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Energy Levels and Electromagnetic Transition of Some Even-Even Xe Isotopes

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Abstract

In this work, the energy levels electromagnetic transition B(E2) and B(M1), branching ratios, mixing ratios and electric quadrupole moment of even-even $^{120\text{-}126}$Xe isotopes have been investigated using Interacting Boson Model (IBM-1). The results were compared with some previous experimental and theoretical values, it was seen that the obtained theoretical results are in agreement with the experimental data.

Keywords: IBM-1, energy levels, electromagnetic transition, mixing ratio

1. Introduction

The interacting boson approximation represents a significant step towards our understanding of nuclear structure. It offers a simple Hamiltonian, capable of describing collective nuclear properties across a wide range of nuclei, and is founded on rather general algebraic group theoretical techniques which have also recently found application to problems in atomic, molecular, and high-energy physics (Firestone, 1996, Pan & Draayer, 1998). The IBM-1 is a valuable interactive model developed by Iachello and Arima (Iachello & Arima, 1974, Arima & Iachello, 1975). It has been successful in describing the collective nuclear structure by prediction of low lying states and description of electromagnetic transition rates in the medium mass nuclei. IBM-1 defines a six-dimensional space described in terms of the unitary group, U(6). Different reductions of U(6) give three dynamical symmetry limits known as harmonic Oscillator, deformed rotator and asymmetric deformed rotor which are labeled by U(5), SU(3) and O(6), respectively (Kumar *et al*., 2010, Cejnar *et al*., 2010).

Xenon isotopes belong to a very interesting but complex region of the periodic Table known as the transition region. The Xe isotopes can exhibit excitation spectra close to the O(6) symmetry. After some theoretical investigations it was concluded that the xenon isotopes should lie in a transitional region from U(5)- to an O(6)-like structure as the neutron number decreases from the closed shell N=82 (Casten & Von Brentano, 1985). The chain of $^{120\text{-}134}$Xe isotopes are interesting because of the existence of transitional nuclei where the nuclear structure changes from rotational to vibrational shapes. Many authors studied this area of isotopes experimentally and theoretically. IBM-1 model has been used in calculating the energy of the positive parity low-lying levels of Xe series of isotopes (Dilling *et al*., 2002). For the neutron number $66 \leq N \leq 72$, the energy ratio $E4^+/E2^+$ is almost 2.5 already pointing to γ-soft shapes. In recent years many works have been done on the structure of Xenon isotopes; Kusakari and M. Sugawara (Kusakari & Sugawara, 1984) calculated the energy levels, the back-bending in the yrast bands , reduced transition probabilities for the positive-parity states of $^{122\text{–}130}$Xe within the framework of the interacting boson model. Xing-Wang Pan *et al* (Pan *et al*., 1996) and Al-Jubbori (Al-Jubbori *et al.,* 2016) described the low-lying energy levels for even-even and even-odd nuclei $^{126\text{-}132}$Xe and $^{131\text{-}137}$Ba by a unified analytical expression with two (three) adjustable parameters of the fermion model. Wang Bao-lin (Bao-lin, 1997) calculated the triaxial deformation parameters β and γ in the O(6)-like nuclei by comparing quadrupole moments between the interacting boson model IBM and the collective model studied. A.D. Efimov *et al* (Efimov *et al*., 2002) studied B(E2; $0_1^+ \rightarrow 2_1^+$) values in the intruder bands of ^{112,114,116,118}Sn within the framework of the interacting boson model (IBM1) and comparison with the ground state bands in the even-mass Xe isotopes to find a similarity not only for the energy spaces, but for the B (E2) values as well. B. Saha *et al* (Saha *et al*., 2004) calculated the B(E2) and B(M1) values for the isotope of ^{124}Xe within the framework of the interacting boson model (IBM1).

Nureddin Turkan (Turkan, 2007) described the quadrupole collective states of the medium-heavy nuclei within the framework of the interacting boson model (IBM) and calculated the energy levels and B(E2) values for the even-even $^{122\text{-}134}$Xe isotopes. The results were compared with the previous experimental and theoretical data and it has been observed that they are in good agreement. Ismail Maras *et al* (Maras *et al*., 2010) studied the ground state, quasi beta and quasi gamma band energies for the ^{114,116,118,120}Xe isotopes using the interacting boson model (IBM) and calculated the energy levels using PHINT and NPBOS program codes. M. A. Jafarizadeh *et al* (Jafarizadeh *et al*., 2013) studied the properties of $^{114–134}$Xe isotopes in the U(5)↔SO(6) transitional region of IBM and calculated the energy levels and B(E2) transition rates. Zhang Da-Li and Ding Bin-Gang (Da-Li & Bin-Gang, 2013) studied the structure evolution of the $^{124\text{-}134}$Xe isotopic chain in the framework of the proton-neutron interacting model (IBM2), they have been calculated the B(E2) transition branching ratios, and the M1 excitations. L. Coquard *et al* (Coquard *et al*., 2011) described the low-lying collective states in ^{126}Xe and calculated the B(E2) values within the framework of the IBM which show a good agreement with the measured values. L Prochniak (Próchniak, 2015) studied the collective properties of the even-even $^{118–144}$Xe isotopes within a model employing the general Bohr Hamiltonian, calculated the low energy spectra and B(E2) transition probabilities and compared the results with the experimental values.

The aim of the present work is study the energy levels, B(E2) and B(M1) values and explore the description of E2/M1 mixing ratios using IBM-1 for $^{120\text{-}128}$Xe isotopes. Furthermore, calculate the value of electric quadrupole (Q_J) of these isotopes within the framework of the IBM-1 and comparing the results with the most recent experimental data and previous studies.

2. Method

The IBM-1 of Arima and Iachello (Arima & Iachello, 1976) has become widely accepted as a tractable theoretical scheme of correlating, describing and predicting low-energy collective properties of complex nuclei. In this model, it was assumed that low-lying collective states of even-even nuclei could be described as states of a given number N of bosons. Each boson could occupy two levels one with angular momentum (L = 0) (s-boson) and another, usually with higher energy, with (L = 2) (d-boson) (Sharrad *et al*., 2012). In the original form of the model known as IBM-1, proton- and neutron-boson degrees of freedom are not distinguished. The model has an inherent group structure, associated with it. The IBM-1 Hamiltonian can be expressed as (Casten & Cizewski, 1978, Abrahams *et al*., 1981, Iachello & Arima, 1987).

$$\begin{aligned} \mathrm{H} = {} & \varepsilon_s(s^\dagger.\tilde{s}) + \varepsilon_d(d^\dagger.\tilde{d}) + \textstyle\sum_{L=0,2,4}\frac{1}{2}(2L+1)^{\frac{1}{2}}\, C_L\left[[d^\dagger\times d^\dagger]^{(L)}\times[\tilde{d}\times\tilde{d}]^{(L)}\right]^{(0)} \\ & +\frac{1}{\sqrt{2}}\upsilon_2\left[[d^\dagger\times d^\dagger]^{(2)}\times[\tilde{d}\times\tilde{s}]^{(2)}+[d^\dagger\times s^\dagger]^{(2)}\times[\tilde{d}\times\tilde{d}]^{(2)}\right]^{(0)} \\ & +\frac{1}{2}\upsilon_0\left[[d^\dagger\times d^\dagger]^{(0)}\times[\tilde{s}\times\tilde{s}]^{(0)}+[s^\dagger\times s^\dagger]^{(0)}\times[\tilde{d}\times\tilde{d}]^{(0)}\right]^{(0)} \\ & +\tfrac{1}{2}u_0\left[[s^\dagger\times s^\dagger]^{(0)}\times[\tilde{s}\times\tilde{s}]^{(0)}\right]^{(0)} + u_2\left[[d^\dagger\times s^\dagger]^{(2)}\times[\tilde{d}\times\tilde{s}]^{(2)}\right]^{(0)} \end{aligned} \quad (1)$$

This Hamiltonian contains two terms of one body interactions, (ε_s and ε_d), and seven terms of two-body interactions [C_L (L =0 ,2,4), ν_L (L =0 ,2), u_L (L =0 ,2)], where ε_s and ε_d are the single-boson energies, and C_L, ν_L and u_L describe the two boson interactions. However, it turns out that for a fixed boson number N, only one of the one-body terms and five of the two body are terms independent, as it can be seen by noting $N = n_s + n_d$. Equation (1) can be rewritten in terms of the Casimir operators of U(6) group. In that case, one says that the Hamiltonian (H) has a dynamical symmetry. These symmetries are called U(5) vibrational, SU(3) rotational and O(6) γ-unstable (Casten & Warner, 1988, Kassim & Sharrad, 2014).

3. Results

The obtained results can be discussed separately for energy levels, transition probabilities B(E2) and quadrupole moment Q_J, B(M1) values and mixing ratio.

3.1 Energy levels

The γ-unstable limit of IBM-1 has been applied for $^{120–126}$Xe nuclei due to the values of the experimental energy ratios (E2 :E4 :E6 :E8 =1:2 .5:4 .5:7). Therefore, these nuclei have γ-unstable dynamical symmetry O(6) with respect to IBM-1. The adopted Hamiltonian is expressing as (Casten & Warner, 1988, Iachello, 2001):

$$\hat{H} = a_0\hat{P}.\hat{P} + a_1\hat{L}.\hat{L} + a_3\hat{T}_3.\hat{T}_3 \quad (2)$$

The Xenon isotopes have a number of proton bosons 2, and number of neutron bosons varies from 7 to 10. The parameters value used in the present work are presented in Table 1.

Table 1. Adopted values for the parameters used for IBM-1 calculations. All parameters are given in MeV, except N.

Isotopes	N	a_0	a_1	a_3
^{120}Xe	10	0.082609	0.012133	0.178435
^{122}Xe	9	0.114918	0.015834	0.168768
^{124}Xe	8	0.14099	0.019322	0.170068
^{126}Xe	7	0.164235	0.022538	0.181002

The calculations of the g-bands, β-bands and γ-bands are compared with the experimental data (http://www.nndc.bnl.gov/chart/getENSDFDatasets.jsp, Kitao *et al*.,2002, Tamura, 2007, Katakura & Wu, 2008, Katakura & Kitao, 2002) for all isotopes under study, and as it given in Table 2. In this Table, one can see a agreement between experimental data and the IBM-1 calculations. Levels with '*' correspond to cases for which the spin and/or parity of the corresponding states are not well established experimentally.

Table 2. The calculated excitation of the g-,β-,and γ-bands and comparison with the experimental data (http://www.nndc.bnl.gov/chart/getENSDFDatasets.jsp, Kitao *et al*.,2002, Tamura, 2007, Katakura & Wu, 2008, Katakura & Kitao, 2002).

^{120}Xe			^{122}Xe	
J^{π}	IBM	EXP	IBM	EXP
2_g^+	0.3226	0.3226	0.3317	0.33128
4_g^+	0.7780	0.79616	0.8240	0.82853
6_g^+	1.3664	1.39733	1.4769	1.46705
8_g^+	2.0876	2.09920	2.2904	2.21769
10_g^+	2.9417	2.87270	3.2645	3.03988
12_g^+	3.9286	3.67651	4.3992	4.1514*
14_g^+	5.0484	4.45892	5.6945	5.2361*
2_γ^+	0.8576	0.87610	0.8387	0.84313*
3_γ^+	1.5369	1.27170	1.5084	1.21434*
4_γ^+	1.4914	1.40134	1.5000	1.40271*
5_γ^+	2.3263	1.81698*	2.3345	1.77450*
6_γ^+	2.2581	1.98563	2.3219	2.05666*
7_γ^+	3.2486	2.46088	3.3212	2.8472*
8_γ^+	2.9962	2.65383*	3.3044	2.8472*
9_γ^+	4.3038	3.17411	4.4685	3.2161*
10_γ^+	3.8503	3.32643*	4.4145	3.6085*
0_β^+	0.9086	0.90870	1.1500	1.14918
2_β^+	1.2312	1.27443*	1.4817	1.49501
4_β^+	1.6867	1.71175*	1.9740	----
6_β^+	2.2750	2.72734*	2.6269	----
8_β^+	3.1577	----	3.4404	----

Continued

^{124}Xe			^{126}Xe	
J^{π}	IBM	EXP	IBM	EXP
2_g^+	0.3538	0.35403	0.3887	0.38863
4_g^+	0.8960	0.87892	0.9940	0.94200
6_g^+	1.6266	1.54846	1.8159	1.63498
8_g^+	2.5456	2.33104	2.8544	2.43571
10_g^+	3.6530	3.17144	4.1095	3.31414
12_g^+	4.9488	4.29914*	5.5812	3.88457

14_g^+	6.4330	5.5188*	7.2659	4.61945
2_γ^+	0.8638	0.84650	0.9317	0.87987
3_γ^+	1.5576	1.24763	1.6824	1.31768
4_γ^+	1.5760	1.43796	1.7180	1.48838
5_γ^+	2.4490	1.83692	2.6675	1.90349
6_γ^+	2.4766	2.14374	2.7209	2.21431
7_γ^+	3.5288	2.57461	3.8692	2.66142
8_γ^+	3.5656	2.91213	3.9404	3.11719*
9_γ^+	4.7970	3.34391*	5.2875	3.38378*
10_γ^+	4.8430	4.0190*	5.3765	3.35971
0_β^+	1.2690	1.26891	1.3136	1.31388
2_β^+	1.6228	1.62857	1.7023	1.678569
4_β^+	2.1650	2.01473*	2.3076	1.90313
6_β^+	2.8956	2.64765*	3.1295	2.9739*
8_β^+	3.8146	3.0132*	4.1680	----

3.2 Reduced transition probabilities B(E2) and quadrupole moment (Q_J)

The general form of the electromagnetic transitions operator in IBM-1 is (Iachello & Arima, 1987, Casten & Warner, 1988, Yazar & Erdem, 2008):

$$\hat{T}(L) = \gamma_0[\hat{s}^\dagger \times \hat{s}^\sim]^{(0)} + \alpha_2[\hat{d}^\dagger \times \hat{s}^\sim + \hat{s}^\dagger \times \hat{d}^\sim]^{(2)} + \beta_L[d^\dagger \times \tilde{d}]^{(L)}, \quad (3)$$

where γ_0,α_2 and β_L (L =0 ,1,2,3,4) are parameters specifying the various terms in the corresponding operators. Equation (3) yields transition operators for E2, and M1 transitions with appropriate values of the corresponding parameters. The E2 transition operator must be a Hermiston tensor of rank two and therefore the number of bosons must be conserved. Since, with these constraints, there are two operators possible in the lowest order, the general E2 operator can be written as (Abrahams et al., 1981,Casten & Warner, 1988):

$$\hat{T}(E2) = \alpha_2[d^\dagger\tilde{s} + s^\dagger\tilde{d}]^{(2)} + \beta_2[d^\dagger\tilde{d}]^{(2)} = \alpha_2([d^\dagger\tilde{s} + s^\dagger\tilde{d}]^{(2)} + \chi[d^\dagger\tilde{d}]^{(2)}) = e_B\hat{Q} \quad (4)$$

where ($s^\dagger, d^\dagger$) and ($\tilde{s}$, $\tilde{d}$) are creation and annihilation operators for s and d bosons, respectively (Casten & Warner, 1988), while α_2 and β_2 are two parameters, and ($\beta_2=\chi\alpha_2$, $\alpha_2 = e_B$ (effective charge) and the quadrupole operator $\hat{Q} = ([d^\dagger\tilde{s}+ s^\dagger\tilde{d}]^{(2)} + \chi[d^\dagger\tilde{d}]^{(2)})$. The electric transition probabilities B(E2) values are defined in terms of reduced matrix elements as (Casten & Warner, 1988, Yazar & Erdem, 2008):

$$B\big((E2), J_i \to J_f\big) = \frac{1}{2J_{i\to}+1}\left|\langle J_f \| \hat{T}(E2) \| J_i\rangle\right|^2 \quad (5)$$

The parameter (e_B) used in present calculated the reduced probability for E2 transitions of the IBM-1 model are tabulated in Table 3. The values of effective charge (e_B) were estimated to reproduce the experimental$B(E2;\ 2_1^+ \to 0_1^+)$.

Table 3. Effective charge (e_B) used in E2 transition calculations for $^{120-126}$Xe nuclei.

A	N	e_B (eb)
^{120} Xe	10	0.111162
^{122} Xe	9	0.109388
^{124} Xe	8	0.103000
^{126} Xe	7	0.100900

The calculated B(E2) values compared with the experimental data (Efimov *et al*., 2002, http://www.nndc.bnl.gov/chart/getENSDFDatasets.jsp, Kitao *et al*.,2002, Tamura, 2007, Katakura & Wu, 2008, Katakura & Kitao, 2002, Coquard, 2010) for all isotopes under study, and as given in Table 4. In this Table, the subscript 1, 2 and 3 numbers represented ground-, γ- and β- states, respectively. Table 4 shows that in general, most of the calculated results in IBM-1(p.w.) reasonably consistent with the available experimental data, except for few cases that deviate from the experimental data. Furthermore, our results have been compared with previous studies (Efimov *et al*., 2002, Coquard, 2010) and it better than those.

A measure of the deviation of a charge distribution from a spherical shape is the electric quadrupole moment of the distribution. Then the quadrupole moment (Q_J) is an important property for nuclei and can be determined if the nucleus spherical (Q = 0), deformed oblate (Q<0) or prolate (Q>0) shapes. The electric quadrupole moments of the nuclei can be derived from the transition rate B(E2,$J_i \rightarrow J_f$) values according to Eq. (5) (Iachello & Arima, 1987, Kassim & Sharrad, 2014):

$$Q_J = (16\pi/5)^{1/2}[(2J-1)/(J+1)(2J+1)(2J+3)]^{1/2}[B(E2,J_i \rightarrow J_f)]^{1/2}, \quad (6)$$

where: J is the total angular momentum. The calculations of the electric quadrupole moment Q_J within the framework of IBM-1 is shown in Table (5) for $^{120-126}$Xe nuclei. This Table shows that the $Q2_2^+$ has a negative value for all interested nuclei. The calculated $Q4_1^+$ has been positive value.

Table 4. The B(E2) values for $^{120-126}$Xe nuclei (in $e^2.b^2$).

$J_i \rightarrow J_f$	^{120}Xe			^{122}Xe		
	P.W.	EXP.(a, b)	Th.(c)	P.W.	EXP.(a, d)	Th.(c)
$2_1^+ \rightarrow 0_1^+$	0.3460	0.346(22)	0.2779	0.2800	0.280(12)	0.230
$2_2^+ \rightarrow 0_1^+$	0.0483	---	---	0.0392	---	---
$2_2^+ \rightarrow 2_1^+$	0.4766	---	---	0.3829	---	---
$4_1^+ \rightarrow 2_1^+$	0.4766	0.412(28)	0.4256	0.3829	0.409 (22)	0.3272
$4_2^+ \rightarrow 2_1^+$	0.0385	---	---	0.0307	---	---
$6_1^+ \rightarrow 4_1^+$	0.5272	0.415(63)	0.4819	0.4188	0.396(144)	0.3524
$6_2^+ \rightarrow 4_1^+$	0.0295	---	---	0.0228	---	---
$6_1^+ \rightarrow 4_2^+$	0.0245	---	---	0.0211	---	---
$8_1^+ \rightarrow 6_1^+$	0.5347	0.341 (59)	0.4925	0.4177	0.288(179)	0.3488
$8_1^+ \rightarrow 6_2^+$	0.0289	---	---	0.0255	---	---
$10_1^+ \rightarrow 8_1^+$	0.5133	0.324(53)	0.4714	0.3912	0.432(179)	0.320
$10_2^+ \rightarrow 10_1^+$	0.0046	---	---	0.0032	---	---
$12_1^+ \rightarrow 10_1^+$	0.4695	0.292(46)	0.4221	0.3446	---	0.2697
$12_2^+ \rightarrow 10_2^+$	0.3042	0.246 (246)	---	0.1532	---	---
$14_1^+ \rightarrow 12_1^+$	0.4070	0.282(42)	---	0.2809	---	---
$16_1^+ \rightarrow 14_1^+$	0.3278	0.42 (14)	---	0.2015	---	---
$18_1^+ \rightarrow 16_1^+$	0.2330	0.598(105)	---	0.1077	---	---
$6_2^+ \rightarrow 6_1^+$	0.1701	---	---	0.1329	---	---
$3_1^+ \rightarrow 4_1^+$	0.1507	---	---	0.1197	---	---
$3_1^+ \rightarrow 2_1^+$	0.0525	---	---	0.0419	---	---
$3_1^+ \rightarrow 2_2^+$	0.3766	---	---	0.2992	---	---

Continued

$J_i \rightarrow J_f$	^{124}Xe			^{126}Xe		
	P.W.	EXP. (a, e)	Th.(f)	P.W.	EXP.(a, g, f)	Th.(f)
$2_1^+ \rightarrow 0_1^+$	0.1920	0.192(12)	0.192	0.1540	0.154(5)	0.154
$2_2^+ \rightarrow 0_1^+$	0.0027	0.0026(5)	0.0054	0.00217	0.002 (7)	0.0024
$4_1^+ \rightarrow 2_1^+$	0.2600	0.2484(7)	0.3126	0.2057	0.267(3)	0.1569
$4_2^+ \rightarrow 2_1^+$	0.0206	0.00025	0.0012	0.0160	0.0015(3)	0.0005
$4_2^+ \rightarrow 2_2^+$	0.1467	0.254(92)	0.1782	0.1135	0.1355(16)	0.1246
$6_1^+ \rightarrow 4_1^+$	0.2800	0.323 (29)	0.3563	0.2167	0.315 (41)	0.2433
$6_2^+ \rightarrow 4_1^+$	0.0147	---	---	0.0108	---	---
$4_2^+ \rightarrow 6_1^+$	0.0228	---	---	0.0204	---	---
$8_1^+ \rightarrow 6_1^+$	0.2727	0.243(77)		0.2036	---	
$8_1^+ \rightarrow 6_2^+$	0.0195	---	---	0.0179	---	---
$10_1^+ \rightarrow 8_1^+$	0.2462	0.0772(1)	---	0.1731	---	---
$12_1^+ \rightarrow 10_1^+$	0.2040	0.202(44)	---	0.1280	---	---
$2_2^+ \rightarrow 2_1^+$	0.2600	0.118(22)	0.2250	0.2057	0.162 (1)	0.1836
$2_3^+ \rightarrow 2_1^+$	0.0031	0.0022(2)	0.0001	0.0026	0.0004	0.00001
$4_2^+ \rightarrow 4_1^+$	0.1333	0.125(47)	0.1232	0.1032	0.1062(14)	0.0968
$6_2^+ \rightarrow 6_1^+$	0.0868	---	---	0.0648	---	---
$10_2^+ \rightarrow 10_1^+$	0.0394	---	---	0.0247	---	---
$3_1^+ \rightarrow 4_1^+$	0.0799	0.096(44)	0.0761	0.0618	0.0829	0.0596
$3_1^+ \rightarrow 2_1^+$	0.0281	0.0029(6)	0.0081	0.0218	0.0034(2)	0.0034
$3_1^+ \rightarrow 2_2^+$	0.2000	0.3454	0.2414	0.1548	0.2091(24)	0.1689

a- http://www.nndc.bnl.gov/chart/getENSDFDatasets.jsp.

b- (Kitao *et al*.,2002)

c-(Efimov *et al*., 2002)

d-(Tamura, 2007)

e-(Katakura & Wu, 2008)

f-(Coquard, 2010)

g-(Katakura & Kitao, 2002)

Table 5. The electric quadrupole moment Q_J in eb unit.

Q_J	^{120}Xe	^{122}Xe	^{124}Xe	^{26}Xe
2_1^+	0.4823	0.4415	0.3737	0.3443
2_2^+	-0.167 2	-0.154 2	-0.131 7	-0.122 8
4_1^+	0.7804	0.7194	0.6145	0.5730
6_1^+	1.0072	0.9364	0.8085	0.7643
8_1^+	1.2060	1.1319	0.9888	0.9485
10_1^+	1.3971	1.3244	1.1708	1.1397
12_1^+	1.5913	1.5234	1.3628	1.3452

3.3 Reduced transition probabilities B(M1) and E2/M1 mixing ratios

Similarly, from Eq. (3), the M1 operator would be just $\beta1[d^{\dagger}\times\tilde{d}](1)$. For calculating M1 transitions, the IBM-1 rule must be extended to second-order in the U(6) generators (Abrahams et al.,1981, Casten & Warner, 1988). The most general second-order M1 generator can then be written as (Casten & Warner, 1988, Arima & Iachello, 1979):

$$\hat{T}(M1) = (g_B + A_1\hat{N})\hat{L} + B[\hat{T}(E2)\times\hat{L}] + C\,\hat{n}_d\hat{L} \quad (7)$$

where g_B is the effective boson g factor and can be written as (Arima & Iachello, 1979):

$$g_B = Z/A, \quad (8)$$

where Z and A are the atomic and mass numbers, respectively. Indeed from Eq. (7), N is the number of bosons, $\hat{L}$ is angular momentum, $\hat{T}(E2)$ is general matrix element of the E2 transition and $\hat{n}_d$ is d-boson number operators. The first term of Eq. (7) is diagonal and does not contribute to transitions. The last terms yield the M1 matrix element can be written as (Casten & Warner, 1988, Warner, 1981):

$$\langle\acute{\varphi}L_f\|T(M1)\|\varphi L_i\rangle = -Bf(L_iL_f)\langle\acute{\varphi}L_f\|T(E2)\|\varphi L_i\rangle + C[L_i(L_i+1)(2L_i+1)]^{1/2}\times\langle\acute{\varphi}L_f|\hat{n}_d|\varphi L_i\rangle\delta_{L_iL_f} \quad (9)$$

where $\acute{\varphi}$ and φ denote additional quantum numbers. In the first term the spin factors given separately in Refs. (Abrahams *et al*.,1981, Casten & Warner, 1988) for the cases $L \rightarrow L\pm1$ and $L \rightarrow L$, and it has been combined into the single factor $f(L_iL_f)$, given by:

$$f(L_iL_f) = \begin{bmatrix}\frac{1}{40}(L_i+L_f+3)(L_f-L_i+2)\\ \times(L_i-L_f+2)(L_i+L_f-1)\end{bmatrix}^{1/2} \quad (10)$$

The second term of Eq. (9) only contributes to transitions between states of the same spin, since the corresponding operator in Eq. (7) is diagonal in $\hat{L}$. In the O(6) symmetry of the IBM-1, the operator $\hat{n}_d\hat{L}$ contributes to both diagonal and off diagonal matrix elements. The matrix elements between the representations σ = N and σ = N −2 can be written as (Iachello & Arima, 1987,Casten & Warner, 1988):

$$\langle[N],\sigma=N,\tau,\nu_\Delta,L|\hat{n}_d|[N],\sigma=N-2,\tau,\nu_\Delta,L\rangle = -\sqrt{N}\sqrt{\left[\frac{N(N+3)-\tau(\tau+3)}{2N(N+1)}\right]}\sqrt{\left[\frac{(N-1)(N-2)-\tau(\tau+3)}{2N(N+1)}\right]} \quad (11)$$

For $L\pm1\rightarrow L$ transitions, Eq. (9) leads to a particularly simple expression for the reduced $E2/M1$ mixing ratio, namely

$$\Delta(E2/M1) = \langle\acute{\varphi}L_f\|T(E2)\|\varphi L_i\rangle/\langle\acute{\varphi}L_f\|T(M1)\|\varphi L_i\rangle = -1/Bf(L_iL_f) \quad (12)$$

The reduced mixing ratio is related to the quantity normally measured, $\delta(E2/M1)$ by (Casten & Warner, 1988):

$$\delta(E2/M1) = 0.835[E_\gamma/(1Mev)]\,\Delta(E2/M1) \quad (13)$$

where E_γ is in MeV and $\Delta(E2/M1)$ is in eb/μN. The spin dependence of Eq. (10) has already been derived in the framework of the geometrical model by Grechukhin (Grechukhin, 1963) in an analogous way by expressing the relevant part of the M1 operator in terms of the quadrupole coordinates of the nuclear surface. In the IBM-1 model, the value of the constant B in Eq. (9) and the validity of the spin dependence can be investigated by looking at empirical values of $[\Delta(E2/M1)\,Bf(L_iL_f)^{-1}$. To $L \rightarrow L$ transitions, the inclusion of both s and d bosons in the IBM-1 formalism gives rise to the additional contribution in Eq. (12) from the $\hat{n}_d$ operator, and the L → L, M1 matrix element thus, in principal, depends on the relative sizes and Signs of the two terms.

Table (6) shows that the parameters g_B, A_1, B and C were used in the present work for the T(M1) transitions. The calculated reduced probability for M1 transitions and the experimental data (http://www.nndc.bnl.gov/chart/getENSDFDatasets.jsp, Kitao *et al*., 2002, Katakura & Wu, 2008, Coquard, 2010) are given in Table (7) for all isotopes under study.

Table 6. The parameters of T(M1) used in the present work. All parameters are given in (μ_N), except N.

A	N	g_B	A_1	B	C
^{120}Xe	10	0.45	- 0.0447	- 0.0571	0.0000
^{122}Xe	9	0.4426	- 0.0492	- 0.2379	0.0000
^{124}Xe	8	0.4355	- 0.0257	- 0.0353	- 0.0000065
^{126}Xe	7	0.4286	- 0.00835	- 0.0295	0.0000

Table 7. The B(M1) values for Xenon nuclei (in μ_N^2).

$J_i \rightarrow J_f$	^{120}Xe		^{122}Xe	
	IBM-1	EXP.(a, b)	IBM-1	EXP.
$2_2^+ \rightarrow 2_1^+$	0.0011	--	0.0292	--
$4_1^+ \rightarrow 4_1^+$	0.0003	--	0.0253	--
$4_2^+ \rightarrow 4_1^+$	0.0021	--	0.0557	--
$3_1^+ \rightarrow 4_1^+$	0.0008	--	0.0195	--
$2_4^+ \rightarrow 3_1^+$	0.0000	0.000007	0.0000	--
$6_1^+ \rightarrow 6_1^+$	0.0012	--	0.0758	--
$8_1^+ \rightarrow 8_1^+$	0.0030	--	0.1735	--
$10_1^+ \rightarrow 10_1^+$	0.0063	--	0.3435	--
$12_1^+ \rightarrow 12_1^+$	0.0120	--	0.6218	--
$3_1^+ \rightarrow 2_1^+$	0.0001	--	0.0036	--
$3_1^+ \rightarrow 2_2^+$	0.0010	--	0.0261	--
$J_i \rightarrow J_f$	^{124}Xe		^{26}Xe	
	IBM-1	EXP.(a, e, f)	IBM-1	EXP. (a, f)
$2_1^+ \rightarrow 2_1^+$	0.3014	---	0.8125	--
$2_2^+ \rightarrow 2_1^+$	0.0003	0.0003	0.0001	--
$2_3^+ \rightarrow 2_1^+$	0.0000	0.0025(3)	0.0000	0.000435 (90)
$2_3^+ \rightarrow 2_2^+$	0.0000	0.0055(11)	0.0000	--
$2_4^+ \rightarrow 2_1^+$	0.0000	0.0014(5)	0.0000	≥ 0.025 ($^{+7}_{-4}$)
$4_1^+ \rightarrow 4_1^+$	0.9970	--	2.6985	--
$4_2^+ \rightarrow 4_1^+$	0.0005	0.0052(36)	0.0003	--
$3_1^+ \rightarrow 4_1^+$	0.00013	--	0.00013	--
$6_1^+ \rightarrow 6_1^+$	2.0801	--	5.6480	--
$8_1^+ \rightarrow 8_1^+$	3.5428	--	9.6493	--
$10_1^+ \rightarrow 10_1^+$	5.3756	--	14.6878	--
$12_1^+ \rightarrow 12_1^+$	7.5669	--	20.7457	--
$3_1^+ \rightarrow 2_1^+$	0.0000	0.0030 (5)	0.0000	--
$5_1^+ \rightarrow 4_1^+$	0.0001	0.0011(7)	0.0000	--
$5_1^+ \rightarrow 4_2^+$	0.0002	0.00054(54)	0.0001	--

a- http://www.nndc.bnl.gov/chart/getENSDFDatasets.jsp.

b- (Kitao *et al*.,2002)

e-(Katakura & Wu, 2008)

f-(Coquard, 2010)

The IBM-1 has been rather successful at describing the collective properties of several medium and heavy nuclei. The δ (E2/M1) multipole mixing ratios of the electromagnetic transitions between the energy states of $^{120-126}Xe$ nuclei were calculated using Eq. (13). The calculated values are given in Table 8. The IBM-1 model, particularly in its extended consistent-δ formalism, has been applied to the calculation of E2/M1 mixing ratios over a wide range of nuclei. In general, it can be seen from the Table that calculated results are not in good agreement with the available experimental data (http://www.nndc.bnl.gov/chart/getENSDFDatasets.jsp, Kitao *et al*.,2002, Katakura & Wu, 2008, Coquard, 2010) in Xe isotopes under study. The mixing ratio found for ^{122}Xe the 0.8829, 0.574 and 0.946 MeV transition are 3.353, 1.068 and 2.462 that value is in agreement with the experimental value of $-3\ (+1\ \text{or}\ -3)$, >1.9 and $+0.9\ (+20\ \text{or}\ -4)$. For ^{124}Xe nucleus, the 0.4925 and 0.5591 MeV transitions are 15.355 and 10.365 that values are in agreement with the experimental values of $+8\ (+7\ \text{or}\ -3)$ and $+2.3\ (+8\ \text{or}\ -4)$. For ^{126}Xe nucleus, the 0.5464 MeV transition is 13.74 that value is in agreement with the experimental value of $+3.0\ (+10\ \text{or}\ -9)$.

Table 8. The IBM-1 values and the experimental data of E2/M1 mixing ratios for $^{120\text{-}126}Xe$ isotopes.

$J_i \rightarrow J_f$	E_γ(MeV).	δ(E2/M1)		E_γ(MeV)	δ(E2/M1)	
		IBM-1	EXP.(a, b)		IBM-1	EXP.(a, d)
		^{120}Xe			^{122}Xe	
$2_2^+ \rightarrow 2_1^+$	0.55345	9.298	---	0.512	3.2	---
$2_3^+ \rightarrow 2_1^+$	0.95193	Unknown	---	1.1636	6.15	---
$2_4^+ \rightarrow 3_1^+$	0.4535	0.0000	4.2 **(38)**	0.8511	Unknown	---
$3_1^+ \rightarrow 2_1^+$	0.949	23.24	---	0.8829	7.11	$-3\ (^{+1}_{-3})$
$3_1^+ \rightarrow 2_2^+$	0.395	11.08	---	0.3714	3.948	---
$3_1^+ \rightarrow 4_1^+$	0.4755	8.866	---	0.3857	2.947	---
$4_2^+ \rightarrow 4_1^+$	0.60527	6.514	---	0.574	2.228	>1 . 9
$5_1^+ \rightarrow 4_1^+$	1.02093	15.162	---	0.946	5.176	$+0.9\ (^{+20}_{-4})$
$J_i \rightarrow J_f$		(a, e, f)	^{124}Xe		(a, g, f)	^{126}Xe
$2_2^+ \rightarrow 2_1^+$	0.49254	15.355	$+8\ (^{+7}_{-3})$	0.491243	10.28	$+9.1\ (^{+43}_{-23})$
$2_3^+ \rightarrow 2_1^+$	1.27438	Unknown	---	1.28987	Unknown	---
$3_1^+ \rightarrow 2_1^+$	0.89369	Unknown	+0.73 (**6**)	0.92908	Unknown	$+1.6\ (^{+3}_{-7})$
$3_1^+ \rightarrow 2_2^+$	0.40132	18.327	+0.32 **(5)**	0. 43785	14.24	$+8\ (^{+3}_{-2})$
$3_1^+ \rightarrow 4_1^+$	0.36809	13.809	$+3.85\ (^{+0.57}_{-0.45})$	0.37566	8.914	$+5.4\ (^{+3}_{-2})$
$4_2^+ \rightarrow 4_1^+$	0.5591	10.365	$+2.3\ (^{+8}_{-4})$	0.5464	13.74	$+3.0\ (^{+10}_{-9})$
$5_1^+ \rightarrow 4_1^+$	0.95825	18.615	$+1.0\ (^{+5}_{-3})$	0.9616	17.17	+0.8 (**3**)
$5_1^+ \rightarrow 4_2^+$	0.399	13.152	$+5.2\ (^{+26}_{-13})$	0.4151	10.09	$+9\ (^{+50}_{-4})$

a- http://www.nndc.bnl.gov/chart/getENSDFDatasets.jsp.

b- (Kitao *et al*.,2002)

c-(Efimov *et al*., 2002)

d-(Tamura, 2007)

e-(Katakura & Wu, 2008)

f-(Coquard, 2010)

g-(Katakura & Kitao, 2002)

3.4 Participant Flow

For experimental and quasi-experimental designs, there must be a description of the flow of participants (human, animal, or units such as classrooms or hospital wards) through the study. Present the total number of units recruited into the study and the number of participants assigned to each group. Provide the number of participants who did not complete the experiment or crossed over to other conditions and explain why. Note the number of participants used in the primary analyses. (This number might differ from the number who completed the study because participants might not show up for or complete the final measurement.)

4. Conclusion

The energy levels and electromagnetic transitions of moderately deformed even–even Xe nuclei have been studied in the present work. It can be described in terms of the O(6) limit in the framework of the IBM. The B(E2) , B(M1) transition probabilities, the E2/M1 mixing ratios, branching ratios, and electric quadrupole moment Q_J for the even– even $^{120–126}$Xe isotopes with neutron numbers between 66 and 72 are calculated using the IBM. The calculated B(E2) and B(M1) values all in good agreements with experimental data.

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