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# **Energy-efficient scheduling for a flexible flow-shop using improved genetic-simulated annealing algorithm**

**Abstract:** The traditional production scheduling problem considers performance indicators such as processing time, cost and quality as optimization objectives in manufacturing systems; however, it does not take energy consumption and environmental impacts into account completely. Therefore, this paper proposes an energy-efficient model for flexible flow-shop scheduling (FFS). First, a mathematical model for a FFS problem, which is based on an energy-efficient mechanism, is described to solve multi-objective optimization. Since FFS is well known as a NP-hard problem, an improved genetic-simulated annealing algorithm is adopted to make a significant trade-off between the makespan and the total energy consumption for implementing a feasible scheduling. Finally, a case study of production scheduling problem for metalworking workshop in a plant is simulated. The experimental results show the relationship between the makespan and the energy consumption is apparently conflicting. Moreover, an energy saving decision is performed in a feasible scheduling. Using the decision method, there can be a significant potential to minimize energy consumption while complying with the conflicting relationship.

**Keywords:** flexible flow-shop scheduling (FFS), energy consumption, energy saving, makespan, genetic-simulated annealing algorithm

## **1. Introduction**

Nowadays manufacturing enterprises are not only facing complex and diverse economic trends of shorter product life cycles, quick changing science and technology, increasing customer demand diversity, and production activities globalization, but also enormous and heavy environmental challenges of global climate change (e.g. greenhouse effect), rapid exhaustion of various non-renewable resources (e.g. gas, oil, coal), and decreasing biodiversity. Statistical data shows the Germany industrial sector was responsible for approximately 47% of the total national electricity

consumption, and the corresponding amount of CO<sub>2</sub> emissions generated by this electricity summed up to 18%-20% [1]. Thus, manufacturing companies are responsible for the environmental outcome, and forced to have manufacturing systems that show major potential to reduce the environmental impacts [2, 3].

Nowadays there has been a growing interest in the development of energy savings due to a sequence of serious environmental impacts and the rising energy costs. Research on minimizing the energy consumption of manufacturing systems has been focused on three levels' perspective. From the machine-level perspective, developing and designing more energy-efficient machines and equipment to reduce power and energy demands of machine components is an important strategic target for manufacturing companies [4-6]. Unfortunately, previous studies show that the share of energy demand for removal of metal material, compared to the share of energy needed for supporting various functions of manufacturing systems, is quite small (less than 30%) on total energy consumption [7-9]. From the product-level perspective, modeling embodied product energy framework based on a product design viewpoint for energy reduction approach is beneficial to support the improvements of product design and operational decisions [10-13]. It requires strong commercial simulation software to facilitate the analysis and evaluation of the embodied product energy. The results could not be applied easily in most of manufacturing companies, especially in small and medium sized enterprises due to enormous financial investments. In the manufacturing system-level perspective, thanks to decision models for supporting energy savings, it is feasible to achieve a significant reduction of energy consumption in manufacturing applications.

In the specialized literature about production scheduling there has been widely discussed the key production objectives for production scheduling decision models, such as cost, time and quality. However, decreasing energy consumption in manufacturing systems through production scheduling has been rather limited. One of the most related researches is the work by Mouzon *et al.*[14], who developed several algorithms and a multiple objective mathematical programming model for

investigating the problem of scheduling jobs on a single CNC machine to reduce the energy consumption and total completion time. They pointed out that there was a significant amount of energy savings when non-bottleneck machines were turned off until needed; such the relevant savings share on the total energy consumption would sum up to 80%. In addition, they reported that the inter-arrivals were forecasted and more energy-efficient dispatching rules could be adopted for scheduling. In further research, Mouzon and Yildirim[15] proposed a greedy randomized adaptive search algorithm for solving a multi-objective optimization schedule that minimized the total energy consumption and the total tardiness on a machine. Fang *et al.*[16] provided a new mixed integer linear programming model for scheduling a classical flow shop that combined the peak total power consumption and associated carbon footprint with the makespan. Bruzzone *et al.* [17] presented an energy-aware scheduling algorithm based on a mixed integer programming formulation to realize energy savings for a given flexible flow shop which was required to keep fixed original jobs' assignment and sequencing.

Although the majority of the research on production scheduling so far has not considered energy saving strategies completely, the efforts mentioned above provide a starting point for exploring an energy-aware schedule optimization from the viewpoint of the energy consumption. In this paper, a multi-objective optimization problem of minimizing the maximum completion time and the total energy consumption in a flexible flow-shop is considered.

The outline of this paper is organized as follows. In Section 2, a general FFS problem is presented, and a multi-objective schedule is described. In Section 3, a novel energy-aware model for flexible flow-shop scheduling is illustrated. In Section 4, a heuristic approach based on improved genetic-simulated annealing algorithm for solving the multi-objective optimization problem is used. In Section 5, a case study on production scheduling problem for metalworking workshop in a plant is simulated. In Section 6, the conclusions are detailed.

## 2. FFS problem description

A flexible flow-shop scheduling (FFS) problem is the further development of the classical flow shop scheduling [18]. The FFS is a multi-stage production process which consists of two or more production stages in series. At each production stage there is at least one machine tool, and at least one stage has more than one machine tool. All jobs have to pass every production stage in the same order. The FFS has infinite intermediate storage between machine tools [19]. One instance of the FFS problem consists of a set of  $J$  jobs and a set of  $M$  machine tools. Each job  $j \in J$  on machine  $i \in M$  has corresponding processing time and power consumption at a given speed. All jobs are available to be processed sequentially and nonpreemptively at different machine stages as illustrated in Fig. 1. The FFS problem is considered as NP-hard in essence and difficult to solve [20].

The constraints of FFS are made as follows:

- (1) One job can be processed by only one machine at each production stage.
- (2) One machine can process at most one operation at a time.
- (3) For the first stage, all jobs are available at time  $t=0$ .
- (4) There are no precedence relationships between operations of different jobs while there are precedence relationships between different operations of one job;
- (5) Preemption is not allowed for processing each job, i.e., once an operation is started, it must be finished without interruption.
- (6) Every operation of one job can be machined at a given speed for every stage.
- (7) For the same operation, the processing time differs at different unrelated parallel machines in a production stage.
- (8) At each stage, power consumption and energy consumption of different states for each machine can be metered and calculated.

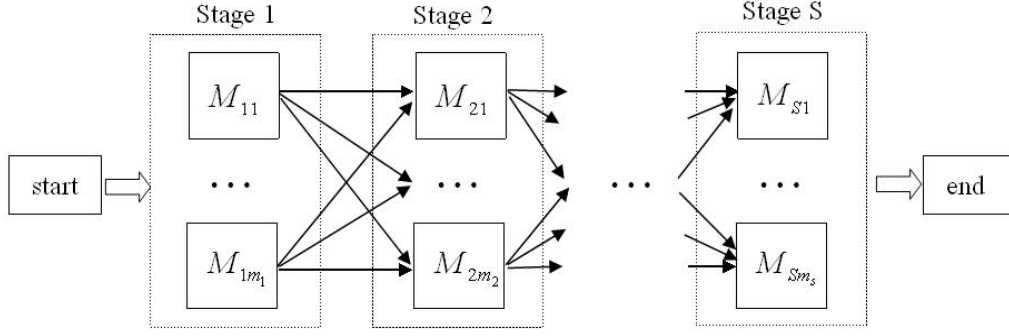


Fig. 1. A flexible flow-shop layout

The scheduling objective of FFS is to assign jobs to machine tools at the corresponding stages and determine the processing sequence of operations on each machine in order to minimize the maximum completion time and the total energy consumption.

### 3. Mathematical model of FFS based an energy-efficient mechanism

#### 3.1 Energy-efficient mechanism

Suppose that M machine tools are at some production shop, and N jobs are to be scheduled on a set of machine tools. By referring to the decomposition of energy consumption types of machining systems [4, 21], the energy balance equation for manufacturing systems is constructed as follows:

$$\sum_{m=1}^M \sum_{j=1}^N \int_0^{t_{mj}} P_{mj}^i(t) dt = \sum_{m=1}^M \sum_{j=1}^N \int_0^{t_{mj}} (P_{mj}^a(t) + P_{mj}^u(t) + P_{mj}^c(t)) dt + E_a \quad (1)$$

where:

$P_{mj}^i(t)$  represents the input power of manufacturing systems, i.e. the total power consumption

$P_{mj}^a(t)$  represents the load power consumption of manufacturing systems, which is composed of the load power consumption of motor drives components, main spindle drives components and servo feed drives components

$P_{mj}^u$  represents the unload power consumption of manufacturing systems, which is involved in the unload power consumption of motor drives components, main spindle drives components and servo feed drives components

$P_{mj}^c$  represents the output power of manufacturing systems, i.e. the cutting power

$E_a$  represents the energy consumption of machine tools' auxiliary systems, such as hydraulic system, cooling and lubrication system, control system, and periphery system

$t_{mj}$  represents the time when the machine tool is at the usage phase, which includes non-production time (e.g. start-up, idle, readiness, off) and production time.

Within the range of the permitted load, considering the load power consumption of manufacturing systems which is related with the actual load, the relationship between the load power and the cutting power of manufacturing systems can be expressed as:

$$P_{mj}^a(t) = \delta P_{mj}^c(t) \quad (2)$$

where  $\delta$  is the coefficient of the load power consumption.

In general, the energy efficiency of a system is defined as the ratio of output energy to input energy. According to the Eqs. (1)-(2), the energy efficiency of the manufacturing system  $\eta_U$  can be formulated as:

$$\eta_U = \frac{\sum_{m=1}^M \sum_{j=1}^N \int_0^{t_{mj}} P_{mj}^c(t) dt}{\sum_{m=1}^M \sum_{j=1}^N \int_0^{t_{mj}} P_{mj}^u(t) dt + \sum_{m=1}^M \sum_{j=1}^N \int_0^{t_{mj}} (1 + \delta) P_{mj}^c(t) dt + E_a} \quad (3)$$

According to the Eq. (3), increasing the energy efficiency is in favor of energy savings in the manufacturing system. On the one hand, improve and optimize the structure of machine tools, in particular auxiliary components, due to high-energy consumption. On the other hand, from the perspective of production decisions a production scheduling will require to take into account the amount of energy consumption in the production process. To this end it is required an energy-aware

scheduling model, in order to minimize the total energy consumption as far as possible.

In addition, Mouzon *et al.* [14] observed that there could be an important decrease in energy consumption by changing its operational state when a single CNC machine was left running idle for a long time. The considered scheduling problem arises in a flexible flow-shop with multiple sleeping mode states, reducing the total energy consumption while not delaying the processing of jobs on selected machine tools. On the basis of the ratio between the turn off + turn on energy usage and the idle running energy usage, this paper gives an energy saving model to determine if machine tools should be on or not.

Assume that the inter-arrival time between jobs is  $T_0$  and the time which is required for turning off and then turning on the machine is  $T_{off-on}$ . Let  $T_s$  be the break-even duration for the turn off + turn on energy usage ( $SE_{sm}$ ) divided by the idle power usage ( $P_{jm}^u$ ). If the running idle energy usage ( $E_{sm}^u$ ) is greater than the turn off + turn on energy usage ( $SE_{sm}$ ), it can be a significant amount of energy savings when machines are shut down, i.e.  $T_0 > T_s$ . Due to the frequent conversion and the limited life of a machine controller, an energy saving allowance  $K$  is considered. Hence, implementing the above energy saving decision model requires  $T_0 \geq (1+K)T_s$ . Besides, it is needed to meet  $T_0 > T_{off-on}$ . The schematic diagram of an energy-efficient decision model could be the one displayed in Fig. 2.

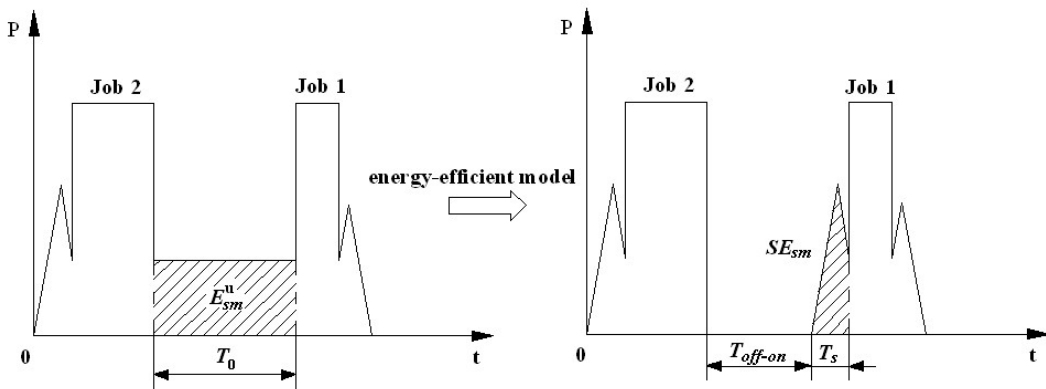


Fig. 2. The schematic diagram of an energy-efficient model



### 3.2 Energy-efficient model of FFS

An energy-efficient model for flexible flow-shop that minimizes the maximum completion time, while limiting the possible worsening of the total energy consumption within a manufacturing system, is proposed in this section. The parameters are given below:

- $S$  is the set of spindle speeds for one machine tool;
- $J$  is the set of jobs;
- $M$  is the set of machine tools;
- $T_{jmv}$  is the processing time when job  $j$  is processed on machine tool  $m$  with speed  $v$ ,  $j \in J, m \in M, v \in S$ ;
- $S_{jm}$  is the starting time when job  $j$  is processed on machine tool  $m$ ,  $j \in J, m \in M$ ;
- $C_{jm}$  is the finishing time when job  $j$  is processed on machine tool  $m$ ,  $j \in J, m \in M$ ;
- $C_{\max}$  is the makespan of the schedule, i.e. the completion time of the last job in the schedule;
- $T_D$  is due date;
- $E_c$  is the total energy consumption when machine tools are at the run-production mode stage;
- $E_b$  is the basic energy consumption when machine tools are at the run-production mode stage;
- $E_f$  is the friction energy consumption when machine tools are at the production mode stage;
- $E_u$  is the total energy consumption when machine tools are at the idle running mode stage, i.e. idle energy consumption;
- $E_u'$  is the total energy consumption after utilizing the energy-aware decision model at the idle running mode stage;
- $E_{jmv}$  is the cutting energy consumption when job  $j$  is processed on machine tool  $m$  with speed  $v$ ,  $j \in J, m \in M, v \in S$ ;

- $P_{jm}^u$  is the idle power consumption when job  $j$  is processed on machine tool  $m$ ,  $j \in J, m \in M$  ;
- $SE_{sm}$  is the energy consumption for turning off machine tool  $m$  and then turning on machine tool  $m$  at the idle running mode stage  $s, m \in M$  ;
- $X_{jmv}$  is an integer variable that can have two possible values: 0 or 1, it is set to 1 if job  $j$  is required to process on machine tool  $m$  with speed  $v$ , and 0 otherwise,  $j \in J, m \in M, v \in S$  .

As mentioned above, the energy consumption of the manufacturing system in a flexible flow-shop is composed of cutting energy usage for removing material process, basic energy usage for maintaining normal operation of system components and non-value added energy consumption due to machine load and friction when jobs are processed on machine tools during the production time. It can be expressed as:

$$E_c = \sum_{j \in J} \sum_{m \in M} \sum_{v \in S} (1 + \delta) X_{jmv} E_{jmv} + E_b + E_f \quad (4)$$

Notice that  $E_{jmv}$ ,  $E_b$  and  $E_f$  are defined as follow, respectively.

$$E_{jmv} = T_{jmv} (F_z v + \frac{F_x n f}{60000}) \times 10^{-3} \quad (5)$$

$$E_b = \sum_{j \in J} \sum_{m \in M} \sum_{v \in S} X_{jmv} P_{jm}^u T_{jmv} \quad (6)$$

$$E_f = V (\beta_0 + \frac{\beta_1}{MRR}) \quad (7)$$

where:

$F_x$  and  $F_z$  represent the axial force and tangential force respectively.

$v$  represents the cutting speed.

$n$  represents the spindle speed.

$f$  represents the feed rate.

$V$  represents the cutting volume.

$MRR$  represents the material removal rate.

$\beta_0$  and  $\beta_1$  represent relevant coefficients.

$F_x$  and  $F_z$  can be metered by a force sensor, and the other parameters of the Eq. (5) can be obtained by referring to standard handbooks [22]. The Eq. (7) is the energy

equation converted to heat. Li and Kara [23] pointed out that the material removal process is efficient when the  $MRR$  is greater than  $0.3\text{cm}^3/\text{s}$  and  $\beta_0 = -0.452, \beta_1 = 0.746$ .

When the manufacturing system is at idle running mode stage, system components which implement activities such as loading or unloading work piece, positioning and clamping, and changing cutting tools will consume much energy. The energy demand for the flexible flow-shop at the non-production time can be given as:

$$E_u = \sum_{j \in J} \sum_{m \in M} \sum_{v \in S} ((C_{(j+1)m} - T_{(j+1)mv}) - C_{jm}) X_{jmv} P_{jm}^u \quad (8)$$

If the inter-arrival time between job  $j$  and job  $j+1$  meets the aforementioned condition of the energy saving decision model in Section 3, then instead of  $E_u$  and  $E_u'$  is adopted as follows:

$$E_u' = \sum_{j \in J} \sum_{m \in M} \sum_{v \in S} (((C_{(j+1)m} - T_{(j+1)mv}) - C_{jm}) X_{jmv} P_{jm}^u - SE_{sm}) \quad (9)$$

According to the Eqs. (4)-(9), the total energy consumption for the manufacturing system in the flexible flow-shop can be calculated as below:

$$E_{total} = E_u + E_c + E_a \quad (10)$$

When the  $MRR$  is determined on the basis of material attributes, the cutting energy consumption ( $E_{jmv}$ ) and friction energy consumption ( $E_f$ ) are constants. Owing to the limitation of length, the energy demand of auxiliary systems ( $E_a$ ) is not considered in this paper. Therefore, within the machining time, in order to reduce the total energy consumption it is required to minimize, as far as possible, the basic energy consumption ( $E_b$ ) and the idle energy consumption ( $E_u$  or  $E_u'$ ).

To sum up, the multi-objective optimization model for a flexible flow-shop scheduling problem to realize the trade-off between makespan ( $f_1$ ) and total energy consumption ( $f_2$ ) is presented by synthesizing two factors, i.e. time and energy as shown below:

$$\begin{cases} \min f_1 = \max_m \sum_{j \in J} C_{jm} \\ \min f_2 = E_b + E_u - E_u' \end{cases} \quad (11)$$

subject to

$$C_{\max} \geq C_{jm}, j \in J, m \in M \quad (12) C_{\max} \leq T_D \quad (13)$$

$$\sum_{m \in M} X_{jmv} = 1, j \in J, v \in S \quad (14)$$

$$\sum_{s \in S} X_{jmv} = 1, j \in J, m \in M \quad (15)$$

$$C_{jm} = S_{jm} + X_{jmv} T_{jmv}, j \in J, m \in M, v \in S \quad (16)$$

$$C_{jm} \leq S_{jh}, m, h \in M, m \neq h, j \in J \quad (17)$$

$$C_{jm} \leq S_{(j+1)m}, j \in J, m \in M \quad (18)$$

$$E_u - E_u' = \begin{cases} SE_{sm} & \text{if } (C_{(j+1)m} - T_{(j+1)mv}) - C_{jm} \geq \max\{(1+K)SE_{sm} / P_{jm}^u, T_{off-on}\} \\ \sum_{j \in J} \sum_{m \in M} \sum_{v \in S} ((C_{(j+1)m} - T_{(j+1)mv}) - C_{jm}) X_{jmv} P_{jm}^u & \text{if } T_0 < \min\{(1+K)SE_{sm} / P_{jm}^u, T_{off-on}\} \end{cases} \quad (19)$$

Constraints (12)-(13) define that the makespan, which requires arrival before the due date, is equal to the completion time of the last job in the schedule. Constraint (14) means that one job can be assigned to only one machine tool at each production stage. Constraint (15) imposes that one job can be processed on one machine tool with one chosen speed. Constraint (16) points out that the completion time of job  $j$  is composed of the processing time and starting time on machine tool  $m$ . Constraint (17) gives the precedence constraints between the operations of job  $j$ , i.e. one operation of the job cannot be processed at next production stage until it has been finished at the current stage. Constraint (18) ensures that one machine can process next job only after it has finished the current one. Constraint (19) determines if the manufacturing system will implement energy savings strategy or not at the idle running mode stage.

The above mathematical model is a multi-objective functions with constraints. Although no optimal or near-optimal solution exists in multi-objective optimization problem (MOP), a set of non-dominated solutions (Pareto optimal solutions) to make the trade-off between the maximum completion time and total energy consumption

are obtained. There have been diverse approaches developed to solve multiple-objective optimization problems. One of the most well-known methods for solving MOP is the weighted additive utility function [24-26], which is employed due to its simplicity, wide-spread use and the ability to identify non-dominated solutions in this paper. Let  $f_i$  be the  $i$ th objective function. Then, the weighted additive utility function with 2 objectives can be described as:

$$U(2) = w_1 f_1 + w_2 f_2 \quad U(k) = \sum_{j=1}^k w_j f_j, \sum_{j=1}^k w_j = 1 \quad (20)$$

where  $w_1$  and  $w_2$  are the importance weights of each objective function. The sum of weights is usually required to be equal to one, i.e.  $w_1 + w_2 = 1$  and each weight is positive number, i.e.  $w_1 \geq 0, w_2 \geq 0$ . The decision maker defines all weights and they reflect decision maker's preference for each objective. Using the utility function, the objective values of multiple objectives are combined to form a single objective function that can be solved easily. In fact, evaluating weights of importance will be hard when the performance measurements are on different scales. By normalizing different criteria values to comparable units, all objectives are assessed in the same scale. Hence, the weighted additive utility function with normalized objectives is described as:

$$U = w_1 f_1' + w_2 f_2' \quad (21)$$

where  $f_1'$  and  $f_2'$  are normalized values of  $f_1$  and  $f_2$ , respectively. Notice that each normalized objective  $f_i'$  is defined as:

$$f_i' = \frac{f_{i,\max} - f_i}{f_{i,\max} - f_{i,\min}} \quad (22)$$

where  $f_{i,\min}$  and  $f_{i,\max}$  represents the given minimum and maximum values for objective function  $f_i$ , respectively.

#### **4. Improved genetic-simulated annealing algorithm for an energy-aware FFS**

In this section, an improved genetic-simulated annealing algorithm is proposed for solving energy-aware scheduling in a flexible flow-shop. There are many meta-heuristic algorithms that have been implemented in an FFS, such as genetic algorithm (GA), simulated-annealing algorithm (SA), particle swarm optimization, ant colony optimization, etc. Among these approaches, GA can quickly approach to the optimization solution, but a fatal shortcoming is that it is liable to be trapped in a local optima, i.e. premature convergence. Fortunately, SA has the ability to jumping out of the local optima and searching for the best solution. Therefore, this paper proposes to incorporate the strengths of a genetic algorithm into a simulated annealing algorithm. GA is developed to rapidly search for an optimal or near-optimal solution among the solution space and then SA is utilized to seek a better one on the base of that solution. In addition, a novel annealing rate function, which is inspired from hormone modulation mechanism, is adopted for further improving the efficiency of the exploration. The proposed genetic-simulated annealing algorithm for an energy-aware FFS is illustrated in Fig. 3.

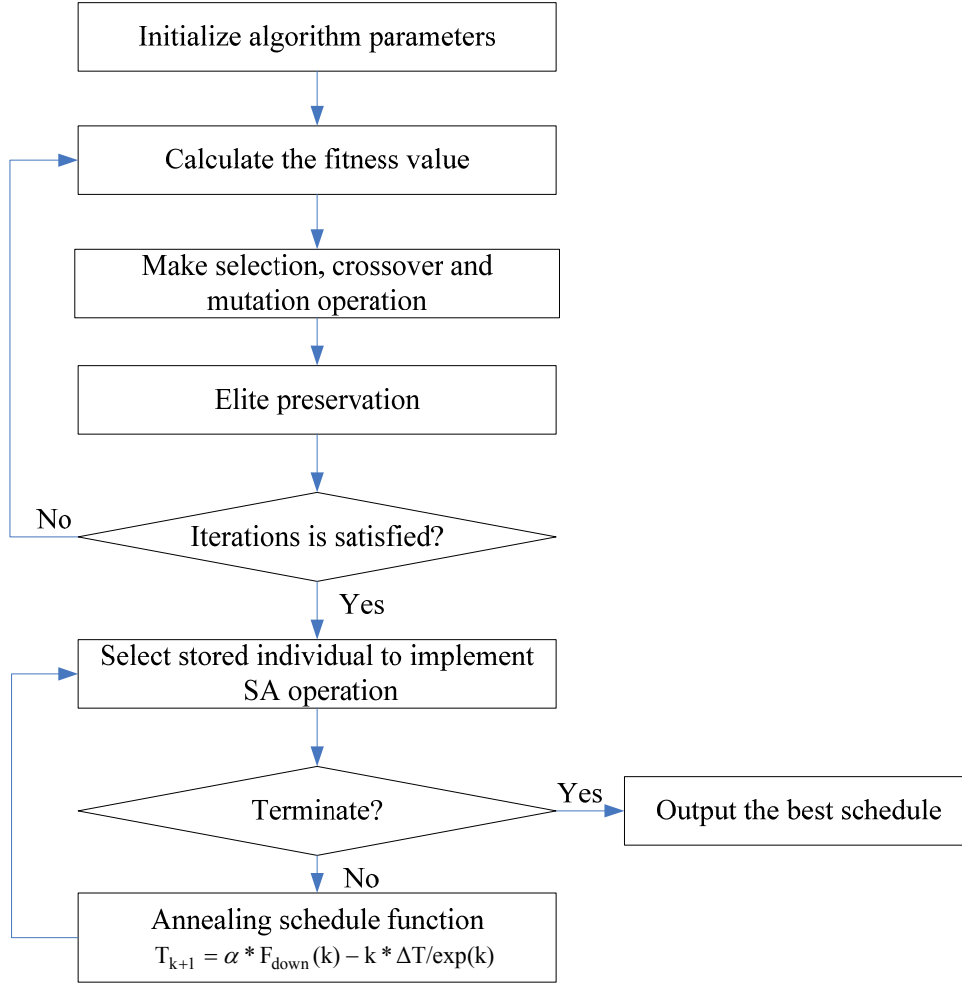


Fig. 3. Summary of genetic-simulated annealing algorithm

#### 4.1 Encoding representation

Based on the elements and their corresponding positions in a matrix describing the constraints between jobs, an encoding approach for an energy-aware FFS is presented. In this representation each chromosome implements a relative and feasible schedule. Suppose that  $N$  jobs are to be processed on a set of machine tools, and each job is required to pass  $S$  stages. There are  $M_s$  ( $s = 1, 2, \dots, S$ ) unrelated parallel machine tools at each production stage. An encoding matrix  $N \times S$  is constructed as follow:

$$A_{N \times S} = \begin{bmatrix} a(1,1) & a(1,2) & \cdots & a(1,S) \\ a(2,1) & a(2,2) & \cdots & a(2,S) \\ \vdots & \vdots & a(i,j) & \vdots \\ a(N,1) & a(N,2) & \cdots & a(N,S) \end{bmatrix} \quad (23)$$

The elements  $a(i, j): (a(i, j) \in (1, M_s + 1), i = 1, 2, \dots, N, j = 1, 2, \dots, S, s = 1, 2, \dots, S)$  are random real numbers.  $\text{Int}(a(i, j))$  is the integer of  $a(i, j)$ , and it indicates the machine tools' identifier that deals with the  $j$ th process of job  $i$ . If the condition  $\text{Int}(a(h, j)) = \text{Int}(a(i, j))$  and  $h \neq i$  is satisfied, then it means that there are several jobs waiting for being processed on the same machine tool for the same process. When the process is the first one, these jobs are arranged to operate in accordance with the ascending sequence of  $a(i, 1): (i=1, 2, \dots, N)$ . When the process number is greater than one, these jobs are determined by their completion time of previous process. In other words, the shorter the finishing time of previous process is, the earlier the next process can be operated. If the completion time is the same, jobs are operated according to the ascending sequence of  $a(i, j), (i=1, 2, \dots, N, j=2, 3, \dots, S)$ .

According to the aforementioned encoding method, a chromosome that consists of  $S$  segments and  $N$  genes included at each segment can be written as:

$$\begin{aligned} chrom = & [\text{Int}(a(1,1)), \text{Int}(a(2,1)), \dots, \text{Int}(a(N,1)), \text{Int}(a(1,2)), \text{Int}(a(2,2)) \dots, \\ & \text{Int}(a(N,2)), \dots, \text{Int}(a(1,S)), \text{Int}(a(2,S)), \dots, \text{Int}(a(N,S))] \dots \end{aligned} \quad (24)$$

For instance, assume that 3 jobs are scheduled at 3 production stages in a flexible flow-shop. Each job has 3 processes and the number of parallel machines for each stage is 3, 2 and 2. An encoding matrix based on the encoding rule is generated randomly using Matlab simulation software as follows:

$$A = \begin{bmatrix} 1.1316 & 2.3234 & 2.1123 \\ 3.4342 & 1.1453 & 2.4357 \\ 3.9015 & 2.6456 & 1.2137 \end{bmatrix} \quad (25)$$

Note that each column of the matrix (25) describes the situation of different jobs at the corresponding production stage. For example, the first column presents the machine number of jobs being processed at production stage 1, i.e. job 1 is processed on machine 1, job 2 on machine 3 and job 3 on machine 3. Due to  $3.9015 > 3.4342$ , the sequence of jobs being waited on machine 3 at stage 1 is job 2 prior to job 3. According to the encoding matrix, the chromosome can be expressed  $chrom A = [1, 3, 3, 2, 1, 2, 2, 2, 1]$ . Also the initial population is produced.



## 4.2 Fitness function

The genetic-simulated annealing algorithm assesses the solutions based on the fitness function. The greater fitness an individual has, the higher chance it has to be chosen into the next generation. In general, the fitness is relative to the objective function. In this paper, the above-mentioned objective function, i.e. Eq. (21) can be transformed into the fitness function for solution  $k$  as follows:

$$F(k) = 1/U(k) \quad (26)$$

## 4.3 GA operation phase of the FFS

In the GA operation phase, an initial population is yielded randomly. Using basic genetic operations, i.e. selection, crossover and mutation, the GA operates to produce new population. Three operations are described in detail as follows:

- Selection operation: On the base of the fitness of the individual, the selection operator chooses individuals used for crossover and mutation. Often the fitness value is not the fittest one. Several selection schemes are developed to determine good solution space. A '2/4 selection' is adopted to preserve fittest individuals at each generation, and maintain the diversity of the population as well [27]. At the same time, roulette-wheel-selection is used to create a new population.
- Crossover operation: According to the aforementioned encoding rule, only if the condition  $a(i, j) \in (1, M_s + 1)$  is satisfied, combining genes of selected solution to generate a new solution is legal. Thus, it is required to implement two-point crossover at each segment of a chromosome with a crossover probability, and pick intersections randomly.
- Mutation operation: As crossover operation cannot yield solutions with new information, it is required mutation operation with a specified probability ( $P_m$ ) for every segment in order to obtain the solutions with greater fitness. If the specified probability ( $P_m$ ) is greater than a random number generated on the interval 0 to 1 using uniformly distributed rule, the mutation operation is executed.

#### ***4.4 SA operation phase of the FFS***

In the process of the genetic-simulated annealing algorithm, the good individuals generated by the GA are sent to the SA for improvement. The SA can avoid falling into a local optimum accepting some probability. However, the search efficiency of the SA is not high. Therefore, some parameters connected to the SA should be studied, including the neighborhood structure, the initial temperature, the annealing rate and the termination condition. These factors play a significant role in the performance of the SA and should be implemented carefully as follow.

##### **(1) The neighborhood structure**

Neighborhood structures have a direct impact on the efficiency of local search. One of the most effective neighborhood strategies for the SA regarding a production scheduling problem is based on the critical path. One critical path which consists of a number of blocks corresponds to one feasible solution. One block represents a maximal sequence of several operations required to be processed on the same machine tool. In this study, moving an operation of one critical block to the end of the block or the beginning of the block is adopted to generate the neighborhood.

##### **(2) The initial temperature**

The initial temperature ( $T_0$ ) should be set to a high enough temperature. In the first iteration of the SA this temperature will be minimized until the probability of accepting the undesired solutions is greater than 0.8; from this point until the end, the temperature will decrease slowly during the iterations of the algorithm. The initial temperature function in the SA can be set as

$$T_0 = (U_{\max} - U_{\min}) \times 100 \quad (27)$$

where  $U_{\max}$  is the maximum sum of all jobs' processing time and  $U_{\min}$  is the minimum sum of all jobs' processing time.

##### **(3) Annealing rate function**

The performance of the SA has a significant relation with Annealing rate [28]. In order to enhance the search efficiency of the SA, a novel annealing rate method, which is inspired from hormone modulation mechanism, is developed. Farhy [29]

pointed out that the modulation of hormone had characteristics with monotone and nonnegative and obeyed up-regulatory and down-regulatory Hill functions, as shown below:

$$\begin{cases} F_{\text{up}}(X) = \frac{X^n}{T^n + X^n} \\ F_{\text{down}}(X) = \frac{T^n}{T^n + X^n} \end{cases} \quad (28)$$

where  $T$  is a threshold,  $T > 0$ ,  $X$  is an independent variable,  $n$  is a Hill coefficient,  $n \geq 1$ . Note that  $F_{\text{up}} + F_{\text{down}} = 1$  and  $0 \leq F_{\text{up}}(X) \leq 1, 0 \leq F_{\text{down}}(X) \leq 1$ . The Hill functions can realize a quick stability, which keeps hormone modulation adaptive and stable. If one hormone  $a$  is controlled by another hormone  $b$ , the secretion of the former  $V_a$  is determined by the concentration of the latter  $C_b$ , which can be described as:

$$V_a = c_0 F(C_b) + V_{a0} \quad (29)$$

where  $V_{a0}$  is the basal secretion of hormone  $a$ , and  $c_0$  is a constant.

Based on the above hormone modulation mechanism, an annealing rate function can be designed as follows:

$$T_{k+1} = \alpha * F_{\text{down}}(k) - k * \Delta T / \exp(k) \quad (30)$$

subject to

$$F_{\text{down}}(k) = 1 / (1 + k^n) \quad (31)$$

$$\Delta T = T_{\text{current}} - T_{\text{previous}} \quad (32)$$

where  $\alpha$  is a small constant, and  $k$  is the number of iterations.  $\Delta T$  is the difference between the current temperature ( $T_{\text{current}}$ ) and the previous temperature ( $T_{\text{previous}}$ ), and  $\Delta T < 0$ .

#### (4) Terminating condition

In the SA, the terminating criterion consists of the Markov chain stability criterion and the external circulation stopping criterion. In this study, calculating the iterations at each given temperature decides if the condition of the Markov chain stability criterion is satisfied or not. The end temperature value is used as the termination condition, and when a temperature is less than the last one the algorithm ends.

## 5. Evaluation

To verify the effectiveness and feasibility of the approach, the improved genetic-simulated annealing algorithm is used to solve a multi-objective scheduling problem. The simulation was carried out utilizing Matlab programming language. The experimental tests were carried out on a personal computer with Intel Pentium (R) with 1 GB Ram and 3.20 GHz frequency, and Windows XP.

Consider the following flexible flow-shop scheduling in which there are 12 jobs waiting for being scheduled and processed at 3 production stages. Each job has 3 processes and the number of parallel machines for each stage is 3, 2 and 4, respectively. The relative data including job number, spindle speed, processing time, unload power are shown in Table 1. Due to the relationship between the makespan and energy consumption, the importance weights of each objective function are determined by the decision-maker's preference. When the higher importance weight is assigned to the objective function of makespan, the solution could lead to the lower makespan. However, the energy consumption could be greater. On the contrary, when the higher importance weight is assigned to the objective function of energy consumption, the solution could lead to the lower energy consumption with higher makespan. The sets of the importance weights of each objective function were tested during the ranging interval  $[0, 1]$ , and the improved genetic-simulated annealing algorithm was run 15 times for each set of  $w_1$  and  $w_2$ .

We will analyze four different scenarios corresponding to different combinations of  $w_1$  and  $w_2$ :

Table 1 Relative data of jobs and machines

Job number	Process 1			Process 2			Process 3			
	machine (M1)	machine (M2)	machine (M3)	machine (M4)	machine (M5)	machine (M6)	machine (M7)	machine (M8)	machine (M9)	
1	Spindle speed/rpm	600	600	400	300	250	1500	1200	1500	1300
	Processing time/min	2	2	3	4	5	2	3	2	3
	Unload power/kW	2.26	1.36	1.43	1.46	1.20	4.03	3.80	3.72	3.43
2	Spindle speed/rpm	400	350	250	350	300	1200	1100	950	1000
	Processing time/min	4	5	4	3	4	3	4	5	4
	Unload power/kW	1.86	0.98	0.90	1.68	1.30	3.42	3.32	2.30	2.68
3	Spindle speed/rpm	200	350	250	300	400	1200	1100	1500	900
	Processing time/min	6	5	4	4	2	3	4	2	5
	Unload power/kW	1.00	0.98	0.90	1.46	1.55	3.42	3.32	3.72	2.25
4	Spindle speed/rpm	400	400	250	150	250	1200	850	950	800
	Processing time/min	4	3	4	6	5	3	6	5	7
	Unload power/kW	1.86	1.12	0.90	1.32	1.20	3.42	2.94	2.30	1.99
5	Spindle speed/rpm	400	350	400	350	450	1200	1100	900	900
	Processing time/min	4	5	3	3	1	3	4	6	5
	Unload power/kW	1.86	0.98	1.43	1.68	1.70	3.42	3.32	2.12	2.25
6	Spindle speed/rpm	200	350	250	200	350	900	1200	800	900
	Processing time/min	6	5	4	5	3	4	3	9	5
	Unload power/kW	1.00	0.98	0.90	1.46	1.42	2.80	3.80	1.92	2.25
7	Spindle speed/rpm	250	600	250	300	200	900	1100	1200	900
	Processing time/min	5	2	4	4	6	4	4	3	5
	Unload power/kW	1.18	1.36	0.90	1.46	1.10	2.80	3.32	3.26	2.25
8	Spindle speed/rpm	500	350	250	125	250	1200	1200	900	1000
	Processing time/min	3	5	4	7	5	3	3	6	4
	Unload power/kW	2.10	0.98	0.90	1.22	1.20	3.42	3.80	2.12	2.68
9	Spindle speed/rpm	600	350	250	500	400	750	750	900	900
	Processing time/min	2	5	4	1	2	7	8	6	5
	Unload power/kW	2.26	0.98	0.90	2.14	1.55	2.14	2.60	2.12	2.25
10	Spindle speed/rpm	500	200	250	350	300	900	750	900	800
	Processing time/min	3	6	4	3	4	4	8	6	7
	Unload power/kW	2.10	0.80	0.90	1.68	1.30	2.80	2.60	2.12	1.99
11	Spindle speed/rpm	250	600	250	350	250	850	800	900	900
	Processing time/min	5	2	4	3	5	6	7	6	5
	Unload power/kW	1.18	1.36	0.90	1.68	1.20	2.54	2.76	2.12	2.25
12	Spindle speed/rpm	200	350	250	200	300	1200	1100	850	900
	Processing time/min	6	5	4	5	4	3	4	7	5
	Unload power/kW	1.00	0.98	0.90	1.46	1.30	3.42	3.32	2.04	2.25

(1) When the decision-maker wants to consider the minimum makespan, the importance weights can be set to:  $w_1=1$  and  $w_2=0$ . The improved genetic-simulated annealing algorithm was run 15 times for the FFS. Fig.4 describes plots of makespan versus energy consumption for a flexible flow-shop problem ( $12 \times 9$ ) with importance weights  $w_1=1$  and  $w_2=0$ . In addition, the quantitative analysis of energy consumption is obtained to analyze the makespan changes as shown in Table 2.

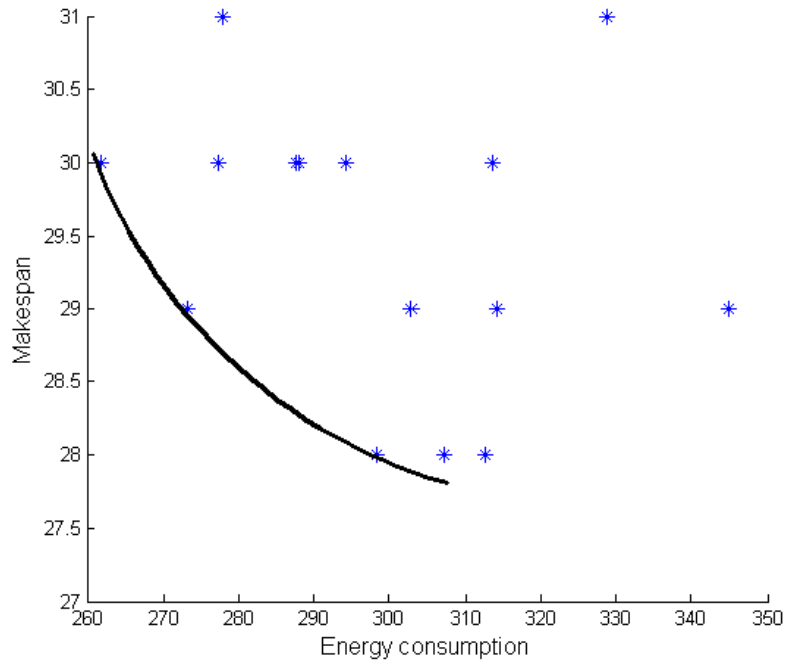


Fig. 4. Plots of makespan versus energy consumption for a FFS with  $w_1=1$  and  $w_2=0$

Table 2 Data of makespan and energy consumption with  $w_1=1$  and  $w_2=0$

<i>Number</i>	<i>Makespan</i> $f_1$	<i>Total energy consumption</i> $f_2$	<i>Idle energy consumption</i> $E_u$	<i>Energy consumption ratio</i> $E_u / f_2$
1	28	307.20	51.05	16.62
2	31	328.91	68.10	20.70
3	30	277.23	20.20	7.29
<b>4</b>	<b>28</b>	<b>298.36</b>	<b>44.64</b>	<b>14.96</b>
5	29	314.13	53.13	16.91
6	29	273.23	15.92	5.83
7	30	261.86	13.02	4.97
8	30	287.55	13.65	4.75
9	30	288.11	29.98	10.41
10	30	294.24	32.12	10.92
11	28	312.53	59.67	19.09
12	31	277.89	22.90	8.24
13	29	302.83	54.00	17.83
14	30	313.54	47.31	15.09
15	29	344.76	90.73	26.32

It can be observed that there exists a conflicting relationship in the Pareto frontier between the makespan and energy consumption.. When the makespan reaches the minimum value 28, the corresponding minimum value of the total energy

consumption is 298.36; on the contrary, when the total energy consumption gets one best near-optimal value 261.86, the makespan increases to 30. In other words, lower makespan will consume more energy and higher energy consumption will reduce the completion time of jobs. Besides, it is found that machine tools will consume a certain amount of energy during the idle running time. As time increases, idle energy consumption increases. The worst ratio between idle energy and total energy is near 26.32% and the average ratio reaches 13.32% in Table 2. Therefore, when only makespan is considered in the manufacturing system, the idle energy consumption cannot be neglected. The decision-maker should employ an energy saving model to determine if machine tools should be turned on or turned off.

(2) When the decision-maker wants to minimize energy consumption, the importance weights can be set to:  $w_1=0$  and  $w_2=1$ . The improved genetic-simulated annealing algorithm was run 15 times for the FFS. Fig. 5 describes plots of makespan versus energy consumption for a flexible flow-shop problem ( $12 \times 9$ ) with importance weights  $w_1=0$  and  $w_2=1$ . At the same time, the quantitative analysis of energy consumption is given for the analysis of the makespan changes as shown in Table 3.

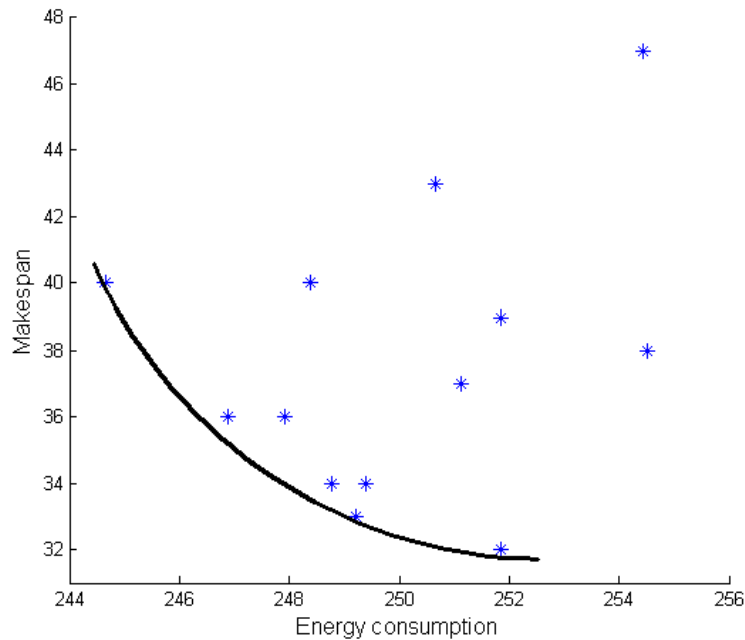


Fig. 5. Plots of makespan versus energy consumption for a FFS with  $w_1=0$  and  $w_2=1$

Table 3 Data of makespan and energy consumption with  $w_1=0$  and  $w_2=1$

Number	Makespan	Total energy consumption	Idle energy consumption	Energy consumption ratio
	$f_1$	$f_2$	$E_u$	$E_u / f_2$
1	33	249.22	0	0
2	34	249.40	0	0
3	39	251.84	1.70	0.68
4	37	251.12	3.42	1.36
5	40	244.64	3.78	1.55
6	43	250.64	1.46	0.58
7	36	246.88	1.46	0.59
8	38	254.52	10.73	4.22
9	48	249.97	3.10	1.24
10	36	247.92	0	0
11	48	255.53	0	0
12	34	248.77	10.50	4.22
13	47	254.44	4.92	1.93
14	40	248.38	4.38	1.76
15	32	251.84	0	0

From the results, on the one hand it can be observed that the relationship between the makespan and energy consumption is conflicting as in the first scenario in Section 5. On the other hand, machine tools' usage efficiency is higher obviously. The maximum idle energy consumption is not more than 5% on the total energy consumption and the average value sums up to 1.21% as illustrated in Table 3.

(3) When the decision-maker takes the makespan and energy consumption into account simultaneously, the importance weights can be set to:  $w_1=0.5$  and  $w_2=0.5$ . The improved genetic-simulated annealing algorithm was run 15 times for the FFS. Fig. 6 describes plots of makespan versus energy consumption for a flexible flow-shop problem ( $12 \times 9$ ) with importance weights  $w_1=0.5$  and  $w_2=0.5$ . Fig. 7 also illustrates that there exists a trade-off between the makespan and energy consumption. If the makespan can be realized as short as possible, the corresponding energy consumption increases. On the other hand, if the maximum completion time is less than due date, the energy consumption can be minimized.



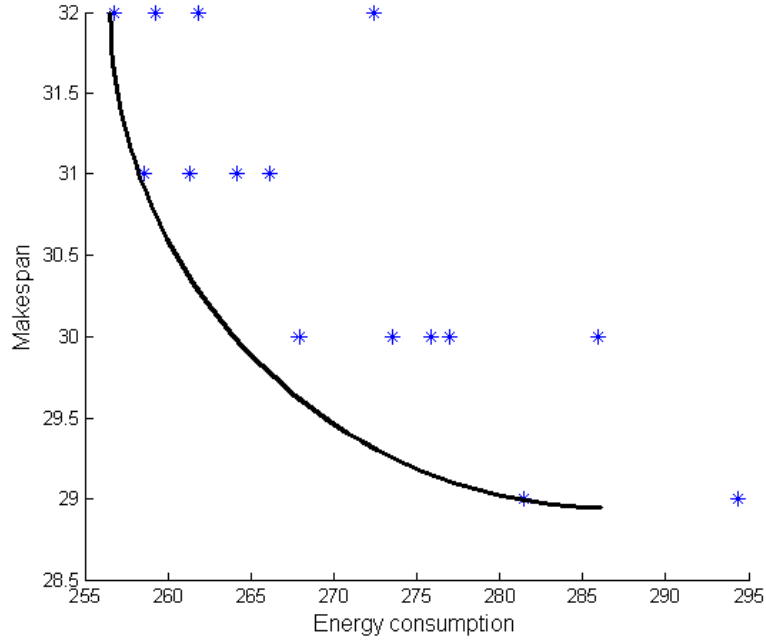


Fig. 6. Plots of makespan versus energy consumption for a FFS with  $w_1=0.5$  and  $w_2=0.5$

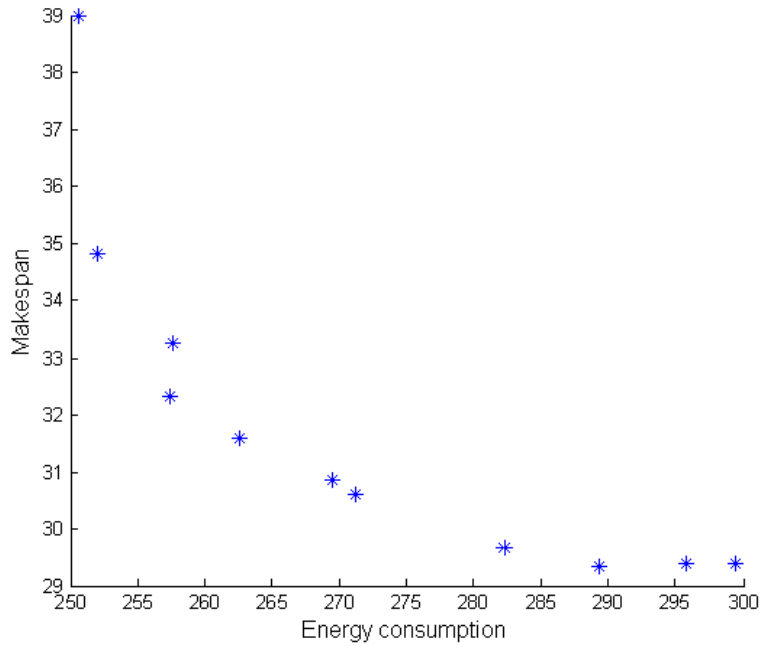


Fig. 7. Plots of makespan versus energy consumption with two important weights ranging from 0 to 1

(4) Furthermore, all sets of two important weights  $w_1$  and  $w_2$  are studied ranging from  $(w_1, w_2) = (0,1)$  to  $(w_1, w_2) = (1,0)$  with an increment of 0.1 and  $w_1+w_2=1$ . For each pair of important weights set, the improved genetic-simulated annealing algorithm was run 15 times for the FFS. Experimental result can be seen as shown in Fig. 6 and each point denotes the average value of these 15 runs. The result

demonstrates that different important weights generate different results on the basis of multi-objective function values. In a word, as  $w_1$  increases with an increment of 0.1, the makespan decreases and the energy consumption increases; as  $w_2$  increases with an increment of 0.1, the makespan decreases and the energy consumption increases. Therefore, it ensures to make a significant trade-off between makespan and total energy consumption for performing a feasible scheduling.

At the same time, due to the serious energy waste of the first scenario in Section 5 for the manufacturing system at the idle running mode stage, this paper implements an energy saving model to determine if machine tools should be on or off. For instance, in Fig.4 it is considered a Pareto optimal solution, i.e. the makespan value is 28 and the corresponding energy consumption value is 298.36. The production scheduling result is shown in Fig. 8. We can find that machine tools, such as M5, M6, M7, M8 and M9 are in the state of waiting for processing jobs when the interval time between jobs on the same machine is much longer. Hence, the mentioned energy saving model in Section 3 is executed to determine if the machine should be shut down or started up, and relative energy date calculation regarding this production scheduling is illustrated in Table 4.

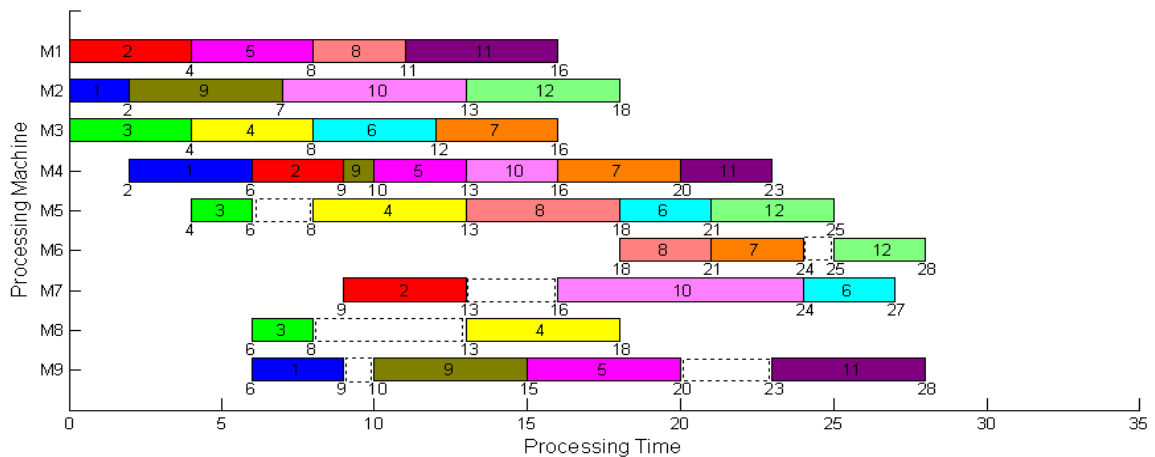


Fig. 8. The Gantt chart of the production scheduling

Table 4 Relative energy date of the production scheduling

<i>Number</i>	<i>Unload time/s</i>	<i>Spindle speed/rpm</i>	<i>Idle power/kW</i>	$SE_{sm} / J$	$1.2T_s / s$	<i>Energy saving ratio/%</i>
M5	120	250	1.1952	8065.4	8.4	94.37
M6	60	1200	3.4901	76782.2	35.2	63.33
M7	180	750	2.6675	32010.0	14.4	93.33
M8	300	950	2.3942	41898.5	21.0	94.17
M9	60 180	900 900	2.3146 2.3146	35876.3	18.6 18.6	74.17 91.39

We can conclude that the energy saving decision model, which is applied in a feasible scheduling, has the distinct potential to reduce the total energy consumption. As well we calculate that the average energy ratio sums up to 85.13% on the idle energy consumption. The idle energy consumption value decreases to 6.64 and the total energy consumption value becomes 260.36. In summary, the share of the idle energy consumption to the total energy consumption decreases by 12.72%. It is clear that performing an energy saving for a feasible scheduling is viable and efficient.

## 6. Conclusion

In this paper, we have explored the multi-objective energy-efficient scheduling problem with two objectives: makespan and energy consumption in manufacturing systems. To solve the multi-objective optimization problem a mathematical model based on an energy-efficient mechanism was proposed, which arises in a flexible flow-shop scheduling (FFS) problem. The establishment of the energy-aware model was only the first step and a theoretical work was conducted to generate Pareto efficient solutions using the weighted additive utility function technique. Moreover, an improved genetic-simulated annealing algorithm, inspired from hormone modulation mechanism, was employed to solve the multi-objective scheduling problem. Several optimization problems with importance weights ranging from 0 to 1, in a plant of metalworking workshop, were tested. For different scenarios, all the experimental results showed that the algorithm can identify a set of Pareto optimal solutions in the solution space, and on the other hand the relationship between the

makespan and the energy consumption was distinctly conflicting. Through making a trade-off between two objective functions, there will be a feasible scheduling.

At the same time, due to much energy waste in Section 5, this paper proposed an energy saving model to determine if machine tools should be on or off when they will be idle for an amount of time. The test results showed that the decision model has the significant potential to minimize energy consumption turning off and then turning on idle machines, if the inter-arrival time between jobs on the same machine is greater than the break-even duration. In future research, uncertainty events such as machine breakdown, new jobs arrival and existing jobs cancellation should be considered in an energy-aware flexible flow-shop scheduling problem. Moreover, an energy-efficient dynamic scheduling model will be included in the future.

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