Energy Efficient Sleep/Wake Scheduling for Multihop Sensor Networks: Non-convexity and Approximation Algorithm

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Presentation Outline

- Introduction
- System Model
- Optimal Sleep/Wake Scheduling Problem
  - PART 1: Single-hop Intra-Cluster
  - PART 2: Multi-hop
- Conclusion

Introduction

- Contribution of the Paper
  - When to wakeup/sleep?
    - Single-hop
    - Multi-hop
  - Uniqueness of the problem
    - Consider synchronization error
  - Technical merit of the paper
    - Non-convex Optimization
      - ☐ Formulate into Convex Optimization
      - ☐ Approximation Method (0.73-approximation algorithm)

Introduction

- Motivation
  - Idle listening consumptions a proportion of total energy
  - Impact of synchronization error is non-negligible

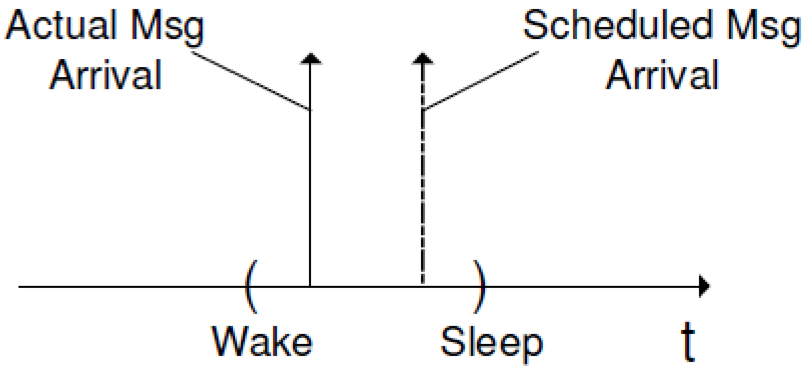

Fig. 3. Wake up interval to capture the message

- Tradeoff
  - Energy consumption vs. Message delivery performance

Introduction

- Optimal sleep/wake scheduling algorithm
  - Goal: min energy consumption (max lifetime)
  - Condition: Satisfies QoS
    - Single-hop: per-hop message capture probability threshold
    - Multi-hop: source (sensors) to destination (BS)

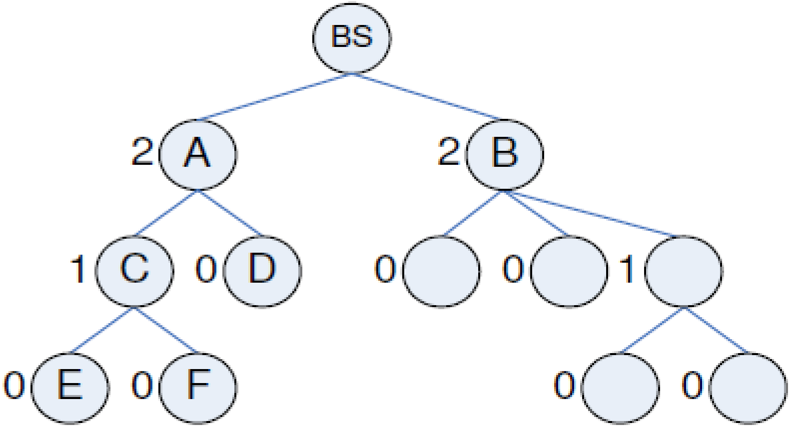

Fig. 1. A three-level cluster hierarchy

System Model

- Cluster Hierarchy
  - Existing clustering algorithms

- Scheduling
  - TDMA (Sync + M Tx)
  - CH - CMs

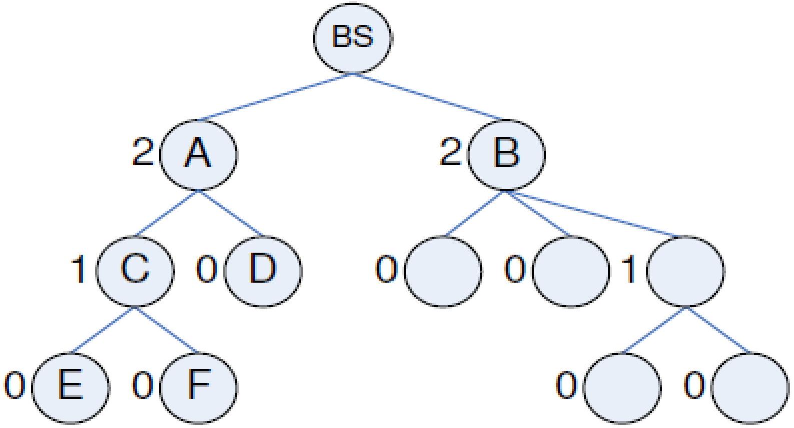

Fig. 1. A three-level cluster hierarchy

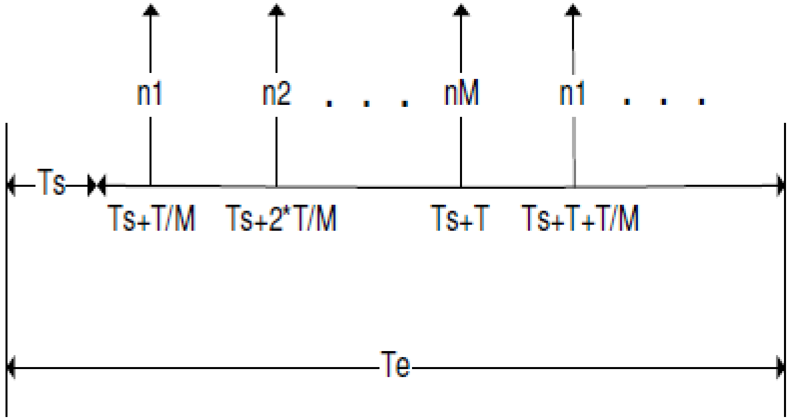

Fig. 2. Equispaced upstream transmissions

System Model

- Assumptions
  - Neighboring clusters do not interfere with each other
    - Orthogonal frequency channel (1000's of channels available)
  - Data aggregation
    - Length of the aggregated message:

$$M \text{ members}, i = 1, \dots, M$$

$$\chi(L_0, \dots L_M) = r \sum_{i=0}^{M} L_i + c. \tag{1}$$

CH "compression ratio" "length of message" "overhead"

-
  - Negligible collision probability
    - large separation distance between transmissions
  - Negligible propagation delay

Problem Formulation

- Synchronization
  - Phase offset
  - Clock skew

$$\underset{\text{“node i time”}}{\underset{\searrow}{t_i(j,k)}} = a_i(j)\underset{\text{“CH time”}}{\underset{\swarrow}{C(j,k)}} + b_i(j) + e_i(j,k), \tag{2}$$

Problem Formulation

- Goal
  - When to wake/sleep?

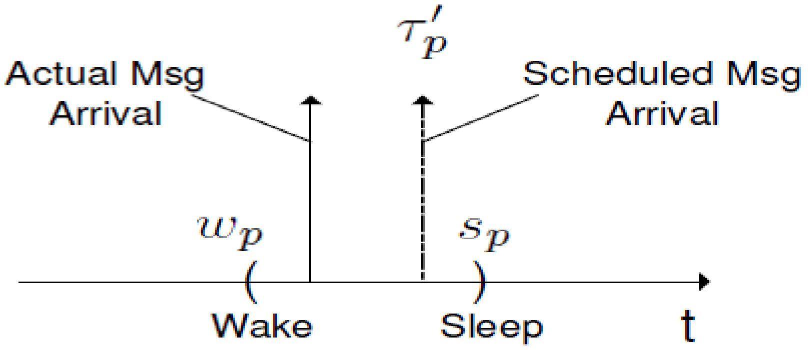

Fig. 3. Wake up interval to capture the message

E = (if arrival outside wp and sp → E_idle) + (if arrival inside wp and sp →E_idle + E_receive)

$$\text{(A)}\ \ \text{Min}\ \ E = (s_p - w_p)\alpha_I Prob\{\tau'_p \notin (w_p, s_p)\} + \longrightarrow \text{Outside}$$

$$\int_{w_p}^{s_p} \{(x - w_p)\alpha_I + \frac{L_p}{R}\alpha_r\} f_{\tau'_p}(x)dx \longrightarrow \text{Inside}$$

$$\text{such that } Prob\{\tau'_p \in (w_p, s_p)\} \geq th,$$

$f_{\tau'_p}(\cdot)$ (PDF) of τ'_p

Single-hop

- Simplification
  - Normal distribution: $f_{\tau'_p}(\cdot)$ (PDF) of τ'_p

$$\text{(A) Min } E = (s_p - w_p)\alpha_I Prob\{\tau'_p \notin (w_p, s_p)\} + \int_{w_p}^{s_p} \{(x - w_p)\alpha_I + \frac{L_p}{R}\alpha_r\} f_{\tau'_p}(x)dx$$

$$\text{such that } Prob\{\tau'_p \in (w_p, s_p)\} \geq th,$$

$$E(\tau'_p) = \tau_p, \qquad (4)$$

$$VAR(\tau'_p) \equiv \sigma_p^2 = \frac{\sigma_0^2}{a_i^2(j)} \frac{1}{N_s}[1 + \frac{(\tau_p - \overline{C(j,k)})^2}{\overline{C^2(j,k)} - (\overline{C(j,k)})^2}],$$

$$\text{where } \overline{C(j,k)} = \frac{\sum_{k=1}^{N_s} C(j,k)}{N_s}, \overline{C^2(j,k)} = \frac{\sum_{k=1}^{N_s} C^2(j,k)}{N_s}.$$

$$\hat{\tau} = \frac{\tau'_p - \tau_p}{\sigma_p}, w = \frac{w_p - \tau_p}{\sigma_p}, s = \frac{s_p - \tau_p}{\sigma_p}$$

$$\text{(A1) Min } F(w,s) = (s - w)\sigma_p\alpha_I - [Q(w) - Q(s)]s\sigma_p\alpha_I + [g(w) - g(s)]\sigma_p\alpha_I + [Q(w) - Q(s)]\frac{L_p}{R}\alpha_r,$$

$$\text{such that } Q(w) - Q(s) \geq th,$$

$g(\cdot)$ is the PDF for the standard normal distribution

$Q(\cdot)$ is the complementary cumulative distribution function

Part 1: Single-hop

- Non-Convexity of (A1)
  - Hessian Matrix

$$\text{(A1) Min } F(w,s) = (s-w)\sigma_p\alpha_I - [Q(w)-Q(s)]s\sigma_p\alpha_I + [g(w)-g(s)]\sigma_p\alpha_I + [Q(w)-Q(s)]\frac{L_p}{R}\alpha_r,$$
$$\text{such that } Q(w)-Q(s) \geq th,$$

Proposition 1: For Problem (A1), any optimal solution (w^*, s^*) satisfies $Q(w^*) - Q(s^*) = th$.

$$\text{(A2) Min } G(w) = (1-th)s(w) - w + g(w) - g(s(w)),$$
$$\text{such that } s(w) = Q^{-1}(Q(w)-th) \text{ and } w < Q^{-1}(th).$$

(Convex Optimization) Second derivative of G(w) > 0

Single-hop

- Finally!

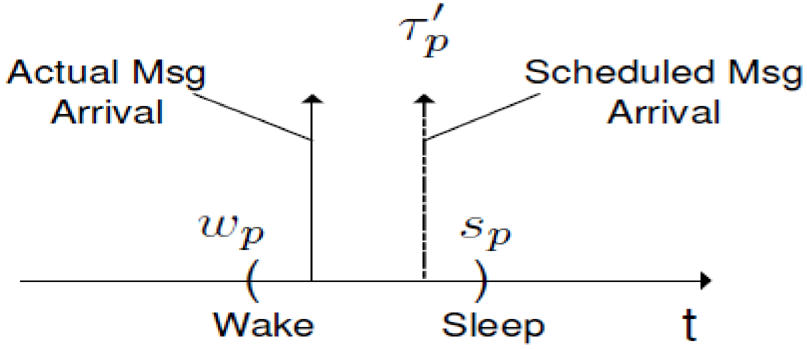

Fig. 3. Wake up interval to capture the message

After we obtain w^*, s^*, we compute the optimal sleep/wake schedule as $(w^*_p = \tau_p + w^*\sigma_p, s^*_p = \tau_p + s^*\sigma_p)$.

Part 2: Multi-hop

- Different from the single hop
  - Per-hop threshold → Src. to Dest. QoS

- System Model (Modified)
  - Cluster Head of node n: H(n)
  - d(n): hop distance to node n from H
  - M(n): members of node n
  - D(n): descendants of node n

$$H^{(d(n))}(n) \equiv \underbrace{H(H(\ldots H(n)\ldots))}_{d(n)} = BS.$$

Ex) node A = n
H(A) = BS
d(n) = 1
M(n) = C, D
D(A) = C, D, E, F

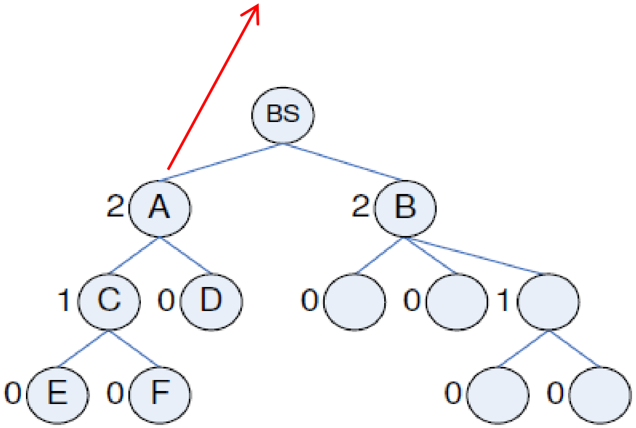

Fig. 1. A three-level cluster hierarchy

Multi-hop

- Goal

$$\text{(B) Max } T_L$$
$$\text{such that } \prod_{i=0}^{d(n)-1} z(H^{(i)}(n)) \geq \Lambda, \forall n \in S,$$

z(n): capture probability threshold of H(n)

- Solution

Satisfy "leaf node" → Satisfy "all ancestors" (on the path to the leaf node)

$$\text{(B) Max } T_L$$
$$\text{such that } \prod_{i=0}^{d(n)-1} z(H^{(i)}(n)) \geq \Lambda, \forall n \in \underline{LF},$$

Multi-hop

- System Model
  - Message Aggregation

$$L^{avg}(n) = r(l + \sum_{i \in M(n)} z(i) L^{avg}(i)) + c.$$

$$L^{avg}(n) = rl + c + \sum_{i \in D(n)} (rl + c) \prod_{k=0}^{d(i)-d(n)-1} [rz(H^{(k)}(i))]. \qquad (8)$$

  - Energy Consumption
    - Avg. Consumption by node n

$$\begin{aligned} \eta(n, \overrightarrow{z}) &= \frac{\varepsilon_s(n) + \varepsilon_{syn}(n) + \varepsilon_t(n) + \varepsilon_r(n)}{T_e} \qquad (12) \\ &= A(n) + \sum_{i \in M(n)} P(n,i)\gamma(z(i)) + \\ &\quad \sum_{i \in D(n)} Q(n,i) \prod_{k=0}^{d(i)-d(n)-1} z(H^{(k)}(i)), \end{aligned}$$

where

$$A(n) = \frac{1}{T_e}[\varepsilon_s(n) + \varepsilon_{syn}(n) + N\alpha_t(n)\frac{rl+c}{R}],$$

$$P(n,i) =$$

$$\frac{1}{T_e}\sum_{h=1}^{N}\alpha_I\sqrt{\sigma_0^2\frac{1}{N_s}[1 + \frac{(T_s + \frac{\theta(i)T}{|M(n)|} + hT - \frac{1+N_s}{2}\frac{T_s}{N_s})^2}{\frac{\sum_{k=1}^{N_s}(k\frac{T_s}{N_s} - \frac{1+N_s}{2N_s}T_s)^2}{N_s}}]},$$

$$Q(n,i) = \frac{1}{T_e}\frac{rN\alpha_t(n) + N\alpha_r}{R}(rl+c)r^{d(i)-d(n)-1}.$$

Multi-hop

- Formulation + Energy Conditions

$$\text{(B) Max } T_L$$
$$\text{such that } \prod_{i=0}^{d(n)-1} z(H^{(i)}(n)) \geq \Lambda, \forall n \in LF,$$

Add

$$\text{Max } T_L$$
$$\text{such that } \prod_{i=0}^{d(n)-1} z(H^{(i)}(n)) \geq \Lambda, \forall n \in LF,$$
$$\eta(n, \overrightarrow{z}) \leq \xi(n)/T_L, \forall n \in S.$$

"avg. energy consumption" "initial energy"

Introduce life-time penalty function: $\Psi(1/T_L)$ $u = 1/T_L$

Equivalent problem

$$\text{(B) Min } \Psi(u)$$
$$\text{such that } \prod_{i=0}^{d(n)-1} z(H^{(i)}(n)) \geq \Lambda, \forall n \in LF,$$
$$\eta(n, \overrightarrow{z}) \leq \xi(n)u, \forall n \in S.$$

Multi-hop

- Non-Convex Optimization Problem

$$
\begin{aligned}
&\text{(B) Min } \Psi(u)\\
&\quad \text{such that } \prod_{i=0}^{d(n)-1} z(H^{(i)}(n)) \geq \Lambda, \forall n \in LF,\\
&\quad\quad \eta(n,\overrightarrow{z}) \leq \xi(n)u, \forall n \in S.
\end{aligned}
$$

$$
\begin{aligned}
\eta(n,\overrightarrow{z}) &= \frac{\varepsilon_s(n)+\varepsilon_{syn}(n)+\varepsilon_t(n)+\varepsilon_r(n)}{T_e} \qquad (12)\\
&= A(n) + \sum_{i\in M(n)} P(n,i)\gamma(z(i)) +\\
&\quad \sum_{i\in D(n)} Q(n,i) \prod_{k=0}^{d(i)-d(n)-1} z(H^{(k)}(i)),
\end{aligned}
$$

Non-convex

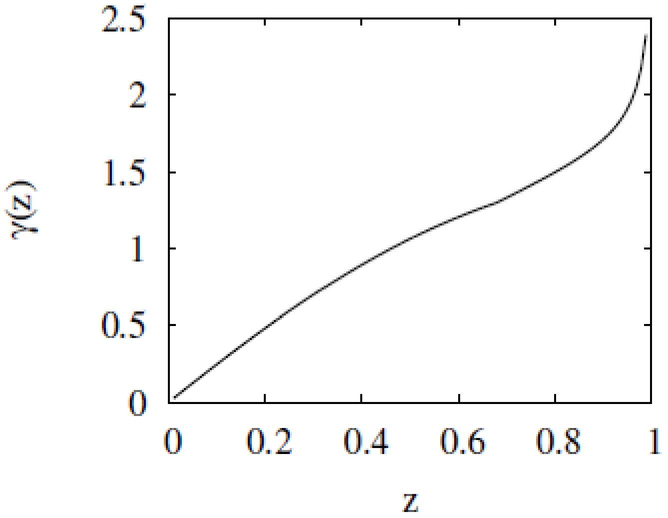

Fig. 4. $\gamma(z)$

$$\gamma(th) = \min\{G(w) : w < Q^{-1}(th)\} \qquad (7)$$

Multi-hop

- Approximation

Proposition 3: (1) For $z \geq 0.86$, $\gamma(z)$ is strictly convex; (2) For $z \in [0, 0.99], 1.86z < \gamma(z) < 2.52z$.

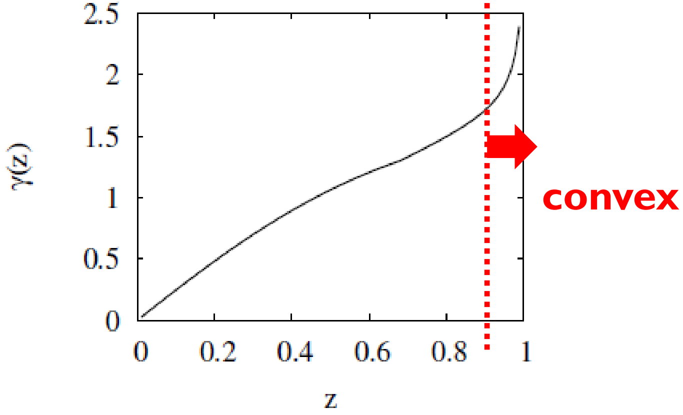

Fig. 4. $\gamma(z)$

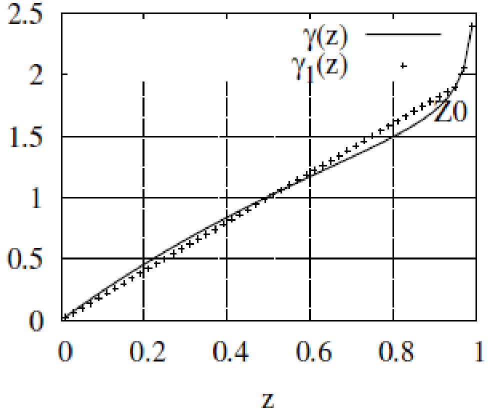

Fig. 5. Approximating $\gamma(z)$

Approximation (Fig. 5): $\gamma_1(z) = \begin{cases} 2z + 0.001z^2 & 0 \leq z \leq Z_0 \\ \gamma(z) & Z_0 \leq z < 1 \end{cases}$ $Z_0 \approx 0.95$

Multi-hop

- How close is the approximation to the optimal?
  - Achievable approximation ratio: **0.73**

Proposition 4: (1) $0.929 \leq \gamma(z)/\gamma_1(z) \leq 1.26$; (2) $\gamma_1(z)$ is strictly convex.

$$T_L(\overrightarrow{z^*}) \leq 1/(0.929u_1^*)$$

$$T_L(\overrightarrow{z_1^*}) \geq 0.73T_L(\overrightarrow{z^*})$$

Multi-hop

- Solve B1 …

(B1') Min $\Psi(u)$

such that $\sum_{i=0}^{d(n)-1} v(H^{(i)}(n)) \geq \ln \Lambda, \forall n \in LF,$

$$\eta_1'(n, \overrightarrow{v}) = A(n) + \sum_{i \in M(n)} P(n,i)\gamma_1(e^{v(i)}) +$$

$$\sum_{i \in D(n)} Q(n,i) e^{\sum_{k=0}^{d(i)-d(n)-1} v(H^{(k)}(i))} \leq \xi(n)u, \forall n \in S.$$

convex objective

convex constrains

dual formulation

$$\max_{\overrightarrow{\lambda} \geq 0, \overrightarrow{\mu} \geq 0} \Phi(\overrightarrow{\lambda}, \overrightarrow{\mu}),$$

$\overrightarrow{\lambda}, \overrightarrow{\mu}$ are Lagrange multipliers

$$\Phi(\overrightarrow{\lambda}, \overrightarrow{\mu}) = \min_{u \geq 0, \overrightarrow{v} < 0} \Psi(u) + \sum_{n \in LF} \lambda_n (\ln \Lambda - \quad (15)$$

$$\sum_{i=0}^{d(n)-1} v(H^{(i)}(n))) + \sum_{n \in S} \mu_n (\eta_1'(n, \overrightarrow{v}) - \xi(n)u).$$

- Then, use "subgradient method" to solve dual problem.

Simulation Results

- Vs. scheme with "equal" thresholds
- Capture probability: 0.7

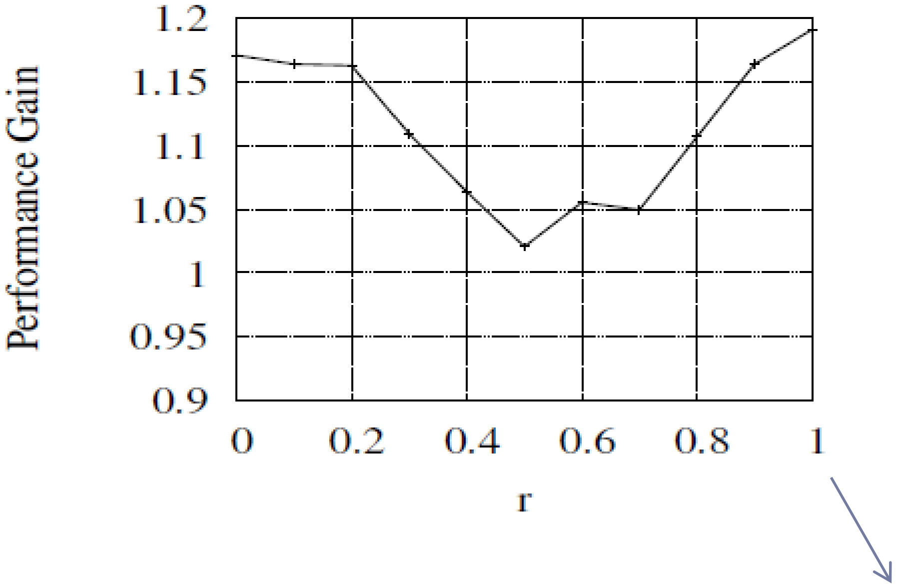

Fig. 7. Performance gain

r: compression ratio
r=1, no compression

Simulation Results

- r=1(no compression)
  - Node 1 becomes bottleneck, z(2) and z(3) should be small
- r=0 (high compression)
  - Node 2 and 3 becomes bottleneck (to receive from more member nodes)

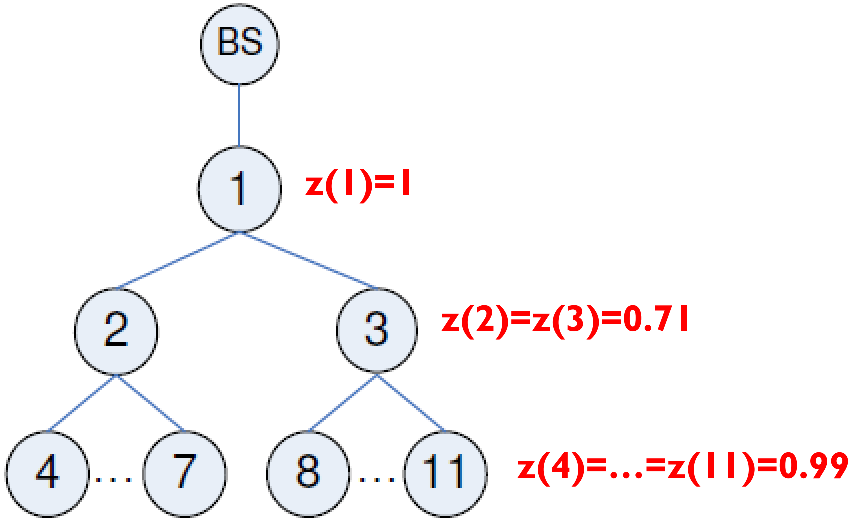

Fig. 6. Simulation topology

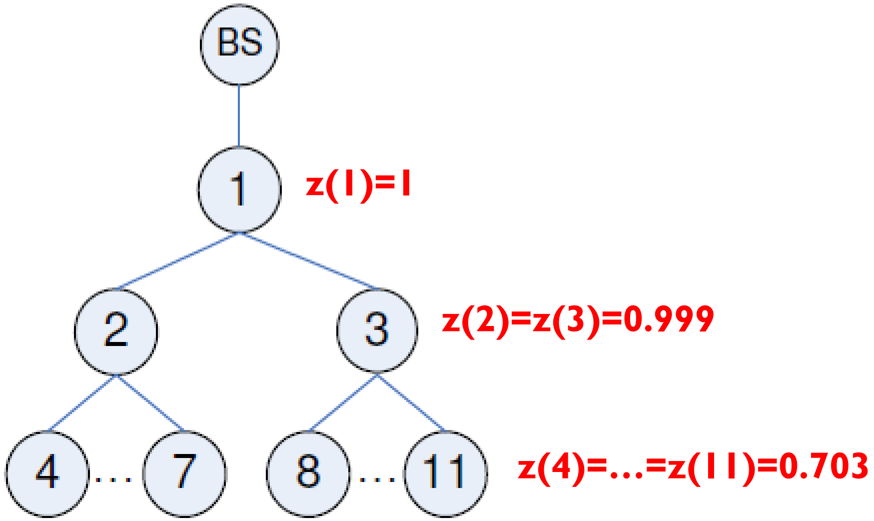

Fig. 6. Simulation topology

Conclusion

- Sleep/wake scheduling for low duty-cycle sensor networks
- Consider synchronization error
- Achieve given capture probability threshold with min energy consumption
  - Non-convex optimization
  - 0.73-approximation algorithm
  - Correctly identifies the bottlenecks to extend the network lifetime