption of the network. Then, the optimal set of cluster heads is selected to undertake the tasks of data collection, fusion, and transmitting in WSN. The protocol of the BFGO-C consists of a system model, an energy consumption model, and a cluster head election model. The system model introduces the assumptions for the HWSN simulation [30]. The energy consumption model describes the detailed processing of the cluster head selection by the BFGO-C. The working process of the protocol based on BFGO-C is shown in Figure 6. The pseudo-code Algorithm 1 of BFGO-C implemented in the routing protocol is as follows.

Algorithm 1 BFGO-C Implemented in Routing Protocol. 1: //Initialization Initialize node information, the energy of nodes in HWSN, the deployment of the nodes in WSN, and parameters in BFGO-C. Initialize the maximum number of running rounds (R_m) , the current number of running rounds (R_c) , the set of cluster heads (CH_s) , the numbers of nodes (N), the number of dead advanced nodes (N_a) , the number of dead normal nodes (N_n) , and survival state of the network (N_s) 2: while $(R_c \leq R_m)$ do 3: if N_s==false then end the network 4: 5: end if for j = 1; $j \le N$; j + + do6: 7: if energy ≤ 0 then 8: N_a ++; $N_n++;$ 9: end if 10: end for 11: //Phase 1: Cluster head election 12: Initialize the nummber of current iteration (t), the nummber of max iteration (T), the individual (X_t) and calculate the individual objective function value (f_t) by Equations (15) and (16) 13: while $(t \leq T)$ do update X_t using Equations (1)–(5), sort and update $X_G, X_{P(k)}$, and C(k)14: update X_t using Equations (6)–(9), sort and update $X_G, X_{P(k)}$, and C(k)15: for $j = 1; j \le k; j + + do$ 16: 17: if $X_{P(k)}$ not updated then count++; 18: 19: end if end for 20: if count==k then 21: Do dynamic updates 22: end if 23: $CH_s=X_G;$ 24: 25: end while //Phase 2: Elected cluster head for data transmission 26: Calculate the number of cluster heads (N_{ch}) 27: for $i = 1; j \le N; j + + do$ 28: 29: if Node(i)==cluster head; then 30: Data transmission Calculate consumed energy, remaining energy, and transfer volume using 31: Equations (11)–(16)else Send data to cluster head 32: end if 33. end for 34: 35: end while

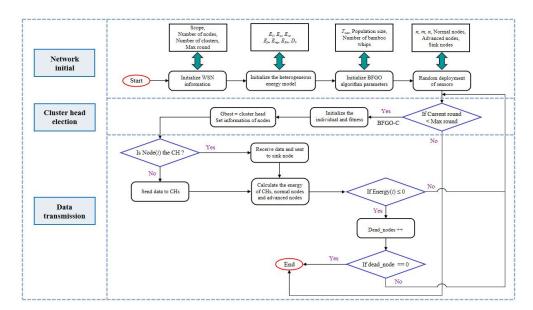


Figure 6. The working process of the protocol based on BFGO-C in HWSN.

4.1. System Model

The Heterogeneous network is a two-level network [38], and the nodes include advanced nodes and normal nodes. It is assumed that *n* is the number of nodes, *m* is the proportion of advanced to all nodes, and its energy is α times higher than that of normal nodes. Let the initial energy of normal nodes be *E*₀, then the initial energy of advanced nodes is *E*₀ × (1 + α). Then the total node energy of the entire HWSN is:

$$E_{total} = n \times (1 - m) \times E_0 + n \times m \times (1 + \alpha) \times E_0 = n \times E_0 \times (1 + \alpha \times m).$$
(10)

There is a WSN randomly and uniformly deployed by *N* nodes to collect data [39]. The following assumptions are made within this simulation environment:

- (a) Nodes are static. The base station node is unique and Located centrally in the area of the network;
- (b) All nodes have unique identification numbers;
- (c) The CH node is responsible for data fusion and transmits the fused data to the base station;
- (d) The energy of sensor nodes is limited. Once they die, they can no longer participate in the network;
- (e) Nodes can calculate and store data, and obtain their residual energy and distance from other nodes;
- (f) The sink node uses a fixed power supply and does not die;
- (g) Only the energy heterogeneity of nodes is considered, and other heterogeneity characteristics are not considered.

4.2. Energy Consumption Model

The energy consumption of the network mainly comes from the wireless communication between nodes. According to the first-order radio model, this study uses the energy loss formula to calculate the energy consumption of nodes. The model diagram is shown in Figure 7, The sensor node consists of a transmitter and a receiver [40]. The left module is the transmitter of the sensor node, and the right module is the receiver. *l* is the length (in bits) of the information received or transmitted by the sensor node. *d* is the distance between the transmitter and the receiver.

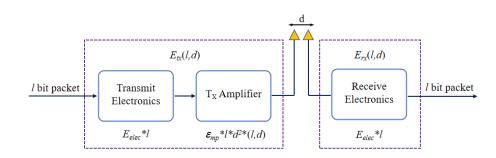


Figure 7. Energy consumption model.

As shown in Figure 7, the data are transmitted from the transmitting electronics to the amplifier. IThey are transmitted to the receive electronics at a distance of *d* meters through wireless communication [41]. When sending data, the energy consumption produced by the node is the sum of the energy consumption produced by the transmitting electrons and the amplifier circuit [42]. The calculation of transmitting volume of routing protocol based on BFGO-C is shown in Equation (11).

$$E_{tx}(l,d) = \begin{cases} E_{elec} \times l + \epsilon_{fs} \times l \times d^2, d < d_0 \\ E_{elec} \times l + \epsilon_{mp} \times l \times d^4, \text{else} \end{cases}$$
(11)

where E_{elec} represents the power used by the transmitter and receiver, and ϵ_{fs} and ϵ_{mp} are the power amplifier coefficients under the free space and multipath fading models, respectively. d_0 represents the distance threshold from the transmitter to the receiver, as shown in Equation (12).

$$d_0 = \sqrt[2]{\frac{\epsilon_{fs}}{\epsilon_{mp}}}.$$
(12)

If $d > d_0$, it is a multipath fading model; conversely, it is a free space model. The received data energy consumption E_{rx} is shown in Equation (13).

$$E_{rx}(l) = E_{elec} \times l. \tag{13}$$

The energy consumption of data fusion is shown in Equation (14).

$$E_m(l) = E_{DA} \times l \times (1+n), \tag{14}$$

where E_{DA} is the unit energy consumption of data fusion.

4.3. Cluster Head Election Model

4.3.1. Encoding Method

The correspondence between the concepts of the clustering mechanism and the BFGO algorithm is shown in the Table 1. As shown in the first row of Table 1, a set of CH nodes in the clustering mechanism is equivalent to a bamboo individual in the BFGO algorithm. In the second row, the number of CH nodes is equivalent to the dimension of the individual, and so on.

Table 1. The correspondence between the concepts of the clustering mechanism and the BFGO algorithm.

| Clustering Mechanism | BFGO Algorithm |
|---|--|
| a set of CH nodes | a bamboo individual |
| the number of cluster heads | the dimensions of individual |
| the effectiveness of a set of CH nodes the optimal set of CH nodes | the fitness of a bamboo individual the global optimal individual |

4.3.2. Fitness Function

From the description of the coding method, It can be known that the fitness function determines the quality of the set of CH nodes by evaluating the individual [43]. The fitness function in this paper is designed according to three factors: the compactness within the cluster, the separation between the clusters, and the relationship between the initial energy and the remaining energy. Among them, the relationship between the initial energy and the remaining energy is an essential factor, because the difference between HWSN and WSN lies in the setting of the initial energy of the sensor node. In advanced nodes, the energy factor accounts for a more significant proportion [44], So that the high-energy nodes as the CH nodes can effectively extend the lifetime of the network. At the same time, the low-energy CH nodes lead to a faster node death rate. The fitness function is shown in Equation (15).

$$Fitness = \sum_{C=1}^{CHs} \frac{E_0}{(E_r)^p} + \sum_{i=1}^{CHs} \sum_{j=1}^{N_i} dis(S_j, CH_i) + \sum_{i=1}^{CHs} \sum_{j=1}^{CHs} dis(CH_i, CH_j)$$
(15)

$$dis(X,Y) = \sqrt[2]{\sum_{i=1}^{n} (x_i - y_i)^2},$$
(16)

where E_0 is the initial energy of the node, E_r is the current remaining energy of node *i*, and *p* is the weight coefficient. The ratio of the residual energy of the node to the initial energy can reflect the current residual energy of the CH node. N_i represents the number of nodes in the *i*-th cluster, and S_j is the non-cluster head node in the *j*-th cluster, $dis(CH_i, CH_j)$ represents the distance between any two cluster heads. The Euclidean distance is used to calculate both compactness and separation, as shown in Equation (16).

5. Performance Test of BFGO Algorithm

The CEC test suite is a set of functions commonly used for testing and evaluating the performance of the metaheuristic algorithm, including unimodal functions, simple multimodal functions, hybrid functions, and composite functions. The tests of multi-type functions can show the performance of the algorithm more comprehensively. To test the optimization performance of the BFGO algorithm, tests and analysis are conducted in the CEC2013 benchmark function set [45], CEC2017 benchmark function set [46], and three engineering optimization problems [47]. Table 2 summarizes the relevant parameters, 'own' represents the parameters set by the algorithm itself, and 'unified' is the public parameter in the experiment.

| Algorithm | Parameter (Own) | Parameter (Unified) |
|---------------------|---|--|
| BA [13] | $A_i = 0.6, r = 0.7, A_f = 0.9,$ $R_f = 0.9, Q_{min} = 0, Q_{max} = 1$ | |
| PSO [9] GWO [15] | $c_1 = 2, c_2 = 2$ none | Runs = 30, Population = 100, iterations = 500, $lb = -100,$ |
| SOA [14] BFGO | none $bamboowhips = 5, sita = 2$ | <i>ub</i> = 100, <i>dimension</i> = 10D/50D |

Table 2. Parameters of the relevant algorithm.

5.1. Test of CEC2013 Benchmark Function

The experimental results of this part are shown in the Appendix A, Figure 8 shows the number of functions the BFGO algorithm outperforms in other algorithms in 10-dimensional (10D) and 50-dimensional (50D) spaces.

From Table A1, among the 28 functions of the CEC2013 test set, the BFGO algorithm is superior to the BA algorithm, PSO algorithm, GWO algorithm, and SOA algorithm in 23, 18, 16, and 26 functions. It can be seen from Table A2 that the BFGO algorithm is superior to the BA algorithm, PSO algorithm, GWO algorithm, and SOA algorithm in 20, 21, 13,

and 25 functions, respectively. It can be seen that the BFGO algorithm has advantages over other algorithms in both low and high dimensions, especially in low dimensional space, showing strong competitiveness.

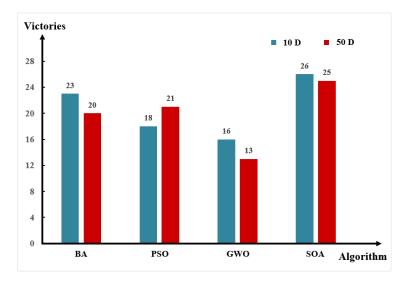


Figure 8. Comparison of BFGO algorithm getting better times in CEC2013 test.

The scalability of the algorithm in CEC2013 can be analyzed in Figure 8. The performance of the BFGO algorithm decreases with the increase of dimension. However, it is still better than the other algorithms in most functions.

5.2. Test of CEC2017 Benchmark Function

The experimental results of this part are shown in the Appendix A, Figure 9 shows the number of functions the BFGO algorithm outperforms in other algorithms in 10D and 50D space.

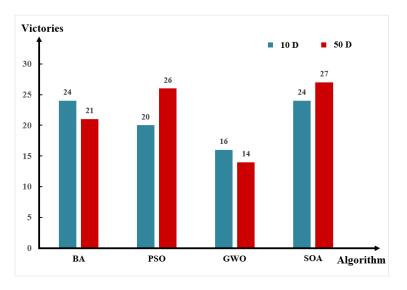


Figure 9. Comparison of BFGO algorithm getting better times in CEC2017 test.

It can be seen from Table A3 that for the 30 functions of CEC2017, the overall performance of BFGO is better than other algorithms on 10D. The BFGO algorithm is superior to the BA algorithm, PSO algorithm, GWO algorithm, and SOA algorithm in 24, 20, 16, and 24 functions. It can be seen from Table A4 that the BFGO algorithm also has good performance on 50-D. The BFGO algorithm is superior to the BA algorithm, PSO algorithm, GWO algorithm, and SOA algorithm in 21, 26, 14, and 27 functions. The scalability of the algorithm in CEC2017 can be analyzed in Figure 9. The performance of the BFGO algorithm decreases slightly with the increase of dimension. However, it is still better than the other algorithms in most functions.

According to the test results of five algorithms in the CEC2013 and CEC2017, the BFGO algorithm is significantly better than the BA, PSO, and SOA algorithms in both low-dimensional and high-dimensional space, and its performance is similar to the GWO algorithm, only slightly lower than the GWO algorithm in high-dimensional space. To sum up, the BFGO algorithm has good optimization performance and strong competitiveness.

5.3. Test of Engineering Optimization

Three classical engineering optimization problems are used to test the performance of the BFGO algorithm in finding appropriate parameters to optimize the solution of practical problems, including compression spring design, welded beam design, and speed reducer design [48,49]. The three issues are optimized with parameters 3, 4, and 7 to minimize cost dissipation.

Tables 3–5 compare the results of the BFGO algorithm with the BA, GWO, PSO, and SOA algorithms tested on the three engineering design problems 30 times, respectively. The table shows the optimal parameters, the mean (Mean), the standard deviation (Std), and the minimum (Min) values for the 30 tests. Where underline represents the same optimum and bold represents the optimum.

Table 3. Comparison of BFGO algorithm and other algorithms in optimizing compression spring design.

| Algorithm | O | ptimize Variał | ole | Mean | Std | Min | |
|-----------|----------|----------------|----------|-----------------|-------------------------|----------|--|
| Algorithm | d | D | Ν | wiedli | 310 | IVIII | |
| BA | 0.050000 | 0.282000 | 2.000000 | 0.002824 | 3.3397×10^{-6} | 0.002820 | |
| PSO | 0.052700 | 0.277500 | 4.136000 | 13,186.75 | 7.2227×10^{4} | 0.004731 | |
| GWO | 0.050000 | 0.282000 | 2.000000 | 0.002820 | 1.5421×10^{-8} | 0.002820 | |
| SOA | 0.050000 | 0.282000 | 2.000000 | 0.002820 | $1.1723	imes10^{-7}$ | 0.002820 | |
| BFGO | 0.050000 | 0.282000 | 2.000000 | <u>0.002820</u> | 4.6049×10^{-8} | 0.002820 | |

Table 4. Comparison of BFGO algorithm and other algorithms in optimizing welded beam design.

| Algorithm | | Optimize | e Variable | | Mean | Std | Min |
|------------|----------|----------|------------|-----------|-----------|----------------------|---------|
| Aigontinii | T_s | T_h | R | L | wiean | Siu | IVIIII |
| BA | 0.196600 | 0.101700 | 10.193600 | 67.907400 | 3074.41 | 7.7449×10^3 | 119.658 |
| PSO | 1.584700 | 20.84740 | 10.100800 | 104.54950 | 103,174.1 | $9.9926	imes10^4$ | 6158.07 |
| GWO | 0.192900 | 0.095300 | 10.000000 | 64.124600 | 108.910 | $5.688	imes10^{-3}$ | 108.902 |
| SOA | 0.192900 | 0.095200 | 10.000000 | 64.197300 | 5617.420 | $1.3710	imes10^4$ | 109.001 |
| BFGO | 0.192800 | 0.095400 | 10.000000 | 64.270600 | 112.824 | 4.5868 | 109.097 |

Table 5. Comparison of BFGO algorithm and other algorithms in optimizing speed reducer design.

| Algorithm | | | Mean | Std | Min | | | | | | |
|-----------|----------|-----------------------|-----------|----------|----------|----------|----------|------------|-----------|------------|--|
| Aigonum | x_1 | <i>x</i> ₂ | x_3 | x_4 | x_5 | x_6 | x_7 | Wiedii | Siu | IVIIII | |
| BA | 3.600000 | 0.800000 | 28.000000 | 7.300000 | 7.800000 | 3.900000 | 5.283700 | 201,614.63 | 1.592047 | 201,613.20 | |
| PSO | 3.492600 | 0.792700 | 25.847700 | 7.691400 | 8.274700 | 3.841700 | 5.315900 | 540,799.84 | 108,075.4 | 369,151.63 | |
| GWO | 3.600000 | 0.800000 | 28.000000 | 7.300000 | 7.800000 | 3.900000 | 5.284700 | 201,613.24 | 0.101490 | 201,613.19 | |
| SOA | 3.600000 | 0.800000 | 28.000000 | 7.300000 | 7.800000 | 3.900000 | 5.284900 | 201,616.66 | 4.204730 | 201,613.20 | |
| BFGO | 3.600000 | 0.800000 | 28.000000 | 7.300000 | 7.800000 | 3.900000 | 5.284700 | 201,613.19 | 0.181397 | 201,613.19 | |

In the test of compression spring design, the BGFO algorithm obtains the same optimal value as the GWO and SOA algorithms in the mean and minimum values. The standard deviation is slight, indicating that the algorithm is stable. In the test of welded beam design, the mean value of 30 tests obtained by the BFGO algorithm ranks second, second only to the GWO algorithm, and other algorithms are still far behind the BFGO algorithm. Similarly, in the speed reducer design problem, the mean value of the BFGO algorithm in the 30 tests

is the first, the minimum value is tied for the first best with the GWO algorithm, and the standard deviation is still tiny, indicating that the algorithm is relatively stable.

Therefore, compared with other algorithms, the BFGO algorithm also has powerful optimization ability for engineering optimization problems.

6. Simulation and Analysis of Protocol Based on BFGO-C in HWSN

This study simulates the protocol based on the BFGO-C and compares the results with the SEP, HCR, and ERP clustering protocols. Table 6 describes the parameters in the simulation experiments. In this paper, four indicators are used as measures to analyze the experimental results: the life of the network, the life of the network until the first node dies, energy consumption, and the data transmission volume. Figure 10 shows the assignment of initial CH nodes in the 100×100 area, where the red dot in the middle is the sink node, the blue dot is the normal node, the purple dot is the advanced node, and the green pentagram is the initial CH node [50]. Figure 11 is the initial node interaction diagram. The black arrows indicate that the CH nodes transmit data to the sink node, and the orange arrows indicate that the nodes in the cluster transmit data to the CH nodes [51].

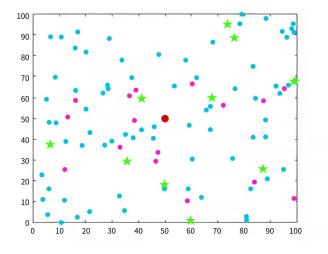


Figure 10. Initial node distribution in the simulation environment.

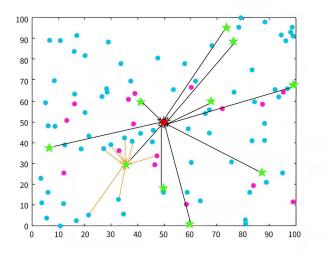


Figure 11. Initial node interaction in the simulation environment.

| Simulation Parameters | Value |
|---|-----------------------------|
| Network area | $100 	imes 100 \text{ m}^2$ |
| Number of nodes (<i>N</i>) | 100 |
| Base station position | (50,50) |
| Packet size (l) | 4000 bits |
| Initial energy of node (E_0) | 0.5 J |
| Advanced node scale (m) | 0.1 or 0.2 |
| Transmitter/Receiver electronics (E_{elec}) | 50 n J/bit |
| Transmit amplifier (free space) ϵ_{fs} | 10 nJ/bit/m^2 |
| Transmit amplifier (multipath) ϵ_{mv} | $0.0013 \text{ nJ/bit/m}^4$ |
| Data aggregation energy cost E_{DA} | 5 nJ/ bit |
| Number of optimized individuals | 20 |
| Number of iterations | 20 |
| The weight of the fitness function | 4 |

Table 6. Simulation parameters of protocol based on BFGO-C in HWSN.

6.1. Comparison of the Lifetime of Network

The number of rounds in which the last node dies represents the life of the network. Figure 12 shows the changes in the surviving nodes of the SEP protocol, the HCR protocol, the ERP protocol, and the protocol based on BFGO-C with the number of rounds.

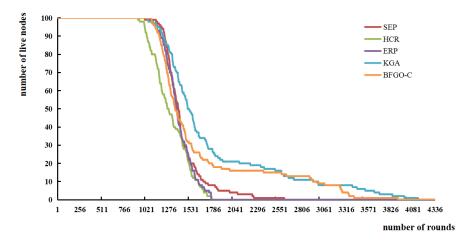


Figure 12. Variation of the number of live nodes with the number of rounds.

The survival trend of the four clustering protocols SEP, HCR, ERP, and the protocol based on BFGO-C in Figure 12. As the network runs, the consume energy of nodes for data transmission and forwarding. After several operation rounds, some nodes run out of energy and become dead nodes. The graph shows that the nodes of the protocol based on BFGO-C die a little slower, and the network lives a little longer than all three other protocols.

There are also application scenarios that pay more attention to the comprehensiveness of the node information. Once the first node in the network dies, the data are missing by default. Hence the paper records the number of rounds when the first node dies, half of the nodes die, and the last node dies, as shown in Figure 13.

In Figure 13, the growth cycles of the four cluster protocols SEP, HCR, ERP, and the protocol based on BFGO-C are 2602, 1771, 1763, and 3909 rounds, respectively. The protocol based on BFGO-C has the longest lifetime of the network. In SEP protocol, the network ran 999 rounds when the first node died and 1385 rounds when half of the nodes died. The first node of the HCR and ERP protocol died in rounds 919 and 1078, and half of the nodes died in rounds 1771 and 1763. The number of rounds when the first node of the protocol based on BFGO-C died was 1134, and the number of rounds when the intermediate node died was 1570.

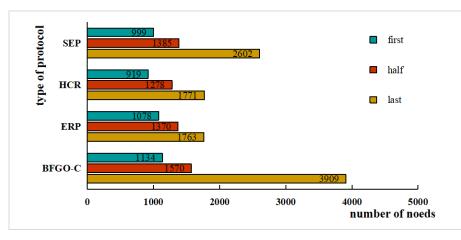


Figure 13. Number of surviving nodes for rounds when the first node dies, half of the nodes die, and the last node dies.

It can be shown that the protocol based on BFGO-C has a higher number of rounds at the death of the first node and a higher number of rounds at the end of half of the nodes than the other three protocols.

6.2. Comparison of Remaining Energy

The energy consumption during network operation reflects the performance of the network. More remaining energy indicates less energy consumption and better network performance. The variation of remaining energy with the number of rounds for the protocols SEP, HCR, ERP, and the protocol based on BFGO-C running in the network is shown in Figure 14, and the remaining energy for the rounds when the first node dies and half of the nodes die is shown in Figure 15.

The trend in energy consumption from Figure 14 shows that the protocol based on BFGO-C has more remaining energy than SEP, HCR, and ERP protocols. At the death of the first node in the network, the HCR protocol has the most residual energy and consumes the least, and the protocol based on BFGO-C is second. At the death of half of the nodes, the protocol based on BFGO-C and HCR protocol have similar residual energy, and both consume less energy than the SEP and ERP protocols. At the death of all nodes, the protocol based on BFGO-C consumes the least total energy than SEP, HCR, and ERP protocols. Protocol based on BFGO-C reduces network energy consumption to a certain extent.

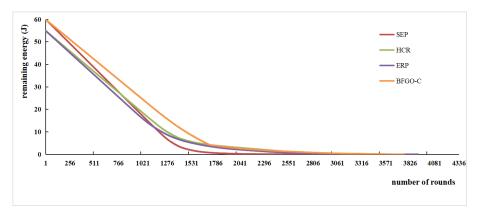


Figure 14. Variation of remaining energy with the number of rounds.

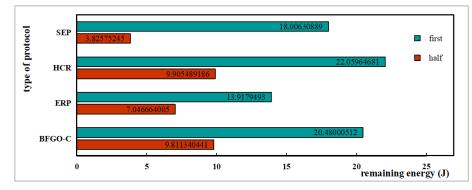


Figure 15. Remaining energy for key rounds.

6.3. Comparison of the Data Transmission Volume

The surviving nodes in the network transmit a data packet to the CH nodes every round, and then the CH nodes transmit it to the base station, and the base station counts the number of data packets collected. The data transmission volume represents the total number of packets sent, it reflects the throughput of the network. The data transmission volume of the four protocols is shown in Figure 16, and Figure 17 also shows the data transmission volume when the first node dies, half of the nodes die and the last node dies.

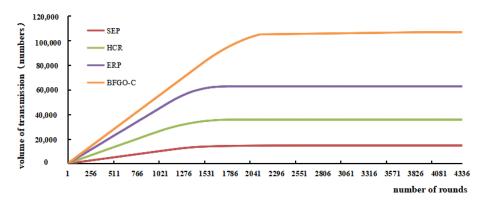


Figure 16. Variation of the volume of transmission with the number of rounds.

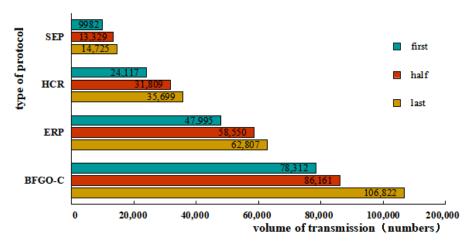


Figure 17. The volume of transmission for key rounds.

The trend of the data transmission volume shows that the protocol based on BFGO-C has the fastest and highest transmission volume. With all the nodes dead, the number of packets stopped growing, and the SEP protocol, HCR protocol, ERP protocol, and protocol based on BFGO-C had 14,725, 35,699, 628,07, and 122,068 data transmissions, respectively, with a protocol based on BFGO-C eventually transmitting the most data. The protocol

based on BFGO-C was consistently ahead of the other protocols regarding the number of packets transmitted in the early and middle stages of the network.

The protocol based on BFGO-C has achieved good performance in HWSN network clustering. Compared with the SEP, HCR and ERC protocols, it extends network life to a certain extent, reduces network energy consumption, and effectively improves network performance.

6.4. Comprehensive Evaluation Based on Entropy Weight Method

Information entropy can measure the discreteness of indicators, and the larger the discreteness, the more significant the impact of the index on a comprehensive evaluation. The entropy weight method determines the index's weight in the comprehensive evaluation according to the variability of the information entropy reaction of the index.

From the simulation results and analysis of the protocol based on BFGO-C in Section 5, the entropy weight method is used to comprehensively evaluate the four protocols in combination with four metrics: the lifetime of the network, the death time of the first node, remaining energy, and the volume of transmission. The radar chart of the comprehensive analysis is shown in Figure 18.

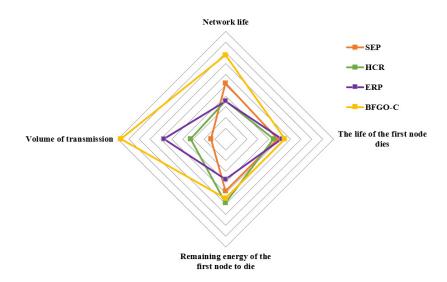


Figure 18. The radar chart of the comprehensive performance.

As can be seen from Figure 18, the comprehensive performance of the protocol based on BFGO-C covers the most extensive range, and it has outstanding performance in the two indicators of network life and volume of transmission, although it is slightly worse in the indicator of the remaining energy of the first node death. Compared with the HCR protocol, the energy of the protocol based on BFGO-C lasts longer in the network. It can be seen from the comprehensive evaluation that the protocol based on BFGO-C can effectively extend the network lifetime by saving energy.

7. Conclusions

This paper proposes an energy-efficient clustering mechanism of routing protocol for heterogeneous WSN based on the BFGO algorithm. Its core concept is to use the optimization ability of the BFGO algorithm to conduct cluster head selection, find the optimal set of CH nodes, guarantee the rationality of the cluster allocation, and maximize the network performance. First, based on the growth characteristics of a bamboo forest, a bionic intelligent optimization algorithm is proposed for the optimization problem. The algorithm has been shown to be highly competitive in both low-dimensional and high-dimensional spaces for CEC2013 and CEC2017 test functions and engineering optimization problems. The fitness function is redesigned when the BFGO algorithm is applied to the clustering mechanism of the routing protocol in heterogeneous WSN. Not only are the

intra-cluster compactness and inter-cluster separation of the clusters considered, but the

ratio of the initial to remaining energy is also taken as an important measure. This study compared the protocol based on BFGO-C with the SEP, HCR, and ERP protocols using four indicators in simulation experiments. The experimental results show that the algorithm can reduce the network energy consumption, extend the network life, and significantly improve the data transmission volume. Finally, to evaluate the clustering performance of these protocols comprehensively, the study used the entropy weight method to give weights and comprehensive analysis, and the results prove that the comprehensive performance of the protocol based on BFGO-C is greater than the other protocols.

This paper does not consider other types of node performance heterogeneity, such as computational or link heterogeneity. In the future, we will continue to investigate how to improve and optimize the cluster routing protocols in heterogeneous and complex environments.

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Appendix A. Test Results on CEC Test Set

| Table A1. Comparison of BFGO algorithm and other algorithms on CEC2013 benchmark function | under 10D. |
|---|------------|
| | |

| £ | | BA | P | so | G | NO | S | DA | BF | GO |
|-----------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|-------------------------|------------------------|------------------------|
| f_x | Mean | Std | Mean | Std | Mean | Std | Mean | Std | Mean | Std |
| f_1 | -1.399×10^{3} | $1.1848 	imes 10^{-1}$ | -1.339×10^{3} | $2.3095 	imes 10^2$ | $-1.385 	imes 10^3$ | $4.7240 	imes 10^1$ | -1.068×10^3 | 1.9838×10^2 | -1.400×10^3 | 3.1599×10^{2} |
| f_2 | $1.7685 	imes 10^5$ | $1.3611 	imes 10^5$ | $6.5680 	imes 10^5$ | $1.7462 	imes 10^6$ | $1.0902 	imes 10^6$ | $9.4889	imes10^5$ | $2.1364	imes10^6$ | $1.7465	imes10^6$ | $2.9837 	imes 10^5$ | $2.4558	imes10^5$ |
| f_3 | $2.1322 	imes 10^7$ | 3.9879×10^{7} | $7.7272 	imes 10^8$ | 1.2511×10^9 | $4.3836	imes10^7$ | $9.7444 	imes 10^7$ | $6.4612	imes10^8$ | $6.1898 	imes 10^8$ | $2.7794	imes10^8$ | $4.4521 	imes 10^8$ |
| f_4 | $1.5171 	imes 10^4$ | $7.0814 	imes 10^3$ | $7.9318 	imes 10^2$ | $7.9543 	imes 10^3$ | $7.1724 	imes 10^3$ | 3.8745×10^{3} | $6.1317 	imes 10^3$ | 2.2208×10^3 | $2.1336 	imes 10^3$ | 1.7939×10^{3} |
| f_5 | -9.996×10^{2} | $6.7815 	imes 10^{-2}$ | -9.759×10^{2} | $4.5922 	imes 10^1$ | $-9.778 	imes 10^2$ | $1.4966 	imes 10^1$ | -8.781×10^2 | 1.6622×10^{2} | -9.999×10^{2} | $4.8037 	imes 10^{-2}$ |
| f ₆ | $-8.916	imes10^2$ | $1.9843	imes10^1$ | $-8.783	imes10^2$ | $2.6334 	imes 10^1$ | $-8.748	imes10^2$ | 2.5992×10^{1} | -8.372×10^{2} | $4.0619 	imes 10^1$ | $-8.837	imes10^2$ | $2.7752 	imes 10^1$ |
| f ₇ | $-6.442 	imes 10^2$ | $1.0048 	imes 10^2$ | -7.591×10^{2} | $2.4188 	imes 10^1$ | $-7.898 	imes 10^2$ | 8.1917 | -7.655×10^{2} | $1.1585 	imes 10^1$ | -7.592×10^{2} | $3.3546	imes10^1$ |
| f_8 | $-6.795 	imes 10^2$ | $9.6019 	imes 10^{-2}$ | -6.796×10^{2} | $9.4408 	imes 10^{-2}$ | -6.796×10^{2} | $7.8637 	imes 10^{-2}$ | -6.797×10^{2} | 7.8866×10^{-2} | -6.797×10^{2} | $9.3192 	imes 10^{-2}$ |
| f_9 | $-5.914	imes10^2$ | 1.7614 	imes 10 | $-5.946 	imes 10^2$ | 1.3188 | $-5.963	imes10^2$ | 1.3082 | $-5.934	imes10^2$ | 1.2878 | $-5.938	imes10^2$ | 1.3414 |
| f_{10} | $-4.990	imes10^2$ | $1.0014	imes10^{-1}$ | $-4.638	imes10^2$ | $3.9716 	imes 10^1$ | $-4.896	imes10^2$ | $1.0867 	imes 10^1$ | $-4.447	imes10^2$ | $3.4506 	imes 10^1$ | $-4.987	imes10^2$ | $7.5568 	imes 10^{-1}$ |
| f_{11} | $-3.194	imes10^2$ | $3.4114 	imes 10^1$ | -3.801×10^2 | $1.0161 	imes 10^1$ | -3.890×10^{2} | 7.2581 | $-3.643	imes10^2$ | $1.2287 	imes 10^1$ | -3.793×10^{2} | $1.1004 	imes 10^1$ |
| f_{12} | $-2.072 	imes 10^2$ | $3.8047	imes10^1$ | $-2.743	imes10^2$ | $1.0737 	imes 10^1$ | $-2.835	imes10^2$ | 8.9362 | -2.609×10^{2} | $1.3191 	imes 10^1$ | $-2.748	imes10^2$ | 9.4686 |
| f_{13} | $-9.033	imes10^1$ | $2.9759 	imes 10^1$ | -1.571×10^{2} | $1.3060	imes10^1$ | $-1.749	imes10^2$ | $1.2035 	imes 10^1$ | $-1.560	imes10^2$ | $1.0963	imes10^1$ | $-1.654	imes10^2$ | $1.4522 	imes 10^1$ |
| f_{14} | 1.2313×10^3 | 2.8635×10^{2} | $4.7548 	imes 10^2$ | $1.9886 	imes 10^2$ | 3.5539×10^{2} | 2.2408×10^{2} | $8.4257 	imes 10^2$ | 3.0412×10^2 | 2.1266×10^{2} | $1.6465 	imes 10^2$ |
| f_{15} | $1.3584	imes10^3$ | $3.7638 	imes 10^2$ | 9.9741×10^{2} | 2.9524×10^2 | 6.8236×10^{2} | 3.2280×10^{2} | 1.2930×10^{3} | 2.8935×10^{2} | $1.0151 	imes 10^3$ | 3.0688×10^{2} |
| f_{16} | $2.0108 	imes 10^2$ | $1.6404	imes10^{-1}$ | 2.0067×10^{2} | $3.0234 	imes 10^{-1}$ | 2.0132×10^{2} | $2.8742 	imes 10^{-1}$ | 2.0130×10^{2} | $3.1604 	imes 10^{-1}$ | $2.0074 	imes 10^2$ | $2.9147 	imes 10^{-1}$ |
| f_{17} | $4.5492 	imes 10^2$ | $3.7348 	imes 10^1$ | $3.3114 	imes 10^2$ | $1.0234 	imes 10^1$ | 3.2727×10^{2} | 6.0447 | 3.5076×10^{2} | 9.0782 | 3.2181×10^{2} | 5.0804 |
| f_{18} | $5.5529 	imes 10^2$ | 5.2666×10^{1} | $4.3184	imes10^2$ | $1.3605 	imes 10^1$ | $4.3805 	imes 10^2$ | 7.5082 | $4.5445 	imes 10^2$ | $1.1120 	imes 10^1$ | $4.2980	imes10^2$ | 6.6620 |
| f_{19} | $5.0812 	imes 10^2$ | 3.3968 | $5.0157 	imes 10^2$ | $7.9477 	imes 10^{-1}$ | $5.0156 	imes 10^2$ | $6.9150 	imes 10^{-1}$ | $5.0377 	imes 10^2$ | 1.0163 | $5.0115 	imes 10^2$ | $4.5671 	imes 10^{-1}$ |
| f ₂₀ | 6.0421×10^2 | $3.1416	imes10^{-1}$ | $6.0333 	imes 10^2$ | $5.4633 	imes 10^{-1}$ | $6.0285 	imes 10^2$ | $5.0574	imes10^{-1}$ | 6.0363×10^{2} | $2.7957 	imes 10^{-1}$ | $6.0334 	imes 10^2$ | $5.2710 	imes 10^{-1}$ |
| f ₂₁ | 1.0551×10^3 | $9.4839	imes10^1$ | 1.0702×10^3 | $7.0293 	imes 10^1$ | 1.1006×10^3 | $3.7800 	imes 10^{-1}$ | $1.1033 	imes 10^3$ | $4.1465	imes10^1$ | $1.0836 	imes 10^3$ | $5.2855 	imes 10^1$ |
| f ₂₂ | 2.4726×10^3 | 3.7527×10^2 | 1.6389×10^3 | $3.0005 	imes 10^2$ | $1.4895 	imes 10^3$ | $3.4980 	imes 10^2$ | $1.9743 	imes 10^3$ | $3.5885 	imes 10^2$ | 1.4281×10^3 | $2.2944 	imes 10^2$ |
| f ₂₃ | $2.5074 	imes 10^3$ | 3.3179×10^2 | $2.0434 	imes 10^3$ | 3.4324×10^2 | $1.7044 	imes 10^3$ | 4.4608×10^2 | $2.1163 	imes 10^3$ | 3.5285×10^{2} | $2.0472 	imes 10^3$ | 3.6717×10^{2} |
| f ₂₄ | $1.2288 	imes 10^3$ | 5.0275 | 1.2160×10^3 | $1.9146 	imes 10^1$ | $1.2115 	imes 10^3$ | 6.2685 | 1.2202×10^3 | 3.6276 | $1.2178 	imes 10^3$ | 5.0555 |
| f25 | 1.3231×10^3 | 4.9373 | 1.3211×10^3 | 5.0100 | $1.3107 	imes 10^3$ | 5.1177 | $1.3214 	imes 10^3$ | 2.7814 | $1.3092 	imes 10^3$ | $2.4628 	imes 10^1$ |
| f ₂₆ | 1.4041×10^3 | $3.8663 	imes 10^1$ | $1.4307 	imes 10^3$ | $6.7151 	imes 10^1$ | $1.3919 	imes 10^3$ | 7.3556×10^1 | $1.4005 	imes 10^3$ | $5.5332 	imes 10^{-1}$ | $1.3861 	imes 10^3$ | $4.8071 	imes 10^1$ |
| f ₂₇ | $1.9462 	imes 10^3$ | $9.9494	imes10^1$ | 1.8572×10^3 | $6.9135	imes10^1$ | 1.7129×10^3 | $8.7656 	imes 10^1$ | 1.8730×10^3 | $3.5100 	imes 10^1$ | $1.7836 	imes 10^3$ | $3.9799 	imes 10^1$ |
| f ₂₈ | 2.3872×10^{3} | 1.3042×10^2 | 1.8734×10^3 | 2.0762×10^{2} | $1.7500 	imes 10^3$ | 1.1019×10^2 | $1.9724 	imes 10^3$ | 1.7008×10^2 | $1.9112 	imes 10^3$ | 2.4722×10^2 |
| =/ | 23/0/5 | | 18/0/10 | | 16/0/12 | | 26/1/1 | | - | |

| £ | | BA | P | so | GI | VO | S | OA | BF | GO |
|------------------------|------------------------|------------------------|------------------------|-------------------------|------------------------|------------------------|------------------------|-------------------------|------------------------|------------------------|
| f_x | Mean | Std | Mean | Std | Mean | Std | Mean | Std | Mean | Std |
| f_1 | -1.383×10^{3} | 1.8103 | 6.6163×10^{2} | 1.9507×10^{3} | 1.7472×10^{3} | $1.8465 	imes 10^3$ | 2.0374×10^4 | 5.9767×10^{3} | -1.096×10^{3} | 3.2342×10^{2} |
| f_2 | $1.0290	imes10^7$ | $2.6234	imes10^6$ | $6.9639	imes10^7$ | $3.3532 	imes 10^7$ | $5.5385 	imes 10^7$ | 1.6617×10^{7} | $1.3813	imes10^8$ | $4.7348	imes10^7$ | $3.9416 	imes 10^7$ | $1.3015 	imes 10^7$ |
| f_3 | $1.4509 	imes 10^9$ | 1.2689×10^{9} | $9.6740 	imes 10^{10}$ | $5.0501 	imes 10^{10}$ | $1.9432	imes10^{10}$ | 5.8728×10^{9} | $4.9613	imes10^{10}$ | 8.9406×10^{9} | $1.9021 	imes 10^{10}$ | $8.0204 	imes 10^9$ |
| f_4 | $1.0877 	imes 10^5$ | $2.8748 	imes 10^4$ | $5.2031 	imes 10^4$ | 1.5635×10^4 | 5.6337×10^4 | 9.5518×10^{3} | $6.7888 	imes 10^4$ | 8.4160×10^{3} | $5.6341 	imes 10^4$ | $7.4857 	imes 10^3$ |
| f_5 | $-9.952	imes10^2$ | $3.9260 	imes 10^{-1}$ | 1.2258×10^3 | $1.9889 	imes 10^3$ | -8.676 | $3.3203 	imes 10^2$ | 2.4606×10^{3} | $1.4971 	imes 10^3$ | $-7.340	imes10^2$ | $5.6941 	imes 10^1$ |
| f_6 | $-8.238	imes10^2$ | $3.8683 	imes 10^1$ | -5.831×10^{2} | 1.3695×10^{2} | -6.070×10^{2} | 7.5124×10^1 | 2.5303×10^{2} | 3.1369×10^{2} | -7.070×10^{2} | $5.2098 	imes 10^1$ |
| f7 | $5.9940 	imes 10^3$ | $1.1640	imes10^4$ | -5.661×10^{2} | $9.8719 	imes 10^1$ | $-7.248	imes10^2$ | $1.4614	imes10^1$ | -6.668×10^{2} | $1.6613 	imes 10^1$ | -6.711×10^{2} | $2.3436	imes10^1$ |
| f_8 | $-6.787	imes10^2$ | $4.6004 	imes 10^{-2}$ | -6.788×10^{2} | 3.2081×10^{-2} | -6.787×10^{2} | $3.9574 	imes 10^{-2}$ | -6.788×10^{2} | 3.9434×10^{-2} | -6.788×10^{2} | $4.6423 	imes 10^{-2}$ |
| f9 | -5.325×10^{2} | 5.0558 | -5.386×10^{2} | 4.5053 | $-5.584	imes10^2$ | 3.5605 | -5.364×10^{2} | 5.2949 | -5.384×10^{2} | 4.7204 |
| <i>f</i> ₁₀ | $-4.931	imes10^2$ | 1.3669 | 5.8897×10^{2} | 5.6839×10^{2} | 2.3073×10^{2} | 2.7002×10^{2} | 1.8301×10^{3} | 5.3997×10^{2} | -3.022×10^{2} | $4.9227	imes10^1$ |
| f_{11} | 7.0216×10^{2} | 1.6976×10^{2} | $2.4000 	imes 10^2$ | 7.7858×10^{1} | -1.741×10^{2} | 3.3734×10^1 | 2.0925×10^{2} | 6.7253×10^{1} | $8.7185 	imes 10^1$ | 1.0942×10^2 |
| f_{12} | $8.5553 	imes 10^2$ | $1.8454 	imes 10^2$ | 3.6411×10^2 | 1.1549×10^2 | -2.296×10^{1} | $7.7324 	imes 10^1$ | $3.1815 	imes 10^2$ | $7.8326 	imes 10^1$ | $2.4418 	imes 10^2$ | 1.4352×10^2 |
| f ₁₃ | $1.1097 	imes 10^3$ | $1.5346 	imes 10^2$ | $5.3618 	imes 10^2$ | 1.0069×10^{2} | 2.1441×10^{2} | $9.1499	imes10^1$ | $4.9592 	imes 10^2$ | $5.3566 	imes 10^1$ | 3.6570×10^2 | $1.3450 	imes 10^2$ |
| f_{14} | $8.6727 	imes 10^3$ | $6.1909 	imes 10^2$ | $8.9854 	imes 10^3$ | 1.0823×10^3 | 6.2244×10^{3} | 1.4656×10^{3} | $1.1741 	imes 10^4$ | $9.0535 	imes 10^2$ | 6.6351×10^3 | $1.1034 	imes 10^3$ |
| f_{15} | 9.3666×10^{3} | $1.0530 	imes 10^3$ | $9.9386 	imes 10^3$ | $1.2284 	imes 10^3$ | $1.0322 	imes 10^4$ | 3.5891×10^3 | $1.3357	imes10^4$ | $9.4375 	imes 10^2$ | $1.0179 	imes 10^4$ | $1.0865 	imes 10^3$ |
| f_{16} | $2.0365 	imes 10^2$ | $2.7577 	imes 10^{-1}$ | 2.0272×10^{2} | $6.7189 	imes 10^{-1}$ | 2.0388×10^{2} | $3.6358 	imes 10^{-1}$ | 2.0401×10^2 | $5.6717 	imes 10^{-1}$ | 2.0262×10^{2} | $5.5048	imes10^{-1}$ |
| f ₁₇ | $2.6983 	imes 10^3$ | $3.5116 	imes 10^2$ | 1.1792×10^3 | 1.6710×10^{2} | 6.7354×10^{2} | $8.5138 	imes 10^1$ | $1.2198 	imes 10^3$ | $8.1104	imes10^1$ | 8.2842×10^2 | $7.8580	imes10^1$ |
| f_{18} | $2.7289 	imes 10^3$ | $3.2267 	imes 10^2$ | 1.2596×10^{3} | $1.7365 	imes 10^2$ | $9.3555 	imes 10^2$ | $5.0618 	imes 10^1$ | $1.3605 	imes 10^3$ | $1.0343 	imes 10^2$ | $9.1815 	imes 10^2$ | $7.0886 	imes 10^1$ |
| f ₁₉ | 5.7588×10^2 | 8.0687 | $2.6246 	imes 10^4$ | $5.5941 	imes 10^4$ | $1.3409 	imes 10^3$ | 1.1356×10^{3} | $3.5449 	imes 10^4$ | $4.9445 	imes 10^4$ | 5.5108×10^2 | $1.8663 	imes 10^1$ |
| f ₂₀ | $6.2489	imes10^2$ | $2.1231 	imes 10^{-1}$ | 6.2361×10^{2} | $8.2959 	imes 10^{-1}$ | $6.2204 	imes 10^2$ | $9.0362 	imes 10^{-1}$ | $6.2342 	imes 10^2$ | $7.3848	imes10^{-1}$ | 6.2380×10^{2} | $6.6203 	imes 10^{-1}$ |
| f ₂₁ | $1.6182 	imes 10^3$ | $2.9018 	imes 10^2$ | $1.8543 	imes 10^3$ | 3.8333×10^{2} | $2.9124 	imes 10^3$ | $6.1084 	imes 10^2$ | $4.4888 	imes 10^3$ | $1.7404 	imes 10^2$ | 1.8462×10^3 | 2.6452×10^{2} |
| f ₂₂ | $1.2914 	imes 10^4$ | $1.1921 	imes 10^3$ | $1.1209 	imes 10^4$ | $1.0923 	imes 10^3$ | $8.0121 	imes 10^3$ | 1.1179×10^{3} | $1.3705 	imes 10^4$ | $1.0819 	imes 10^3$ | 9.1425×10^{3} | 1.4366×10^{3} |
| f ₂₃ | $1.2468	imes10^4$ | $1.0566 	imes 10^3$ | $1.2326 	imes 10^4$ | $1.3194	imes10^3$ | $1.0486	imes10^4$ | 2.9243×10^{3} | $1.4278	imes10^4$ | $9.0028 	imes 10^2$ | $1.2177 	imes 10^4$ | 1.7124×10^3 |
| f_{24} | $1.4731 	imes 10^3$ | $3.9179	imes10^1$ | $1.3883 	imes 10^3$ | $1.4005 	imes 10^1$ | 1.3106×10^{3} | $1.2248 	imes 10^1$ | $1.3817 	imes 10^3$ | $1.3103 	imes 10^1$ | 1.3708×10^{3} | $1.3201 	imes 10^1$ |
| f25 | $1.4754 	imes 10^3$ | 9.6601 | 1.5060×10^3 | $1.5542 	imes 10^1$ | $1.4574 	imes 10^3$ | $1.1841 	imes 10^1$ | $1.5020 	imes 10^3$ | $1.4279	imes10^1$ | 1.4651×10^3 | $1.3123 	imes 10^1$ |
| f_{26} | 1.6909×10^3 | $1.5062 	imes 10^1$ | $1.6429 	imes 10^3$ | $6.4205	imes10^1$ | 1.6000×10^3 | $3.8194 	imes 10^1$ | $1.6669 	imes 10^3$ | $1.1536 	imes 10^1$ | 1.6590×10^3 | $1.1516	imes10^1$ |
| f ₂₇ | 3.8321×10^{3} | 2.0766×10^{2} | 3.2727×10^3 | $1.0808 	imes 10^2$ | $2.6967 	imes 10^3$ | 1.2526×10^{2} | $3.3107 	imes 10^3$ | $1.4174 	imes 10^2$ | 3.3245×10^3 | $1.3656 	imes 10^2$ |
| f ₂₈ | $1.0205 	imes 10^4$ | 1.0577×10^3 | 5.2892×10^{3} | 2.1351×10^{3} | $2.9264 	imes 10^3$ | $1.1033 	imes 10^3$ | $5.9023 	imes 10^3$ | $9.7219 	imes 10^2$ | 4.3700×10^3 | $2.3070 	imes 10^3$ |
| =/ | 20/0/8 | | 21/1/6 | | 13/0/17 | | 25/1/2 | | — | |

Table A2. Comparison of BFGO algorithm and other algorithms on CEC2013 benchmark function under 50D.

| C | | BA | PS | 50 | G | WO | S | DA | BF | GO |
|-----------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|--------------------------|
| f_x | Mean | Std |
| f_1 | $4.2293 	imes 10^5$ | $8.5118 	imes 10^4$ | 1.1585×10^{8} | 3.4578×10^{8} | 1.9774×10^{6} | 7.0625×10^{6} | 3.4701×10^{8} | 2.3867×10^{8} | 2.5290×10^{4} | 1.9533×10^{4} |
| f_2 | 2.0057×10^2 | 2.1880 | $1.5486	imes10^7$ | $8.4716	imes10^7$ | $6.4564	imes10^6$ | $1.4605	imes10^7$ | $4.0476	imes10^7$ | $1.0966 	imes 10^8$ | $4.8763 	imes 10^3$ | 8.9014×10^{3} |
| f ₃ | 3.0101×10^{2} | $3.0580 	imes 10^{-1}$ | 3.0000×10^2 | 5.4499×10^{9} | 1.2754×10^3 | $1.2935 	imes 10^3$ | 1.6546×10^3 | $1.9474 	imes 10^3$ | 3.0115×10^2 | 1.1444 |
| f_4 | $4.0219 	imes 10^2$ | $9.6308	imes10^{-1}$ | $4.1182 	imes 10^2$ | $2.2868 	imes 10^1$ | 4.0839×10^{2} | 4.3872 | $4.4300 	imes 10^2$ | $2.7242 	imes 10^1$ | $4.0915 	imes 10^2$ | 1.5054×10^{1} |
| f5 | $5.6011 	imes 10^2$ | 1.7863×10^{1} | $5.2498 	imes 10^2$ | 1.1711×10^1 | 5.1177×10^2 | 7.4132 | 5.2212×10^2 | 7.0485 | 5.2665×10^{2} | 1.1387×10^{1} |
| f_6 | 6.3898×10^{2} | 8.4694 | 6.0747×10^{2} | 5.0815 | 6.0035×10^{2} | $4.3927 	imes 10^{-1}$ | 6.0995×10^{2} | 4.8354 | 6.0593×10^{2} | 4.7531 |
| f ₇ | $8.3113 	imes 10^2$ | $3.3091 	imes 10^1$ | 7.3166×10^{2} | $1.1849 	imes 10^1$ | 7.2683×10^{2} | $1.0954	imes10^1$ | 7.5549×10^2 | $1.3435 	imes 10^1$ | 7.2469×10^{2} | 6.4241 |
| 8 | 8.4906×10^{2} | $1.4585 	imes 10^1$ | 8.2121×10^{2} | 7.4713 | 8.1326×10^{2} | 4.4746 | 8.2367×10^{2} | 7.2746 | 8.1852×10^{2} | 8.1085 |
| f.9 | 1.5951×10^{3} | 3.9579×10^{2} | 9.2195×10^{2} | $3.4230 	imes 10^1$ | 9.0607×10^{2} | 1.7239×10^{1} | $1.0042 	imes 10^3$ | $6.7349 	imes 10^1$ | 9.0717×10^{2} | 1.1684×10^{-1} |
| f ₁₀ | $2.1547 	imes 10^3$ | 3.9922×10^{2} | 1.7283×10^{3} | 3.4155×10^2 | 1.5018×10^3 | 3.3580×10^{2} | $1.7834 	imes 10^3$ | 2.7596×10^{2} | 1.9155×10^{3} | 3.1717×10^{2} |
| f ₁₁ | 1.2084×10^3 | $7.4265 	imes 10^1$ | $1.1459 	imes 10^3$ | 2.9616×10^{1} | $1.1213 	imes 10^3$ | $1.1663 	imes 10^1$ | 1.2571×10^{3} | $7.2155 	imes 10^1$ | 1.1571×10^{3} | 4.8062×10^{-10} |
| 12 | $8.3494 	imes 10^5$ | 6.5698×10^{5} | 1.0832×10^{6} | $4.5495 	imes 10^6$ | 7.4796×10^{5} | $8.3136 	imes 10^5$ | 3.7491×10^{6} | 4.1740×10^{6} | 1.3268×10^{5} | 5.3167×10 |
| 13 | $1.5430 	imes 10^4$ | 1.2175×10^4 | 4.2316×10^{3} | 7.9373×10^{3} | $1.0964	imes10^4$ | 7.0922×10^{3} | $2.1674	imes10^4$ | $2.1984 	imes 10^4$ | $1.5318 	imes 10^4$ | 1.1571×10 |
| r 14 | 3.7182×10^3 | 3.1837×10^3 | 1.4759×10^{3} | $4.3835 	imes 10^1$ | 2.3807×10^{3} | 1.5995×10^{3} | 1.6212×10^3 | 2.2511×10^{2} | 1.4725×10^{3} | 3.3370×10 |
| f ₁₅ | 1.1571×10^4 | 9.4442×10^3 | 1.6048×10^3 | $8.6485 	imes 10^1$ | 4.1679×10^{3} | 4.0458×10^3 | 2.8544×10^{3} | 1.4257×10^{3} | 1.7594×10^{3} | 3.0605×10 |
| 16 | 2.0112×10^{3} | 1.8594×10^2 | 1.7337×10^3 | 1.2692×10^{2} | $1.6868 	imes 10^3$ | $9.7140 	imes 10^1$ | 1.7691×10^{3} | $1.0430 	imes 10^2$ | $1.7687 	imes 10^3$ | 1.0646×10 |
| f ₁₇ | 1.8141×10^3 | 6.5555×10^{1} | 1.7714×10^{3} | 3.8969×10^{1} | 1.7621×10^{3} | $3.6485 	imes 10^1$ | 1.7770×10^{3} | 3.7898×10^1 | 1.7522×10^{3} | 3.2698×10 |
| f ₁₈ | $1.4378 	imes 10^4$ | $1.1066 	imes 10^4$ | $2.2198 	imes 10^4$ | 1.6268×10^4 | 2.3569×10^{4} | $1.5890 	imes 10^4$ | $4.1488	imes10^4$ | 1.5111×10^4 | 2.0561×10^{4} | 1.4616×10 |
| 19 | 5.2659×10^{3} | 3.0924×10^{3} | 1.9590×10^{3} | $5.8781 	imes 10^1$ | $7.6128 	imes 10^3$ | $5.9054 	imes 10^3$ | $1.0028 	imes 10^4$ | 7.8380×10^{3} | 1.9604×10^3 | 1.1959×10 |
| 20 | 2.1436×10^{3} | 7.3136×10^{1} | 2.0889×10^{3} | $6.1595 	imes 10^1$ | 2.0752×10^{3} | $5.9730 	imes 10^1$ | 2.0969×10^{3} | 5.7679×10^{1} | 2.0824×10^{3} | 5.4668×10 |
| 21 | 2.2988×10^{3} | 7.1270×10^{1} | 2.3170×10^{3} | $4.0461 	imes 10^1$ | 2.3018×10^3 | $3.4492 	imes 10^1$ | $2.2033 	imes 10^3$ | 1.6705 | 2.2551×10^{3} | 6.1269×10 |
| f ₂₂ | 2.3110×10^{3} | $1.4069 	imes 10^1$ | 2.3106×10^{3} | 1.9266×10^{1} | 2.3072×10^{3} | 6.5405 | $2.8108 	imes 10^3$ | 5.8824×10^{2} | 2.3044×10^{3} | 1.1164×10 |
| f ₂₃ | 2.6821×10^{3} | 3.0873×10^{1} | 2.6371×10^{3} | 1.2561×10^{1} | 2.6155×10^{3} | 8.3301 | 2.6296×10^{3} | 1.1686×10^1 | 2.6364×10^{3} | 1.8128×10 |
| 24 | 2.7916×10^{3} | 1.0332×10^{2} | 2.7578×10^{3} | $6.1950 	imes 10^1$ | 2.7430×10^{3} | $1.1482 	imes 10^1$ | 2.7569×10^{3} | $1.0514 	imes 10^1$ | 2.7378×10^{3} | 8.1547×10 |
| 25 | 2.9181×10^{3} | 6.5990×10^{1} | 2.9385×10^{3} | $3.5059 	imes 10^1$ | 2.9338×10^{3} | $1.6730 	imes 10^1$ | 2.9349×10^{3} | 1.4572×10^{1} | 2.9445×10^{3} | 3.3154×10 |
| 26 | 3.4224×10^{3} | 3.9858×10^{2} | $3.0388 	imes 10^3$ | 1.3173×10^2 | $2.9442 	imes 10^3$ | 1.9990×10^{2} | $3.2888 	imes 10^3$ | $4.8915 	imes 10^2$ | 3.0242×10^3 | 2.0469×10 |
| 27 | 3.1578×10^{3} | $3.9004 	imes 10^1$ | 3.1101×10^{3} | 2.2392×10^{1} | 3.0979×10^{3} | 9.0433 | 3.0940×10^3 | 2.5020 | 3.0910×10^3 | 3.3723×10 |
| 28 | 3.2206×10^{3} | $7.9718 	imes 10^1$ | 3.3422×10^3 | 1.3226×10^{2} | $3.3913 	imes 10^3$ | $5.1933	imes10^1$ | 3.2515×10^3 | $1.0093 	imes 10^2$ | 3.2425×10^3 | 5.8119×10^{-10} |
| 29 | 3.3214×10^3 | 1.2027×10^{2} | $3.2174 	imes 10^3$ | $4.7685 	imes 10^1$ | $3.1853 	imes 10^3$ | $4.5780 	imes 10^1$ | $3.1903 	imes 10^3$ | $3.2599 	imes 10^1$ | $3.2467 	imes 10^3$ | 6.0966×10^{-10} |
| 30 | $4.4106 	imes 10^4$ | 5.0891×10^{4} | $1.4518 	imes 10^6$ | 1.7254×10^{6} | 5.1162×10^{5} | 6.5062×10^{5} | 1.2757×10^{5} | 1.7577×10^{5} | 1.8622×10^{4} | 5.3608×10^{-10} |
| =/ | 24/0/6 | | 20/0/10 | | 16/0/14 | | 24/0/3 | | _ | |

Table A3. Comparison of BFGO algorithm and other algorithms on CEC2017 benchmark function under 10D.

| c | | BA | P | 50 | GI | VO | S | DA | BF | GO |
|----------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|--------------------------|
| f_x | Mean | Std |
| f_1 | 2.5996×10^{7} | 2.9346×10^{6} | 5.2824×10^{9} | 5.7281×10^{9} | 5.5419×10^{9} | 2.9747×10^{9} | $3.3474	imes10^{10}$ | 8.0725×10^{9} | 7.4546×10^8 | 6.4551×10^{8} |
| f_2 | $2.0360 	imes 10^{18}$ | $4.4725	imes10^{18}$ | $5.6044 	imes 10^{71}$ | $3.0696 	imes 10^{72}$ | $1.1665 	imes 10^{51}$ | $4.4867 	imes 10^{51}$ | $8.9101 	imes 10^{62}$ | $3.2693 	imes 10^{63}$ | $4.0636 	imes 10^{51}$ | $2.2239 	imes 10^{52}$ |
| f ₃ | $1.4364 	imes 10^5$ | $4.8168	imes10^4$ | $8.6954	imes10^4$ | $3.4116 	imes 10^4$ | 1.1160×10^5 | $1.8904 	imes 10^4$ | $1.2009 	imes 10^5$ | $1.7223 	imes 10^4$ | $9.5425 	imes 10^4$ | 1.7171×10^{4} |
| f_4 | $5.5308 	imes 10^2$ | $5.9032	imes10^1$ | $1.4340 	imes 10^3$ | 8.0630×10^{2} | $9.3657 	imes 10^2$ | 2.2268×10^{2} | $3.4781 	imes 10^3$ | 1.2856×10^{3} | $7.7404 	imes 10^2$ | 1.0894×10^{2} |
| 5 | 1.1093×10^{3} | 1.2335×10^2 | $8.8483 	imes 10^2$ | $4.7980 	imes 10^1$ | 7.1229×10^{2} | $4.9750 	imes 10^1$ | 9.1724×10^{2} | $5.2095 	imes 10^1$ | $8.3948 	imes 10^2$ | 5.5701×10^{-10} |
| f ₆ | 6.9040×10^{2} | 8.7596 | 6.5982×10^{2} | 6.6482 | 6.1370×10^{2} | 3.5955 | 6.6170×10^{2} | 7.0973 | 6.5589×10^{2} | 9.3846 |
| f ₇ | $2.8054 	imes 10^3$ | $2.4397 	imes 10^2$ | $1.5680 	imes 10^3$ | 1.3551×10^2 | 1.0441×10^3 | $7.5877	imes10^1$ | 1.5758×10^3 | $9.8844	imes10^1$ | $1.1553 	imes 10^3$ | 8.6659×10^{-10} |
| r 8 | 1.4327×10^3 | $9.8047	imes10^1$ | $1.1784 	imes 10^3$ | $5.9460 	imes 10^1$ | 1.0033×10^{3} | 3.1279×10^{1} | 1.2231×10^{3} | $3.5670 	imes 10^1$ | 1.1470×10^{3} | 5.7759×10^{-10} |
| f ₉ | $3.6998 	imes 10^4$ | $8.4008 	imes 10^3$ | $1.3080 	imes 10^4$ | 2.7610×10^{3} | 5.4608×10^3 | 2.4573×10^{3} | $2.1399 	imes 10^4$ | $4.4859 	imes 10^3$ | $1.2223 	imes 10^4$ | 3.5224×10^{3} |
| 10 | 9.1541×10^{3} | $9.9009 	imes 10^2$ | $9.6018 	imes 10^3$ | 1.0159×10^3 | 7.9802×10^{3} | 2.5234×10^{3} | 1.2672×10^4 | $1.3785 	imes 10^3$ | $8.9844 	imes 10^3$ | 1.2649×10^{-1} |
| 11 | 1.4787×10^3 | $8.7459	imes10^1$ | $1.8884 	imes 10^3$ | 4.9847×10^2 | 3.7474×10^{3} | 1.5741×10^{3} | 6.5202×10^{3} | 2.1345×10^{3} | 1.8712×10^{3} | 2.3450×10^{-10} |
| 12 | 7.0553×10^{7} | $3.5334 	imes 10^7$ | 2.3810×10^{9} | 2.6378×10^{9} | 7.3794×10^8 | 9.1607×10^{8} | 5.3287×10^{9} | 2.1329×10^{9} | 2.2972×10^{8} | 2.7394×10 |
| 13 | 2.4616×10^{6} | $4.9332 	imes 10^5$ | $1.3173 	imes 10^9$ | 2.3459×10^{9} | 1.2681×10^8 | 1.3877×10^8 | $1.2046 	imes 10^9$ | 1.3709×10^{9} | 5.1631×10^{5} | 3.7632×10 |
| 14 | $1.4477 	imes 10^5$ | $7.5507 	imes 10^4$ | 1.1591×10^{6} | 2.0245×10^{6} | 6.5289×10^{5} | 5.0739×10^{5} | 1.4236×10^{6} | 9.6042×10^{5} | 5.6328×10^{5} | 5.0871×10 |
| 15 | 8.3509×10^{5} | $1.9403 	imes 10^5$ | $7.4841 	imes 10^7$ | 3.2644×10^{8} | 1.2036×10^{7} | 1.4955×10^{7} | 6.3604×10^{7} | 5.1677×10^{7} | 1.0734×10^5 | 1.1059×10 |
| 16 | $4.8433 	imes 10^3$ | $6.2042 	imes 10^2$ | 3.9830×10^{3} | $4.8984 	imes 10^2$ | 3.1431×10^{3} | 3.9416×10^{2} | $4.2678 	imes 10^3$ | 4.8146×10^2 | 4.0669×10^{3} | 5.1607×10 |
| 17 | $4.0175 	imes 10^3$ | $4.6202 	imes 10^2$ | $3.8427 	imes 10^3$ | $3.7463 	imes 10^2$ | 2.9043×10^3 | $4.2594	imes10^2$ | 3.6441×10^3 | 3.6229×10^2 | 3.5526×10^3 | 3.8962×10 |
| 18 | 1.7657×10^{6} | $1.0588 	imes 10^6$ | 7.3006×10^{6} | $1.3247 	imes 10^7$ | 5.2799×10^{6} | $4.1014	imes10^6$ | $7.5737 	imes 10^6$ | $6.1990 	imes 10^6$ | 3.0399×10^{6} | 2.2408×10 |
| 19 | 5.0522×10^{6} | $2.5594 	imes 10^6$ | $3.4389 	imes 10^7$ | 1.5369×10^{8} | 3.1693×10^{6} | $6.9178 	imes 10^6$ | $6.1074 	imes 10^7$ | 1.8552×10^{8} | 6.3622×10^{5} | 9.1839×10 |
| 20 | 3.8384×10^3 | 4.0362×10^2 | 3.3310×10^{3} | 3.3364×10^{2} | 2.9993×10^{3} | 3.2801×10^{2} | 3.5795×10^{3} | 3.7528×10^{2} | 3.5222×10^{3} | 3.0390×10 |
| 21 | $3.0118 	imes 10^3$ | 1.0722×10^2 | 2.7035×10^{3} | $6.8215 	imes 10^1$ | $2.5003 	imes 10^3$ | $2.2103 	imes 10^1$ | 2.7309×10^{3} | $4.5679	imes10^1$ | 2.7083×10^{3} | 7.6226×10 |
| 22 | $1.1129 	imes 10^4$ | $9.9276 	imes 10^2$ | $1.1279 	imes 10^4$ | 1.1399×10^{3} | 9.9172×10^{3} | 2.0693×10^{3} | $1.4385 	imes 10^4$ | $1.0818 	imes 10^3$ | 1.0522×10^4 | 7.8315×10 |
| 23 | 4.0802×10^3 | $1.7815 	imes 10^2$ | 3.4727×10^3 | 1.1112×10^2 | $2.9834 	imes 10^3$ | $9.8832 	imes 10^1$ | 3.2475×10^{3} | $6.7624	imes10^1$ | 3.4397×10^{3} | 2.5472×10 |
| 24 | 4.2572×10^{3} | 1.2529×10^{2} | 3.7247×10^{3} | 1.6005×10^{2} | $3.1374 	imes 10^3$ | $9.8964 	imes 10^1$ | 3.3203×10^{3} | $4.5576 	imes 10^1$ | 3.7152×10^{3} | 1.6440×10 |
| 25 | 2.9957×10^{3} | $4.4782 	imes 10^1$ | 3.5564×10^3 | 6.4807×10^{2} | 3.4140×10^{3} | $1.5436 	imes 10^2$ | 5.2906×10^{3} | 8.1324×10^2 | 3.2062×10^{3} | 8.0942×10 |
| 26 | 1.5330×10^4 | 2.7522×10^3 | $1.0359 	imes 10^4$ | $1.5911 	imes 10^3$ | $6.3828 	imes 10^3$ | $1.0589 	imes 10^3$ | $8.5240 	imes 10^3$ | $5.4931 	imes 10^2$ | 8.7267×10^{3} | 3.0592×10 |
| .~ 27 | $4.0892 	imes 10^3$ | $1.0635 	imes 10^3$ | 4.0757×10^3 | 2.7373×10^{2} | 3.5532×10^3 | $8.1559 	imes 10^1$ | 3.8512×10^3 | 1.8150×10^2 | 3.2213×10^3 | 5.3457×10 |
| 28 | 3.3000×10^3 | $5.7224	imes10^{-5}$ | $5.1795 	imes 10^3$ | 2.2277×10^{3} | 3.9701×10^{3} | 3.4515×10^2 | 9.0966×10^{3} | 1.3974×10^3 | 3.5687×10^3 | 1.8689×10 |
| 29 | 6.2416×10^3 | $7.1671 	imes 10^2$ | $6.2585 	imes 10^3$ | 7.2500×10^2 | 4.4561×10^3 | 3.2569×10^2 | $6.5129 	imes 10^3$ | $7.1693 	imes 10^2$ | $6.0243 	imes 10^3$ | 7.8572×10^{-10} |
| 30 | 5.4560×10^{7} | $7.6516 	imes 10^6$ | $5.6907 	imes 10^7$ | $1.1099 	imes 10^8$ | $1.2225 	imes 10^8$ | 4.0766×10^{7} | 2.2969×10^{8} | $1.1178 	imes 10^8$ | $1.1685 	imes 10^7$ | 1.3323×10^{-1} |
| =/ | 21/0/9 | | 26/0/2 | | 14/0/16 | | 27/0/3 | | — | |

Table A4. Comparison of BFGO algorithm and other algorithms on CEC2017 benchmark function under 50D.

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