CHAPTER 32

ENERGY LOSS AND SET-UP DUE TO BREAKING OF RANDOM WAVES

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ABSTRACT

A description is given of a model developed for the prediction of the dissipation of energy in random waves breaking on a beach. The dissipation rate per breaking wave is estimated from that in a bore of corresponding height, while the probability of occurrence of breaking waves is estimated on the basis of a wave height distribution with an upper cut-off which in shallow water is determined mainly by the local depth. A comparison with measurements of wave height decay and set-up, on a plane beach and on a beach with a bar-trough profile, indicates that the model is capable of predicting qualitatively and quantitatively all the main features of the data.

INTRODUCTION

Quantitative predictions of wave-induced mean sea level variations and currents in the nearshore region require a specification of the variation of the mean wave energy density (E) in that region. Battjes (1972) and Goda (1975) have presented methods to that effect applicable to random, breaking waves. In these models, E is forced to follow in some sense, to be described below, the variation of the local depth. The energy dissipation rate due to breaking is not estimated independently in these models. This is a shortcoming, since it immediately precludes the use of an approach based on the energy balance. Such an approach is physically more sound, and would e.g. be needed in applications to profiles where the depth is not monotonically decreasing shoreward, such as in the commonly occurring bar-trough profiles. For these reasons an attempt was made to develop a model for the dissipation of wave energy in random waves, breaking on a beach.

The outline of the paper is as follows. Existing models for the prediction of the energy variation across the surfzone are discussed first. Following that, the various elements of a new dissipation model for random waves are described. The results are combined with a conventional model for the prediction of wave-induced variations in mean water level. Subsequently, experiments are described which were carried out to test the model, and to determine the magnitude of a coefficient which in the theory can be estimated in order of magnitude only. Finally, a comparison is given of the results of the theoretical model to the experimental data.

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EXISTING MODELS

We consider a two-dimensional situation of waves normally incident on a beach with straight and parallel depth contours.

For given incident wave parameters and beach profile, the variation of mean wave energy density (E) with distance to the shoreline can in principle be calculated from the wave energy balance, written as

$$\frac{\partial P_x}{\partial x} + D = 0, \qquad (1)$$

in which P_x is the x-component of the time-mean energy flux per unit length, x is a horizontal coordinate, normal to the still-water line, and D is the time-mean dissipated power per unit area. Knowing the dependence of P_x and D on E and on known parameters such as local mean depth (h), wave frequency (f), mass density (ρ) and gravity acceleration (g), it is possible to integrate (1), subject to the initial condition of the given incident wave, to find E as a function of x.

Outside the surfzone, the wave decay is rather weak, and D can often be neglected entirely. If not, the only contributions to it are from such mechanisms as the formation of a boundary layer near the bed, percolation in a porous bottom, etc., for which the dependence of D on E can be estimated reasonably well. In either case, (1) can in fact be used outside the surf zone.

Inside the surf zone, the dissipation of wave energy in the breaking process is dominant. Because the details of these processes are so little understood, the energy balance is usually abandoned in the surf zone. An exception must be made for LeMéhauté (1962), Divoky et al (1970), and Hwang and Divoky (1970), who do apply an energy balance to breaking solitary or periodic waves, by using a bore model for the dissipation rate. Usually, however, instead of solving the energy balance equation, a self-similarity in the surf zone is assumed (for solitary or periodic waves), such that the wave height (H) decays in constant proportion to the mean depth, or

$$H(x) = \gamma\, h(x), \qquad (2)$$

in which γ is a coefficient of O(1), whose actual magnitude can vary with beach slope and incident wave steepness.

For *random* waves breaking on a beach, a model based on (1) does not seem to have been published. An approach based on (2), using it as an upper bound to the local wave height distribution, has been presented by Battjes (1972) and Goda (1975). Goda's model is similar to Battjes' model in essence, though different in the details.

The use of (1), rather than (2), is in principle much to be preferred for a number of reasons, the most fundamental one being that (1) has a sound physical basis, whereas (2) is assumed ad hoc. Associated with this are the following items:

- Assumption (2) relates the local wave height to the local mean depth. This introduces an unrealistically large dependence of dissipation rate on local bottom slope (Battjes, 1978b). In (1), the local wave height is found from an integration, so that it not only depends on the local depth but also on those further seaward, which is more realistic.
- If necessary, other dissipation mechanisms than that due to breaking can be incorporated in (1) in a straightforward manner, whereas this is not the case for (2).
- Eq. (2) is restricted to profiles in which the depth decreases monotonically in the shoreward direction. This is not the case for (1), which in principle can be applied in bar-trough profiles as well.

A DISSIPATION MODEL FOR RANDOM WAVES

Introduction

As noted above, no model based on the energy balance seemed to be available for the prediction of the energy variation in random waves, breaking on a beach. The preceding comments should make it clear that such a model is in fact desirable. One such model which has been developed will be described in the following.

Before turning to the specifics, some general remarks will be made about the philosophy of approach which has been adopted.

Firstly, since at present there is virtually no systematic quantitative knowledge of the internal structure of breaking waves, the approach used herein is based on knowledge of external, macroscopic properties of breakers only.

Secondly, it was deemed prudent to build a rather crude model at first, containing the simplest possible elements which would still represent those aspects of the problem which were considered essential. A confrontation of such model with real data should serve to indicate whether the model is sufficiently realistic to warrant further refinements. Examples of choices made on this basis are an abrupt upper cut-off of the wave height distribution, the use of a simple, linear approximation for the energy flux P, and the omission of all dissipation mechanisms other than that due to breaking. The results presented in the following represent only the first steps in the sense just described. However, at several places it will be indicated how refinements could possibly be made.

A third point to be mentioned has to do with terminology. When reference is made to a "breaking wave", this could be interpreted as a wave at the moment of incipient breaking, defined in one way or another, or it could perhaps be taken to refer to the turbulent, aerated bore-like structure some time after the onset of breaking. To avoid this possible semantic confusion, we shall here refer to the latter as a broken wave; the breaking itself is then taken to correspond to the transition of the unbroken mode to the broken mode, according to the definition adopted for such transition.

Wave height distribution

Consider a fixed point on a beach, with mean depth h, in the presence of a random incident wave field. It is impossible for waves with heights considerably in excess of h to pass the point being considered, since those which otherwise would do so are reduced in height as a result of breaking. The limited depth effectively limits the larger wave heights in the distribution. A simplified model of this is obtained by defining for each depth h a maximum possible wave height H_m (to be specified subsequently), and to assume that the heights of all the waves which are breaking or broken at the point considered, and only those, are equal to H_m.

The assumption just stated is of course a simplification. Not all the heights of broken waves passing a fixed point are equal, nor are they all necessarily larger than those of the non-broken waves. The first of these two aspects can be remedied by providing for a smoother cut-off, as in Goda's model (1975). For the time being the simpler model described here was used. After all, our purpose is not to estimate the details of the wave height distribution in the range near H_m; it is to derive mean square values from a distribution of wave heights which is somehow suppressed in its upper region by the local depth.

The assumption stated above will now be written in terms of the probability distribution of the wave heights, F(H). The shape of F(H) for the lower, non-broken wave heights is assumed to be the same as it is in absence of wave breaking, i.e. of the Rayleigh-type, with modal value $\hat{H}$, say. This leads to

$$\begin{aligned} F(H) \equiv \Pr\{\underline{H} \leq H\} &= 1 - \exp(-\tfrac{1}{2}H^2/\hat{H}^2) \qquad \text{for } 0 \leq H < H_m \\ &= 1 \qquad\qquad\qquad\qquad\qquad \text{for } H_m \leq H, \end{aligned} \tag{3}$$

in which the underscore indicates a random variable. The local value of the parameter $\hat{H}$ is left unspecified for the moment.

Eq. (3) represents a probability distribution with two parameters, $\hat{H}$ and H_m. All the statistics of the wave heights can therefore be expressed in terms of $(\hat{H}, H_m)$. Among those are the rms value (H_{rms}), defined by

$$H_{rms} = \{\int_0^\infty H^2 dF(H)\}^{\frac{1}{2}}, \tag{4}$$

and the probability that at a given point a height is associated with a breaking or broken wave (Q_b), which on the assumption stated above equals

$$Q_b = \Pr\{\underline{H} = H_m\}. \tag{5}$$

Substitution of (3) into (4) and (5) gives

$$H_{rms}^2 = 2(1 - Q_b)\hat{H}^2 \tag{6}$$

and

$$Q_b = \exp(-\tfrac{1}{2}H_m^2/\hat{H}^2). \tag{7}$$

Instead of using $(\hat{H}, H_m)$ as the two parameters of F(H), it is equally possible to use (H_{rms}, H_m), which have a clearer physical meaning. This can be achieved by eliminating $\hat{H}$ between (6) and (7), which yields for Q_b

$$\frac{1 - Q_b}{\ln Q_b} = -\left(\frac{H_{rms}}{H_m}\right)^2. \tag{8}$$

From this transcendental equation, Q_b can be solved as a function of H_{rms}/H_m:

$$Q_b = f(H_{rms}/H_m). \tag{9}$$

This is a key element in the whole model. It expresses the fraction of waves which at any one point are breaking or broken, in terms of the ratio of the rms wave height (equivalent to mean energy density, E) actually present, to the maximum wave height which the given depth can sustain. The local value of H_{rms} is not known a priori; it is found by integrating the differential equation (1). The importance of Q_b for this equation is due to the fact that the average local energy dissipation rate D is proportional to it, at least in the dissipation model to be described below. It is mainly through Q_b that this model reacts to changes in depth.

In very deep water, where $H_{rms}/H_m \to 0$, (8) gives $Q_b \to 0$, as expected. If the waves are shoaling then the ratio H_{rms}/H_m tends to increase, and so does Q_b. In the limit $H_{rms}/H_m \to 1$, $Q_b \to 1$, which is a degenerate case since it corresponds to all the waves being broken, and all the wave heights being equal to H_m.

Breaker height

So far the quantitative estimation of H_m was left unspecified. This will be considered next.

The form chosen is based on Miche's criterion for the maximum height of periodic waves of constant form:

$$H_m \simeq 0.14\ L \tanh(2\pi h/L) \simeq 0.88\ k^{-1} \tanh kh, \qquad (10)$$

in which $k = 2\pi/L$ is the positive real root of the dispersion equation

$$(2\pi f)^2 = gk \tanh kh. \qquad (11)$$

Eq. (10) as it stands would predict $H_m \simeq 0.88\ h$ in shallow water. In application to waves on beaches we want to use a similar functional relationship as in (10) but we also want some freedom of adjustment, to allow for effects of beach slope and of the transformation to random waves, such that in shallow water our expression for H_m reduces to $H_m = \gamma h$, in which γ is a (slightly) adjustable coefficient. In order that the deep-water limit shall not be influenced by the bottom slope the following form was finally adopted:

$$H_m = 0.88\ k^{-1} \tanh(\gamma kh/0.88). \qquad (12)$$

In application to random waves, (12) will be used, with f in (11) being given a single, representative value, such as $\bar{f}$, a mean frequency defined as the ratio of the first moment of the surface elevation spectrum about f = 0 to the zeroth moment.

It should be pointed out that neither the frequency-dependent transition from deep water to shallow water, as expressed by the tanh-function in (10) or (12), nor the representative frequency to be used, needs to be estimated with great accuracy, inasmuch as most of the wave breaking takes place in shallow water anyway, where $H_m \simeq \gamma h$, regardless of the frequency used (within reasonable limits). For the same reason, the use of only one representative value of f is deemed sufficient. In principle it is possible to use a distribution of f-values, leading to a distribution of H_m-values for given h. (A distribution of H_m has in fact been used by Goda (1975), though it was chosen ad hoc.)

Energy dissipation in a broken wave

Following LeMéhauté (1962), the energy dissipation rate in a broken wave will here be estimated from that in a bore of correponding height.

Consider a bore connecting two regions of uniform flow, with depths Y_1 and Y_2, respectively (fig. 1a). The macroscopic bore properties are determined by the conservation of mass and momentum across the bore. The power dissipated in the bore per unit span, written as D', can then be calculated, with the result (Lamb, 1932)

$$D' = \frac{1}{4} \rho g (Y_2 - Y_1)^3 \left\{\frac{g(Y_1 + Y_2)}{2\, Y_1 Y_2}\right\}^{\frac{1}{2}}. \qquad (13)$$

This expression will be used to estimate the power dissipated in the crest region of a broken wave. If, as is the case here, the broken wave is one of a sequence on a sloping beach, then the flow conditions on either side of it are non-uniform. Thus, (13) cannot be expected to apply in any exact sense, but at most in order of magnitude. For this reason it is felt to be justified to use further order-of-magnitude estimates in (13) (written as $\sim$), if applied to broken waves on a beach. In this respect the treatment here deviates from that of LeMéhauté (1962). These order-of-magnitude estimates are (see fig. 1b)

$$Y_2 - Y_1 \sim H \qquad (14)$$

and

$$\left\{\frac{g(Y_2 + Y_1)}{2\, Y_1 Y_2}\right\}^{\frac{1}{2}} \sim \left\{\frac{g}{h}\right\}^{\frac{1}{2}}. \qquad (15)$$

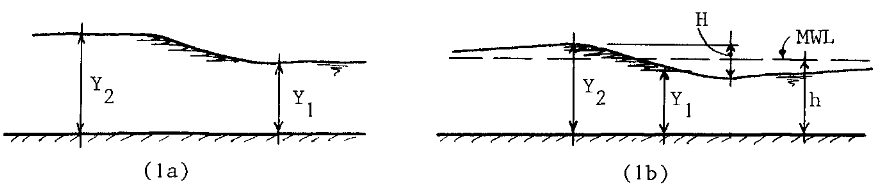

Fig. 1 - Sketch of a single, steady bore (1a) and of one out of a sequence of broken waves on a beach.

Substitution of these in (13) yields the following estimate for the power dissipated per broken wave, per unit span:

$$D' \sim \frac{1}{4}\, \rho g\, H^3 \left(\frac{g}{h}\right)^{\frac{1}{2}}. \qquad (16)$$

If the waves were periodic with frequency f, then the average power dissipated in the breaking process, per unit area, would be

$$D = \frac{D'}{L} = \frac{fD'}{c} \sim \frac{fD'}{(gh)^{\frac{1}{2}}} \sim \frac{1}{4}\, f \rho g\, \frac{H^3}{h}\ , \qquad (17)$$

where L and c denote wavelength and phase speed, respectively.

A more refined estimate for the relation between bore strength (Y_2-Y_1) and wave height H than that in (14) has been given elsewhere, and applied to solitary and periodic broken waves (Battjes, 1978a). However, that refinement has not yet been implemented in the present model for random waves.

Mean energy dissipation in a random breaking wave field

In application to random waves, we are interested in the expected value of the dissipated power per unit area. This can be estimated by applying (17) to the broken waves, and to those only. In the model adopted above these have a height equal to H_m, and a probability of occurrence (at a fixed point) equal to Q_b. Furthermore, the mean frequency $(\bar{f})$ of the energy spectrum is used again as a representative value of f. If lastly the ratio H_m/h is dropped from the order-of-magnitude relationship, as being of order one in the region where most of the dissipation occurs, we obtain for the overall mean dissipation rate per area

$$D \sim \frac{1}{4}\, Q_b \bar{f} \rho g\, H_m^2, \qquad (18)$$

or, written as an equality

$$D = \frac{\alpha}{4}\, Q_b \bar{f} \rho g\, H_m^2, \qquad (19)$$

in which - if the model is good - α is a constant of order one.

In interpretations of (19), it is important to note that Q_b, representing the fraction of broken waves passing any one point, was seen to be a function of H_{rms}/H_m. Thus, the combination of (8) and (19) determines the power dissipated in the breaking process, D, as a function of the unknown local H_{rms} (or the local energy density E), the known local depth (through H_m), and some constants. This is a key result of the present model, enabling the application of the energy balance (1) to the surf zone.

At first sight, it might seem from (19) as if D were decreasing with decreasing H_m (decreasing h), but this is not normally the case. As long as a random wave train of low or moderate steepness is in relatively deep water, Q_b will be virtually zero, and therefore also D. When, upon approaching the beach, the depth becomes less than 2 to 3 times the rms wave height, Q_b increases strongly, such that its increase in fact more than compensates for the reduction in H_m^2, so that D also increases. Near the limit of very shallow water, Q_b ultimately approaches unity (except for extremely mild slopes), in which case D decreases, ultimately in proportion to H_m^2.

Likewise, if the wave train passes a bar, then the increase in depth (and consequently in H_m) shoreward of the bar crest causes a reduction in Q_b which more than offsets the increase in H_m^2, so that the dissipation rate in the model can become virtually zero again in the trough shoreward of the bar. Examples of this situation will be given below.

Energy balance

Having established a dependence of the dissipation rate D on the rms wave height, it remains to do the same for the energy flux P in order to be able to integrate the energy balance, eq. (1). A simple linear approximation will be used, viz.

$$P_x = P = Ec_g, \tag{20}$$

in which

$$E = \frac{1}{8} \rho g H_{rms}^2 \tag{21}$$

and

$$c_g = \left[\frac{2\pi f}{k} \left(\frac{1}{2} + \frac{kh}{\sinh 2kh}\right)\right]_{f = \bar{f}} . \tag{22}$$

This closes the system of equations for H_{rms}. For given depth profile h(x), given incident wave parameters, and a (suitable) choice of the model parameters α and γ, eq. (1) can be integrated to find $H_{rms}(x)$.

From trial calculations it was found that the damping contained in the model appeared to be insufficient to prevent the ratio H_{rms}/H_m from tending to blow up as the waterline (zero depth) was approached. This phenomenon is reminiscent of the classical shoreline singularity for dissipationless progressive waves. As such it need not be of great concern in the sense that it would invalidate the model away from the waterline. However, the wave height distribution assumed here limits H_{rms} to values not exceeding H_m, so that an internal incompatibility arises.

The situation could have been remedied by simply terminating the calculation where the condition $H_{rms} \leq H_m$ is first violated, or, rather more fundamentally, by incorporating additional physical mechanisms of energy dissipation in eq. (1), which would be predominant near the waterline, and of such strength that $H_{rms}/H_m \leq 1$ as $h \to 0$. In the preliminary results to be given below, neither of these was done. Instead, the integration of (1) was ceased at the point where the condition $H_{rms} \leq H_m$ is first violated. Shoreward of this point, $H_{rms} = H_m$ was assumed.

MOMENTUM BALANCE

We have so far assumed that the mean-depth profile h(x) was given. Normally, only the depth of the bottom below some reference plane, d(x), is given. In the experiments to be described subsequently, the still-water level was chosen as the reference plane for d, which therefore represents the still-water depth.

The height of the mean water level above the reference plane, written as $\bar{\eta}(x)$, can be determined from the mean momentum balance. Following Longuet-Higgins and Stewart (1962) the mean balance of x-momentum is written as

$$\frac{dS_{xx}}{dx} + \rho g h \frac{d\bar{\eta}}{dx} = 0, \qquad (23)$$

in which

$$h = d + \bar{\eta} \qquad (24)$$

and

$$S_{xx} = \left(\tfrac{1}{2} + \frac{2\,kh}{\sinh\,2kh}\right)E, \qquad (25)$$

the component of the radiation stress tensor normal to planes x = constant. As an initial condition for the integration of (23), it is common to choose $\bar{\eta} = 0$ in a reference point in deep water.

In the applications given below, the energy balance and the momentum balance have been integrated simultaneously, by a process of iteration.

The most important reason for the inclusion of the wave-induced variations in $\bar{\eta}$ in the model is not to correct d (the energy dissipation model could be checked by working from the outset with h), but to provide an independent check on the validity of the model. As noted in the introduction, the variation of the wave energy density E in the surf zone is important because the radiation stresses are proportional to E, and the gradients of these stresses provide driving forces for the mean flow. It is therefore very useful to have a check of the capability of the present model to predict such driving forces. A limited but nevertheless useful way to do this is to calculate the wave-induced set-up $\bar{\eta}$, and to compare the results with measurements. This then is one purpose of the experiments to be described in the following chapter, the other one being a more direct check on the model through measurements of H_{rms}.

EXPERIMENTS

Arrangements and procedures

The experiments referred to above were carried out in a flume in the Laboratory of Fluid Mechanics, Department of Civil Engineering, Delft University of Technology. The flume has an overall length 45 m, width 0.8 m, and height 1 m.

The flume is equipped with a hydraulically driven random-wave generator. The control signal was obtained by filtering a random noise signal.

At the end of the flume opposite the wave generator, two beaches with different profiles have been built, viz. a plane beach with a 1:20 slope, and a beach with an idealized bar-trough profile, consisting of two 1:20 plane sections sloping seawards, connected by a 1:40 plane section, sloping shoreward, 4.4 m in length, (Fig. 2).

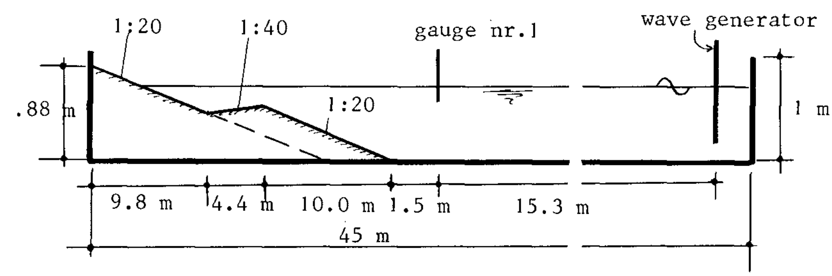

Fig. 2 - Sketch of wave flume with bar-trough beach profile (plane slope indicated by dashed line)

The height of the bar crest was about 0.50 m above the flume bottom, and the depth of the trough below the bar crest was about 0.11 m. The beaches occupied the entire flume width, one at a time. They consisted of compacted sand, finished with a smooth cement-sand mortar layer.

Measurements of the free surface fluctuations were made with 7 parallel-wire resistance gauges. One of these was placed in the constant-depth section, 1.5 m seaward of the toe of the beach; this gauge served as the reference gauge for the incident waves. The other gauges, shoreward of the beach toe, were placed at intervals of 2 m. After a recording time of about 90 min., they were moved by 1 m, so as to obtain an effective interval between adjacent measurement points of 1 m.

The gauges were calibrated immediately before and after each run. Deviations from linearity were less than 0.5%, provided the minimum submergence exceeded 4 cm. This was guaranteed by placing the gauges in the beach, in those points where the minimum required submergence was not available above the beach face (i.e., in the points relatively high on the slope).

The gauge signals were analyzed with respect to the variance (σ^2) and its spectral distribution. To this end they were fed to an eight-channel Pulse Code Modulation recorder, and afterwards from there, at 16 times the recording speed, to a digital correlator and a hybrid spectral analyzer. A total of 2^{15} points per signal were sampled, at intervals of 0.01 s (0.16 s true time). The corresponding Nyquist frequency (f_N) is 50 Hz (50/16 Hz true frequency). Per spectrum, 100 spectral estimates were made, at a separation of 0.01 f_N, each with about $2^{15}/100 \simeq 328$ degrees of freedom.

Values of σ^2, obtained from the digital correlator, were used to estimate H_{rms} according to

$$H_{rms} = 8^{\frac{1}{2}}\sigma, \tag{26}$$

which is consistent with (21) and with

$$E = \rho g \sigma^2. \tag{27}$$

Paper chart recordings were made incidentally for visual inspection; they were not used in the quantitative analyses.

Wave set-down and set-up were measured indirectly, through the mean piezometric level at the beach face, which in turn was detected by means of 1.5 mm ID tappings placed at intervals of 1 m.

The tappings were flush with the beach face. However, in addition a deviating arrangement was made for some of the higher points, which were above the instantaneous waterline during parts of the time (during draw-down). During such time intervals, conventionally placed tappings at these points would experience a piezometric level equal to the local level of the beach face, even though the point would be momentarily "high and dry" above the water. This would lead to a systematic overestimation of the wave set-up in that region.

To remedy this situation, a narrow slot, 1 cm wide and 10 cm deep, was gouged into the beach face, parallel to the side walls of the flume. The slot covered an interval of the beach face from a point above the highest run-up expected, to a point which was always submerged. Because the water in the slot can draw down to below the beach face, tappings which were provided in the bottom of the slot can experience piezometric levels lower than the beach face at the local cross section. A comparison of the set-up so obtained with that obtained from a conventional tapping in the same cross section, in a cross section where the latter reading was not suspect, showed them to agree to within the random experimental error (0.1 to 0.2 mm).

The system of measuring the mean water level with tappings in the slot is somewhat similar to another conceivable way of achieving the same objective, viz. through measurements of the mean piezometric level in the pores of a porous beach. If the permeability of such beach is low enough, though not zero, then the beach is virtually impermeable to water motions in the frequency band of the waves. The wave motion would then be the same as on a strictly impermeable beach, while the beach would be permeable to the mean component, making a MWL-measurement possible.

All the piezometer tappings were connected via 9 mm ID plastic tubes to 86 mm ID stilling wells. The waterlevel in the wells was read with a point gauge to 0.1 mm accuracy.

The energy in the frequency band of the incident waves was filtered out to imperceptible values by the system tube-stilling well. However, some lower-frequency oscillations were noticeable in the wells, particularly for the higher points on the beach. For this reason about 10 readings per well were made for each run, with intervals of about 15 min. The arithmetic average of these readings was taken to represent the local mean water level, and the height of this above the still-water level, which was read in the stilling wells before and after each run, was taken as the local set-up. The difference between individual readings was largest for points high on the slope, but even there the maximum deviation from the average was less than 10% of the local set-up.

Tests and results

As mentioned above, tests have been carried out on a plane beach and on a bar-trough beach. A summary of the independent parameters used is given in the following table.

Run	Profile	d_1 (m)	d_c (m)	$\bar{f}_1$ (Hz)	σ_1 (m)	H_{rms1} (m)	$H_{rms\,o}$ (m)	s_o (-)	∂_c (-)
2	plane	.705	-	.544	.051	.144	.157	.030	-
3	plane	.697	-	.407	.043	.121	.126	.013	-
4	plane	.701	-	.463	.050	.142	.152	.021	-
11	bar	.702	.209	.479	.051	.143	.148	.022	1.41
12	bar	.645	.150	.471	.043	.121	.128	.018	1.17
13	bar	.762	.267	.497	.037	.104	.113	.018	2.35
14	bar	.732	.236	.512	.042	.118	.129	.022	1.83
15	bar	.616	.120	.530	.051	.143	.154	.028	0.78

The subscript 1 refers to the reference gauge, 1.5 m seaward of the toe of the beach; d_c is the still-water depth above the bar crest; H_{rms_o} is a deep-water rms wave height, defined as

$$H_{rms_o} = H_{rms1}/K_{s1} = 8^{\frac{1}{2}}\sigma_1/K_{s1} \ , \qquad (28)$$

in which K_{s1} is the linear shoaling coefficient for depth d_1 and frequency $\bar{f}_1$; s_o is a deep-water steepness defined by

$$s_o \equiv 2\pi\bar{f}_1^{-2}H_{rms_o}/g \ , \qquad (29)$$

and

$$\tilde{d}_c \equiv d_c/H_{rms_o} \ . \qquad (30)$$

The incident wave spectrum was rather narrow (half-power bandwidth about 25% of peak frequency), and virtually unimodal, except for a bulge in the range of frequencies about twice the peak frequency.

For the tests on the plane beach, of given slope which was not varied, the deep-water steepness is the most important independent variable. Three such runs were made, with s_o-values of 1%, 2% and 3% approximately. The results for the run with 2% steepness were intermediate between those of the runs with 1% and 3% steepness. Only the results of the latter two are presented here, the wave heights in figs. 3a and 4a and the set-up in figs. 3b and 4b. These have been plotted against the still-water depth. The variables have been normalized as follows:

$$\tilde{H} \equiv H_{rms}/H_{rms_o}$$

$$\tilde{\bar{\eta}} \equiv \bar{\eta}/H_{rms_o} \qquad (31)$$

$$\tilde{d} \equiv d/H_{rms_o} \ .$$

Five runs were made with a bar-trough profile. The most important independent variable in this case is $\tilde{d}_c$, the relative depth at the bar crest. It was varied from 2.35 to 0.78. In the first case relatively few waves break on the bar, and in the latter case the majority does. Virtually no waves were breaking in the trough region, regardless of the relative depth over the bar crest.

Only the results of the two runs with the largest and the smallest value of $\tilde{d}_c$ are presented here, the wave heights in the figs. 5a and 6a, the set-up in the figs. 5b and 6b. These have been plotted against the relative distance seaward of the still-water line, denoted by

$$\tilde{x} \equiv x/H_{rms_o} \qquad (32)$$

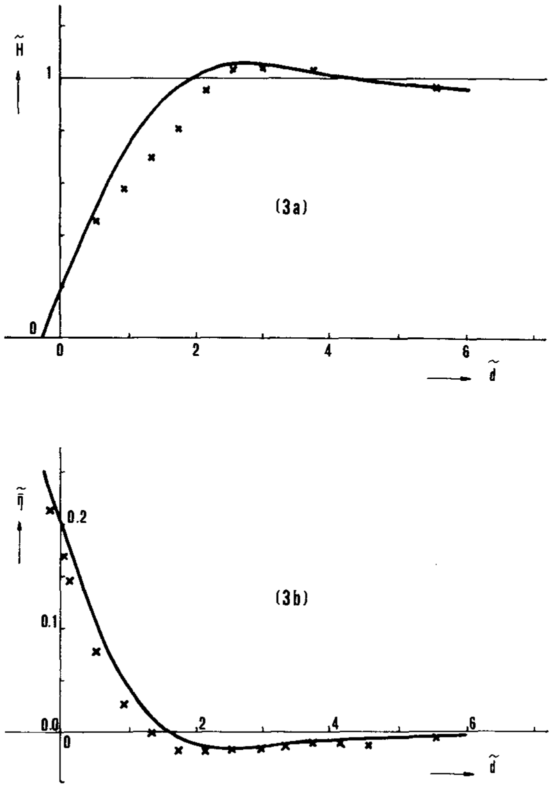

Fig. 3 - Run 3; experimental values (x); theoretical values for $\alpha = 1$, $\gamma = 0.8$ (———)

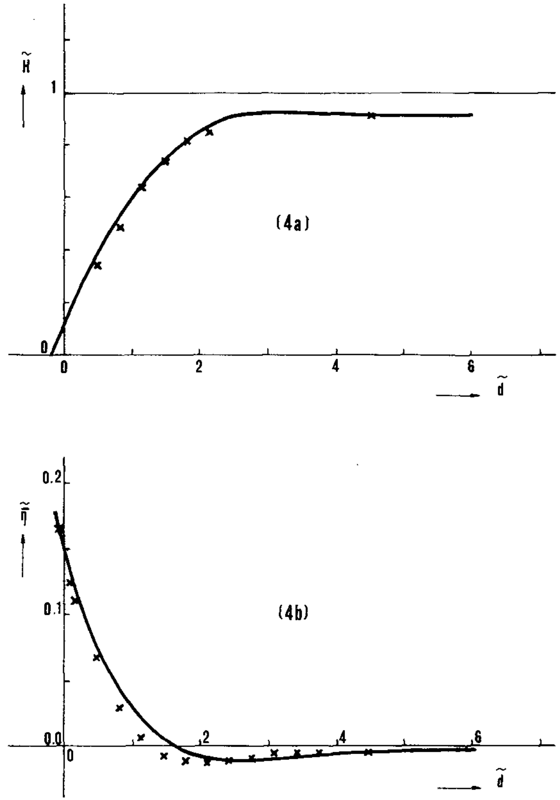

Fig. 4 - Run 2; experimental values (x); theoretical values for $\alpha = 1$, $\gamma = 0.8$ (———)

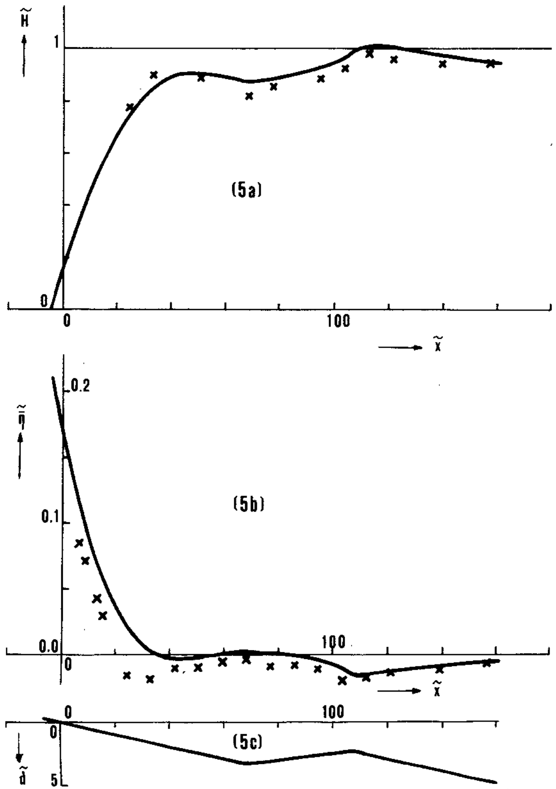

Fig. 5 - Run 13; experimental values (x); theoretical values for $\alpha = 1$, $\gamma = 0.8$ (———)

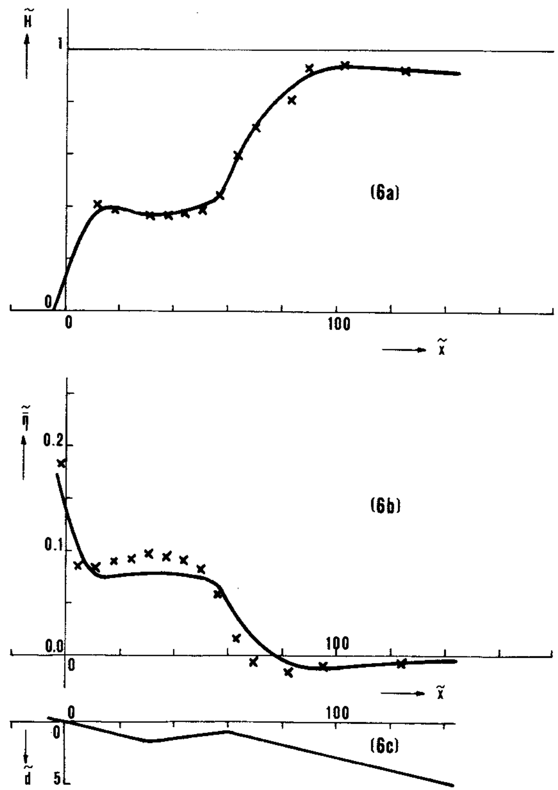

Fig. 6 - Run 15; experimental values (x) theoretical values for $\alpha = 1$, $\gamma = 0.8$ (———)

As an aid in the interpretation, the beach profile has also been plotted, in the figs. 5c and 6c.

In all the plots, except figure 5, the most seaward data point is that of the reference gauge in the constant-depth section of the flume.

Discussion of experimental results

The wave height decay on the plane slope (figs. 3a and 4a) shows a slight steepness effect, in the sense which might be expected. For the lower steepness ($s_o \simeq 0.01$), there is a noticeable enhancement, in fact to values exceeding the deep-water wave height, before the decay due to breaking sets in. No enhancement has been measured for the steeper waves.

For both steepnesses, the maximum set-down occurs near $\tilde{d} = 2$. The maximum relative set-up, occurring at the waterline, and estimated by extrapolation of measured values, decreases with increasing incident wave steepness.

The wave height data for the bar-trough profile (figs. 5a and 6a) show a slight enhancement, followed by a more or less severe decay on the bar, depending on the relative depth over the bar crest. In both cases there is a wave height minimum in the trough of the beach, as would be the case for dissipationless waves in shallow water. This pattern is consistent with the visual observation that there was virtually no breaking going on in the trough region. The fact that the mean water level has a maximum near the trough is also consistent with this.

The mean water level gradient just shoreward of the bar crest is far greater than further toward the trough, but still on the same slope. (This is particularly clear in fig. 6b.) This is due to the fact that in that region there is still dissipation, though the depth is already increasing. The dissipation contributes to the mean water level gradient in the same sense as the increasing depth (both tending to decrease S_{xx}).

COMPARISON OF THEORETICAL MODEL TO EXPERIMENTAL DATA

Results of theoretical model

The equations of the theoretical model described in previous chapters were programmed for numerical evaluation on a digital computer.

The nearshore beach profiles used in the calculations were the same as those used in the experiments. The seaward portion of the profiles was a constant-depth section in the experiments, but in the calculations this section was replaced by a 1:20 slope extending to deep water. The calculations were started in a point of effectively deep water, using experimental values of $\bar{f}_1$ and H_{rms_o} as input parameters.

Due to the finite size of the flume, the wave-induced set-up on the beach causes a lowering of the mean water level in the constant-depth section. Thus, instead of having an initial value for the integration of the set-up equation (23), the constancy of mass in the flume must provide the integration constant. However, there were indications that water was not only stored on the beach, but also behind the wave generator. The latter amount was not predicted. For this reason, the measured set-down in the constant-depth section was used as the reference value for the calculations of the mean water level.

The calculations cannot be initiated until the parameters (α,γ) have been assigned certain values. It is planned to determine a single pair of (α,γ)-values such that the fit of the calculated results for H_{rms} and $\bar{\eta}$ to the complete data set is optimised in some sense. This has not yet been done. In the calculations performed so far, the coefficients α and γ were given some plausible values a priori.

For γ, which indicates a breaker height-to-depth ratio, the value 0.8 was chosen.

The coefficient of proportionality in the expression for the dissipation rate is expected to be of order unity. A logical a priori choice for it would therefore be 1. This has in fact been done in a previous formulation, in which however a coefficient has been used which was a factor 2 smaller than the coefficient α used here, defined in eq. 19. Thus, the initial calculations were in fact performed for $\gamma = 0.8$, $\alpha = 0.5$. The calculated decay rate proved to be too low. The calculations were then repeated with $\gamma = 0.8$, $\alpha = 1$. The results of these calculations have been superimposed on the plots of the experimental data in the figs. 3 through 6.

Discussion of results

A comparison of the theoretical results to the experimental data leads to the following conclusions.

In general, the wave height variation across the surf zone is predicted reasonably well. In some cases, the agreement between model predictions and experimental data is quite good, such as in the figs. 4a and 6a. In other cases the agreement is less good, though still fair.

The set-down and set-up appear to be less well predicted than the wave heights. Although the overall agreement is fair, the model consistently predicts the transition from set-down to set-up in regions somewhat too far seaward. The fact that this discrepancy exists even where the wave height variation is predicted well, suggests that in the breaking process S_{xx} decreases not as fast as the potential energy of the waves, perhaps due to a local relative surplus of kinetic energy of organized wave motions and turbulent motions together. Data obtained previously have given similar indications (Battjes, 1974).

The probability that an arbitrary wave passing a given point shall be a breaking or broken wave, is an important quantity in the description of random waves on a beach. It has not been measured quantitatively.

However, a comparison of theoretical results with qualitative observations is possible, particularly for the runs with a bar-trough profile. In all cases, the calculated value of Q_b in the trough was virtually zero, even though it could be considerable on the bar crest (Q_b = 0.06 in run 13, with $\tilde{d}_c$ = 2.35, and Q_b = 0.46 in run 15, with $\tilde{d}_c$ = 0.78). These variations agree at least qualitatively with the visual observations.

Summarising: the model, when given a single pair of plausible values for the coefficients α and γ, appears capable of predicting qualitatively and quantitatively the main features of the complete set of data, both with respect to the wave heights and to the mean water level variations, both on a plane beach and on a beach with a bar-trough profile. This lends strong support to the usefulness of the model proposed herein.

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