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ENERGY LOSS OF SLOWLY MOVING MAGNETIC MONOPOLES IN MATTER*

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ABSTRACT

We calculate the cross section for exciting simple atoms by slowly moving magnetic monopoles. Including the effects of the monopole magnetic field on the atomic energy levels, we obtain an energy loss per unit density much larger than previous studies. For helium $1/\rho\, dE/dx = 15(\beta/10^{-4})(1-(9.29\times 10^{-5}/\beta)^2)^{3/2} MeV cm^2/g$ for β in the range 10^{-4} to 10^{-3}. The possibility of using helium as a monopole detector is discussed.

During the last few years there has been a great deal of theoretical interest in the supermassive ($\sim 10^{16} GeV$) magnetic monopoles of grand unified theories. Recently Cabrera[1] intensified this interest with his report of a possible monopole event in the superconducting loop experiment at Stanford. The fact that Cabrera's apparatus is sensitive to monopoles of any speed (βc) together with the absence of monopole events in non-induction experiments that are much more sensitive to a flux of fast monopoles ($\beta > 10^{-3}$) indicates that if Cabrera's event was caused by a monopole it was moving slowly. Theoretical models[2] of how supermassive monopoles enter and move in the galactic magnetic fields and in the solar system suggest that their velocities at the earth's surface would be of the order of the earth's orbital velocity about the sun ($\beta \sim 10^{-4}$), and in any event no less than the escape velocity from the earth ($\beta \sim 3 \times 10^{-5}$). Therefore a quantitative understanding of the mechanisms by which slowly moving monopoles lose energy when passing through matter is important both for a description of monopole interactions in the solar system and for the interpretation and design of non-induction experiments.

In this letter we calculate this energy loss for simple atoms. We find that when one includes Zeeman splittings, diamagnetic shifts and crossings of the energy levels caused by the interaction of the atomic electrons with the monopole magnetic field, the calculated energy loss is larger by an order of magnitude or more than that found in previous studies[3] which ignored this effect.

When a monopole passes through matter the time varying pulse of its field can excite electrons in (or ionize) nearby atoms and molecules. The monopole's energy loss can be observed in the form of subsequent electromagnetic radiation when the excited electrons cascade down to their ground states. For the problem of interest we treat the very heavy monopoles classically as moving on straight line trajectories with velocity β. Atoms of size a will "see" the time varying field of the monopole as it traverses as a pulse with frequencies $\omega_m \sim \beta/a$. Thus excitations of frequencies $\omega_x \leq \omega_m$ will be induced in the atom. In the case of very slow passage (β much smaller than the velocity of atomic electrons), ω_m might be too small to excite the atoms; e.g. for $\beta \sim 10^{-4}$ and $a \sim 0.5A$, $\omega_m \sim 0.4eV$. We would then expect the adiabatic approximation to be valid and the resulting excitation probability for atoms with ω_x greater than a few eV to be exponentially small.

This picture is, however, inadequate for very slow monopoles because the strength of a Dirac magnetic charge[4] $g = 1/2e$ is so great that its magnetic field will cause very large changes in the energy levels when it passes through an atom. For example, the characteristic energy shift for a monopole at distance a from an atomic electron of mass m is $eg/2ma^2 \sim 7eV$. Hence, for a monopole passing within the atom substantial level mixings and some level crossings will occur, and the adiabatic approximation can fail badly. In particular, if the ground state and an initial excited state are shifted close together for a monopole near the center of the atom, the two levels will be mixed. There will then be a good chance of finding the atom in an excited state after the monopole has passed. (The transit time of a monopole through an atom $a/\beta \sim 2 \times 10^{-15}$sec for $\beta \sim 10^{-4}$ is very much shorter than the radiative lifetime of an excited atom, $\tau \sim 10^{-8} - 10^{-9}$sec.)

The essential features of the atomic problem can be illustrated by considering passage of a monopole through the center of a simple loop of radius a around which a spinless electron circulates with angular coordinate ϕ. When the monopole is far away the Hamiltonian is $H = L^2/2ma^2; L = -i(\partial/\partial\phi)$ and the eigenvalues and eigenfunctions are $E = \ell^2/2ma^2$, where $\ell = 0, \pm 1, \pm 2, \ldots$ As the monopole approaches along the axis of the loop the doubly degenerate excited states, $\ell \neq 0$, are split by the magnetic field. In particular the $\ell = -1$ excited state shifts down and the $\ell = 0$ ground state up. When the monopole reaches the center of the loop the system is described by the Hamiltonian $H = J^2/2ma^2; J = L - eg = L - 1/2$. The energy levels are all doubly degenerate in this case, with eigenvalues $E = p^2/2ma^2$ and $p = \pm 1/2, \pm 3/2, \ldots$ As the monopole goes through the loop the two states originally with $\ell = 0$, and -1 *cross* - i.e. they are not mixed by the perturbing monopole field since $[A_\mu, L] = 0$. Thus the electron on the loop, initially in the $\ell = 0$ state, is excited and a current flows.

In general the monopole will not pass through the center of the loop but will miss it by a finite impact parameter b. For a loop whose plane is held fixed in space, the result will be the same as that above for all impact parameters $b < a$. (This is just a model of Cabrera's superconducting loop.) This is easily generalized to a fixed loop of any shape.

If on the other hand the loop is free to turn, or rotate its plane, the results will differ. When $b \neq 0$, the component of angular momentum along the z-axis is no longer

conserved. If β is small enough, the loop will rotate out of the way of the passing monopole and no current will be excited in the loop. In this adiabatic limit there will be no energy loss by the monopole. The critical value β_{crit} for this to occur clearly depends on the impact parameter b since for $b = 0$ there is no adiabatic limit - i.e. $\beta_{crit}(b = 0) = 0$. For any β there is a critical $\bar{b}_\beta$ such that for $b < \bar{b}_\beta$ the loop cannot adjust and rotate away. This value defines the effective cross section $\pi\bar{b}_\beta^2$ for excitation and energy loss by a traversing monopole.

To see what occurs in an atom when a slow monopole passes through it we consider first a hydrogen atom[5] including the effects due to electron spin. If the monopole impinges with zero impact parameter along the z-axis the z-component of angular momentum,

$$J_z \equiv \left[\vec{r} \times (\vec{p} - e\vec{A}) + \vec{\sigma}/2 - \hat{n}/2\right]_z \tag{1}$$

is conserved; $\vec{r}$ is the electron coordinate relative to the proton *fixed* at the origin and $\hat{n}$ is the unit vector from the monopole to the electron. Since $\hat{n}_z$ changes sign as the monopole moves from the far left ($\hat{n}_z = -1$) to the far right ($\hat{n}_z = +1$) the z-component of the electron's angular momentum must change by a compensating amount. Consider for example the doubly degenerate ground state of the H atom with principal quantum number $n = 1$ and $m_j = \pm 1/2$ when the monopole is far away. An electron initially with $m_j = -1/2$ will flip spin to $m_j = +1/2$ as the monopole traverses left to right, while one with $m_j = +1/2$ will be raised to an excited state with $n > 1$ and $m_j = +3/2$. On the way up, this level will necessarily cross one moving down from $m_j = -3/2$ to the ground state with $m_j = -1/2$.

In order to map out this level crossing we consider the two extremes of the monopole at the origin and at large distance from the atom and interpolate by a perturbative calculation. As the monopole approaches the atom from a large distance the energy levels split in the characteristic Zeeman pattern for a uniform magnetic field. In particular three of the excited n=2 octet of levels start to move down in energy and the n=1 level with $m_j = +1/2$ starts up toward them. When the monopole is at the origin of coordinates the exact non-relativistic eigenvalues and eigenvectors for the electron are known.[6] The eigenstates of this system are classified in terms of a principal quantum number $n = 0, 1, 2...$ and a spin $j = 0, 1, 2....$ The singlet states with

$j = 0$ have the same energies as the s-states of the H atom: $E_n = -m\alpha^2/2(n+1)^2$. For $\mu \equiv \sqrt{j(j+1)} > 0$, the states form two sequences with eigenvalues $E_n = -m\alpha^2/2(n+\mu)^2$ and $E_n = -m\alpha^2/2(n+\mu+1)^2$. Thus the energy eigenstates are in the following sequence of multiplets starting from the ground state: $1, 3, 1, 3^2, 5, 1..$ where the superscript gives the number of such multiplets. The lowest triplet state with $n = 0$, $j = 1$ is the state of most interest to us and has energy $-m\alpha^2/4$.

When the monopole is near the origin, its effect can be found using the multipole expansion about the origin. If the monopole is at a distance small compared to the atomic size, the terms higher than dipole can be neglected and we can treat the dipole with first order perturbation theory. The results of this calculation can be joined to the Zeeman shifted levels for large separation of the monopole. The solid lines in Fig. 1give the energy levels for the lowest relevant states for arbitrary separation of a stationary monopole along the z axis. The energy levels actually *cross* as the monopole "passes" through the origin because they have different eigenvalues of J_z.

The energy level diagram for a stationary monopole along a path of non-zero impact parameter b can be obtained in a similar manner. This is also displayed in Fig. 1 and follows the solid lines except near the point of closest approach for which the dotted lines are applicable. In this case the symmetry axis rotates as the monopole approaches the atom. Hence J_z is not conserved, and the levels mix and do not cross. The minimum interval between them, ω_{min}, increases with increasing values of b. For sufficiently large impact parameters and slow enough monopole velocities, such that $(\beta/b) < \omega_{min}(b)$, the adiabatic approximation is applicable and the electron would just follow these levels. Then an electron in either of the two degenerate ground states would remain in the ground state. For smaller values of b there will be level mixing, the adiabatic approximation will break down, and the electron will be excited. The region of the monopole's trajectory where the probability of level transition is greatest occurs when the monopole is closest to the origin where the interval between the energy levels is the smallest. In this region we can model the effects of a monopole by using the dipole approximation and first order degenerate perturbation theory amongst the triplet of levels obtained with the monopole at the origin. The time-dependent Schroedinger equation for this system is given by

$$i\frac{d}{dt}\begin{pmatrix} c_1 \\ c_0 \\ c_{-1} \end{pmatrix} = \gamma \begin{pmatrix} \beta t & b/\sqrt{2} & 0 \\ b/\sqrt{2} & 0 & b/\sqrt{2} \\ 0 & b/\sqrt{2} & -\beta t \end{pmatrix} \begin{pmatrix} c_1 \\ c_0 \\ c_{-1} \end{pmatrix} \tag{2}$$

where $c_i(t)$ is the amplitude for the electron to be in eigenstate i in a J_z - diagonal basis and $\gamma = (2-\sqrt{2}) < r^{-3} >_{j=1} /4m = m\alpha^2/4(4-\sqrt{2})a_0$ where a_0 is the Bohr radius. These coupled differential equations can be solved[7] in terms of Weber functions.

Starting with the initial condition $|c_1(t\rightarrow -\infty)|^2 = 1$, we find for large positive times $|c_1(\infty)|^2 = x^2, |c_0(\infty)|^2 = 2x(1-x)$ and $|c_{-1}(\infty)|^2 = (1-x)^2$ where $x \equiv exp[-3b^2/2\bar{b}_\beta^2]$ and $\bar{b}_\beta^2 = 3\beta/\pi\gamma$. The large b limit of $|c_{-1}|^2 \rightarrow 1$ corresponds to the adiabatic approximation of no energy loss. The $b\rightarrow 0$ limit coincides with the sudden approximation for the non-excitation probability: $|c_{-1}|^2 \propto b^4$. Integrating over all impact parameters we obtain for the cross section for exciting the electron to an $n = 2$ level

$$\sigma_{n=2} = (1/2)\int db\, 2\pi b(|c_1|^2 + |c_0|^2) = \pi\bar{b}_\beta^2/2 \tag{3}$$

where the factor 1/2 recognizes that 50% of the time the electron is initially in the state with $m_j = -1/2$ that is not excited by the monopole as shown in Fig. 1. The corresponding energy loss per atom[8] is $\Delta E = (3/8)m\alpha^2\sigma_{n=2}$.

The peak contribution to the cross section occurs when $b = 0.70\bar{b}_\beta \simeq 0.095(\beta/10^{-4})^{1/2} < r >_{j=1}$ where $< r >_{j=1}$ ($\simeq 2.7a_0$) is the radius of the triplet state of the H-atom with the monopole at the center. We therefore expect the dipole expansion to be a reasonable approximation up to $\beta \sim 1 \times 10^{-3}$. The process has a threshold at $\beta = 1.47 \times 10^{-4}$ due to recoil of the H atom. On account of recoil, the values of β and b when the energy level transfers are taking place are different from the asymptotic values. Taking this into account, we find that Eq. (3) should be corrected by a recoil factor $(\beta_0/\beta)^3$ where the subscript 0 refers to quantities when the pole is at the atomic center. This correction factor is obtained by introducing β_0 and b_0 and for β and b in Eq. (2) and using angular momentum conservation, $\beta_0 b_0 = \beta b$. Expressing β_0 in terms of the energy when the levels cross, $1/2\, M\beta_0^2 + \Delta E_0 = 1/2\, M\beta^2$, we can write a threshold corrected formula

$$\sigma_{n=2} = (3\beta/2\gamma)(1-\beta_c^2/\beta^2)^{3/2} \tag{4}$$

with $\beta_c = \sqrt{2M\Delta E_0} = 1.20 \times 10^{-4}$. The energy loss per unit density is $37(\beta/10^{-4})(1-\beta_c^2/\beta^2)^{3/2} MeV cm^2/g$. Comparison is made with previous calculations[3] in Fig. 2.

Atomic helium is of interest as a practical substance for a non-induction experiment that is also amenable to analysis in terms of simple calculations. The energy level diagram for He is also shown in Fig. 1 including the shielding effects between the two electrons. The level mixing for a monopole trajectory with non-zero impact parameter can be calculated in the same way as for H except that in Eq. (2), $\gamma \propto < r^{-3} >_{j=1}$, is changed as a result of shielding due to the presence of a second electron. This change can be calculated by standard variational techniques leading to $\gamma \rightarrow Z_{eff}^3 \gamma$ where $Z_{eff} \simeq 1.33$ for this state. Furthermore, the factor 1/2 appearing in Eq. (3) and (4) should be omitted. This yields the value $5 \times 10^{-18} (\beta/10^{-4})(1 - \beta_c^2/\beta^2)^{3/2}\ cm^2$ for the excitation cross section. Here $\beta_c = 9.29 \times 10^{-5}$ and the threshold values of $\beta \times 10^4$ are 1.06 and 1.03 for $n = 2$ ^3P and ^3S excitations respectively. The energy loss per unit density is $15(\beta/10^{-4})(1-\beta_c^2/\beta^2)^{3/2} MeV\ cm^2/g$ and the relative populations of ^3P to ^3S is approximately 1:2.

The fact that the excitation cross-section in He is large and exclusively to the triplet $n = 2$ levels should provide a unique signature for the passage of the monopole. The ^3P state will decay to ^3S with emission of a 1.15 eV photon, which is not self absorbed, but which may be difficult to detect efficiently. For pure He the metastable ^3S states are likely to remain excited until the atoms reach the walls of the He container, where the electron ejection is likely to occur. The ^3S states may be rendered optically active by the addition of Ne, which is readily excited to the nearby resonant $4s$ levels by collision with the metastable He atoms. There are also a large variety of additives which will be collisionally ionized, producing electrons and ions which could be collected. Some of the uniqueness of signature, which may be lost as the result of the additives, may be regained by the use of timing measurements.

Analogous effects are likely to occur in other atomic and molecular systems. Of particular interest are spherically symmetric systems such as higher Z noble gas atoms. When a monopole passes through the precise center of such a system it must leave the atom with Z units of angular momentum, a circumstance which implies multiple electron excitations likely to decay by auto-ionising Auger processes and to involve

large excitation energies. However, there are also likely to be excitation inducing level crossings for non-zero impact parameter, which can lead to smaller angular momentum transfer, smaller excitation energies, and single electron excitations. The situation is complex and our analysis is in the most preliminary stage, but the likelihood of substantial energy loss and observable excitation appears to us to be high. Provided that the probability of single electron excitation is substantial the kinematic constraint is less severe than for He with thresholds below $\beta = 5 \times 10^{-5}$.

We have also considered some molecular systems in a preliminary way. The H_2 molecule is clearly the simplest case. There are two positions on the molecular axis, outside the pair of protons, at which the monopole causes a crossing of the singlet (bonding) and triplet (anti-bonding) states. Thus passage of the monopole near these points will induce a substantial fraction of transitions which cause the dissociation of H_2 into ground state atoms. The cross section for this process is likely to be comparable to those estimated above. There is also the probability of a dissociation which produces one excited atom, a process which is caused when a monopoles which passes through the molecule nearly parallel to the molecular axis, but we have not estimated the size of this effect. Organic molecules of the sort used in scintillators as well as scintillating inorganic crystals also have intriguing possibilities, which are being investigated. The π electrons on benzene rings, whose excitation is crucial for the functioning of organic scintillators such as polystyrene plastic, are similar in many ways to electrons on the simple spatially fixed loops discussed above except that angular momentum can be transferred to the benzene ring in integral multiples of 6 $\hbar$. Unknown matrix elements for multi-electron transitions of this kind have so far eluded a quantitative calculation.

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4. We will use the Dirac value but all our results are easily generalized to integer multiples.

5. For our calculations the following interactions of the proton with the monopole are negligible on the atomic scale; hyperfine splitting $\sim 0.01eV$, radiative capture $\sim \mu barns$ and the Rubakov-Callan effect.

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8. The probability of radiative de-excitation of the electron as the monopole approaches the atom is $\alpha^6/30\beta$ and is therefore negligible.

FIGURE CAPTIONS

1. The energy levels for atomic hydrogen and helium.

2. The energy loss verses velocity for a Dirac monopole. The curves for atomic hydrogen and helium are the results of this calculation including recoil effects.

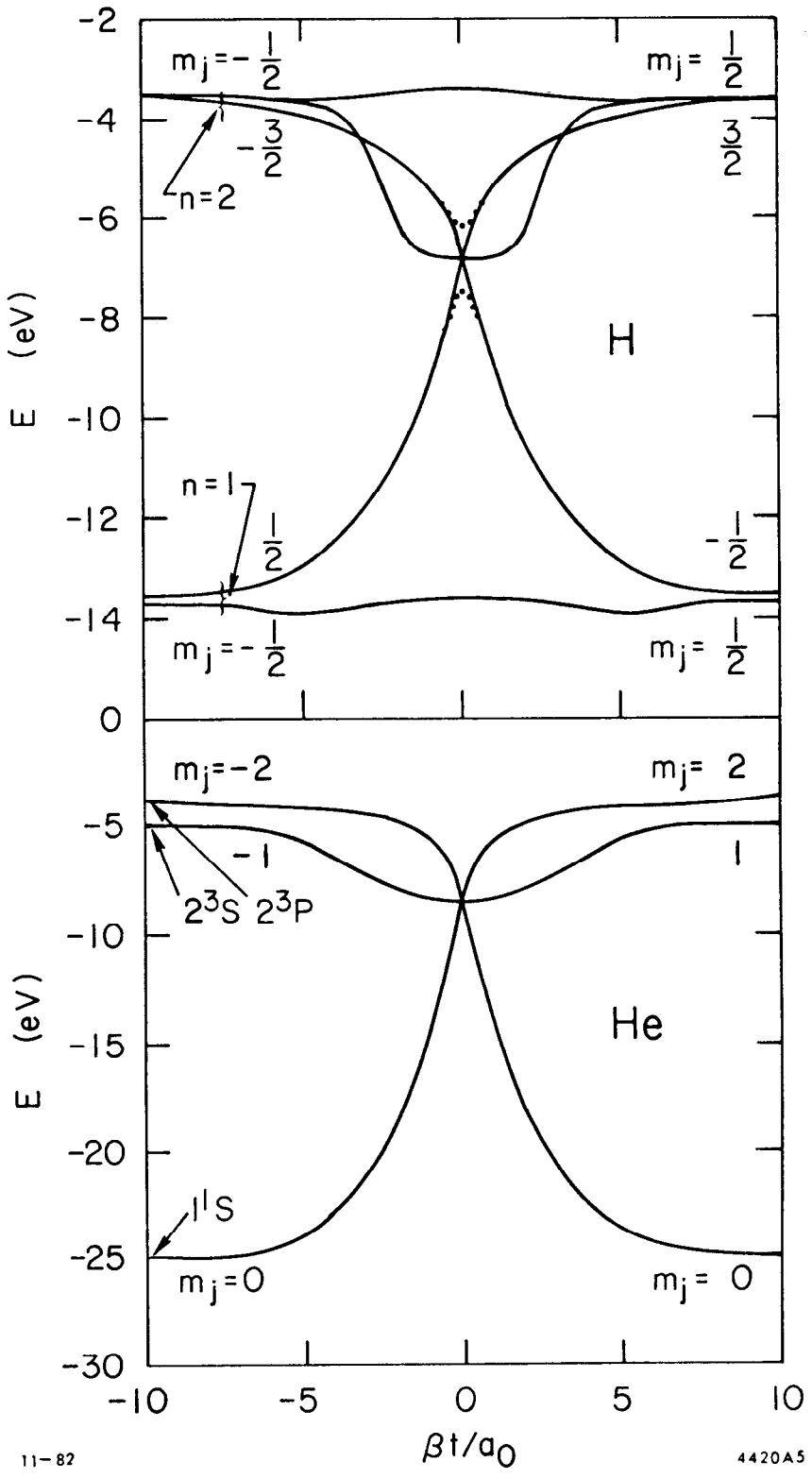
-2
-4
-6
-8
-10
-12
-14
0
-5
-10
-15
-20
-25
-30
E (eV)
E (eV)
$m_j = -\frac{1}{2}$
$m_j = \frac{1}{2}$
$-\frac{3}{2}$
$\frac{3}{2}$
n=2
H
n=1
$\frac{1}{2}$
$-\frac{1}{2}$
$m_j = -\frac{1}{2}$
$m_j = \frac{1}{2}$
$m_j = -2$
$m_j = 2$
-1
1
2^3S 2^3P
He
1^1S
$m_j = 0$
$m_j = 0$
-10
-5
0
5
10
$\beta t/a_0$
11-82
4420A5

Fig. 1

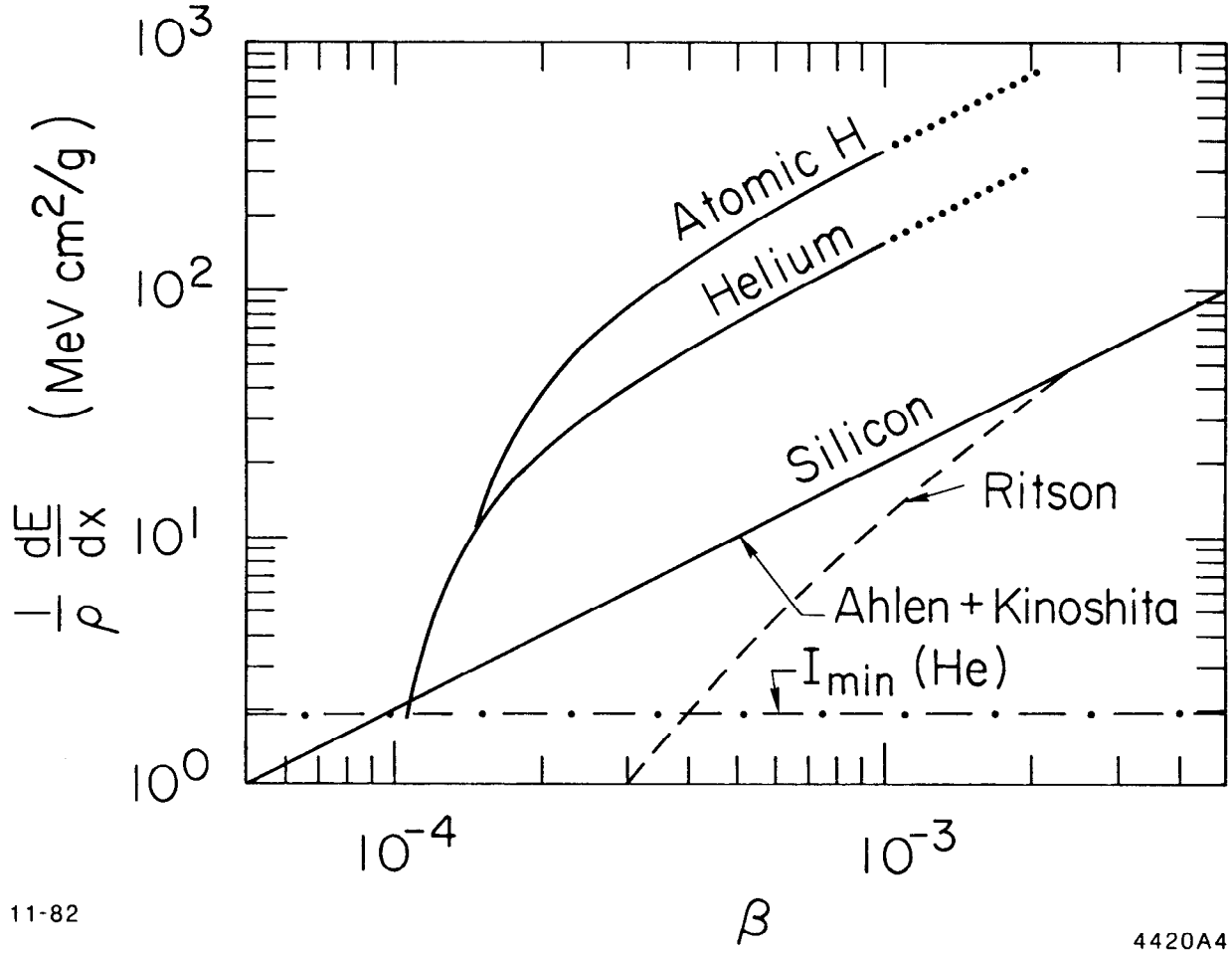
10^3
10^2
10^1
10^0
$-\frac{1}{\rho}\frac{dE}{dx}$ (MeV cm^2/g)
Atomic H
Helium
Silicon
Ritson
Ahlen + Kinoshita
I_{min} (He)
10^{-4}
10^{-3}
β
11-82
4420A4

Fig. 2