

Energy-Efficient Design for Downlink Cloud Radio Access Networks

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Abstract—This work aims to maximize the energy efficiency of a downlink cloud radio access network (C-RAN), where data is transferred from a baseband unit in the core network to several remote radio heads via a set of edge routers over capacity-limited fronthaul links. The remote radio heads then send the received signals to their users via radio access links. We formulate a new mixed-integer nonlinear problem in which the ratio of network throughput and total power consumption is maximized. This challenging problem formulation includes practical constraints on routing, predefined minimum data rates, fronthaul capacity and maximum RRH transmit power. By employing the successive convex quadratic programming framework, an iterative algorithm is proposed with guaranteed convergence to a Fritz John solution of the formulated problem. Significantly, each iteration of the proposed algorithm solves only one simple convex program. Numerical examples with practical parameters confirm that the proposed joint optimization design markedly improves the C-RAN's energy efficiency compared to benchmark schemes.

Index Terms—C-RAN, energy efficiency, limited-capacity fronthaul, precoding design, user association

I. INTRODUCTION

Cloud radio access networks (C-RANs) are considered as a promising solution for the fifth generation of mobile communication systems [1]. In a C-RAN, low-cost low-power remote radio heads (RRHs) replace traditional high-cost high-power base stations, resulting in lower energy consumption for a dense network implementation [2]. A central base band unit (BBU) in the core network is connected to the RRHs via wireline fronthaul links, whereas the RRHs are connected to users via radio access links. The most important advantage of C-RANs is that large-scale allocation of radio and computing resources across all the RRHs can be centrally processed at the same BBU pools, which enable significant spectral and energy efficiency gains over the single-cell processing [3]. However, C-RANs require a tremendous amount of data sharing on the fronthaul links due to fully joint processing, making the finite-capacity fronthaul link a main bottleneck of practical C-RANs [4].

To improve energy efficiency of the downlink C-RANs, one may opt to (i) increase data rate, (ii) decrease RRH transmit power, (iii) turn off RRHs, (iv) reduce the fronthaul rate for power saving, and (v) implement any combination thereof. The authors in [5] fix user rates and transforms the energy efficiency maximization problem into a power minimization problem. However, fronthaul capacity constraint is not taken into account. The work of [6] addresses a different power minimization problem for downlink C-RANs, where users are put into several multicast groups. Applying random matrix theory, [7] proposes heuristic user association (UA) schemes that maximize the equivalent energy efficiency. These works show that the UA and RRH activation play important roles in energy efficiency enhancement.

It is worth noting that the aforementioned solutions focus on single-hop fronthaul networks only, while a practical BBU is

typically connected to RRHs via a number of edge routers over a multi-hop fronthaul network [8]. Very recently, [8] attempts to maximize the network throughput of a downlink multi-hop C-RAN, where network coding is shown to be necessary for a better utilization of the finite-capacity fronthaul links in the multi-hop fronthaul case. However, to the best of our knowledge, energy efficiency maximization has not yet been addressed for this practical network scenario.

Paper Contribution: This paper considers the general downlink of a C-RAN with multi-hop and capacity-limited fronthaul. The aim is to maximize the network energy efficiency by jointly optimizing user association (UA), RRH activation, data rate allocation and signal precoding. Since these optimization variables are strongly interrelated, it is not straightforward to even formulate this problem in a suitable form, let alone solving them effectively and optimally. We formulate a new problem of energy efficiency maximization, subject to routing constraints, limited fronthaul capacities, predefined minimum rates and maximum transmit power at each RRH. Practically, the total power consumption resulting from data transmission, RRH and fronthaul operations is included in the formulations.

We propose a new iterative algorithm to solve the challenging mixed-integer nonlinear problem formulation, where each iteration involves solving only one simple convex program. Specifically, the original problem is first transformed into an epigraph form. To deal with the binary nature of UA and RRH activation decisions, it is further recast to an equivalent problem that includes continuous variables only. This problem is finally solved by the successive convex quadratic programming. We prove theoretically and verify by numerical examples that the proposed algorithm converges to a solution that satisfies the Fritz John conditions¹ of the formulated problem once initialized from a feasible point. Simulation results with practical parameter settings show that our joint optimization approach significantly improves the energy efficiency over existing methods in both single-hop and multi-hop cases.

II. SYSTEM MODEL AND PROBLEM FORMULATION

Fig. 1 illustrates the downlink of a general C-RAN model, where the baseband unit (BBU) in the core network connects to a set of RRHs $\mathcal{K}_R \triangleq \{1, \dots, K_R\}$ via a fronthaul network of M routers and N noiseless fronthaul links [5], [8], where $M > 0$, $N > K_R$ for a multi-hop C-RAN and $M = 0$, $N = K_R$ for a single-hop C-RAN. Denote by $\mathcal{M} \triangleq \{1, \dots, M\}$ and $\mathcal{N} \triangleq \{1, \dots, N\}$ the sets of routers and fronthaul links, respectively. Assume that a fronthaul link $n \in \mathcal{N}$ has a limited capacity of $C_n > 0$ (in bits per second). The RRHs then serve a set of users $\mathcal{K}_U \triangleq \{1, \dots, K_U\}$ via radio access links, where a user

¹The Fritz John conditions are necessary conditions for a solution in nonlinear programming to be optimal [9].

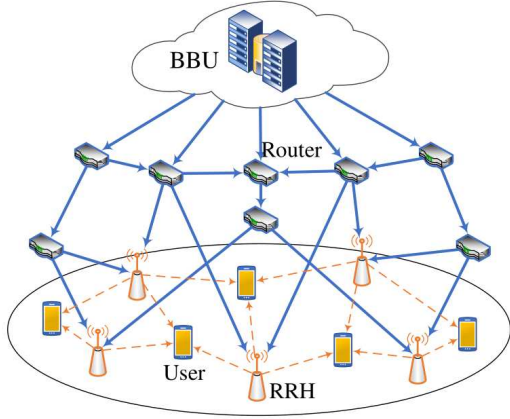


Fig. 1. A general C-RAN with multi-hop fronthaul links.

is allowed to connect to multiple RRHs. Each user $k \in \mathcal{K}_U$ is equipped with N_u antennas while each eRRH $i \in \mathcal{K}_R$ is equipped with N_r antennas.

At the BBU, a message M_k intended for the user k is uniformly distributed in the set $\{1, \dots, 2^{uR_k}\}$, where u is the block length and R_k (in bits per second) is the data rate of message M_k [10]. The message M_k is then encoded into symbol $\mathbf{s}_k \in \mathbb{C}^{d \times 1}$, where \mathbf{s}_k is taken from a Gaussian channel codebook $\mathcal{C}_k^{\text{CH}}$ so that $\mathbf{s}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$ and $d \triangleq \min(N_u, N_r)$ is the number of data streams. The network throughput is defined as the following sum rate:

$$R_{\text{sum}} \triangleq \sum_{k \in \mathcal{K}_U} R_k. \quad (1)$$

Each user's intended message symbol is routed through the fronthaul and delivered to a set of RRHs. At the radio access links, the RRH-user associations and RRH activation respectively are expressed by the following binary variables

$$a_{k,i} \triangleq \begin{cases} 1, & \text{if eRRH } i \text{ serves user } k, \\ 0, & \text{otherwise,} \end{cases} \quad (2)$$

$$b_i \triangleq \begin{cases} 0, & \text{if eRRH } i \text{ serves no user,} \\ 1, & \text{otherwise.} \end{cases} \quad (3)$$

Denote the set of RRHs that serves user k as $\mathcal{D}_k \triangleq \{i | a_{k,i} = 1, i \in \mathcal{K}_R\}$. Then the BBU sends \mathbf{s}_k to \mathcal{D}_k over the multi-hop fronthaul network at the rate R_k .

Following [8], this paper employs a network coding scheme that consists of a flow routing scheme and a code assignment to determine the rate and content of each data flow being delivered across the fronthaul network. The study of [11] shows that intersession coding only provides marginal throughput gain over independent coding while making the routing problem an NP-hard one. Therefore, this paper assumes that each multicast session is routed and coded independently but not jointly.

In our model, there are K_U multicast sessions for K_U users' messages. Denote by $r_{k,n}$ the routing variable that determines the flow rate on a link n for a multicast session k . If $r_{k,n} = 0$, the multicast session k is not routed on the link n . The network coding theorem in [12] shows that if the rate R_k is achievable at each destination in \mathcal{D}_k independently, it is also achievable for the entire multicast session k . Hence, the multicast flow on the link n to an RRH $i \in \mathcal{D}_k$ can be viewed as including the independent conceptual flows $f_{k,i,n} \leq r_{k,n}, \forall k \in \mathcal{K}_U, i \in \mathcal{K}_R, m \in \mathcal{M}$ [8]. The routing constraints for the multi-hop

fronthaul network can be formulated as follows [8]

$$f_{k,i,n} \leq r_{k,n}, \forall k \in \mathcal{K}_U, i \in \mathcal{K}_R, n \in \mathcal{N} \quad (4)$$

$$r_{k,n} \leq a_{k,i} C_n, \forall k \in \mathcal{K}_U, n \in \mathcal{I}_i^{\mathcal{K}_R} \quad (5)$$

$$a_{k,i} R_k \leq \sum_{n \in \mathcal{I}_i^{\mathcal{K}_R}} f_{k,i,n}, \forall k \in \mathcal{K}_U, i \in \mathcal{K}_R \quad (6)$$

$$\sum_{n \in \mathcal{O}_m^{\mathcal{M}}} f_{k,i,n} = \sum_{n \in \mathcal{I}_m^{\mathcal{M}}} f_{k,i,n}, \forall k \in \mathcal{K}_U, i \in \mathcal{K}_R, m \in \mathcal{M} \quad (7)$$

$$\sum_{k \in \mathcal{K}_U} r_{k,n} \leq C_n, \forall n \in \mathcal{N} \quad (8)$$

$$R_k \geq R_{\text{QoS}}, r_{k,n} \geq 0, f_{k,i,n} \geq 0, \forall k \in \mathcal{K}_U, i \in \mathcal{K}_R, n \in \mathcal{M}, \quad (9)$$

where $\mathcal{I}_i^{\mathcal{K}_R}$ denotes the set of incoming links at an RRH i while $\mathcal{I}_m^{\mathcal{M}}$ and $\mathcal{O}_m^{\mathcal{M}}$ are the sets of incoming and outgoing links at a router m , respectively. Constraint (4) shows that the actual flow rate on link n for multicast session k is a MAX operation, i.e., $r_{k,n} = \max_{i \in \mathcal{D}_k} f_{k,i,n}$, instead of a SUM operation, i.e., $r_{k,n} = \sum_{i \in \mathcal{D}_k} f_{k,i,n}$, of the conceptual flows. This is the benefit of network coding in which the amount of information conveyed on a fixed-capacity link is increased by splitting each multicast session into subsessions and sending only an XOR version of the subsessions on this link. Constraint (5) makes sure there is no data transmission to the unassigned or inactive RRHs. Constraint (6) guarantees that each RRH $i \in \mathcal{D}_k$ can receive the information flow at rate R_k when $a_{k,i} = 1$. Constraint (7) follows the law of flow conservation for conceptual flows at a router m . Constraint (8) ensures that the information flow for all K multicast sessions does not exceed each link capacity. Constraint (9) guarantees a quality-of-service (QoS) rate $R_{\text{QoS}} \geq 0$ for each user as well as nonnegative flow rates on all links for all the multicast sessions. For any given flow rates that satisfy the routing constraints (4)-(9), a code assignment scheme can be found to design the content of each flow [8], [13].

Signal Model: After receiving the message symbols from all K multicast sessions via the multi-hop fronthaul network, RRH i generates the transmitted baseband signal $\mathbf{x}_i \in \mathbb{C}^{N_r \times 1}$ as

$$\mathbf{x}_i = \sum_{k \in \mathcal{K}_U} \mathbf{F}_{k,i} \mathbf{s}_k, \quad (10)$$

where $\mathbf{F}_{k,i} \in \mathbb{C}^{N_r \times d}$ is the precoding matrix for \mathbf{s}_k at RRH i . Each RRH i is assumed to be subjected to the average transmit power constraint expressed as

$$\mathbb{E} \{ \|\mathbf{x}_i\|^2 \} \leq P_i. \quad (11)$$

Denote by $\mathbf{H}_{k,i} \in \mathbb{C}^{N_u \times N_r}$ the flat-fading channel matrix from RRH i to user k and by $\mathbf{H}_k \triangleq [\mathbf{H}_{k,1}, \dots, \mathbf{H}_{k,K_R}] \in \mathbb{C}^{N_u \times N_R}$ the channel matrix from all RRHs to user k , where $N_R \triangleq K_R N_r$. Assume that channel states $\mathbf{H}_{k,i}, k \in \mathcal{K}_U, i \in \mathcal{K}_R$ remain unchanged during the transmission interval and are available at the BBU and RRHs [8]. Upon defining $\bar{\mathbf{F}}_k \triangleq [(\mathbf{F}_{k,1})^H, (\mathbf{F}_{k,2})^H, \dots, (\mathbf{F}_{k,K_R})^H]^H \in \mathbb{C}^{N_R \times d}$, the received signal $\mathbf{y}_k \in \mathbb{C}^{N_u \times 1}$ at user k can thus be written as

$$\mathbf{y}_k = \mathbf{H}_k \bar{\mathbf{F}}_k \mathbf{s}_k + \underbrace{\sum_{\ell \in \mathcal{K}_U \setminus \{k\}} \mathbf{H}_\ell \bar{\mathbf{F}}_\ell \mathbf{s}_\ell}_{\text{interference}} + \mathbf{n}_k, \quad (12)$$

where $\mathbf{n}_k \in \mathbb{C}^{N_u \times 1}$ is the additive noise term with $\mathbf{n}_k \sim \mathcal{CN}(\mathbf{0}, \Sigma_k)$. By treating the interference in (12) as additive Gaussian noise, the rate R_k of message symbol \mathbf{s}_k is always

achievable in the Shannon sense as follows

$$R_k \leq g_k(\bar{\mathbf{F}}) \triangleq W \log_2 \left| \mathbf{I}_{N_u} + \mathbf{\Pi}_k \mathbf{\Pi}_k^H \bar{\mathbf{\Xi}}_k^{-1} \right|, \quad (13)$$

where W is the total available bandwidth, $\bar{\mathbf{F}} \triangleq \{\bar{\mathbf{F}}_k\}_{k \in \mathcal{K}_U}$, $\mathbf{\Pi}_k \triangleq \mathbf{H}_k \bar{\mathbf{F}}_k$, and

$$\bar{\mathbf{\Xi}}_k \triangleq \sum_{\ell \in \mathcal{K}_U \setminus \{k\}} \mathbf{H}_k \bar{\mathbf{F}}_\ell \bar{\mathbf{F}}_\ell^H \mathbf{H}_k^H + \mathbf{\Sigma}_k. \quad (14)$$

Define $\mathbf{a} \triangleq \{a_{k,i}\}_{k \in \mathcal{K}_U, i \in \mathcal{K}_R}$ and $\mathbf{b} \triangleq \{b_i\}_{i \in \mathcal{K}_R}$. The interdependence among \mathbf{a} , \mathbf{b} and $\bar{\mathbf{F}}$ is modeled as

$$a_{k,i} = \begin{cases} 0, & \text{if } \langle \bar{\mathbf{E}}_i^H \bar{\mathbf{F}}_k \bar{\mathbf{F}}_k^H \bar{\mathbf{E}}_i \rangle = 0, \forall k \in \mathcal{K}_U, i \in \mathcal{K}_R \\ 1, & \text{otherwise,} \end{cases} \quad (15)$$

$$b_i = \begin{cases} 0, & \text{if } a_{k,i} = 0, \forall k \in \mathcal{K}_U, \forall i \in \mathcal{K}_R, \\ 1, & \text{otherwise,} \end{cases} \quad (16)$$

where (15) and (16) are indeed (2) and (3) in the sense that RRH i is assigned to serve user k if and only if the corresponding precoder of message symbol \mathbf{s}_k is not a zero matrix, i.e., $\bar{\mathbf{F}}_{k,i} = \bar{\mathbf{E}}_i^H \bar{\mathbf{F}}_k \neq \mathbf{0}$. Following (15) and (16), the relationship between \mathbf{a} and \mathbf{b} implies that

$$a_{k,i} \leq b_i \leq \sum_{k \in \mathcal{K}_U} a_{k,i}, \forall k \in \mathcal{K}_U, i \in \mathcal{K}_R, \quad (17)$$

which guarantees no user being assigned to an inactive RRH.

Power Model: This paper adopts a practical power consumption model that is applicable to different types of BSs [5]. The power consumed by RRH $i \in \mathcal{K}_R$ in the given transmission interval is expressed as

$$P_i^{\text{RRH}} \triangleq \begin{cases} \beta_i P_i^{\text{Tx}} + P_{i,a}, & \text{if } 0 < P_i^{\text{Tx}} \leq P_i, \\ P_{i,s}, & \text{if } P_i^{\text{Tx}} = 0, \end{cases} \quad (18)$$

where constant $\beta_i > 0$, $i \in \mathcal{K}_R$ reflects the power amplifier efficiency, feeder loss and other loss factors due to power supply and cooling for RRH i [5]; P_i^{Tx} is the transmit power required to deliver all requested files from RRH i ; $P_{i,a}$ is the power required to support RRH i in the active mode; $P_{i,s} < P_{i,a}$ is the power consumption in the sleep mode; and P_i is the maximum transmit power at RRH i . The transmit power at RRH i is computed as

$$\begin{aligned} P_i^{\text{Tx}} &= b_i \sum_{k \in \mathcal{K}_U} a_{k,i} \langle \bar{\mathbf{E}}_i^H \bar{\mathbf{F}}_k \bar{\mathbf{F}}_k^H \bar{\mathbf{E}}_i \rangle \\ &= b_i \sum_{k \in \mathcal{K}_U} \langle \bar{\mathbf{E}}_i^H \bar{\mathbf{F}}_k \bar{\mathbf{F}}_k^H \bar{\mathbf{E}}_i \rangle = \sum_{k \in \mathcal{K}_U} \langle \bar{\mathbf{E}}_i^H \bar{\mathbf{F}}_k \bar{\mathbf{F}}_k^H \bar{\mathbf{E}}_i \rangle. \end{aligned} \quad (19)$$

On the other hand, fronthaul link $n \in \mathcal{N}$ is modeled as a set of communication channels with a total capacity C_n and total power dissipation $P_{n,\text{max}}^{\text{FH}}$. Its power consumption is given by [5]

$$P_n^{\text{FH}} \triangleq \frac{\sum_{k \in \mathcal{K}_U} r_{k,n}}{C_n} P_{n,\text{max}}^{\text{FH}} = \alpha_n \sum_{k \in \mathcal{K}_U} r_{k,n}, \quad (20)$$

where $\alpha_n \triangleq P_{n,\text{max}}^{\text{FH}}/C_n$, $r_{k,n}$ is defined above as the actual flow rate on link n for multicast session k . From (18)-(20), the total network power consumption is computed as follows

$$\begin{aligned} P_{\text{total}}(\bar{\mathbf{F}}, \mathbf{r}, \mathbf{b}) &\triangleq \sum_{i \in \mathcal{K}_R} P_i^{\text{RRH}} + \sum_{n \in \mathcal{N}} P_n^{\text{FH}} \\ &= \sum_{i \in \mathcal{K}_R} (\beta_i P_i^{\text{Tx}} + b_i P_{i,\Delta}) + \sum_{n \in \mathcal{N}} \alpha_n \sum_{k \in \mathcal{K}_U} r_{k,n} + P_s, \end{aligned} \quad (21)$$

where $\bar{\mathbf{F}} \triangleq \{\bar{\mathbf{F}}_{k,i}\}_{k \in \mathcal{K}_U, i \in \mathcal{K}_R}$; $P_i^{\text{Tx,DS}}$ is defined in (19); $P_{i,\Delta} \triangleq P_{i,a} - P_{i,s}$; $P_s \triangleq \sum_{i \in \mathcal{K}_R} P_{i,s}$; and $\mathbf{r} \triangleq \{r_{k,n}\}_{k \in \mathcal{K}_U, n \in \mathcal{N}}$.

Problem Formulation: The energy efficiency in wireless

network can be measured by the area power consumption metric (watts/unit area) or the economical energy efficiency metric (effective bits/Joule) [14]. Here, we aim to maximize a more widely adopted metric, i.e., the network energy efficiency defined as the ratio of the achievable sum rate and the total power consumption (bits/Joule) [5]. The optimization problem for the multi-hop C-RAN is formulated as follows

$$\max_{\mathbf{a}, \mathbf{b}, \mathbf{R}, \bar{\mathbf{F}}, \mathbf{f}, \mathbf{r}} \mathcal{P}_1 \triangleq \frac{R_{\text{sum}}}{P_{\text{total}}} \quad (22a)$$

$$\text{s.t.} \quad (4) - (9), (13), (15), (16) \quad (22b)$$

$$\sum_{k \in \mathcal{K}_U} \langle \bar{\mathbf{E}}_i^H \bar{\mathbf{F}}_k \bar{\mathbf{F}}_k^H \bar{\mathbf{E}}_i \rangle \leq P_i, \forall i \in \mathcal{K}_R \quad (22c)$$

$$\sum_{i \in \mathcal{K}_R} a_{k,i} \geq 1, \forall k \in \mathcal{K}_U \quad (22d)$$

where $\mathbf{R} \triangleq \{R_k\}_{k \in \mathcal{K}_U}$; $\mathbf{f} \triangleq \{f_{k,i,n}\}_{k \in \mathcal{K}_U, i \in \mathcal{K}_R, n \in \mathcal{N}}$; R_{sum} and $P_{\text{total}}^{\text{DS}}$ are defined in (1) and (21), respectively; constraint (22c) is the per-RRH power constraint (11) via (10) and (19); constraint (22d) guarantees that each user is served by at least one active RRH.

In the case of single-hop C-RANs which does not require the network coding, the formulation in (22) does not cover this case. Instead, (22) has to be modified by replacing the constraints (4)-(9) with the following constraints:

$$0 \leq r_{k,i} \leq a_{k,i} C_i, \forall k \in \mathcal{K}_U, i \in \mathcal{K}_R \quad (23)$$

$$a_{k,i} R_k \leq r_{k,i}, \forall k \in \mathcal{K}_U, i \in \mathcal{K}_R \quad (24)$$

$$\sum_{k \in \mathcal{K}_U} r_{k,i} \leq C_i, \forall i \in \mathcal{K}_R \quad (25)$$

$$R_k \geq R_{\text{QoS}}, r_{k,i} \geq 0, \forall k \in \mathcal{K}_U, i \in \mathcal{K}_R. \quad (26)$$

It is noteworthy that the mathematical structure of constraints (23)-(26) are similar to that of (4)-(9). Therefore, the algorithm devised in the next section for (22) can be straightforwardly adapted to solve this corresponding single-hop problem too.

III. PROPOSED ALGORITHM

First, we rewrite problem (22) in an epigraph form [15] as

$$\max_{\mathbf{t}, \mathbf{p}, \mathbf{f}, \mathbf{r}, \mathbf{a}} t \quad (27a)$$

$$\text{s.t.} \quad (4) - (9), (13), (17), (22d) \quad (27b)$$

$$tz \leq \sum_{k \in \mathcal{K}_U} R_k \quad (27c)$$

$$\begin{aligned} z &\geq \sum_{i \in \mathcal{K}_R} (\beta_i \sum_{k \in \mathcal{K}_U} \langle \bar{\mathbf{E}}_i^H \bar{\mathbf{F}}_k \bar{\mathbf{F}}_k^H \bar{\mathbf{E}}_i \rangle + b_i P_{i,\Delta}) \\ &\quad + \sum_{n \in \mathcal{N}} \alpha_n \sum_{k \in \mathcal{K}_U} r_{k,n} + P_s \end{aligned} \quad (27d)$$

$$\langle \bar{\mathbf{E}}_i^H \bar{\mathbf{F}}_k \bar{\mathbf{F}}_k^H \bar{\mathbf{E}}_i \rangle \leq u_{k,i}, \forall k \in \mathcal{K}_U, i \in \mathcal{K}_R \quad (27e)$$

$$u_{k,i} \leq a_{k,i} P_i, \forall k \in \mathcal{K}_U, i \in \mathcal{K}_R \quad (27f)$$

$$a_{k,i} \in \{0, 1\}, b_i \in \{0, 1\}, \forall k \in \mathcal{K}_U, i \in \mathcal{K}_R \quad (27g)$$

$$\sum_{k \in \mathcal{K}_U} u_{k,i} \leq P_i, \forall i \in \mathcal{K}_R \quad (27h)$$

where $\mathbf{p} \triangleq (\mathbf{R}, \bar{\mathbf{F}}, z, \mathbf{u})$; $\mathbf{u} \triangleq \{u_{k,i}\}_{k \in \mathcal{K}_U, i \in \mathcal{K}_R}$; (17) and (27e)-(27g) follow from (15) and (16), which guarantees no power is consumed if $a_{k,i} = 0$; (27h) is indeed (22c) via (27e). Still, problem (27) is challenging due to the nonconvex constraints (6), (13), (27c), (27d) and (27g).

To deal with the binary nature of constraint (27g), we note that $\forall a_{k,i}, b_i \in \{0, 1\}$ then $\sum_{i \in \mathcal{K}_R} \sum_{k \in \mathcal{K}_U} (a_{k,i} - a_{k,i}^2) +$

$\sum_{i \in \mathcal{K}_R} (b_i - b_i^2) = 0$. In contrast, for all $a_{k,i}, b_i \in [0, 1]$, $\sum_{i \in \mathcal{K}_R} \sum_{k \in \mathcal{K}_U} (a_{k,i} - a_{k,i}^2) + \sum_{i \in \mathcal{K}_R} (b_i - b_i^2) \geq 0$. Therefore, (27g) can be rewritten as

$$\sum_{i \in \mathcal{K}_R} \sum_{k \in \mathcal{K}_U} (a_{k,i} - a_{k,i}^2) + \sum_{i \in \mathcal{K}_R} (b_i - b_i^2) \leq 0 \quad (28)$$

$$0 \leq a_{k,i} \leq 1, 0 \leq b_i \leq 1, \forall k \in \mathcal{K}_U, i \in \mathcal{K}_R. \quad (29)$$

With (28) and (29), (27d) also becomes a convex constraint. Problem (27) is now transformed to the following problem with continuous variables $a_{k,i}, b_i \in [0, 1], \forall k \in \mathcal{K}_U, i \in \mathcal{K}_R$

$$\min_{(t, \mathbf{p}, \mathbf{f}, \mathbf{r}, \mathbf{a}, \mathbf{b}) \in \mathcal{H}} -t \quad (30)$$

where $\mathcal{H} \triangleq \{(t, \mathbf{p}, \mathbf{f}, \mathbf{r}, \mathbf{a}, \mathbf{b}) | (4) - (9), (13), (17), (22d), (27c) - (27f), (27h), (28), (29)\}$.

Without including the nonconvex constraint (28), let $\widehat{\mathcal{H}} \triangleq \{(t, \mathbf{p}, \mathbf{f}, \mathbf{r}, \mathbf{a}, \mathbf{b}) | (4) - (9), (13), (17), (22d), (27c) - (27f), (27h), (29)\}$ be the compact, feasible set of problem (30). The Lagrangian of (30) is given as

$$\mathcal{L}(t, \mathbf{a}, \mathbf{b}, \lambda) \triangleq -t + \lambda \left(\sum_{i \in \mathcal{K}_R} \sum_{k \in \mathcal{K}_U} (a_{k,i} - a_{k,i}^2) + \sum_{i \in \mathcal{K}_R} (b_i - b_i^2) \right), \quad (31)$$

where $\lambda \geq 0$ is the Lagrangian multiplier to handle the nonconvex constraint (28). Problem (30) can then be expressed as $\min_{(t, \mathbf{p}, \mathbf{f}, \mathbf{r}, \mathbf{a}, \mathbf{b}) \in \widehat{\mathcal{H}}} \max_{\lambda \geq 0} \mathcal{L}(t, \mathbf{a}, \mathbf{b}, \lambda)$ and its dual problem as $\max_{\lambda \geq 0} \min_{(t, \mathbf{p}, \mathbf{f}, \mathbf{r}, \mathbf{a}, \mathbf{b}) \in \widehat{\mathcal{H}}} \mathcal{L}(t, \mathbf{a}, \mathbf{b}, \lambda)$. The property of (30) is stated in the following result.

Proposition 1. Strong Lagrangian duality holds for problem (30), i.e.,

$$\begin{aligned} & \min_{(t, \mathbf{p}, \mathbf{f}, \mathbf{r}, \mathbf{a}, \mathbf{b}) \in \widehat{\mathcal{H}}} \max_{\lambda \geq 0} \mathcal{L}(t, \mathbf{a}, \mathbf{b}, \lambda) \\ & = \max_{\lambda \geq 0} \min_{(t, \mathbf{p}, \mathbf{f}, \mathbf{r}, \mathbf{a}, \mathbf{b}) \in \widehat{\mathcal{H}}} \mathcal{L}(t, \mathbf{a}, \mathbf{b}, \lambda). \end{aligned} \quad (32)$$

Problem (30) is thus equivalent to the following problem

$$\begin{aligned} & \min_{(t, \mathbf{p}, \mathbf{f}, \mathbf{r}, \mathbf{a}, \mathbf{b}) \in \widehat{\mathcal{H}}} \mathcal{L}(t, \mathbf{a}, \mathbf{b}, \lambda) \\ & = -t + \lambda \left(\sum_{i \in \mathcal{K}_R} \sum_{k \in \mathcal{K}_U} (a_{k,i} - a_{k,i}^2) + \sum_{i \in \mathcal{K}_R} (b_i - b_i^2) \right) \end{aligned} \quad (33)$$

at the optimal $\lambda^* \in [0, +\infty)$ of the max-min problem in (32).

The proof of Proposition 1 is similar to [16] and, hence, omitted. Proposition 1 implies that the optimal solution of problem (30) can be found by solving problem (33) for an appropriately chosen value of λ .

To deal with constraints (6) and (27c), we rewrite them respectively as

$$\begin{aligned} & (R_k + a_{k,i})^2 - (R_k - a_{k,i})^2 - 4 \sum_{n \in \mathcal{I}_i^{\mathcal{K}_R}} f_{k,i,n} \leq 0, \\ & \forall k \in \mathcal{K}_U, i \in \mathcal{K}_R \end{aligned} \quad (34)$$

$$(t+z)^2 - (t-z)^2 - 4 \sum_{k \in \mathcal{K}_U} R_k \leq 0. \quad (35)$$

Note that a function $f_1(x, y) \triangleq (x - y)^2$ is jointly convex in (x, y) . Upon applying the first-order Taylor series expansion at a given point $(x^{(\kappa)}, y^{(\kappa)})$, its convex lower bound is given as $2(x^{(\kappa)} - y^{(\kappa)})(x - y) - (x^{(\kappa)} - y^{(\kappa)})^2 \leq (x - y)^2$. Therefore, constraints (34) and (35) can be approximated at a given point

$(t^{(\kappa)}, \mathbf{p}^{(\kappa)}, \mathbf{a}^{(\kappa)})$ by the following convex constraints:

$$\begin{aligned} & (R_k + a_{k,i})^2 - 2(R_k - a_{k,i})(R_k - a_{k,i}) + (R_k - a_{k,i})^2 \\ & - 4 \sum_{n \in \mathcal{I}_i^{\mathcal{K}_R}} f_{k,i,n} \leq 0, \forall k \in \mathcal{K}_U, i \in \mathcal{K}_R \end{aligned} \quad (36)$$

$$\begin{aligned} & (t+z)^2 - 2(t^{(\kappa)} - z^{(\kappa)})(t-z) + (t^{(\kappa)} - z^{(\kappa)})^2 \\ & - 4 \sum_{k \in \mathcal{K}_U} R_k \leq 0 \end{aligned} \quad (37)$$

in the sense that every point $(t, \mathbf{p}, \mathbf{a})$ that satisfies constraints (36) and (37) would also satisfy constraints (34) and (35).

To deal with constraint (13), it is observed that the nonconvex part $g_k(\bar{\mathbf{F}})$ of (13) has a concave lower bound $\Gamma_k^{(\kappa)}(\bar{\mathbf{F}})$ at a specific point $\bar{\mathbf{F}}^{(\kappa)}$ as (38) (see the top of the next page) where $\Phi_k \triangleq \mathbf{\Pi}_k \mathbf{\Pi}_k^H + \mathbf{\Xi}_k$. The derivation of $\Gamma_k^{(\kappa)}(\bar{\mathbf{F}})$ in (38) and the proof of its concavity follow from the results of [17] and thus are omitted for brevity. Constraint (13) can then be approximated at a given point $\bar{\mathbf{F}}^{(\kappa)}$ by the following convex constraint

$$R_k \leq \Gamma_k^{(\kappa)}(\bar{\mathbf{F}}), \forall k \in \mathcal{K}_U. \quad (39)$$

As such, for a given point $(t^{(\kappa)}, \mathbf{p}^{(\kappa)}, \mathbf{f}^{(\kappa)}, \mathbf{r}^{(\kappa)}, \mathbf{a}^{(\kappa)}, \mathbf{b}^{(\kappa)})$, problem (33) can be approximated as

$$\min_{(t, \mathbf{p}, \mathbf{f}, \mathbf{r}, \mathbf{a}, \mathbf{b}) \in \widehat{\mathcal{H}}^{(\kappa)}} \mathcal{L}(t, \mathbf{a}, \mathbf{b}, \lambda) \quad (40)$$

where $\widehat{\mathcal{H}}^{(\kappa)} \triangleq \{(t, \mathbf{p}, \mathbf{f}, \mathbf{r}, \mathbf{a}, \mathbf{b}) | (4), (5), (7) - (9), (17), (22d), (27d) - (27f), (27h), (29), (36), (37), (39)\}$ is a convex feasible set.

Upon applying $a_{k,i}^2 \geq 2a_{k,i}^{(\kappa)} a_{k,i} - (a_{k,i}^{(\kappa)})^2$ and $b_i^2 \geq 2b_i^{(\kappa)} b_i - (b_i^{(\kappa)})^2$, problem (33) can be further approximated at a given point $(t^{(\kappa)}, \mathbf{p}^{(\kappa)}, \mathbf{f}^{(\kappa)}, \mathbf{r}^{(\kappa)}, \mathbf{a}^{(\kappa)}, \mathbf{b}^{(\kappa)})$ by the convex problem (41) (see the top of the next page) in the sense of minimizing the upper bound $\tilde{\mathcal{L}}(t, \mathbf{a}, \mathbf{b}, \lambda)$ of $\mathcal{L}(t, \mathbf{a}, \mathbf{b}, \lambda)$.

Now, we are ready to outline the steps to find the solution of problem (33) in Algorithm 1. For an empirically chosen λ and starting from a feasible initial point, we solve problem (41) to obtain the optimal solution $(t^*, \mathbf{p}^*, \mathbf{f}^*, \mathbf{r}^*, \mathbf{a}^*, \mathbf{b}^*)$. This solution is then used as an initial point for the next iteration. The loop terminates when there is no improvement in the objective function $\tilde{\mathcal{L}}$ of problem (41). With the above analysis, Algorithm 1 is guaranteed to converge to a solution that satisfies the Fritz John conditions of problem (33). The proof of this fact follows from [9] and [18], and hence is omitted here for brevity.

The initial point $(t^{(0)}, \mathbf{p}^{(0)}, \mathbf{f}^{(0)}, \mathbf{r}^{(0)}, \mathbf{a}^{(0)}, \mathbf{b}^{(0)}) \in \widehat{\mathcal{H}}^{(\kappa)}$ of Algorithm 1 can be found by random methods. Here, to further improve the performance of the solution, we propose Subroutine 1 that aims to solve problem (30) without constraint (28), which can be approximated by the following convex problem

$$\min_{(t, \mathbf{p}, \mathbf{f}, \mathbf{r}, \mathbf{a}, \mathbf{b}) \in \widehat{\mathcal{H}}^{(\kappa)}} -t. \quad (42)$$

Starting from a random feasible point $(t^{(0)}, \mathbf{p}^{(0)}, \mathbf{f}^{(0)}, \mathbf{r}^{(0)}, \mathbf{a}^{(0)}, \mathbf{b}^{(0)}) \in \widehat{\mathcal{H}}$, the initial point obtained by Subroutine 1 is located close to a solution of problem (30). Since (30) and (33) are equivalent, the initial point obtained by Subroutine 1 will improve the solution obtained by solving (41) which is the approximation of (33) via (40).

IV. NUMERICAL EXAMPLES

We consider a C-RAN with a multi-hop fronthaul in Fig. 2(a) and a single-hop fronthaul Fig. 2(b). The radio access part of

$$\Gamma_k^{(\kappa)}(\bar{\mathbf{F}}) \triangleq g_k(\bar{\mathbf{F}}^{(\kappa)}) + \frac{2W}{\ln 2} \Re \left\{ \left\langle \left((\Phi_k^{(\kappa)} - \Pi_k^{(\kappa)} (\Pi_k^{(\kappa)})^H)^{-1} \Pi_k^{(\kappa)} \right)^H (\Pi_k(\bar{\mathbf{F}}_k) - \Pi_k^{(\kappa)}) \right\rangle \right\} - \frac{W}{\ln 2} \left\langle \left((\Phi_k^{(\kappa)} - \Pi_k^{(\kappa)} (\Pi_k^{(\kappa)})^H)^{-1} - (\Phi_k^{(\kappa)})^{-1} \right)^H (\Phi_k(\bar{\mathbf{F}}) - \Phi_k^{(n)}) \right\rangle \leq g_k(\bar{\mathbf{F}}) \quad (38)$$

$$\min_{(t, \mathbf{p}, \mathbf{f}, \mathbf{r}, \mathbf{a}, \mathbf{b}) \in \hat{\mathcal{H}}^{(\kappa)}} \tilde{\mathcal{L}}(t, \mathbf{a}, \mathbf{b}, \lambda) \triangleq -t + \lambda \left(\sum_{k \in \mathcal{K}_U} \sum_{i \in \mathcal{K}_R} \left((1 - 2a_{k,i}^{(\kappa)})a_{k,i} + (a_{k,i}^{(\kappa)})^2 \right) + \sum_{i \in \mathcal{K}_R} \left((1 - 2b_i^{(\kappa)})b_i + (b_i^{(\kappa)})^2 \right) \right) \quad (41)$$

Algorithm 1 Energy efficiency maximization for the downlink C-RANs

- 1: **Initialization:** Set $\kappa := 1$. Set a value of λ and choose an initial point $(t^{(0)}, \mathbf{p}^{(0)}, \mathbf{f}^{(0)}, \mathbf{r}^{(0)}, \mathbf{a}^{(0)}, \mathbf{b}^{(0)})$ by Subroutine 1.
 - 2: **repeat**
 - 3: Update $\kappa := \kappa + 1$
 - 4: Find the optimal solution $(t^*, \mathbf{p}^*, \mathbf{f}^*, \mathbf{r}^*, \mathbf{a}^*, \mathbf{b}^*)$ by solving convex problem (41)
 - 5: Update $(t^{(\kappa)}, \mathbf{p}^{(\kappa)}, \mathbf{f}^{(\kappa)}, \mathbf{r}^{(\kappa)}, \mathbf{a}^{(\kappa)}, \mathbf{b}^{(\kappa)}) := (t^*, \mathbf{p}^*, \mathbf{f}^*, \mathbf{r}^*, \mathbf{a}^*, \mathbf{b}^*)$
 - 6: **until** convergence
-

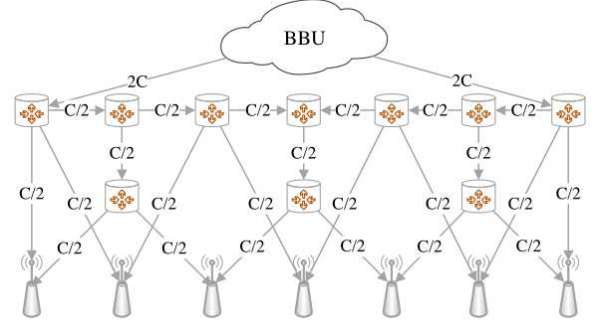
Subroutine 1 Finding a initial point for Algorithm 1

- 1: **Initialization:** Set $\kappa := 1$ and randomly select a point $(t^{(0)}, \mathbf{p}^{(0)}, \mathbf{f}^{(0)}, \mathbf{r}^{(0)}, \mathbf{a}^{(0)}, \mathbf{b}^{(0)}) \in \hat{\mathcal{H}}$
 - 2: **repeat**
 - 3: Update $\kappa := \kappa + 1$
 - 4: Find the optimal solution $(t^*, \mathbf{p}^*, \mathbf{f}^*, \mathbf{r}^*, \mathbf{a}^*, \mathbf{b}^*)$ by solving convex problem (42)
 - 5: Update $(t^{(\kappa)}, \mathbf{p}^{(\kappa)}, \mathbf{f}^{(\kappa)}, \mathbf{r}^{(\kappa)}, \mathbf{a}^{(\kappa)}, \mathbf{b}^{(\kappa)}) := (t^*, \mathbf{p}^*, \mathbf{f}^*, \mathbf{r}^*, \mathbf{a}^*, \mathbf{b}^*)$
 - 6: **until** convergence
-

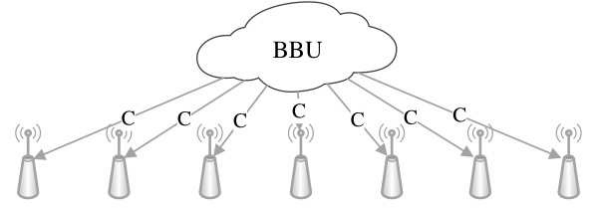
the considered C-RAN is illustrated in Fig. 3. The locations of the $K_R = 7$ RRHs are fixed, while the $K_U = 5$ users are uniformly and independently placed within the RRHs' coverage area, excluding the circular area of 50 m around each RRH [6]. The LTE parameters used in our numerical examples are listed in Table I. Each RRH is assumed to be equipped with $N_r = 2$ antennas and each user with $N_u = 1$ antennas. The active mode and the sleep mode at each eRRH consume 84W and 56W of power, respectively. The slope of transmit power is set as $\beta_i = \beta = 2.8$ and $\alpha_i = \alpha = 5$ for all $i \in \mathcal{K}_R$ [5]. We take $d = 1$, $P_i = P$, and $\Sigma_k = \sigma^2 \mathbf{I}$ for all $k \in \mathcal{K}_U$. Here, we set $R_{\text{QoS}} = 0.1 \text{ Mbps}$ for the feasibility of the problem (27). All the presented results have been averaged over 100 simulation trials with $\lambda = 100$.

To verify the effectiveness of the proposed algorithm in the multi-hop C-RANs (referred to as Alg. 1-MH in the figures) and in the single-hop C-RANs (referred to as Alg. 1-SH in the figures), we consider the following benchmark schemes:

- HUA-SH: This scheme applies a heuristic UA scheme [8] in the single-hop C-RAN. Here, each user is heuristically assigned to N_c RRHs that have the largest channel gains, where N_c is empirically chosen for the best performance.
- HUA-MH: The HUA scheme of [8] is used in the multi-hop C-RAN.



(a) Multi-hop fronthaul networks with $M = 10$ routers and $L = 25$ fronthaul links



(b) Single-hop fronthaul networks

Fig. 2. Multi-hop and single-hop fronthaul network simulation scenarios with $K_R = 7$ RRHs. Note that the total capacity of the information flow to each RRH is C in both cases.

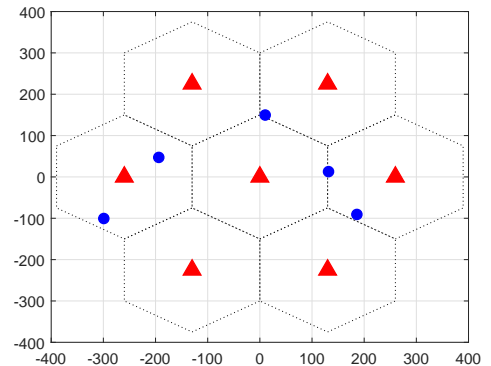


Fig. 3. Wireless network simulation scenario with $K_R = 7$ fixed RRHs and $K_U = 5$ randomly positioned users.

For existing HUA solutions [8], the energy efficiency achieved by these HUA schemes above can be found by only jointly optimizing data rate allocation and signal precoding with the similar manners discussed in Section III.

TABLE I
LTE PARAMETERS USED IN NUMERICAL EXAMPLES [19]

Parameters	Values
Distance between adjacent eRRHs	0.3 km
Total bandwidth	10 MHz
Standard deviation of log-normal shadowing	10 dB
Path loss at distance d (km)	$140.7 + 36.7 \log_{10}(d)$ dB
Noise variance	-174 dBm/Hz
Maximum RRH transmit power	24 dBm

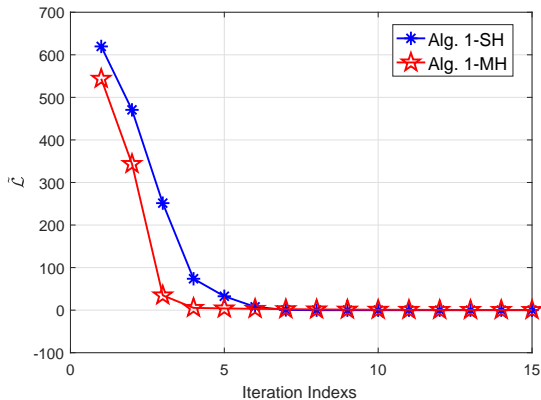


Fig. 4. Convergence process of the proposed Algorithm 1.

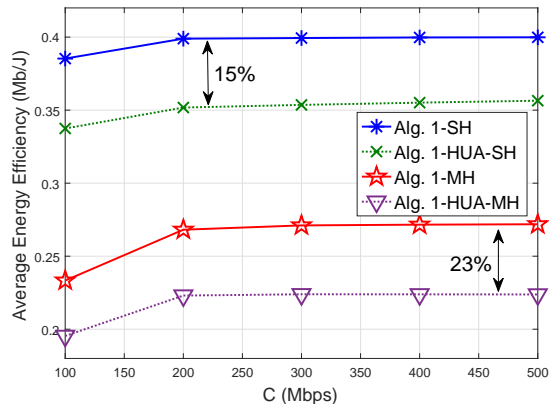


Fig. 5. Performance of the proposed algorithm in comparison with the benchmark schemes.

Fig. 4 plots the convergence process of the proposed algorithm under both single-hop and multi-hop fronthaul cases. In the considered example, they converge quickly in fewer than 15 iterations. It should be emphasized that each iteration of the algorithm corresponds to solving only one simple convex program (41), which results in a low computational cost.

Fig. 5 shows that the proposed algorithm outperforms the benchmark schemes in terms of the average energy efficiency in all cases. From Fig. 5, the improvements by Algorithm 1 are 15% and 23% in the single-hop and the multi-hop fronthaul scenarios, respectively. Such enhancement is brought about by the extra dimension of UA optimization in the proposed joint optimization algorithm compared to the benchmark schemes.

V. CONCLUSION

This paper has jointly designed UA, RRH activation, data rate and signal precoding to maximize the energy efficiency

of a downlink C-RAN. The formulated mixed-integer optimization problem takes into account routing constraints at the fronthaul links, minimum data rate requirements, limited fronthaul capacity and maximum RRH transmit power. Using optimization techniques, we propose a new iterative algorithm with guaranteed convergence to a Fritz John solution of the formulated problem. Numerical results confirm the significant performance advantage of the developed solution over baseline schemes.

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