

ENERGY LEVELS, OSCILLATOR STRENGTHS AND FORBIDDEN TRANSITION PROBABILITIES IN THE Sc II ISOELECTRONIC SEQUENCE

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(Received 1969 March 28)

SUMMARY

Empirical Slater parameters are derived from a study of energy levels in the isoelectronic sequence Sc II–Ni IX. Positions for some unobserved levels are predicted.

Electric dipole oscillator strengths have been calculated for all possible transitions.

Electric quadrupole and magnetic dipole transitions between and within the $3d^2$ and $3d4s$ configurations are given.

1. INTRODUCTION

This study is concerned with making available calculations of oscillator strengths of electric dipole transitions and transition probabilities for electric quadrupole and magnetic dipole transitions in Sc II and the higher isoelectronic ions. All of these ions are of astrophysical interest. Lines of Sc II occur in absorption in the spectra of nearly all types of stars. Ti III lines are observed in absorption in some peculiar hot stars (e.g. HD 168733: Little & Warner 1969). Lines of the higher ions are either identified, or may be expected to occur, in emission in the far ultraviolet spectra of the Sun.

Forbidden lines of Cr V and Mn VI are identified in RR Tel (A. D. Thackeray, private communication). Forbidden lines of Fe VII are observed in planetary nebulae, novae and symbiotic stars. [Co VIII] and [Ni IX] should be detectable in coronal spectra.

We have undertaken the calculation of energy levels, oscillator strengths for permitted lines, and transition probabilities for forbidden lines, for essentially all transitions of likely astrophysical significance in the spectra of the Sc II isoelectronic sequence.

2. ENERGY LEVELS

Throughout this study we have used the single configuration approximation, i.e. we have neglected interactions between configurations. It is for this reason that our coverage starts at Sc II rather than Ca I, in which configuration mixing is very important. Even in Sc II some important configuration interaction is present. This is principally between $3d4p$ and $4s4p$, which lie close together in energy, and shows up in the rather poor prediction of energy levels (see below). Consequently the theoretical f -values involving transitions to $3d4p$ are probably of inferior quality. This is especially true of the weaker intercombination lines.

In the higher ions the energy separation between configurations increases and the mixing is much less important.

Energy matrices for the configurations of interest (ds , dp , d^2 , dd , df , dg , sp) were constructed using a program written by R. D. Cowan. A least squares fit of these energy level schemes to observed energy levels was performed using a program made available by the Argonne National Laboratory. The empirical Slater parameters derived from these solutions enabled us to predict energy levels for some missing levels, and to provide approximate levels in some unobserved configurations (e.g. $3d4s$ in Mn VI–Ni IX and $3d^2$ in Ni IX).

The sources of the energy levels are:

Sc II	: Atomic Energy Levels (AEL) (Moore 1949, 1952)
Ti III	: B. Edlén (private communication)
V IV	: L. Iglesias (1968)
Cr V	: AEL
Mn VI	: AEL
Fe VII	: AEL and B. Edlén (private communication)
Co VIII	: B. Edlén (private communication)

In the case of Ti III and V IV the laboratory analyses are very complete. A new extended analysis of Sc II is sorely needed—there must be many moderately strong unidentified lines in the solar spectrum arising from the unknown $3d^2$ – $3d4f$ and $3d4d$ – $3d4f$ arrays. In the case of Cr V and Fe VII only the lowest two or three configurations are known.

In order to achieve physically realistic solutions, it was occasionally necessary to assign *a priori* values to some of the spin-orbit parameters. These assignments followed the following precept: In general we were required to estimate a value of $\zeta(3d)$ in some of the $3dnl$ configurations. A value of $\zeta(3d)$ can be obtained from the $3d$ configurations of the Sc III isoelectronic sequence by noting the ground state splitting is given by

$$E(^2D_{5/2}) - E(^2D_{3/2}) = 2/5\zeta(3d). \quad (1)$$

The value of $\zeta(3d)$ deduced from this equation gives the value that should apply for large n in the configurations $3dnl$. In general $\zeta(3d)$ in $3dnl$ will be slightly smaller than that deduced from $3d$ because of screening from the nl electron. However, our solutions for the well-behaved $3dns$ sequences indicate that the difference is small, so we usually adopted a value of $\zeta(3d)$ one or two per cent smaller than that found from equation (1).

Our empirically determined Slater parameters, and others (given in brackets) found from use of simple extrapolation formulae are presented in Table I.

TABLE I
Slater and spin-orbit parameters

$3d^2$	E_{av}	F^2	F^4	$\zeta(3d)$	α
Sc	9442.33	36168.78	22841.10	52.04	–2.95
Ti	6132.48	52769.84	30308.60	119.53	61.39
V	7958.88	67401.07	41175.63	204.38	70.21
Cr	9498.48	79784.21	49021.57	319.17	92.17
Mn	11082.34	91440.53	56617.31	464.21	106.32
Fe	12730.53	102680.21	64016.34	645.96	117.55
Co	14705.80	114144.32	73139.86	869.74	111.15
Ni	(15978.00)	(119600.00)	(77000.00)	(1130.00)	(115.00)

TABLE I (continued)

<i>3d4s</i>	E_{av}	$\zeta(3d)$	G^2					
Sc	714.34	71.07	6078.55					
Ti	39134.58	144.74	8535.68					
V	97461.95	240.91	9052.05					
Cr	168725.70	376.00*	9901.97					
Mn	(250000.00)	(536.00)	(11050.00)					
Fe	(355000.00)	(746.00)	(12500.00)					
Co	(475000.00)	(988.00)	(14300.00)					
Ni	(605000.00)	(1270.00)	(16500.00)					
<i>3d5s</i>	E_{av}	$\zeta(3d)$	G^2					
Sc	57809.37	76.78	1427.92					
Ti	134433.04	153.49	3153.43					
V	236780.99	247.27	2613.40					
<i>3d6s</i>	E_{av}	$\zeta(3d)$	G^2					
Ti	170220.40	153.60*	1299.28					
V	292286.66	248.24	1173.83					
<i>3d4p</i>	E_{av}	$\zeta(3d)$	$\zeta(4p)$	F^2	G^1	G^3		
Sc	28560.56	79.0*	30.35	8511.65	5117.43	1345.38		
Ti	78829.83	150.0*	487.25	14997.89	6543.41	3784.03		
V	148850.26	250.0*	1031.66	19170.17	7681.10	5713.81		
Cr	231641.85	380.0*	1537.71	23047.03	8808.94	7442.37		
Mn	326283.91	540.0*	2070.83	26780.07	9889.64	9107.32		
Fe	432838.57	750.0*	2746.28	30641.83	9302.94	8796.28		
<i>3d5p</i>	E_{av}	$\zeta(3d)$	$\zeta(4p)$	F^2	G^1	G^3		
Ti	148359.52	153.60*	254.99	4112.70	1713.61	38.71		
V	256085.19	250.00*	226.80	6681.71	2090.37	1861.28		
<i>3d4d</i>	E_{av}	$\zeta(3d)$	$\zeta(4d)$	F^2	F^4	G^0	G^2	G^4
Sc	61951.73	79.00*	4.45*	3312.44	214.41	2067.65	279.33	-280.26
Ti	131511.14	153.60*	17.36	8378.54	3563.16	3171.79	2930.49	2076.27
V	221112.07	248.00*	31.46	12668.64	6464.99	4355.17	4861.45	3856.63
<i>3d5d</i>								
Ti	169174.42	153.60*	11.49	3229.66	1363.05	1184.09	1088.63	784.22
V	285491.43	248.00*	21.62	5325.30	2952.83	1641.18	2261.38	1891.17
<i>3d4f</i>	E_{av}	$\zeta(3d)$	$\zeta(4f)$	F^2	F^4	G^1	G^3	G^5
Ti	159211.71	155.0*	0.50*	5117.44	2698.25	1269.76	1249.09	3176.48
V	264714.72	250.0*	2.00*	6927.64	3374.24	1967.60	2691.83	3147.18
Fe	664069.34	750.0*	19.12	18721.44	4369.23	8907.26	6483.22	4347.32
Co	818140.07	980.0*	7.76	24726.85	3790.68	14505.00	4989.80	6120.32
<i>3d5f</i>								
Ti	181761.86	153.6*	0.14	1994.62	737.96	717.52	483.98	454.05
<i>3d5g</i>	E_{av}	$\zeta(3d)$	$\zeta(5g)$	F^2	F^4	G^2	G^4	G^6
Ti	182352.84	152.91	0.09	857.80	66.61	3.43	-5.28	35.94
<i>4s4p</i>	E_{av}	$\zeta(4p)$	G^1					
Sc	43351.33	228.88	24725.22					
Ti	142595.05	468.14	29207.10					

* Assumed values.

The energy levels E computed from these parameters, and comparison with observed levels ($O - C$ equals observed minus calculated), are given in Table II. We have had to change the assignment of only one level: in Fe VII, 661176 cm^{-1} described as $3d4f^3F_4$ in AEL is better represented by $3d4f^3H_4$. Also, for sake of uniformity, in Ti III $3d5g$, we have used LS-coupling notation although the pair-coupling notation used in Edlén's analysis is more appropriate.

3. LINE STRENGTHS

We consider the calculation of line strengths for electric multipole and magnetic dipole radiation.

In LS-coupling, the expression for the matrix element of a tensor operator $T^{(k)}$, connecting two states of a two-electron configuration may be written (see e.g. equations 2-48 and 2-49 of Wybourne 1965)

$$\begin{aligned} \langle \beta J \| T^{(k)} \| \beta J' \rangle &= \langle l_1 L_1 S_1, l_2 L S J \| T^{(k)} \| l_1 L_1 S_1, l_2' L' S' J' \rangle \\ &= \delta_{SS'} (-1)^{L_1+l_2'+S+J'+k} [J, J', L, L'] \\ &\quad \times \begin{Bmatrix} L J S \\ J' L' k \end{Bmatrix} \begin{Bmatrix} l_2 L L_1 \\ L' l_2' k \end{Bmatrix} \langle l_2 \| T^{(k)} \| l_2' \rangle \quad (2) \end{aligned}$$

where we use the notation $[x, y, \dots] \equiv (2x+1)(2y+1)\dots$. In equation (2) the final matrix element is evaluated (Wybourne equations 2-37), putting $T^{(k)}$ as the appropriate electric multipole operator, as

$$\langle l_2 \| T^{(k)} \| l_2' \rangle = (-1)^{l_2} [l_2, l_2']^{1/2} \begin{pmatrix} l_2 & k & l_2' \\ 0 & 0 & 0 \end{pmatrix} \int_1^\infty P_{l_2} P_{l_2'} r^k dr \quad (3)$$

$$= Q_k(l_2, l_2') \int_0^\infty P_{l_2} P_{l_2'} r^k dr \quad (4)$$

where $P_l = P_l(r)$ is the normalized radial wave function.

For the case where l_1 and l_2 are equivalent it may be shown that the matrix element is simply twice that given by equation (2).

For transitions within the ds configuration, we need to apply a theorem of Condon & Shortley (1964 p. 216):

$$\langle l_1 l_2 \| T^{(k)} \| l_1 l_2 \rangle = \langle l_1 \| T^{(k)} \| l_1 \rangle + \langle l_2 \| T^{(k)} \| l_2 \rangle.$$

It is readily shown that the matrix element for the s electron is zero, and hence

$$\begin{aligned} \langle d L_1 S_1, s L S J \| T^{(2)} \| d L_1 S_1, s L' S' J' \rangle \\ = \delta_{SS'} (-1)^{L_1+S_1+J'} [J, J']^{1/2} \begin{Bmatrix} J & J' & 2 \\ L' & L & S_1 \end{Bmatrix} \langle d \| T^{(2)} \| d \rangle. \end{aligned}$$

Evaluation of the $3j$ -symbol in equation (3) leads to the following values of $Q_k(l_2, l_2')$:

$$\begin{aligned} Q_2(l, l) &= - \left[\frac{l(l+1)(2l+1)}{(2l-1)(2l+3)} \right]^{1/2} \\ Q_2(l, l+2) &= \left[\frac{3(l+1)(l+2)}{2(2l+3)} \right]^{1/2} \\ Q_1(l_2, l_2') &= (-1)^{l_>} + l_2 + 1 l_>^{1/2} \quad (l_> = \text{greater of } l_2, l_2'). \end{aligned}$$

TABLE II

Energy levels for $3d^2$

	Sc II		Ti III		V IV		Cr V		Mn VI		Fe VII		Co VIII		Ni IX	
	E	O-C	E	O-C	E	O-C	E	O-C	E	O-C	E	O-C	E	O-C	E	O-C
3F_4	4987	1	451	-30	708	27	1141	5	1661	8	2316	11	3135	20	4054	—
3F_3	4883	0	213	-28	302	24	508	5	742	4	1039	8	1422	11	1836	—
3F_2	4804	-1	29	-29	-14	14	10	-10	12	-12	14	-14	27	-27	0	—
3P_2	12157	-2	10731	-9	13471	-12	16068	-16	18648	-20	21300	-25	24109	-39	25988	—
3P_1	12100	1	10598	5	13230	9	15673	11	18042	15	20410	18	22814	16	24203	—
3P_0	12073	1	10535	4	13119	4	15496	4	17778	4	20032	5	22291	19	23498	—
1G_4	14261	0	14375	22	18408	-16	22060	0	25511	0	28916	-1	32384	-1	34399	—
1D_2	10945	0	8373	101	11034	-75	13199	0	15335	1	17480	-5	19965	—	21259	—
1S_0	28176	—	32510	-35	42436	26	50407	—	58117	—	65705	2	74249	1	78431	—

Energy levels for $3d_4s$

	Sc II		Ti III		V IV		Cr V		Mn VI		Fe VII		Co VIII		Ni IX	
	E	O-C	E	O-C	E	O-C	E	O-C	E	O-C	E	O-C	E	O-C	E	O-C
3D_3	178	0	38426	0	96798	0	168111	-12	249431	—	354496	—	474558	—	604620	—
3D_2	68	0	38200	-1	96413	-1	167497	0	248537	—	353226	—	472849	—	602395	—
3D_1	0	0	38064	0	96195	1	167171	12	248091	—	352631	—	472088	—	601445	—
1D_2	2541	0	41704	0	100201	0	171747	—	253405	—	358901	—	479517	—	610270	—

Energy levels for $3d_5s$ and $3d_6s$

	Sc II		Ti III		V IV		Ti III		V IV		
	E	O-C	E	O-C	E	O-C	E	O-C	E	O-C	
$3d_5s$	57743	0	134272	4	236767	0	3d _{6s}	170244	11	292418	0
3D_3	57614	0	134015	-15	236322	0	3D_3	169959	-28	291917	1
3D_2	57551	0	133887	11	236149	0	3D_2	169860	15	291797	-1
3D_1	58252	0	135405	0	237639	0	3D_1	170665	2	292767	0
1D_2							1D_2				

TABLE II (continued)

Energy levels for 3d4p and 3d5p

	Sc II		Ti III		V IV		Cr V		Mn VI		Fe VII		Ti III*		V IV*	
	E	O-C	E	O-C	E	O-C	E	O-C	E	O-C	E	O-C	E	O-C	E	O-C
3F_4	27476	365	78015	143	148326	43	231399	9	326372	-3	433881	-11	148306	105	256191	61
3F_3	27349	253	77373	51	147623	34	230307	21	324852	-4	431577	29	147921	18	255696	52
3F_2	27259	184	77082	86	146963	173	229365	191	323615	180	430020	195	147737	13	255433	30
3D_3	28236	-75	77722	25	146829	26	229099	28	323237	46	420122	-	148149	-38	255452	-6
3D_2	28133	-112	77371	50	146352	77	228422	75	322355	54	427899	-119	147919	12	255155	-9
3D_1	28057	-140	77074	-74	146154	-36	228031	-25	321716	-21	426786	-	147696	-133	254832	-8
3P_2	30027	-203	81036	-12	151488	80	234740	117	329862	133	437373	195	149248	20	257175	-31
3P_1	29917	-175	80998	-59	151470	-43	234655	-30	329645	-10	430896	67	148998	-19	256811	-29
3P_0	29871	-135	81037	-92	151601	-152	234855	-175	329914	-182	437330	-320	149071	-51	256761	-21
$1F_3$	31511	839	82963	153	153898	21	237577	-9	333083	-20	439664	148	149419	236	257576	114
$1D_2$	26342	-261	75371	-173	144482	-209	226340	-210	319997	-186	425466	-78	147191	22	254546	-77
$1P_1$	31356	-541	83896	-99	155578	-13	239932	7	336112	14	443560	-105	149588	-184	258364	-76

* 3d5p.

Energy levels for 4s4p

	Sc II		Ti III	
	E	O-C	E	O-C
3P_2	39344	0	137961	0
3P_1	39114	0	137488	0
3P_0	39002	0	137259	0
$1P_1$	55715	0	157204	0

TABLE II (continued)
Energy levels for 3d4f, 3d5f and 3d5g

	Ti III		V IV		Fe VII		Co VIII		Ti III*		Ti III†	
	E	O-C	E	O-C	E	O-C	E	O-C	E	O-C	E	O-C
³ H ₆	159428	-158	264915	-69	663223	—	815973	—	181835	3	182595	2
³ H ₅	159244	-115	264642	-50	662104	—	814618	—	181553	6	182168	3
³ H ₄	159122	-99	264459	387	661257	-81	813727	—	181433	6	182167	0
³ G ₅	158822	82	264166	-4	664285	198	819628	40	181812	10	182439	0
³ G ₄	158657	34	263921	192	663795	158	818959	4	181752	6	182015	-2
³ F ₃	158504	54	263658	-49	662859	245	817704	138	181501	7	182015	0
³ F ₄	158837	28	264174	228	661737	—	814370	-229	181623	-11	182405	-1
³ F ₂	158685	56	263950	-48	660733	-373	813394	-95	181368	1	182067	2
³ F ₁	158544	-8	263685	-92	660437	-415	813022	-159	181348	-8	182068	0
³ D ₃	159205	-25	264947	-45	666624	39	821556	341	182014	12	182473	1
³ D ₂	159175	-51	265001	67	666014	-90	820769	-172	181903	5	182225	-1
³ D ₁	159171	224	264958	62	665949	-106	820698	-102	181820	—	182227	-2
³ P ₂	159942	49	265856	24	667744	159	822991	88	182205	2	182588	0
³ P ₁	160052	52	266024	-145	668134	131	823516	105	182275	2	182685	-4
³ P ₀	160120	47	266127	—	668386	111	823847	72	182336	-2	182679	1
¹ H ₅	160470	-415	266567	33	670594	-616	827855	-692	182374	-20	182596	6
¹ G ₄	158256	29	263400	-289	659646	277	811362	-144	181231	-12	182438	-2
¹ F ₃	159550	-68	265302	-31	665183	242	819867	598	181858	2	182405	0
¹ D ₂	159353	51	264652	-170	663987	-105	818586	63	181717	-17	182475	-2
¹ P ₁	161621	233	267804	—	671244	226	827367	145	182555	7	182590	-2

* 3d5f.

† 3d5g.

TABLE II (continued)
Energy levels for $3d4d$ and $3d5d$

	Sc II		Ti III		V IV		Ti III*		V IV*	
	E	O-C	E	O-C	E	O-C	E	O-C	E	O-C
$3G_5$	60423	34	129419	51	218364	99	168503	17	284655	44
$3G_4$	60316	32	129200	53	218006	94	168283	24	284292	48
$3G_3$	60238	29	129041	52	217749	87	168132	20	284058	43
$3F_4$	63702	-174	133309	62	223287	17	169904	8	286280	6
$3F_3$	63622	-178	133145	62	223021	12	169758	11	286047	10
$3F_2$	63549	-175	133003	62	222788	7	169600	15	285788	11
$3D_3$	59994	8	128686	4	217339	11	168203	4	284229	-2
$3D_2$	59923	7	128540	6	217098	10	168023	7	283938	3
$3D_1$	59872	3	128429	5	216906	-1	167899	6	283724	-2
$3P_2$	64778	-73	135747	-26	226809	-13	170822	19	287748	-15
$3P_1$	64733	-87	135647	-46	226631	-13	170713	-53	287540	-
$3P_0$	64691	-76	135589	-48	226535	-13	170633	-52	287411	-
$3S_1$	61103	-31	130787	-47	220427	-83	168954	-21	285332	-34
$1G_4$	64523	713	136085	255	227617	96	170904	238	288072	56
$1F_3$	59574	-46	127853	-63	216066	-108	167741	-17	283503	-44
$1D_2$	64006	360	134904	-346	225893	-89	170429	-159	287209	-68
$1P_1$	60435	-35	129314	-60	218101	-111	168383	-40	284423	-57
$1S_0$	65253	-310	139996	24	234125	-3	172401	-28	290998	-

* $3d5d$.

The above equations, together with the formulae for the line strengths S_k :

$$S_1^{1/2} = \langle \beta J \| T^{(1)} \| \beta J' \rangle$$

$$S_2^{1/2} = \sqrt{2/3} \langle \beta J \| T^{(2)} \| \beta J' \rangle$$

enable us to calculate LS -coupled line strengths for electric dipole and electric quadrupole transitions. Finally, transformation to intermediate coupling takes place through the equation

$$\langle \alpha J \| T^{(k)} \| \alpha' J' \rangle = \sum_{\beta \beta'} \langle \alpha J | \beta J \rangle \langle \beta J \| T^{(k)} \| \beta' J' \rangle \langle \beta' J' | \alpha' J' \rangle \quad (5)$$

where the eigenvectors $\langle \alpha J | \beta J \rangle$ are found from the diagonalized energy matrices. The line strengths for magnetic dipole radiation are found to be (Garstang 1962)

$$S^{1/2}(SLJ, SLJ+1) = \left\{ \frac{(J-S+L+1)(J+S-L+1)(J+S+L+2)(S+L-J)}{4(J+1)} \right\}^{1/2}$$

$$S^{1/2}(SLJ, SLJ) = \left\{ \frac{(2J+1)}{4J(J+1)} \right\}^{1/2} \{S(S+1) - L(L+1) + 3J(J+1)\}$$

and transformation to intermediate coupling again takes place through equation (5).

The radial integrals in equation (4) were evaluated using scaled Thomas–Fermi wavefunctions (Stewart & Rotenberg 1965; Warner 1968a). One radial integral was used for each transition array (see discussion by Warner 1968a). The energy eigenvalues adopted were given by the difference between the average energy of a configuration, and the ionization energy of the ion. This suffices for the wavefunction of the $n'l'$ electron in an $nl'n'l'$ configuration. However, in those cases where a wavefunction was required for the nl electron we obtained this by considering the ionization energy of the nl configuration in the next higher ion of the same element.

Thus for the $3d4s$ configuration of Sc II we adopted an energy eigenvalue equal to the ionization potential of Sc III in order to find the $3d$ wavefunction. An alternative method would be to use the difference in energy between $3d4s$ of Sc II and $4s$ of Sc III. However, in this case the absence of $3d$ screening on the $4s$ state of Sc III seriously distorts the eigenvalue. In the method we have adopted, we merely neglect the effect of $4s$ screening on $3d$, which is much less important.

The dipole and quadrupole radial integrals used in the current work are listed in Tables III and IV respectively.

4. OSCILLATOR STRENGTHS AND TRANSITION PROBABILITIES

Weighted oscillator strengths gf were calculated for all possible electric dipole transitions between levels of each ion. Full sets of calculations may be obtained on application to the authors.

Emission transition probabilities were calculated for transitions between and within the two lowest even configurations: $3d^2$ and $3d4s$. This data is given in Table V. Previous studies (Pasternack 1940; Garstang 1964) have only considered transitions within $3d^2$ for the ions Ti III, Mn VI and Fe VII.

5. Ti III LINES IN STARS

Aller & Jugaku (1958) have given a list of spectrum lines in the B2 IV star γ Pegasi. From the study of Ti III made in this paper, we find that a few of the

TABLE III

Dipole radial integrals

	Sc II	Ti III	V IV	Cr V	Mn VI	Fe VII	Co VIII
$3d4s-3d4p$	-3.474	-2.831	-2.394	-2.072	-1.863	-1.730	
$-3d5p$		0.4034	0.5802				
$3d5s-3d4p$	2.345	1.767	1.468				
$-3d5p$		-5.519	-4.436				
$3d6s-3d4p$		0.4963	0.3678				
$-3d5p$		3.763	3.027				
$3d^2-3d4p$	1.699	0.9951	0.7138	0.5627	0.4609	0.3859	
$-3d5p$		0.1758	0.1333				
$-3d4f$		0.9163	0.8541			0.6322	0.5993
$-3d5f$		0.6037					
$3d4d-3d4p$	-3.799	-3.206	-2.676				
$-3d5p$		3.405	2.386				
$-3d4f$		-4.324	-3.130				
$-3d5f$		0.7974					
$3d5d-3d4p$		-0.1459	0.3429				
$-3d5p$		-5.707	-4.709				
$-3d4f$		3.191	2.090				
$-3d5f$		-7.501					
$3d4f-3d5g$		5.119					
$3d5f-3d5g$		-5.078					
$4s4p-3d4s$	1.192	-1.789					
$-4s^2$	-3.106	-2.615					

TABLE IV

Quadrupole radial integrals

	Sc II	Ti III	V IV	Cr V	Mn VI	Fe VII	Co VIII	Ni IX
$3d^2$	7.131	2.198	1.444	1.054	0.8363	0.6921	0.5783	0.4919
$3d^2-3d4s$	-8.881	-3.038	-1.854	-1.267	-0.9497	-0.7359	-0.5790	-0.4683
$3d4s$	2.666	1.711	1.252	0.9747	0.7904	0.6585	0.5398	0.4631

weakest unidentified lines in the spectrum of γ Peg may be attributed to Ti III (e.g. $\lambda\lambda$ 4023.59, 4215.49, 4424.42, 4652.85 and 4874.09). Ti III may also be a significant contributor to some other weak lines. It is possible to make a crude estimate of the abundance of Ti in the atmosphere of γ Peg from these Ti III identifications. The only other abundance for an element of intermediate atomic weight in this star is that of Fe deduced from Fe III lines (Warner 1966).

The suggested Ti III lines in γ Peg mostly have $\log gf \sim 0.5$ and lower excitation potentials ~ 16.5 V. Their equivalent widths, too small to be measurable by Aller & Jugaku, are estimated to be ~ 1 mÅ. A direct comparison with the Fe III lines used previously (Warner 1966) which have $\log gf \sim 0.5$, e.p. ~ 20.5 V and $W_\lambda \sim 25$ mÅ, using an excitation temperature of 18 000°K, and an abundance of Fe of $\log N(\text{Fe}) = 6.55$ (on the scale where $\log N(\text{H}) = 12.00$) immediately provides $\log N(\text{Ti}) \simeq 4.60$ (since both Ti and Fe exist primarily in the second stage of ionization throughout the atmosphere of γ Peg). This is close to the solar abundance of Ti ($\log N(\text{Ti}) = 4.50$, Warner 1968b).

A comparison of Ti III f -values with lines observed in the Ti-rich star HD 168733 has been given elsewhere (Little & Warner private communication).

TABLE V
Transition probabilities (A_q in s^{-1}) for $3d^2-3d4s$

	Sc II	Ti III	V IV	Cr V	Mn VI	Fe VII	Co VIII	Ni IX
$1S_0-1D_2$	3.91 +1	5.53 -3	1.97 +1	3.76 +2	2.27 +3	1.03 +4	3.22 +4	8.13 +4
$-3D_2$	4.62 -2	7.08 -7	5.26 -2	2.38 0	2.37 +1	1.58 +2	6.28 +2	1.78 +3
$3P_0-1D_2$	2.07 -4	3.39 -3	5.76 -1	1.01 +1	6.86 +1	3.60 +2	1.25 +3	3.14 +3
$-3D_2$	8.81 -1	1.34 0	1.23 +2	1.16 +3	5.23 +3	1.96 +4	5.46 +4	1.24 +5
$3P_1-3D_1$	4.13 -1	2.91 0	2.72 +2	2.58 +3	1.17 +4	4.42 +4	1.24 +5	2.82 +5
$-1D_2$	2.92 -5	1.54 -3	2.34 -1	3.96 0	2.70 +1	1.44 +2	5.09 +2	1.38 +3
$-3D_2$	7.42 -2	3.30 -1	3.05 +1	2.87 +2	1.29 +3	4.82 +3	1.34 +4	3.03 +4
$-3D_3$	3.98 -1	1.38 0	1.25 +2	1.18 +3	5.36 +3	2.01 +4	5.65 +4	1.29 +5
$3P_2-3D_1$	8.38 -2	9.37 -1	8.70 +1	8.07 +2	3.54 +3	1.28 +4	3.31 +4	6.94 +4
$-1D_2$	2.00 -3	1.26 -1	2.14 +1	3.37 +2	2.34 +3	1.26 +4	5.19 +4	1.52 +5
$-3D_2$	3.16 -1	2.23 0	2.02 +2	1.83 +3	7.86 +3	2.76 +4	6.92 +4	1.43 +5
$-3D_3$	4.86 -1	2.68 0	2.44 +2	2.27 +3	1.01 +4	3.71 +4	9.90 +4	2.16 +5
$1D_2-3D_1$	4.51 -4	2.34 -2	3.89 0	6.10 +1	4.29 +2	2.36 +3	9.82 +3	3.06 +4
$-1D_2$	1.47 -1	1.66 +1	8.45 +2	6.81 +3	2.82 +4	9.83 +4	2.50 +5	5.27 +5
$-3D_2$	2.18 -3	6.15 -2	1.22 +1	2.04 +2	1.42 +3	7.58 +3	2.95 +4	8.15 +4
$-3D_3$	1.24 -3	2.52 -2	4.86 0	7.70 +1	5.48 +2	3.02 +3	1.33 +4	4.07 +4
$3F_2-3D_1$	5.06 -3	3.08 +1	1.18 +3	8.75 +3	3.53 +4	1.23 +5	3.26 +5	7.11 +5
$-1D_2$	1.76 -8	5.10 -3	6.81 -1	1.09 +1	6.73 +1	3.23 +2	1.01 +3	1.98 +3
$-3D_2$	3.37 -3	1.35 +1	5.14 +2	3.80 +3	1.54 +4	5.36 +4	1.43 +5	3.15 +5
$-3D_3$	2.99 -4	9.87 -1	3.72 +1	2.73 +2	1.10 +3	3.80 +3	1.00 +4	2.16 +4
$3F_3-3D_1$	1.96 -3	1.51 +1	5.82 +2	4.32 +3	1.75 +4	6.08 +4	1.62 +5	3.54 +5
$-1D_2$	1.55 -7	9.16 -2	6.54 0	8.81 +1	5.40 +2	2.65 +3	8.95 +3	2.32 +4
$-3D_2$	4.57 -3	2.29 +1	8.78 +2	6.46 +3	2.59 +4	8.96 +4	2.36 +5	5.13 +5
$-3D_3$	2.45 -3	1.01 +1	3.86 +2	2.85 +3	1.15 +4	4.02 +4	1.07 +5	2.34 +5
$1G_4-1D_2$	7.81 -1	1.13 +1	1.01 +3	9.63 +3	4.40 +4	1.67 +5	4.69 +5	1.08 +6
$-3D_2$	2.21 -3	1.26 -2	4.32 0	8.90 +1	6.60 +2	3.68 +3	1.36 +4	3.62 +4
$-3D_3$	5.14 -5	2.62 -4	6.52 -2	1.19 0	9.24 0	5.61 +1	2.41 +2	8.77 +2
$3F_4-1D_2$	2.38 -8	1.31 -2	1.12 0	1.61 +1	9.67 +1	4.58 +2	1.41 +3	3.02 +3
$-3D_2$	1.70 -3	9.54 0	3.70 +2	2.74 +3	1.11 +4	3.84 +4	1.02 +5	2.24 +5
$-3D_3$	2.58 -3	3.51 +1	1.35 +3	9.99 +3	4.04 +4	1.41 +5	3.74 +5	8.18 +5

TABLE V (continued)
Transition probabilities (s^{-1}) in $3d^2$

	Sc II	Ti III	V IV	Cr V	Mn VI	Fe VII	Co VIII	Ni IX
$1S_0-^3P_1$	1.40	1.00	3.92	1.13	2.75	6.01	1.23	2.19
$1S_0-^3P_2$	3.09	2.00	9.61	2.12	4.30	7.89	1.60	1.96
$1S_0-^1D_2$	9.85	4.89	8.13	9.82	1.22	1.48	1.78	1.59
$1S_0-^3F_2$	2.99	4.22	1.17	2.48	4.91	9.07	1.58	2.16
$3P_0-^1D_2$	1.59	4.26	6.45	8.31	1.15	1.48	1.07	8.28
$3P_0-^3F_2$	5.29	3.22	4.16	5.10	6.40	7.97	9.55	9.00
$3P_1-^3P_0$	3.71	4.98	2.83	1.12	3.73	1.07	2.52	6.27
$3P_1-^1D_2$	1.82	5.69	8.79	1.27	2.02	3.09	2.91	3.27
$3P_1-^3F_2$	1.35	1.18	3.99	9.18	1.94	3.70	5.42	7.92
$3P_1-^3F_2$	1.80	1.10	1.44	1.79	2.28	2.88	3.49	3.36
$3P_1-^3F_3$	1.59	3.45	2.07	1.01	3.80	1.22	3.36	8.63
$3P_1-^3F_3$	3.40	2.02	2.56	3.05	3.71	4.47	5.12	4.62
$3P_2-^3P_0$	1.52	8.78	7.81	4.96	2.59	1.16	4.51	1.76
$3P_2-^3P_1$	4.22	2.18	2.08	1.46	8.10	3.87	1.73	7.32
$3P_2-^1D_2$	1.99	2.18	1.40	6.53	2.40	7.66	2.30	6.60
$3P_2-^3F_2$	8.39	2.06	4.77	8.81	1.84	3.82	6.38	1.23
$3P_2-^3F_2$	2.77	2.48	9.32	2.44	5.94	1.33	2.59	5.13
$3P_2-^3F_3$	1.58	9.87	1.30	1.63	2.07	2.62	3.10	2.97
$3P_2-^3F_3$	1.65	1.20	1.64	8.73	3.53	1.17	4.16	9.99
$3P_2-^3F_3$	1.05	6.40	8.24	1.01	1.26	1.57	1.86	1.79
$3P_2-^3F_4$	6.44	6.83	6.73	3.41	1.32	4.21	1.34	3.20
$1D_2-^3F_2$	3.78	2.20	2.70	3.13	3.68	4.28	4.67	4.08
$1D_2-^3F_2$	7.81	7.28	2.44	5.74	1.21	2.34	4.73	6.47
$1D_2-^3F_3$	7.16	5.37	1.93	5.73	1.40	3.05	6.17	1.09
$1D_2-^3F_3$	1.39	8.12	3.58	8.24	1.69	3.12	6.50	7.97
$1D_2-^3F_4$	1.37	1.00	3.50	1.00	2.36	4.91	9.41	1.55
$3F_3-^3F_2$	6.72	4.19	1.59	3.43	6.52	1.11	2.08	2.29
$3F_3-^3F_2$	2.04	1.23	8.96	4.66	1.91	7.15	2.41	6.06
$1G_4-^3P_2$	1.35	1.62	8.83	3.46	1.06	2.94	7.52	1.58
$1G_4-^1D_2$	1.10	2.51	1.23	3.11	6.22	1.07	1.80	1.89
$1G_4-^3F_2$	2.12	3.66	4.84	6.11	7.52	8.98	8.94	8.10
$1G_4-^3F_3$	7.31	1.61	3.94	8.89	1.81	3.43	5.80	8.31
$1G_4-^3F_3$	1.11	1.97	5.10	1.14	2.35	4.53	8.01	1.17
$1G_4-^3F_4$	5.25	4.28	1.59	4.71	1.16	2.58	5.30	9.69
$3F_4-^3F_2$	4.44	7.68	1.93	4.18	8.25	1.52	2.55	3.49
$3F_4-^3F_3$	8.46	6.78	2.47	7.17	1.73	3.73	7.43	1.30
$3F_4-^3F_3$	4.74	2.70	1.87	8.92	3.53	1.20	3.58	8.00
$3F_4-^3F_3$	5.96	3.33	2.28	1.08	4.46	1.57	4.82	1.24

TABLE V (continued)

Transition probabilities (s^{-1}) in 3d4s

	Sc II		Ti III		V IV		Cr V	
	A_q	A_m	A_q	A_m	A_q	A_m	A_q	A_m
$1D_2-3D_1$	1.05	2.49	5.37	2.08	1.10	4.54	2.50	1.16
$-3D_2$	6.52	1.65	3.17	8.58	5.94	2.98	1.24	8.52
$-3D_3$	8.32	2.07	3.64	1.56	5.59	2.85	9.33	5.96
$3D_2-3D_1$	1.13	7.52	1.45	5.91	8.21	2.44	3.16	7.43
$3D_3-3D_1$	1.15		1.66		1.13		5.57	
$-3D_2$	1.04	2.38	1.61	2.09	1.22	1.02	6.79	3.81
Ni IX								
	Mn VI		Fe VII		Co VIII		Ni IX	
	A_q	A_m	A_q	A_m	A_q	A_m	A_q	A_m
$1D_2-3D_1$	5.49	2.89	1.25	7.17	2.52	1.67	5.23	4.23
$-3D_2$	2.49	1.98	5.29	4.44	1.01	8.94	2.03	1.60
$-3D_3$	1.47	1.20	2.45	2.42	3.81	4.63	6.43	9.95
$3D_2-3D_1$	1.21	2.15	3.52	5.10	8.03	1.07	1.78	2.07
$3D_3-3D_1$	2.47		8.95		2.45		6.33	
$-3D_2$	3.21	1.23	1.28	3.44	3.76	8.05	1.03	1.71

ACKNOWLEDGMENTS

We are grateful to Drs R. D. Cowan, M. Wilson and M. Fred for helpful correspondence. Part of this work was supported by a Research Grant from the Research Institute at the University of Texas. All calculations were carried out on the CDC 6600 computer at the Computational Center of the University of Texas at Austin.

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Received in original form 1968 December 28.

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