ENERGY LOSSES AND MODULATION OF GALACTIC COSMIC RAYS

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Abstract. Numerical solutions of the cosmic-ray transport equation for the interplanetary region and including the effects of diffusion, convection, and energy loss, have been obtained for a galactic differential number-density spectrum of gaussian form, central kinetic energy T_0, and a width at half height maximum of 0.1 T_0. These solutions have been used to show the redistribution in energy, and the reduction in number density, within the solar cavity. The mean energy loss is shown to be usefully approximated by the force-field potential Φ when $\Phi/T_0 \lesssim \frac{1}{2}$. The principal finding is that galactic cosmic-ray protons and helium nuclei, with kinetic energies less than about 80 MeV/nucleon, virtually, are completely excluded from near Earth by convective effects. As a result of this exclusion, it is found that it is not possible to demodulate the proton and helium-nuclei spectra, observed in 1964–65, to obtain information about low-energy galactic cosmic rays.

1. Introduction

The aim of this paper is to illustrate in detail the effects of energy losses on galactic cosmic rays within the solar cavity, and, in particular, to relate these findings to the demodulation problem at low energies. It has been increasingly evident since the formulation of the transport equation including energy losses (Parker, 1965, 1966) that energy losses have significant effects on the modulation of galactic cosmic rays, particularly at kinetic energies T less than 100 MeV. Studies have been made of the spectrum to be expected near Earth from a well distributed spectrum of cosmic rays in the Galaxy, but no general quantitative study has hitherto been made of the redistribution associated with a near-monoenergetic spectrum of particles in galactic space.

Observational evidence for the *necessity* to include energy losses has been provided by Webber (1969), and by Lezniak and Webber (1970), from an examination of a modulation parameter relevant to cosmic-ray fluxes observed at times t_1 and t_2. They find that, at kinetic energies below about 100 MeV, this modulation parameter plotted as a function of rigidity P splits into a separate curve for each species. This splitting is contrary to the predictions of convection-diffusion theory without energy loss; but it is present in approximate solutions including energy losses, and in some numerical solutions that have been obtained (see Webber, 1969 and Lezniak and Webber, 1970).

Theoretical evidence of the *effects* of energy losses on modulation has recently become available for models which, we believe, represent conditions existing in the solar cavity. In these models a regular distribution (e.g. differential intensity $\alpha T^{-2.5}$) is assumed outside the solar cavity, a realistic diffusion coefficient (a function of position and energy) is assumed within the cavity, and approximate analytic solutions or numerical solutions of the transport equation have been obtained (Gleeson and Axford, 1968b; Fisk, 1969, 1971; Fisk and Axford, 1969; Lezniak and Webber, 1970; Goldstein *et al.*, 1970). These solutions show very significant differences in the differential intensities predicted with and without energy losses. But, with one exception, they do not isolate the energy loss of the cosmic ray-particles entering the solar system at a particular kinetic energy, nor do they enable us to determine the response at Earth (say) due to cosmic-ray particles in a particular energy interval in the galactic spectrum. The exception is the force-field solution (Gleeson and Axford, 1968b) which includes a function Φ which can be interpreted as a mean energy-loss; but since this solution is not valid for monoenergetic galactic spectra, and at low energies, the use of the function Φ requires further investigation.

Parker (1965, 1966) has investigated the consequences of injection of monoenergetic particles at a boundary at heliocentric radius R, and determined approximate analytic expressions for the cosmic ray energy distribution *at the origin* under the conditions $VR/\kappa \ll 1$ and $VR/\kappa \gg 1$, with V the solar wind speed and κ the diffusion coefficient assumed constant (see Figure 5, Parker, 1965 and Figure 1, Parker, 1966). These cases correspond to small and large modulation, respectively. The results are very instructive and show clearly the trend of the energy losses, but they are not general enough for our present purpose.

In order to demonstrate in more detail the manner in which the galactic cosmic-ray particles lose energy, and alter their distribution in energy, we have taken a spherically symmetric model, assumed a cosmic-ray differential number-density spectrum with a gaussian distribution (mean T_0 and 10 per cent half-width) at a heliocentric-radius of 10 AU, and determined numerically the resulting distribution within the cavity. A representative range of diffusion coefficients have been used. In each case the mean energy and the number of particles per unit volume have been determined as a function of heliocentric radius.

Seeking to find an analytic function to reproduce the mean energy losses obtained numerically, we have compared them with the force-field 'potential energy' Φ. It is shown that the mean energy loss is well approximated by Φ if $\Phi/T_0 \leqslant \frac{1}{2}$.

It is found for some (but not all) of the diffusion coefficients that galactic particles with $T \leqslant 80$ MeV are excluded almost entirely from regions near Earth by convection. Thus, the galactic particles observed near Earth at these kinetic energies entered the solar cavity with energies greater than that expected from simple considerations of mean energy loss. These effects are very significant for the interpretation and demodulation of cosmic-ray spectra at $T \leqslant 100$ MeV. To show this we take a power-law spectrum at the boundary at 10 AU, remove all the particles below a kinetic energy T_L, and determine the effect on the cosmic-ray spectrum at Earth (say) as T_L is progressively increased.

In the present models the distribution of cosmic-rays within the solar cavity cannot be determined analytically, and we have used numerical solutions of the transport equation. A method of obtaining numerical solutions has been given previously by Fisk (1969, 1971) and used by Lezniak and Webber (1970) and Fisk *et al.* (1969), but here a different method is developed. It avoids the inadvertent source-like behaviour which could occur in the first method given by Fisk (1969), and noted in Fisk (1971), by surrounding the centre of the spherically-symmetric region with a perfectly reflecting spherical surface. This method has the advantage that it can be used with a source of particles at the origin, and thus to investigate propagation of solar cosmic rays in interplanetary space.

The equations and method of solution are given in Section 2 and Appendix A; the redistribution of cosmic-ray particles corresponding to a gaussian galactic spectrum is investigated in Section 3; a comparison of mean energy-loss with Φ is made in Section 4; the effect of a lower-energy cut-off in an otherwise regular galactic spectrum is detailed in Section 5; and finally, in Section 6, the implications of the results are discussed.

2. The Mathematical Equations

The distribution of cosmic rays in a spherically symmetric model which takes account of diffusion, convection, and energy-loss, is governed by the transport equation for the differential number density $U(r, T)$,

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 VU - r^2 \kappa \frac{\partial U}{\partial r}\right) - \frac{1}{3r^2}\frac{\partial}{\partial r}(r^2 V)\frac{\partial}{\partial T}(\alpha T U) = 0, \tag{2.1}$$

and the equation for the differential current density, or streaming, $S(r, T)$,

$$S(r, T) = C(r, T)\, UV - \kappa \frac{\partial U}{\partial r}, \tag{2.2}$$

in which $C(r, T)$ is the Compton-Getting factor (Parker, 1965, 1966; Gleeson and Axford, 1967; Jokipii and Parker, 1967; Gleeson and Axford, 1969). In these equations, r is the heliocentric radius, T is the kinetic energy, $V(r)$ is the solar wind speed, $\kappa(r, T)$ is the diffusion coefficient, and $\alpha = (T + 2E_0)/(T + E_0)$ with E_0 the particle rest energy. Models appropriate to the problem of galactic cosmic rays in the interplanetary region are based on the assumption of a spectrum at an outer boundary at $r = R$, the specification of κ as a function of r and T within the boundary, and the subsequent solution of Equation (1) to determine $U(r, T)$ within the solar cavity.

Many of the general physical consequences of Equation (1) (and also its time dependent form) were demonstrated by Parker (1965, 1966) in a series of illustrative examples using a constant diffusion coefficient. Several approximate analytic solutions have been obtained which further our knowledge of the behaviour of cosmic rays, and some of these can be applied to observations (Jokipii 1967; Gleeson and Axford, 1968a, 1968b; Fisk and Axford, 1969). However, none can be applied to the observational data through the whole energy range of interest and it has been necessary to obtain numerical solutions of Equation (1) for diffusion coefficients throughout the cavity, and spectra at the outer boundary, which are thought to approximate the actual ones (Fisk, 1969, 1971; Webber, 1969; Fisk *et al.*, 1969; Lezniak and Webber, 1970).

The mathematical problem to be solved here is the determination of $U(r, T)$ in the domain $0 \leqslant r \leqslant R$ and $T > T_1$, given $\kappa(r, T)$ and $V(r)$, and having specified $U(R, T)$ and that there be no sources at $r=0$ (i.e. $r^2 S \to 0$ as $r \to 0$). Note that for all spectra of interest $U(R, T) \to 0$ as $T \to \infty$. Furthermore, for all $\kappa(r, T)$ of interest, $\kappa \to \infty$ as $T \to \infty$, and the well known result that $U(r, T) \to U(R, T)$ as $T \to \infty$ follows from the asymptotic solutions of Equation (1).

We have reorganized the problem as follows: First, we have ensured no sources and avoided having to deal numerically with the singularities in Equation (1) at $r=0$ by enclosing the origin in a sphere having a radius R_0 equal to that of the Sun and specifying $S(R_0, T)=0$. Then we have converted this boundary condition by assuming the solar-wind speed to be zero at the surface of the Sun, $V(R_0)=0$, so that, by (2), the boundary condition $S(R_0, T)=0$ requires $(\partial U/\partial r)=0$ at $r=R_0$. The solar-wind speed cannot be exactly zero at $r=R_0$ but it will be small compared with its mean value throughout the solar cavity, and any differences introduced by the approximations $V(R_0)=0$ are expected to be trivial. Finally, we replace T by the alternative independent variable s, defined by $T=\frac{1}{2}E_0 \tan^2 s$, which transforms the infinite range of T into a finite range more suitable for computation; and write $Y(r, s)$ as the differential number density in terms of this variable: $Y(r, s) \equiv U(r, T)$.

The above steps convert the problem into one with a finite domain, $R_0 \leqslant r \leqslant R$, $0 < s \leqslant \frac{1}{2}\pi$, and with boundary conditions

$$Y(R, s) = Y_0(s); \qquad \partial y(R_0, s)/\partial r = 0. \tag{2.3}$$

The equation for $Y(r, s)$ can be solved stepwise by the Crank-Nicholson method modified as shown in Appendix A, and beginning at $s=\frac{1}{2}\pi$ where $Y(r, \frac{1}{2}\pi) = U(r, \infty) = 0$.

For later use (Gleeson and Urch, in preparation) we note that solar sources of cosmic rays can be included in these models by a modification of the boundary condition at $r=R_0$. A solar source releasing $Q(T)\,\mathrm{d}T$ particles per second in the kinetic-energy interval $(T, T+\mathrm{d}T)$, requires

$$4\pi R_0^2 S(R_0, T) = Q(T), \tag{2.4}$$

and, hence, from (2.2), and taking $V(R_0)=0$, the boundary condition becomes

$$\left(\frac{\partial U}{\partial r}\right)_{r=R_0} = \frac{Q(T)}{4\pi R_0^2 \kappa(R_0, T)} \tag{2.5}$$

in place of $(\partial U/\partial r)=0$ at $r=R_0$. The case of $V(R_0) \neq 0$ can also be included; the boundary condition then becomes

$$\left(CVU - \kappa \frac{\partial U}{\partial r}\right)_{r=R_0} = \frac{Q(T)}{4\pi R_0^2}; \tag{2.6}$$

but this refinement appears to be unwarranted at present.

The numerical method given here differs from those used previously, and has the advantages that singularities at $r=0$ are excluded, and that solar sources can be included. Previous work has been based on the method proposed by Fisk (1969, 1971); in it the transformation $u=r^{\gamma}U$, $0<\gamma<1$ is used in order to obtain manageable boundary conditions at $r=0$. The method is suitable for most galactic cosmic ray problems but cannot take into account sources at the origin with the transformation suggested. However, we have shown that sources can be included if the proper transformation is selected for each form of the diffusion coefficient $\kappa(r, T)$ (Gleeson and Urch, 1970).

3. Gaussian Input Spectra

In order to show the redistribution and energy losses of galactic cosmic rays, we have specified a differential number density distribution

$$U(R, T) = A \exp\left[-280 (T/T_0 - 1)^2\right] \tag{3.1}$$

at a boundary situated at $R=10$ AU, and determined the resulting $U(r, T)$ for several forms of the diffusion coefficient $\kappa(r, T)$. This distribution has a width of 0.1 T_0 at half maximum height; the mean energy, T_0, has been taken in the range 19–1860 MeV/nucleon; and the solar wind speed has been taken as

$$V(r) = 400\{1 - \exp[(1 - r)/15]\} \text{ km sec}^{-1}, \tag{3.2}$$

where r is in units of the solar radius R_0.

Typical redistribution of protons are shown in Figures 1 and 2 for the diffusion coefficient $\kappa = 1.9 \times 10^{21}\, r\beta P \text{ cm}^2 \text{ sec}^{-1}$; r is in AU, P (the rigidity) in GV, and β=(velocity/speed of light). This value of κ is considered to be low, and has been chosen to show the redistribution effects clearly. Figure 1 is for $T_0=280$ MeV and Figure 2 for $T_0=930$ MeV, and in each case the distribution on the extreme right is the spectrum imposed at the boundary. The main characteristics are the marked decrease in energy, the spread of the particles in kinetic energy, and the considerable reduction in differential intensity, as the distance from the boundary is increased. Of particular interest is the extensive low-energy 'tail' which develops even quite close to the outer boundary. This tail is more marked for $T_0=930$ MeV than for $T_0=280$ MeV.

The reductions in energy that take place are well indicated by comparing the mean kinetic energy at heliocentric radius r,

$$\langle T(r)\rangle = \int_0^T T' U(r, T')\, \mathrm{d}T' \Big/ \int_0^T U(r, T')\, \mathrm{d}T', \tag{3.3}$$

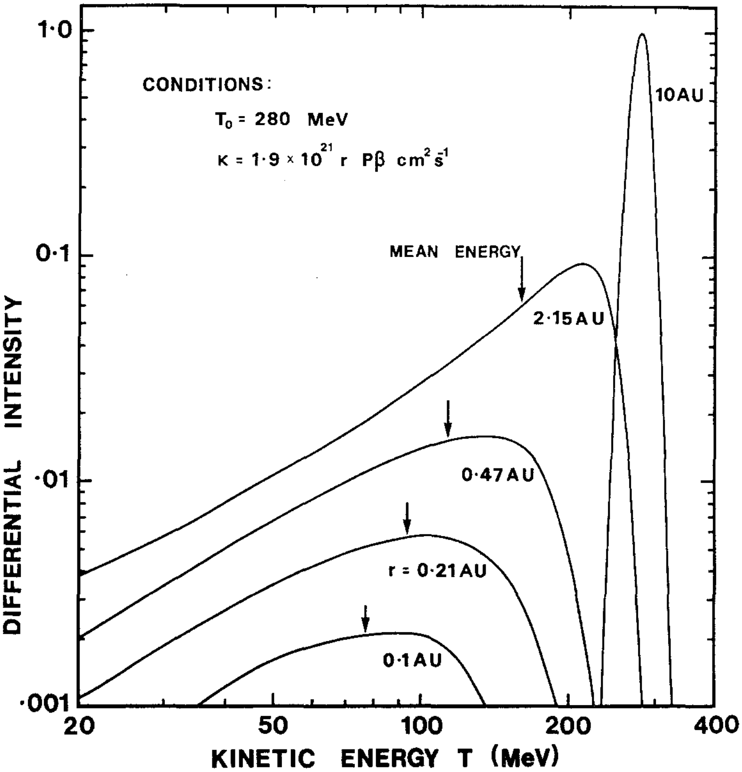

Fig. 1. Illustrating the redistribution of cosmic-ray protons within the solar cavity with a gaussian distribution of the differential number density, specified at $r = R = 10$ AU. The distribution is centred at $T_0 = 280$ MeV and is 28 MeV wide at half maximum density. A diffusion coefficient $\kappa = 1.9 \times 10^{21}\, rP\beta$ cm^2 sec^{-1}, with r in AU and P in GV, has been assumed.

with T_0, its value at the outer boundary. The mean energies are marked in Figure 1, and are shown in terms of the mean energy loss

$$\Delta \langle T(r) \rangle = T_0 - \langle T(r) \rangle \tag{3.4}$$

in Figure 3 for a selection of T_0 values. In the model for which Figure 3 applies a 'realistic' diffusion coefficient, $\kappa = 6.0 \times 10^{21}\, r^{1/2} \beta P$ cm^2 sec^{-1} with r in AU and P in GV, has been assumed (see Section 5 and Figure 7). In general, the lower energy particles lose a greater fraction of their kinetic energy.

The degree by which particles are prevented, by convection, from penetrating to heliocentric radius r can be assessed by counting all particles per unit volume *irrespective of energy*, and comparing this number with the corresponding value at the boundary. The comparison ratio

$$F(r, T_0) = \int_0^\infty U(r, T')\,\mathrm{d}T' \Big/ \int_0^\infty U(R, T')\,\mathrm{d}T' \tag{3.5}$$

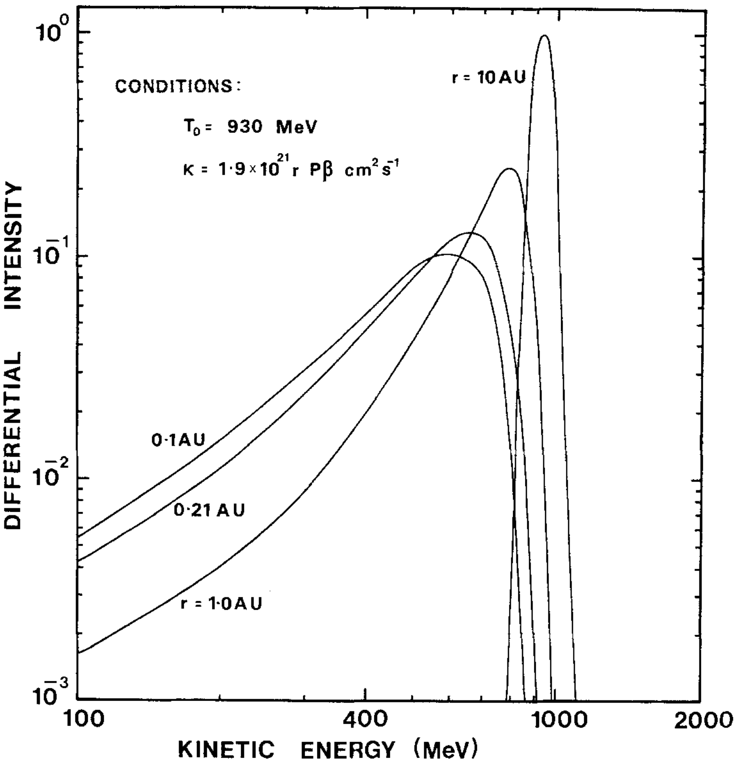

Fig. 2. Illustrating the redistribution of cosmic-ray protons within the solar cavity with a gaussian distribution of the differential number density specified at $r = R = 10$ AU. The distribution is centred at $T_0 = 930$ MeV and is 93 MeV wide at half maximum density. A diffusion coefficient $\kappa = 1.9 \times 10^{21}\, rP\beta$ cm^2 sec^{-1}, with r in AU and P in GV, has been assumed.

is denoted the density reduction factor; it is shown in Figure 4 for the present case (that of Figure 3). Cosmic-ray protons with $T_0 = 45$ MeV, say, are reduced sharply in number density inside the solar cavity, the density reduction factor being $\sim 10^{-3}$ at Earth; but those with $T_0 = 186$ MeV have a density reduction factor of about 3 at Earth.

Helium nuclei lose less energy (per nucleon), and suffer less reduction in number density, than protons because of their higher diffusion coefficient. The energy loss per nucleon is roughly half that of protons, and the redistributions in energy are not as broad.

For any species the details of the energy redistribution and the density reduction factor are very dependent on the diffusion coefficient adopted through the cavity. We have determined the redistribution in energy, the mean energy loss, $\Delta\langle T(r)\rangle$, and $F(r, T_0)$ for the diffusion coefficient forms:

$$\kappa \propto \begin{cases} r\beta P, \\ r^{1/2}\beta P, \\ \exp(r/r_1)\,\beta P; \end{cases} \tag{3.6}$$

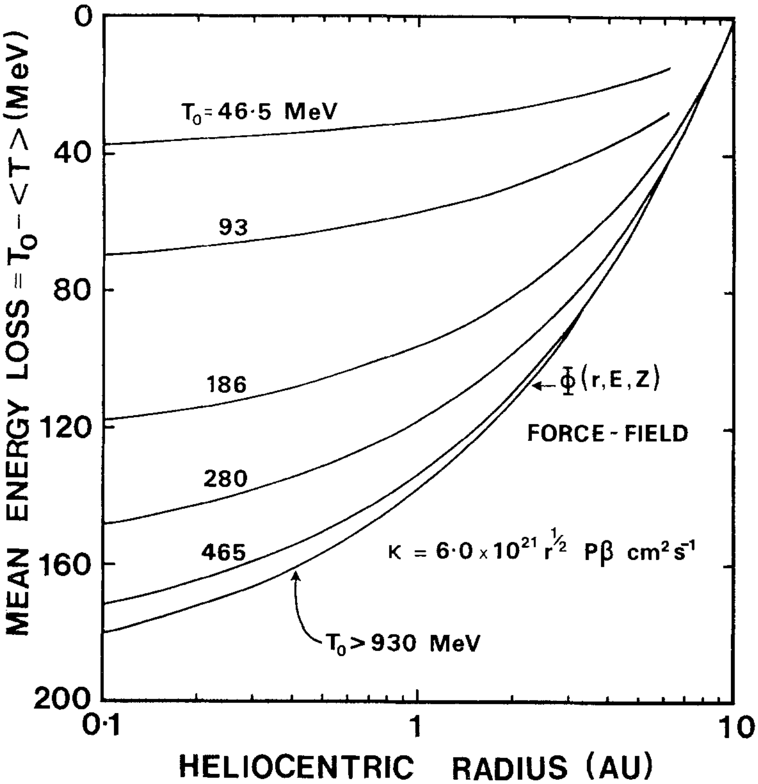

Fig. 3. Illustrating the mean energy loss, $T_0 - \langle T \rangle$, of protons as a function of heliocentric radius. For each curve a gaussian distribution of differential number density, with central energy T_0 and width 0.1 T_0 at half maximum density, is specified at the boundary at $r = R = 10$ AU. A diffusion coefficient $\kappa = 6.0 \times 10^{21}\, r^{1/2} P\beta$, with r in AU and P in GV, has been assumed. The force-field potential $\Phi(r, E, Z)$ has also been shown for comparison (Section 4 of text).

and for a similar set with radial dependence as above, but a rigidity dependence given by

$$\kappa_2(P) = \begin{cases} P, & P \geqslant P_c; \\ (PP_c)^{1/2}, & P \leqslant P_c; \end{cases} \tag{3.7}$$

(see the discussion in Section 5). In each of these cases we have set $\kappa = 4 \times 10^{22}\,\text{cm}^2\,\text{sec}^{-1}$, at Earth for $P = 6$ GeV in agreement with the value deduced from the magnetic power spectra in 1965 (Jokipii and Coleman, 1968), but we have also explored the effects of changing the value of the coefficients. In general, a lowering of the diffusion coefficient increases $\Delta\langle T(r)\rangle$, increases $F(r, T_0)$, and produces a greater spread in energy of the particles; and the distribution within the solar cavity is very sensitive to the rigidity dependence of $\kappa(r, T)$.

In the above we used a gaussian external spectrum with a width of 0.1 T_0 between the half-maximum-density points. If the width is increased to 0.2 T_0 the results presented in Figures 3 and 4 are unchanged; the distributions corresponding to those of Figures 1 and 2 become broader, but the general features are unaltered.

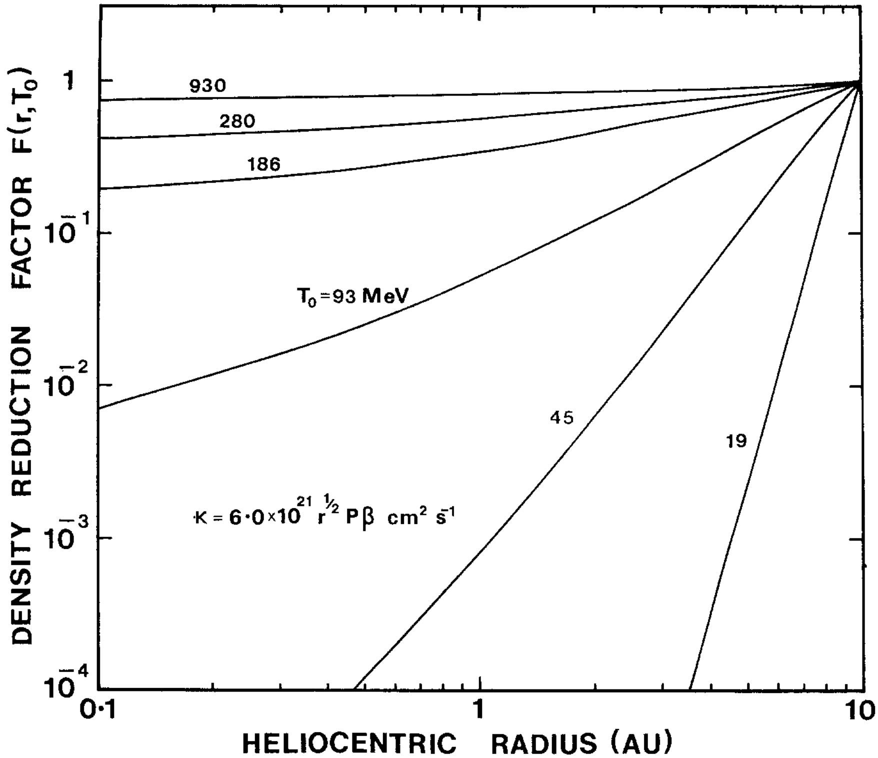

Fig. 4. The curves illustrate the way in which the number density of particles is reduced within the solar cavity. Conditions are the same as for Figure 3. The ordinate $F(r, T_0)$ is the ratio of the *total* number of particles per unit volume at radius r to that at the boundary at $r = R = 10$ AU.

4. The Force-Field Energy Loss

It would be useful to have a general, approximate, expression for the mean energy loss $\Delta\langle T(r)\rangle$ of the distribution of particles for the present models and gaussian galactic spectra. In this context we consider the force-field solution of Gleeson and Axford (1968b). They showed that if $|S/UV| \ll 1$ at each point between r and R and that if the diffusion coefficient was separable, $\kappa = \kappa_1(r)\,\kappa_2(P)\,\beta$, then the differential intensities at r and R of particles with electric charge Ze were related by

$$\frac{j(r, E)}{E^2 - E_0^2} = \frac{j(R, E + \Phi)}{(E + \Phi)^2 - E_0^2}, \tag{4.1}$$

with $E = T + E_0$ the particle energy, and $\Phi(r, T, Z)$ an energy dependent 'potential energy'. This 'potential energy', $\Phi(r, T, Z)$, is a function of the modulation parameter

$$\phi(r) = \int_r^R \frac{V(x)}{3\kappa_1(x)}\,\mathrm{d}x, \tag{4.2}$$

and Φ can be determined if $\phi(r)$ and the function $\kappa_2(P)$ are known. When the diffusion coefficient has the form $\kappa=\kappa_1(r)P\beta$ then Φ has the simple dependence $\Phi=|Ze|\,\phi(r)$. Because this form of κ is appropriate for realistic models (see Section 5), and since in this case Φ, conveniently, is independent of T, we will make use of $\kappa=\kappa_1(r)P\beta$ in the discussion in this section. Note that the parameter $\phi(r)$ is independent of the species, and thus very suitable for generally characterizing modulation.

The above approximate solutions (4.1) have been shown to be in excellent agreement, down to $T\sim 100$ MeV/nucleon, with numerical solutions obtained for models of the galactic cosmic-ray propagation problem in which acceptable forms of the diffusion coefficient, and external spectra, have been assumed; and which match the observed spectra (Fisk *et al.*, 1969; Fisk, 1971). In these cases the galactic spectrum is smoothly varying with kinetic energy, in contrast with the peaked form of the gaussian spectrum used in Section 3. Because of this, the approximate solution (4.1) is not expected to give good results for the redistribution in energy of particles injected according to the gaussian distribution (3.1), but we will show that it does provide useful estimates of $\Delta\langle T(r)\rangle$. The poor representation of the *redistribution* of particles in energy is exemplified in Figure 5; it displays a numerically determined distribution of protons for the case $T_0=930$ MeV in (3.1), and the corresponding force-field solution.

The function $\Phi(r, T, Z)$ has been identified with the mean energy-loss that particles, observed at radius r with energy $E=T+E_0$, have undergone in their passage from outside the boundary at radius R. These particles have originated outside the boundary with kinetic energies in a range about $T+\Phi$ (Gleeson and Axford, 1968b; Jokipii and Parker, 1970). Strictly then, Φ should not be identified with the mean loss of energy of particles observed at radius r and originating from a monoenergetic (or near monoenergetic) distribution imposed at the boundary. However, we note that when the modulation is small ($\Phi \ll T$), to a good approximation

$$\Phi = \frac{\alpha T}{3} \int_r^R \frac{V(x)}{\kappa(x, T)}\, \mathrm{d}x, \tag{4.3}$$

and that this has, at $r=0$, the same value as the mean energy loss determined by Parker (1966) for the special case of κ constant, monoenergetic injection near the boundary, and small modulation (Gleeson and Axford, 1968b). This correspondence arises because the energy losses are a small fraction of the initial kinetic energy, and the particles observed at (r, T) each entered the solar cavity with nearly the same energy. Thus their mean energy loss, Φ, could be expected to be the same as the mean energy loss of particles injected monoenergetically. The distinction between the two cases of mean energy loss was not made clear in the original formulation of Φ.

The discussion in the previous paragraph suggests that the force-field energy loss Φ might be used to estimate $\Delta\langle T(r)\rangle$ in the present calculations, particularly when the modulation is small; and we have represented it in Figure 3 so that a comparison

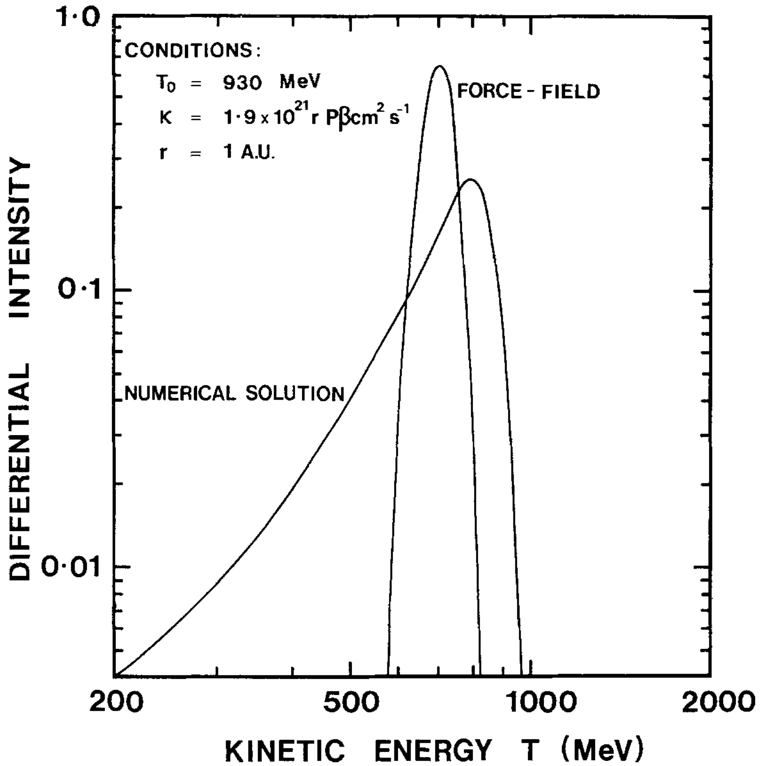

Fig. 5. Illustrating, for protons at $r = 1$ AU, the difference between a force-field solution for the differential intensity and the corresponding numerical solution. Conditions assumed are those given for Figure 2 with a gaussian differential number density having $T_0 = 930$ MeV specified at $r = 10$ AU. The force-field solution does *not* give a good representation of the distribution but in this case it does reproduce the mean energy (also see Figure 3).

can be made. In general Φ is an excellent estimate of $\Delta\langle T(r)\rangle$ when $\Phi/T_0 \ll 1$ (e.g. $\Phi(r)$ corresponds almost exactly with the curve labelled $T_0 > 930$ MeV in Figure 3), and it provides a useful estimate even when $\Phi/T_0 \leqslant \frac{1}{2}$ (e.g., at $r=1$ AU with $T_0 = 280$ MeV we find $\Phi = 138$ MeV and $\Delta\langle T(r)\rangle = 118$ MeV). The value of Φ always exceeds that of $\Delta\langle T(r)\rangle$. This relationship between Φ and $\Delta\langle T(r)\rangle$ has also been verified in models assuming the diffusion coefficients listed in Equations (3.6) and (3.7) and for helium nuclei.

5. The Modulation of Low-Energy Cosmic Rays

This section is concerned, primarily, with the effects of galactic protons and helium nuclei having $T \leqslant 100$ MeV/nucleon on the spectra near $r=1$ AU; and with the demodulation of the near-Earth spectra for $T \leqslant 100$ MeV/nucleon. In the first of three subsections we give examples of the results obtained; in the second we consider observational data pertinent to the diffusion coefficient; and in the third we indicate, briefly, the results obtained for a range of diffusion coefficients and assess the validity of each.

A. MODULATION AT LOW ENERGIES

From Section 3, we recall the extensive low-energy tail inside the solar cavity associated with protons from the gaussian galactic spectrum with $T_0 = 930$ MeV (Figure 2); this tail indicates that there may be a significant contribution to the differential intensity at energies much less than $T_0 - \langle T \rangle$. We also note that protons with $T_0 = 50$ MeV have, typically, $\langle T \rangle \simeq 25$ MeV at $r = 1$ AU and a very large density reduction ($F \sim 10^{-3}$). It is evident from these results that, with a 'reasonable' galactic spectrum, galactic protons with $T \sim 50$ MeV may contribute an insignificant portion of the differential intensity of cosmic-ray protons observed in the vicinity of Earth at kinetic energies $T \leqslant 50$ MeV.

We establish a 'reasonable' galactic spectrum by noting that numerical methods have been used previously to determine spectra near Earth resulting from an assumed galactic spectrum, and assumed variations of $\kappa(r, T)$ within the solar cavity; and agreement with the observed spectra has been obtained for protons, helium nuclei and electrons (Fisk *et al.*, 1969; Lezniak and Webber, 1970, Fisk, 1971; Goldstein *et al.*, 1970). A galactic spectrum of protons and helium nuclei of the form $U(\infty, T) \alpha E^{-2.5}$ is typical of those assumed. In order to examine the degree to which low-energy galactic cosmic rays penetrate to near Earth, we have modified the proton and helium galactic spectra to

$$\begin{aligned} T/T_L > a &: U(\infty, T) = AE^{-2.5}, \\ T/T_L < a &: U(\infty, T) = AE^{-2.5} \exp\left(-\ln 2\,(T/T_L - a)^2/(1-a)^2\right), \end{aligned} \tag{5.1}$$

with the value of a being 1.1 or 1.2. This modification effectively removes particles with kinetic energy less than T_L, and we have determined numerically the spectra at Earth ($r = 1$ AU) for a range of $T_L \leqslant 380$ MeV/nucleon.

Figure 6 shows a typical result for protons; the diffusion coefficient has been taken as

$$\kappa = 6.0 \times 10^{21}\, r^{1/2} P\beta\, \text{cm}^2\, \text{sec}^{-1}, \tag{5.2}$$

(r in AU and P in GV), and this is representative of the diffusion coefficients thought to be present during 1964–65. With this diffusion coefficient, and $R = 10$ AU, we can reproduce the electron modulation, and the proton and helium nuclei spectra, at $r = 1$ AU (see later discussion and Figure 7). It can be seen in Figure 6 that there is no significant reduction in the differential intensity at $r = 1$ AU in the kinetic-energy range $T \leqslant 50$ MeV, for any cut-off energy T_L less than 60 MeV. A more complete set of calculations shows that with $T_L \simeq 85$ MeV there is 10 percent reduction in differential intensity at $r = 1$ AU, and in the range $T \leqslant 50$ MeV. Similar results apply to helium nuclei. These calculations show that, with this diffusion coefficient and a galactic spectrum $U(\infty, T) \alpha E^{-2.5}$, there is, effectively, almost total exclusion of galactic protons with $T \leqslant 85$ MeV and galactic helium nuclei with $T \leqslant 50$ MeV/nucleon from the interplanetary region near $r = 1$ AU.

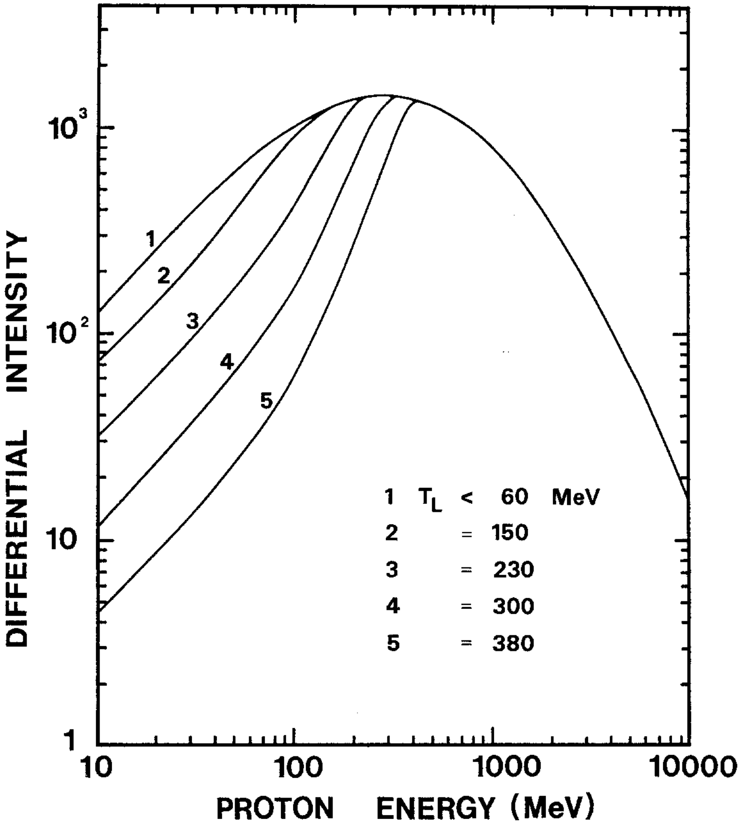

Fig. 6. Illustrating the effect of removing galactic protons with kinetic energies less than T_L. We have put $R = 10$ AU; taken $U(R, T) \alpha E^{-2.5}$; and assumed a diffusion coefficient $\kappa = 6.0 \times 10^{21} r^{1/2} P\beta$ cm^2 sec^{-1}, with r in AU and P in GV.

With other forms of the diffusion coefficient and galactic spectra, the exclusion of cosmic-ray particles may not be as extensive as in the above case. For example, if we assume that

$$\begin{aligned} \kappa &= 6.0 \times 10^{21}\, r^{1/2} P\beta \,\text{cm}^2\,\text{sec}^{-1}, \qquad P > 1\ \text{GV} \\ \kappa &= 6.0 \times 10^{21}\, r^{1/2} \beta \,\text{cm}^2\,\text{sec}^{-1}, \qquad P < 1\ \text{GV} \end{aligned} \tag{5.3}$$

(r in AU and P in GV), and apply a cut-off at $T_L = 18$ MeV to the galactic spectrum, there is a 10 percent reduction in the proton differential intensity near Earth at $T \simeq 10$ MeV and a 40 percent reduction at $T \simeq 5$ MeV.

These results are important in relating the observed spectra near Earth to the corresponding galactic spectra. In view of the difference in the above two cases, it is essential to examine our knowledge about the diffusion coefficient in order, if possible, to establish the kinetic energy at which exclusion is likely to be complete. Two sets of information are relevant to the restriction of the form and magnitude of κ: the magnitude and rigidity dependence of κ found near Earth; and the galactic and near-Earth cosmic-ray electron spectra.

B. THE DIFFUSION COEFFICIENT

At the present time we have available direct knowledge of the diffusion coefficient during 1962 and 1965–66 and only near the orient of Earth; it is derived from observations of the magnetic-field power spectrum. For our present purpose, the most extensive is that given by Jokipii and Coleman (1968) and obtained on Mariner 4 during 1965. Their work yields $\kappa = 4 \times 10^{22}$ cm^2 sec^{-1} (within a factor of about 2) at $P = 6$ GV; shows an approximate $P\beta$ dependence of κ for $P_c \leqslant P \leqslant 10$ GV with $P_c \sim 1$ GV; and shows an approximate $(PP_c)^{1/2}\,\beta$ dependence for $P < P_c$.

Other observations of the magnetic-field power spectrum have been reported from Mariner 2 during 1962 (Coleman, 1966), from Mariner 4 during 1965 (Siscoe *et al.*, 1968), and from Pioneer 6 during 1966 (Sari and Ness, 1969). The frequency dependence of the power spectra obtained in these three investigations are summarized in Figure 6 of Sari and Ness (1969). These spectra show κ having a $P\beta$ dependence at large rigidities; this dependence extended to 0.1 GV in 1962 (Coleman) but changed in the other cases to a $P^{1/2}\beta$ dependence (Siscoe *et al.*) or a $P^0\beta$ dependence (Sari and Ness) at rigidities less than P_c. The values of P_c are uncertain but appear to be less than 1 GV; Sari and Ness give $P_c = 0.4$ GV as appropriate for their data. The differences between these results may show inconsistencies in the results or reflect secular changes in the microstructure of the interplanetary medium (Sari and Ness, 1969).

With the above considerations and, as usual, assuming κ is of the form $\kappa = \kappa_1(r)\,\kappa_2(P)\,\beta$ we adopt a diffusion coefficient

$$\kappa = \kappa_1(r)\,\beta \times \begin{cases} P, & P > P_c\,; \\ P_c^{1-\gamma} P^{\gamma}, & P < P_c\,; \end{cases} \tag{5.4}$$

with $\gamma = 0$ or $\gamma = \frac{1}{2}$, and consider values of P_c in the range $0 \leqslant P_c \leqslant 1$ GV. The functional forms used for $\kappa_1(r)$ are discussed in the next paragraph, and in each case we set, nominally, $\kappa = 4 \times 10^{22}$ cm^2 sec^{-1} at $r = 1$ AU and $P = 6$ GV but permit some variation about this value. This form of $\kappa(r, T)$ includes all of the cases derived from the magnetic-field power spectra.

The radial dependence of $\kappa(r, T)$ is not known except for the substantially constant values, in the small heliocentric range 1.0 AU $\leqslant r \leqslant$ 1.5 AU, reported by Jokipii and Coleman (1968), and deduced from observations of the magnetic field made on Mariner 4. Since $\kappa(r, T)$ is expected to increase with r, in order to investigate the exclusion of cosmic-ray particles under a wide range of possible radial dependence, we have considered each of the functional forms $r^{1/2}$, r, and $\exp(r/r_0)$ for $\kappa_1(r)$. The latter, $\exp(r/r_0)$, has been used extensively in other papers (Gleeson and Axford, 1968a, 1968b; Fisk, 1969; Fisk *et al.*, 1969; Lezniak and Webber, 1970; Goldstein *et al.*, 1970b) but the first two forms have not; and we concentrate on $\kappa_1 \propto r^{1/2}$ in this presentation.

We note that the diffusion coefficients (5.2) and (5.3), used, at the beginning of this section, to exemplify the cosmic ray exclusion are of the form (5.4); and also note that

(5.2) provides the smallest possible diffusion coefficient at low rigidities with a given $\kappa_1(r)$, and (5.3) provides the largest.

Also available are crude estimates of the galactic electron spectrum (based on observation of the galactic radio emission), and observations of the cosmic ray electron spectrum near Earth (Webber, 1968; Goldstein *et al.*, 1970; see Figure 7a). These two spectra provide an important constraint on the radial dependence and rigidity dependence of the diffusion coefficient between Earth and galactic space: any diffusion coefficient which is assumed must provide the observed modulation of the galactic electron spectrum in the rigidity range of the protons and helium nuclei we are considering ($P \geqslant 0.13$ GV) (Lezniak and Webber, 1970; Goldstein *et al.*, 1970b).

In assessing the validity of a diffusion coefficient, we consider if it has the approximate form of (5.4); whether it leads to the required electron modulation; and then what galactic proton and helium nuclei spectra are required in order to reproduce the spectra observed near Earth. Because of the data available, our assessment is restricted to the period about the 1964–65 solar minimum.

C. PARTICULAR CASES AND THEIR VALIDITY

The diffusion coefficient (5.2), which gave a near Earth spectrum which was insensitive to the corresponding galactic spectrum at $T \leqslant 50$ MeV/nucleon (Figure 6), has the form (5.4) with $P_c = 0$, $\gamma = \frac{1}{2}$, and $\kappa_1 \propto r^{1/2}$. There is reasonable agreement between the electron spectrum observed near Earth and those calculated from the estimated galactic electron spectra on the basis of this diffusion coefficient, Figure 7a. The proton and helium spectra observed near Earth are reproduced with this diffusion coefficient and galactic spectra $U(\infty, T) \propto E^{-2.5}$, Figure 7b. Thus the diffusion coefficient (5.2) appears to be acceptable.

In the above example, P_c is less than the value of 1 GV given by Jokipii and Coleman (1968), and already noted. If we take $P_c = 1$ GV, and retain $\kappa_1 \propto r^{1/2}$ and $\gamma = \frac{1}{2}$, the observed proton and helium-nuclei spectra can be readily reproduced with a galactic spectrum slightly modified from $U(\infty, T) \propto E^{-2.5}$. The low-energy proton and helium-nuclei spectra at $r = 1$ AU are still insensitive to the corresponding galactic spectra, but, because the diffusion coefficient for $P < P_c$ has been increased above that of (5.2) (and that used in Figure 6), the exclusion is not as marked. To illustrate this we note that, in this case, there is a 10 percent reduction in the differential intensity at Earth for protons with $T \leqslant 50$ MeV when $T_L = 50$ MeV, whereas with $P_c = 0$ the 10 percent reduction occurs with $T_L = 85$ MeV. However, because of the increased magnitude over that of (5.2) for $P < P_c$, this diffusion coefficient does not lead to a satisfactory match between the near Earth and galactic electron spectra.

With $P_c = 1$ GV, $\kappa_1 \propto r^{1/2}$, and $\gamma = \frac{1}{2}$, the electron spectra can be matched if the magnitude of the diffusion coefficient is decreased from the reported value of 4×10^{22} cm^2 sec^{-1} at $P = 6$ GV to 1.7×10^{22} cm^2 sec^{-1} at $P = 6$ GV. With this κ, galactic proton and helium-nuclei spectra of the form $(T + E_0/2)^{-2.5}$ are required in order to match the observed spectra. Again the low-energy proton and helium-nuclei spectra near Earth remain insensitive to the corresponding galactic spectra, with the

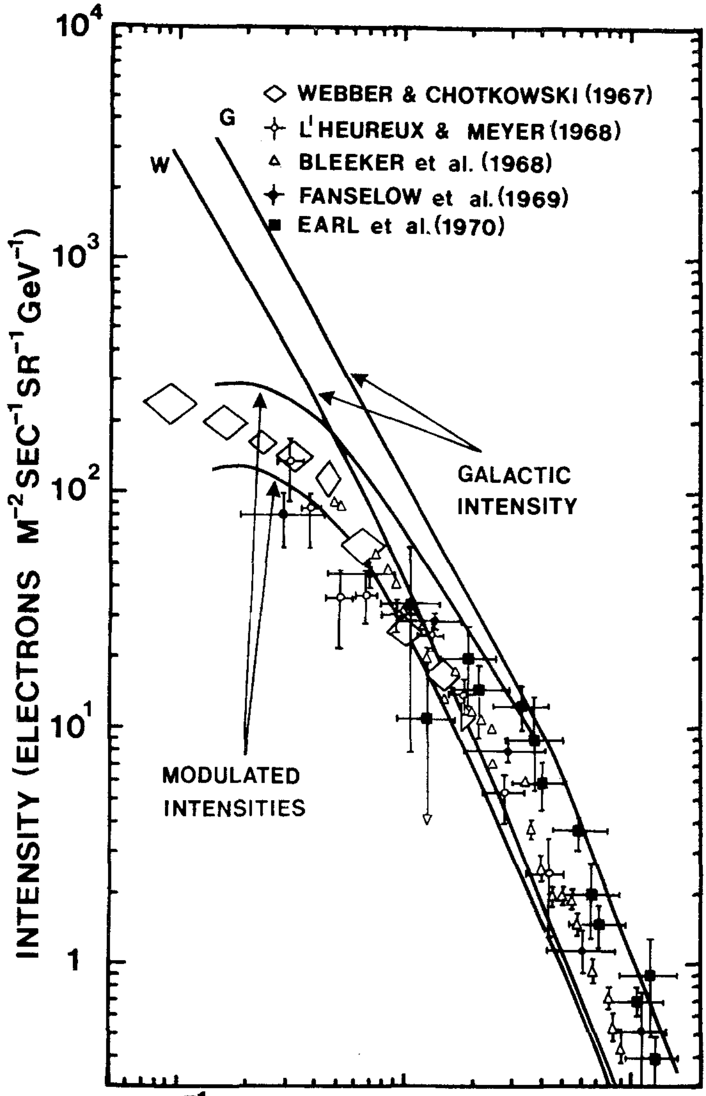

Fig. 7a

Fig. 7. Showing a comparison between observed and calculated proton, helium nuclei, and electron spectra at $r = 1$ AU. The observations are for a period near the 1964–65 sunspot minimum. A diffusion coefficient $\kappa = 6.0 \times 10^{21}\, r^{1/2} P\beta\, \mathrm{cm}^2\, \mathrm{sec}^{-1}$, with r in AU and P in GV, and $R = 10$ AU have been assumed. (a) Electrons: galactic spectra due to Webber (1968), curve W, and due to Goldstein *et al.* (1970b), curve G, are displayed together with the corresponding modulated spectra. (b) Protons and helium nuclei: a galactic spectrum $U\alpha E^{-2.5}$ has been assumed, and the observational data is that compiled from several sources by Gloeckler and Jokipii (1967).

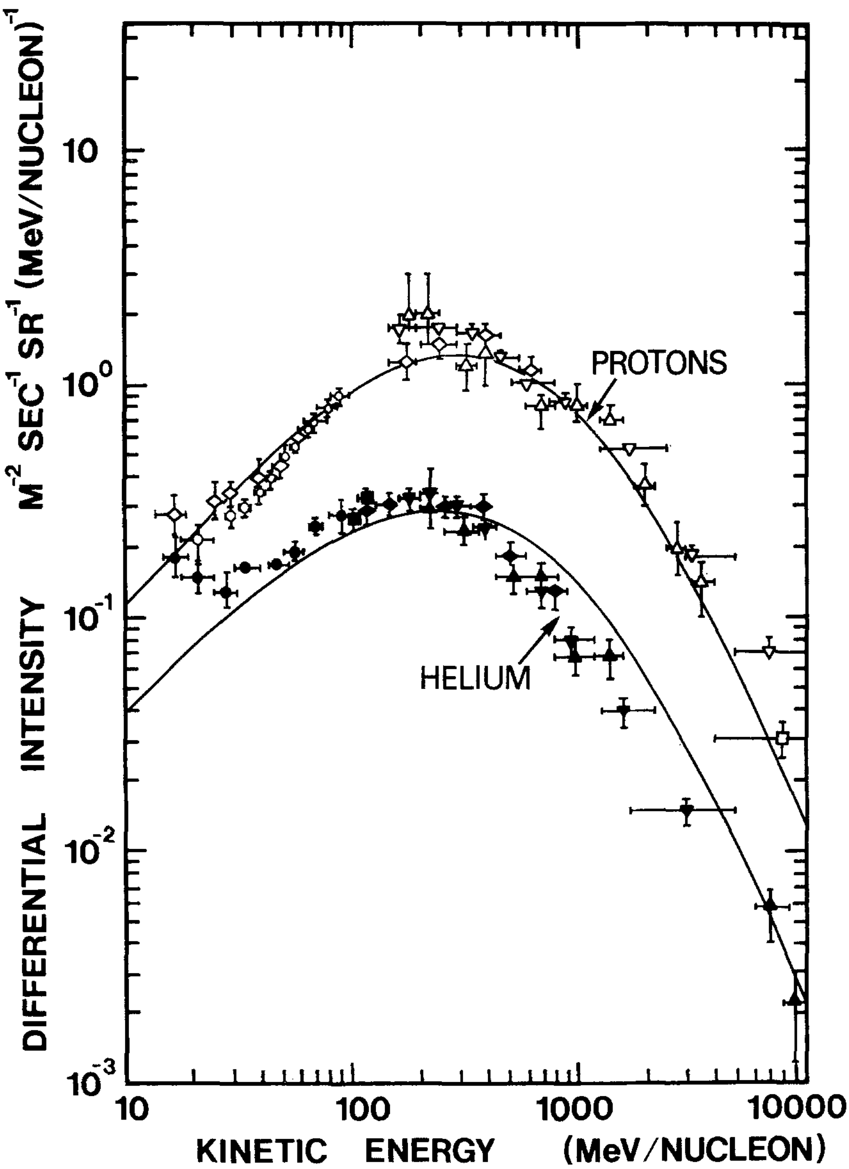

Fig. 7b

differential intensity at $r=1$ AU changed by 10 percent in the kinetic energy range $T \leqslant 50$ MeV/nucleon with $T_L \sim 100$ MeV for protons and $T_L \sim 65$ MeV/nucleon for helium nuclei. Values of P_c intermediate between $P_c=0$ and $P_c=1$ GV require a proportionate change in the magnitude of κ in order to match the two electron spectra.

Similar results to those described in the three preceding paragraphs are found for the alternative forms of $\kappa_1 : \kappa_1 \propto r$ and $\kappa_1 \propto \exp(r/r_0)$.

In the case of the diffusion coefficient (5.3), which gave a near-Earth spectrum sensitive to the corresponding galactic spectrum at $T \leqslant 50$ MeV/nucleon, $\kappa(r, T)$ corresponds with that given in (5.4) with $\gamma = 0$, $P_c = 1$ GV and $\kappa_1(r) \propto r^{1/2}$. We find there is a large discrepancy between the predicted and observed electron spectra near Earth for rigidities less than $P \sim 0.4$ GV (~ 80 MeV protons). Hence this diffusion coefficient is considered as unlikely to be appropriate. Similar sensitivity to the galactic spectra, and incompatibility with the electron spectra, is found if the $r^{1/2}$ radial dependence of (5.3) is replaced with r or $\exp(r/r_0)$.

Agreement between the predicted and observed electron spectra can be obtained with $\gamma = 0$ and $\kappa_1(r) \propto r^{1/2}$ (as above) if we set $P_c = 0.4$ GV (as suggested by Sari and Ness, 1969) and reduce κ to 2×10^{22} cm^2 sec^{-1} at $r = 1$ AU and $P = 6$ GV. With these parameters the low-energy proton and helium nuclei spectra near Earth are again insensitive to the corresponding galactic spectra; specifically, there is a 10 percent reduction in the differential intensity at Earth in the range $T \leqslant 50$ MeV/nucleon with $T_L \simeq 150$ MeV, for protons and with $T \simeq 100$ MeV/nucleon for helium nuclei. Again, similar results are found with the alternative forms of $\kappa_1(r)$.

Noting again that the form of κ, the value of P_c, the magnitude of κ at $r = 1$ AU, and the galactic electron spectrum are each not known precisely, the examples given above show that there is a range of diffusion coefficients which are consistent with the available spectral data. The combinations of $\kappa_1(r)$ and $\kappa_2(P)$ considered here include those put forward by Fisk *et al.* (1969), Fisk (1971), Lezniak and Webber (1970), and Goldstein *et al.* (1970b). In all cases considered to be acceptable (in that the galactic and near Earth electron spectra could be matched), the proton and helium-nuclei spectra at $r = 1$ AU and at low kinetic energies ($T \leqslant 50$ MeV/nucleon, typically), are insensitive to the galactic spectra at these energies. Thus it appears that this will be the general rule, although an unequivocal statement cannot be made without a complete knowledge of $\kappa(r, T)$ throughout the solar cavity.

6. Discussion

The computations presented in this paper show clearly, and quantitatively, the reduction in differential number density, the reduction of mean kinetic energy, and the spread in kinetic energy, of cosmic-ray protons and helium nuclei within the solar cavity after near-monoenergetic injection at $r = 10$ AU.

Assuming a range of diffusion coefficients considered to be representative of those to be found in interplanetary space during 1964–65, it has been determined that galactic protons and helium nuclei with kinetic energies less than (typically) 80 MeV/nucleon are virtually excluded from the inner regions, $r \leqslant 1$ AU. The computations show that, at $r = 1$ AU, the 80 MeV galactic protons would have a mean kinetic energy of about 30 MeV, and that 80 MeV/nucleon galactic helium nuclei would have a mean kinetic energy of about 45 MeV/nucleon. It has also been determined that, with representative models and galactic spectra, the proton and helium-nuclei spectra at $r = 1$ AU and $T \leqslant 50$ MeV/nucleon (typically) are insensitive to the galactic spectra with $T \leqslant 80$ MeV/nucleon (typically).

Two important conclusions follows from the above results: The first is that almost all proton and helium nuclei of galactic origin which are observed near Earth with kinetic energy T_E and $T_E \leqslant 50$ MeV/nucleon had, at the time of their entry into the solar cavity, a much higher kinetic energy than that given by adding the appropriate mean-energy-loss to T_E. The second is that galactic particles with $T \leqslant 80$ MeV/nucleon (typically) cannot be associated closely with particles of some corresponding kinetic energy near Earth; thus, even with $\kappa(r, T)$ accurately and fully known, *it will not be possible to demodulate the proton and helium-nuclei spectra observed at Earth and determine the interstellar spectra at* $T \leqslant 80$ MeV/nucleon.

We remark parenthetically that the above conclusion (in italic) is dependent on the form of the diffusion coefficient adopted, and that we have indicated cases (cf. κ given by (5.3)) for which it would not be valid. These cases have been shown to be inconsistent with the presently available observational data for 1964–65, but they should be kept in mind because of the possibility that this data may subsequently be modified (e.g. the galactic electron spectrum), or that the rigidity dependence of the diffusion coefficient may change during the solar cycle.

It has been noted in Section 4 that the force-field solution with modulation parameter $\phi(r)$, and its related 'potential energy' $\Phi(r, E, Z)$, provides, through (4.1), a good representation of the modulation for protons and helium-nuclei with $T \geqslant 100$ MeV/nucleon. For electrons it is applicable to much lower kinetic energies (Lezniak and Webber, 1970). It has also been shown in Section 4 that $\Phi(r, T, Z)$ is a good estimate of the mean energy loss of particles injected into the solar cavity at kinetic energy T_0, provided $\Phi \leqslant \frac{1}{2}T_0$. Thus it is of considerable interest to ascertain ϕ and Φ for the acceptable diffusion coefficients discussed in Section 5. The values of ϕ and Φ at $r = 1$ AU are of particular interest.

The radial dependence of the acceptable diffusion coefficients given in Section 5 yield a range of $\phi(r = 1\ \text{AU})$ from 150 MV to 320 MV (approx.) as κ_1 at $r = 1$ AU and $P = 6$ GV, is varied from 4×10^{22} cm^2 sec^{-1} to 1.7×10^{22} cm^2 sec^{-1}. For particles with rigidity $P > P_c$, the corresponding Φ are given by $\Phi = |Ze|\phi$. The former value ($\phi = 150$ MV) is appropriate with $P_c = 0$ in (5.4), and the latter value ($\phi \simeq 320$ MV) is appropriate with $P_c = 1$ GV. We note that in matching their galactic and near-Earth electron spectra, Lezniak and Webber (1970) adopt a best value of $P_c = 0$ and $\phi = 140$ MV; Goldstein *et al.* (1970b) adopt $P_c = 350$ MV, and their radial dependence of κ leads to $\phi(r = 1\ \text{AU}) = 210$ MV. We also note that Gleeson and Axford (1968b), on deriving the parameters ϕ and Φ, deduced a value $\phi(r = 1\ \text{AU}) \approx 140$ MV for the 1964 solar minimum from considerations of the change in modulation of protons and helium nuclei in the period 1963–65. Thus the present consensus is away from the upper limit of 320 MV.

As we might anticipate from the force-field solution, the modulation parameter $\phi(r)$ is very useful for the coordination of the present results: given $\kappa_2(P)$, we find that, to a substantial degree, the distributions and spectra calculated at $r = r_i$ are determined by $\phi(r_i)$. Thus, for each form of the radial dependence of the diffusion coefficient which was used ($r^{1/2}$, r, $\exp r/r_0$), it was found that closely similar results were obtained at $r=1$ AU if we maintained the value of $\phi(1\text{ AU})$. These remarks also show that the results presented here are not critically dependent on the position of the boundary of the solar cavity, taken here to be at $R=10$ AU; a change in boundary position being easily offset by a change in $\kappa_1(r)$ to maintain the value of $\phi(1\text{ AU})$.

Finally we wish to bring to the reader's notice the work of Goldstein *et al.* (1970a): they have concurrently carried out an investigation very similar to the present one and reach similar conclusions.

Acknowledgements

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Appendix A

The partial differential equation to be solved is parabolic with the general form

$$p(r,s)\frac{\partial Y}{\partial s}=f(r,s)\frac{\partial^2 Y}{\partial r^2}+g(r,s)\frac{\partial Y}{\partial r}+q(r,s)\,Y\,. \tag{A.1}$$

In the Crank-Nicholson method of solution (Diaz, 1958) the differential equation is replaced by a finite difference equation

$$\frac{p_i^j}{k}\left(Y_i^{j+1}-Y_i^j\right)=\frac{f_i^j}{2h^2}\left(Y_{i+1}^{j+1}-2Y_i^{j+1}+Y_{i-1}^{j+1}+Y_{i+1}^j-2Y_i^j+Y_{i-1}^j\right)$$
$$+\frac{g_i^j}{4h}\left(Y_{i+1}^j+Y_{i+1}^{j+1}-Y_{i-1}^j-Y_{i-1}^{j+1}\right)+q_i^jY_i^j\,, \tag{A.2}$$

on the grid $(r_i s_j;\ i=1,\dots,N_i;\ j=1,\dots,N_j)$. In (A.2), h and k are the grid intervals on r and s respectively and the notation $p_i^j=p(r_i,s_j)$ has been used.

With these algebraic equations and the boundary condition equations of the present problem one can evaluate Y_i^{j+1} on the grid line of constant s, $(r_i, s_{j+1};\ i=1\dots N_i)$ from the adjacent grid line $(r_i, s_j, \dots N_i)$ and proceed stepwise to a complete solution beginning at $s=\frac{1}{2}\pi$. The procedure is unconditionally stable if $p(r,s)$ and $f(r,s)$ are constants and $g(r,s)=q(r,s)=0$ and experience has indicated that in general the procedure is stable providing $p(r,s)$ and $f(r,s)$ remain finite (Diaz, 1958).

In the present problem we have found the system based on (A.2) to be unstable presumably because $p(r,s)=0$ at the starting grid line $s=\frac{1}{2}\pi$. A stable system has been found by replacing (A.2) with a modified form which is equally dependent on the quantities in each of the grid lines s_j and s_{j+1}. This is achieved by writing down the finite difference equation at the point $(r_i, s_{j+1/2})$:

$$\frac{(p_i^j + p_i^{j+1})}{2k}(Y_i^{j+1} - Y_i^j)$$
$$= \frac{f_i^j + f_i^{j+1}}{4h^2}(Y_{i+1}^{j+1} - 2Y_i^{j+1} + Y_{i-1}^{j+1} + Y_{i+1}^j - 2Y_i^j + Y_{i-1}^j)$$
$$+ \frac{g_i^j + g_i^{j+1}}{8h}(Y_{i+1}^j + Y_{i+1}^{j+1} - Y_{i-1}^j - Y_{i-1}^{j+1}) + \tfrac{1}{2}(q_i^j Y_i^j + q_i^{j+1} Y_i^{j+1}). \tag{A.3}$$

Equation (A.3) reverts to (A.2) when $q(r, s)=0$ and $p(r, s)\, f(r, s)$ and $g(r, s)$ are functions of r only.

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