Energy Method in Efficient Estimation of Elastic Buckling Critical Load of Axially Loaded Three-Segment Stepped Column

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This paper treats the elastic stability of three-segment stepped column that is subjected to axial concentrated compressive forces using the strain energy method. Efforts have been made to establish the methodology for the development of a model that is useful for quick, yet quite accurate, estimation of the elastic buckling critical load. By using the load, segments stiffness and lengths ratios, the dimensionless influence coefficient is defined for any combination of parameters. The diagrams of critical load based on geometric parameters variation are given. The results of analytical model are compared with numerical results obtained from finite element method. Within two investigation regimes, influences of bending stiffness and lengths variations are analyzed separately.

Keywords: *elastic buckling, stepped column, axial load, strain energy, finite element method.*

1. INTRODUCTION

In order to accomplish weight reduction and decrease production costs of steel carrying structures, the engineers often design steel columns as multi-segment carriers with a non-uniform cross-section. Since columns are usually compressed by payload, self-weight, etc., one of the most important aspects of using such carriers is their elastic stability. This paper treats the elastic stability of three-segment stepped column that is subjected to axial concentrated compressive forces. Since the issue of elastic stability of multiple-stepped columns is best analyzed by means of computerized structural analysis methods [1], this is the effort towards development of such analytical model for calculating the elastic buckling critical load that possesses a suitable form for building the software application. The basis for building such analytical model of critical buckling load of stepped column is well-known Euler's model [1-3].

The determination of critical buckling load of the columns with non-uniform cross-section can be a complex task that depends on different load and boundary conditions. An exact solution approach to studying the buckling of a non-uniform column with spring supports under combined concentrated and distributed loads is presented in [4]. In paper [5], the governing differential equation for buckling of a multi-step non-uniform beam under several concentrated axial forces is established. Some authors have taken into consideration the cases of an axially compressed non-uniform column subjected to the stepped axial loads that act eccentrically on the column [6,7]. Paper [7] also included the influence of initial imperfections on the stability of non-uniform steel members subjected to eccentrically applied axial loading. In some papers the expressions for describing the distribution of flexural stiffness of a non-uniform column and the distribution of axial forces acting on the column are related [8,9]. The results obtained from the proposed methods are often compared with those determined from finite element method (FEM) [10]. Further, some authors have pointed out the application of effective lengths concept [11]. Considering the complexity of the issue, great efforts have been made to establish the methodology for the development of models that are useful for quick, yet quite accurate, calculations of the critical buckling load of tapered columns [12,13]. On the other hand, some authors have carried out the buckling analysis of non-prismatic columns based on modified vibration modes [14]. In addition, some authors have studied the stability of composite columns and beams of variable cross-sections [15-16].

In general, the issue of elastic stability of a stepped column can be solved through several ways, but most of the previous studies can be sorted into two main approaches.

The first approach means determining the elastic buckling critical force that is based on differential equations of elastic lines for every segment of complex column structure, where the segments are defined either by value change of axial load or by geometric characteristics change of cross section. This approach leads to exact but more complex solution. On the other hand, in order to derive simplified but yet quite accurate solution, it is rational to resort to energy method of calculating the critical force. For the sake of simplicity, the application of this method can be firstly shown on the simple model of column with uniform cross-section that is subjected to the axial compressive force at its top P (Fig. 1a).

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The strain energy accumulates as the column is being bended. At the same time, the potential energy decreases due to certain lowering of acting point of the force. If ΔV represents strain energy and ΔT the work of force P while lowering its acting point, then the elastic stability will be maintained if $\Delta V - \Delta T > 0$, or violated for $\Delta V - \Delta T < 0$. The eigenvalue P_{cr}, called the critical load, which denotes the value of load P for which a nonzero deflection of the perfect column is possible, can be determined from the following condition:

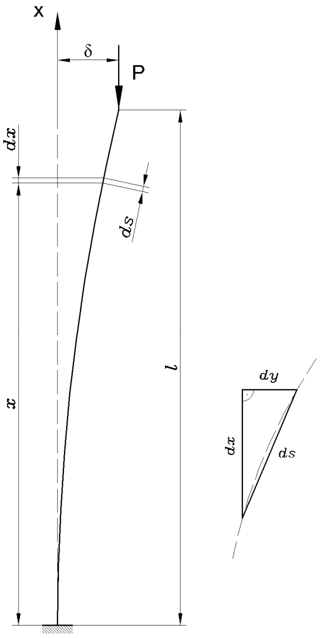
$$\Delta V = \Delta T \,. \tag{1}$$

The displacement of acting point of force P is theremainder between bended column elastic line length and column height. To determine the total displacement, the relation upon an elementary portion of the elastic line is established (Fig. 1b):

$$ds - dx = \sqrt{dx^2 + dy^2} - dx = dx(\sqrt{1 + (\frac{dy}{dx})^2} - 1) \,. \tag{2}$$

By expanding the function into Maclaurin serie, it is derived:

$$ds - dx = dx\left[1 + \frac{1}{2}(\frac{dy}{dx})^2 - 1\right] = \frac{1}{2}(\frac{dy}{dx})^2 dx \,. \tag{3}$$



(a) (b)

Figure 1. a) Cantilever column with constant cross-section with axial compressive force at the top b) Elementary portion of bended column elastic line

The total displacement of force P acting point is obtained by integration [2]:

$$\lambda = \frac{1}{2}\int_0^l (\frac{dy}{dx})^2 dx \,. \tag{4}$$

Work of force P along displacement λ is:

$$\Delta T = P\lambda = \frac{1}{2}P\int_0^l (\frac{dy}{dx})^2 dx \,. \tag{5}$$

Strain energy is calculated as follows:

$$\Delta V = \int_0^l \frac{M^2 dx}{2EI} \,. \tag{6}$$

2. METHODOLOGY OF THE ANALYTICAL MODEL

The case being considered is a three-segment cantilever stepped column subjected to axial compressive force acting at the top. The line that goes through the centroids of all cross-sections is straight. In addition, the following assumptions are made: the column is assumed to be made of homogeneous material that obeys Hooke's law, the load P is concentrated and the deformations of the column are small. Self-weights were also taken into account, which are considered to act at the middle of segments lengths. The model of the considered case is shown in Fig. 2.

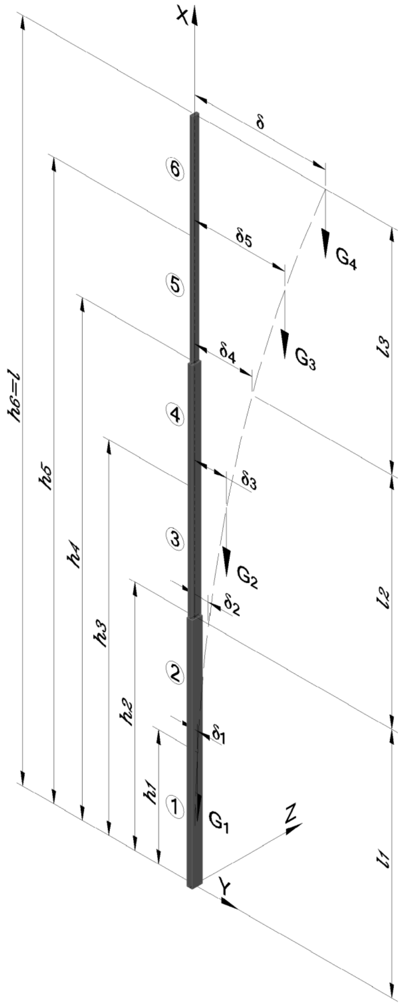


Figure 2. Three-segment cantilever stepped column subjected to axial compressive force and self-weight

As approximate function that satisfies the boundary conditions of support and deflection at the top, the following expression can be adopted:

$$y(x) = \delta(1 - \cos\frac{\pi x}{2l}) \tag{7}$$

where δ is deflection at free end of the column and l is overall column length.

Deflection values at points of forces action, according to adopted approximation of elastic line are:

$$\delta_5 = y(x = h_5) = \delta(1 - \cos\frac{\pi h_5}{2l})$$

$$\delta_3 = y(x = h_3) = \delta(1 - \cos\frac{\pi h_3}{2l}) \quad (8)$$

$$\delta_1 = y(x = h_1) = \delta(1 - \cos\frac{\pi h_1}{2l})$$

where h_i , i=1,...6 are section lengths:
$h_1=l_1/2$; $h_2= l_1$; $h_3= l_1+l_2/2$; $h_4= l_1+ l_2$; $h_5= l_1+ l_2+l_3/2$; $h_6= l_1+ l_2+l_3$.

Total strain energy due to the bending moment is obtained from the expression

$$\Delta V = \sum_{i=1}^{6} \Delta V_i = \int_0^{h_1} \frac{M_1^2(x)dx}{2EI_1} + \int_{h_1}^{h_2} \frac{M_2^2(x)dx}{2EI_1} + \int_{h_2}^{h_3} \frac{M_3^2(x)dx}{2EI_2} + \int_{h_3}^{h_4} \frac{M_4^2(x)dx}{2EI_2} + \int_{h_4}^{h_5} \frac{M_5^2(x)dx}{2EI_3} + \int_{h_5}^{h_6} \frac{M_6^2(x)dx}{2EI_3} \quad (9)$$

Members under integrals $M_i(x)$, $i=1,...,6$ represent bending moments functions by sections:

$$\begin{aligned} M_1(x) &= (G_1 + G_2 + G_3 + G_4)y - \\ &\quad -(G_1\delta_1 + G_2\delta_3 + G_3\delta_5 + G_4\delta) \\ M_2(x) &= (G_2 + G_3 + G_4)y - (G_2\delta_3 + G_3\delta_5 + G_4\delta) \\ M_3(x) &= (G_2 + G_3 + G_4)y - (G_2\delta_3 + G_3\delta_5 + G_4\delta) \quad (10) \\ M_4(x) &= (G_3 + G_4)y - (G_3\delta_5 + G_4\delta) \\ M_5(x) &= (G_3 + G_4)y - (G_3\delta_5 + G_4\delta) \\ M_6(x) &= G_4 y - G_4\delta \end{aligned}$$

After introduction of loads, bending stiffness and lengths ratios $m_1=G_1/G_4$, $m_2=G_2/G_4$, $m_3=G_3/G_4$, $n_2=I_2/I_1$, $n_3=I_3/I_1$, $s_1=l_1/l$, $s_2=l_2/l$ and $s_3=l_3/l$, the members from (9) after integration and many transformations can be written in the following form:

$$\Delta V_1 = \frac{G\delta^2 l}{4\pi EI_1} G_4 [A_1] \qquad \Delta V_2 = \frac{G\delta^2 l}{4\pi EI_1} G_4 [A_2]$$

$$\Delta V_3 = \frac{G\delta^2 l}{4\pi EI_2 \frac{I_1}{I_1}} G_4 [A_3] \quad \Delta V_4 = \frac{G\delta^2 l}{4\pi EI_2 \frac{I_1}{I_1}} G_4 [A_4] \quad (11)$$

$$\Delta V_5 = \frac{G\delta^2 l}{4\pi EI_3 \frac{I_1}{I_1}} G_4 [A_5] \quad \Delta V_6 = \frac{G\delta^2 l}{4\pi EI_3 \frac{I_1}{I_1}} G_4 [A_6]$$

where expressions designated as G and A_i, $i=1,...6$ are given in Table 1. Coefficients which are used within these expressions are given in Table 2.

Total strain energy gets a simpler formulation:

$$\Delta V = \sum_{i=1}^{6} \Delta V_i = \frac{G\delta^2 l G_4}{4\pi EI_1} \left(A_1 + A_2 + \frac{A_3}{n_2} + \frac{A_4}{n_2} + \frac{A_5}{n_3} + \frac{A_6}{n_3} \right) (12)$$

Total deformation work of all forces on corresponding displacements is:

$$\Delta T = \sum_{m=1}^{4} \Delta T_m (G_m) . \quad (13)$$

In accordance with (5), work of force G_1 along its acting point displacement is:

$$\Delta T_1 = \frac{1}{2} G_1 \int_0^{h_1} \left(\frac{dy}{dx} \right)^2 dx . \quad (14)$$

Starting from the implied equation of elastic line (7) after integration it is obtained

$$\Delta T_1 = \frac{G_1 \delta^2 \pi}{16l} \left(\frac{\pi h_1}{l} - \sin\frac{\pi h_1}{l} \right). \quad (15)$$

Other members of the expression for total deformation work have the form as (15), where integration boundaries are changed, so it is derived:

$$\Delta T_2 = \frac{G_2 \delta^2 \pi}{16l} \left(\frac{\pi h_3}{l} - \sin\frac{\pi h_3}{l} \right)$$

$$\Delta T_3 = \frac{G_3 \delta^2 \pi}{16l} \left(\frac{\pi h_5}{l} - \sin\frac{\pi h_5}{l} \right) \quad (16)$$

$$\Delta T_4 = \frac{G_4 \delta^2 \pi}{16l} \left(\frac{\pi l}{l} - \sin\frac{\pi l}{l} \right) = \frac{G_4 \delta^2 \pi^2}{16l}$$

Total deformation work of all forces on corresponding displacements is:

$$\Delta T = \frac{\delta^2 \pi G_4}{16l} [H] . \quad (17)$$

where the expression designated as H is given in Table 1.

The expanded form of (1) from which the buckling critical load is to be calculated is:

$$\frac{G\delta^2 l G_4}{4\pi EI_1} \left(A_1 + A_2 + \frac{A_3}{n_2} + \frac{A_4}{n_2} + \frac{A_5}{n_3} + \frac{A_6}{n_3} \right) = \frac{\delta^2 \pi G_4}{16l} H \ .(18)$$

Considering Euler's formula for buckling critical force of cantilever column, the suitable form in this case would be:

$$P_{cr} = \frac{\pi^2 EI_1}{4l^2} \cdot q = P_{cr}^1 \cdot q . \quad (19)$$

First member of equation P_{cr}^1 is the known Euler's formula for buckling critical force of column with uniform first segment's cross-section and overall height. Parameter q encompasses the influences of segments lengths ratios, bending stiffness ratios and load ratios.

Therefore, the analysis of elastic stability is reduced to the analysis of dimensionless influence parameter q:

$$q = \frac{P_{cr}}{P_{cr}^1} = \frac{H}{A_1 + A_2 + \frac{A_3}{n_2} + \frac{A_4}{n_2} + \frac{A_5}{n_3} + \frac{A_6}{n_3}} = f\left(m_1, m_2, m_3, n_2, n_3, s_1, s_2, s_3\right) \quad (20)$$

Table 1. Expressions in (11) and (17)

$A_1 = \dfrac{2r_1^2 + p_1^2}{2p_1}\pi s_1 - 8r_1 a_1 + p_1 b_1$
$A_2 = \dfrac{2r_2^2 + p_2^2}{2p_1}\pi s_1 - 8\dfrac{p_2}{p_1} r_2 (a_2 - a_1) + \dfrac{p_2^2}{p_1}(b_2 - b_1)$
$A_3 = \dfrac{2r_2^2 + p_2^2}{2p_1}\pi s_2 - 8\dfrac{p_2}{p_1} r_2 (a_3 - a_2) + \dfrac{p_2^2}{p_1}(b_3 - b_2)$
$A_4 = \dfrac{2r_3^2 + p_3^2}{2p_1}\pi s_2 - 8\dfrac{p_3}{p_1} r_3 (a_4 - a_3) + \dfrac{p_3^2}{p_1}(b_4 - b_3)$
$A_5 = \dfrac{2r_3^2 + p_3^2}{2p_1}\pi s_3 - 8\dfrac{p_3}{p_1} r_3 (a_5 - a_4) + \dfrac{p_3^2}{p_1}(b_5 - b_4)$
$A_6 = \dfrac{\pi s_3}{2p_1} - \dfrac{b_5}{p_1}$
$G = G_1 + G_2 + G_3 + G_4$
$H = m_1(0.5\pi s_1 - b_1) + m_2[\pi(s_1 + 0.5s_2) - b_3] + {}$ $+ m_3[\pi(s_1 + s_2 + 0.5s_3) - b_5] + \pi$

Table 2. Coefficients within expressions in Table 1

$a_1 = \sin(0.25\pi s_1)$
$a_2 = \sin(0.5\pi s_1)$
$a_3 = \sin(0.5\pi(s_1 + 0.5s_2))$
$a_4 = \sin(0.5\pi(s_1 + s_2))$
$a_5 = \sin(0.5\pi(s_1 + s_2 + 0.5s_3))$
$b_1 = \sin(0.5\pi s_1)$
$b_2 = \sin(\pi s_1)$
$b_3 = \sin(\pi(s_1 + 0.5s_2))$
$b_4 = \sin(\pi(s_1 + s_2))$
$b_5 = \sin(\pi(s_1 + s_2 + 0.5s_3))$
$c_1 = \cos(0.25\pi s_1)$
$c_2 = \cos(0.5\pi(s_1 + 0.5s_2))$
$c_3 = \cos(0.5\pi(s_1 + s_2 + 0.5s_3))$
$p_1 = m_1 + m_2 + m_3 + 1$
$p_2 = m_2 + m_3 + 1$
$p_3 = m_3 + 1$
$r_1 = m_1 c_1 + m_2 c_2 + m_3 c_3$
$r_2 = m_2 c_2 + m_3 c_3$
$r_3 = m_3 c_3$

By varying the values of dimensionless load ratios and segments stiffness and lengths ratios, the influence coefficient can be defined for any combination of the parameters. On the other hand, segments lengths ratios have the limitation s1+s2+s3=1. If in expression for influence parameter q are put the values m1=m2=m3=0, n2=n3=1 and s1=s2=s3=0, it is obtained that q=1, i.e. Pcr=Pcr1, which is the known Euler's equation.

3. COMPARATIVE RESULTS OBTAINED FROM ANALYTICAL MODEL AND FEM

Since the influence coefficient of critical axial load $q=q(m_1,m_2,m_3,n_2,n_3,s_1,s_2,s_3)$ depends on many parameters, it gives the possibility to perform various analysis. Simplifying the verification, it was considered that segments' self-weight load is much smaller than compressive axial force at the top of the column ($G_1,G_2,G_3 \ll G_4=P$ or $m_1=m_2=m_3\approx 0$), which is the most often case in real life. This means that the influence coefficient q is analyzed in cases of bending stiffness and lengths ratios variation.

In order to verify the analytical model, the FEM analysis is performed by using ANSYS software. Circular hollow sections with constant and equal wall thickness are taken for segments' cross-sections, therefore the bending stiffness variation is done by changing the diameters values, Fig. 3a.

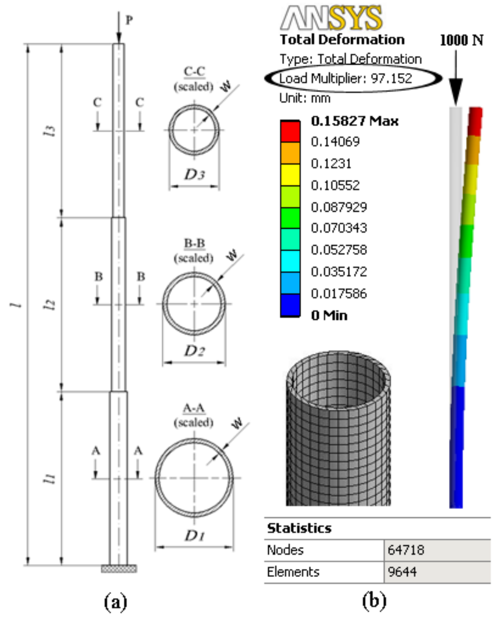


Figure 3. Verification model for analytical solution:a) stepped column with cross-sections; b) FE model and result for well-known case of cantilever column with uniform cross-section.

Wall thickness of pipes is w=5[mm] and overall height of column is l=3000[mm]. In all cases, the diameter of first (lowest) segment is D_1=100[mm], while the diameters of second and third segment D_2 and D_3 are varied. Young's modulus of elasticity for steel is taken to be E=2.1·10^4[kN/cm^2]. In all cases, the compressive axial force P=1000[N] is applied at the top of the column, while its bottom end is fixed.

Since the results of an analytical model are to be compared with FEM results that are considered as true, a preliminary test was done in order to verify the FEM model. A well-known case of cantilever column with uniform cross-section with pipe diameter of D=100[mm], wall thickness w=5[mm] and height l=3000[mm] was discretized by 9644 20-node hexagonal finite elements and 64718 nodes, Fig. 3b.

Analytical solution is well-known Euler's formula:

$$P_{cr}^{1} = \frac{\pi^2 EI}{4l^2} = \frac{\pi^2 2.1 \cdot 10^4 \cdot 168{,}81}{4 \cdot 300^2} = 97.188 kN \quad (21)$$

The result obtained by FEM is a load multiplier (Fig 3b) that scale the loads applied in the static structural analysis, which precedes the buckling analysis. Since the applied load is 1kN, the bucking critical force is load multiplier itself, i.e. 97.152 kN, Fig 3b. Relative error of FEM model in relation to known analytical solution is 0.037% which can be neglected.

Two analysis regimes were established. Firstly, analytical and numerical models were compared through variation of pipe diameters (bending stiffness) while maintaining segments lengths equal. After that, the comparison was done for the regime with variable segments lengths and fixed (adopted) cross sections. Atotal of 40 tests were done, 20 tests per each analysis regime.

3.1 Variation of pipe diameters (bending stiffness) with fixed and equal segments lengths

Bending stiffness variation is accomplished by changing the pipe diameters of the second and the third segment, while the segments lengths are kept equal, i.e. $s_1=s_2=s_3=1/3$. The diameters are changed by 5[mm] gradually, while maintaining the relation $D_1>D_2>D_3$. Table 3 comparatively shows elastic buckling critical force values P_{cr} obtained from analytical and numerical FEM model for various combinations of segments diameters.

Elastic buckling critical force value P_{cr} from the FEM model is calculated by multiplying the load multiplier and the value of compressive axial force acting at top of the stepped column. Since the applied force is P=1[kN] (Fig.3), the load multiplier itself represents the critical force in [kN], Fig. 4.

Fig. 5 represents the diagram of critical force (19) based on variable bending stiffness ratios and constant and equal segments lengths. The results of FEM model are represented through points with designation given in Table 3. A relative error of results obtained from the analytical model in comparison with FEM is listed in a far right-hand column.

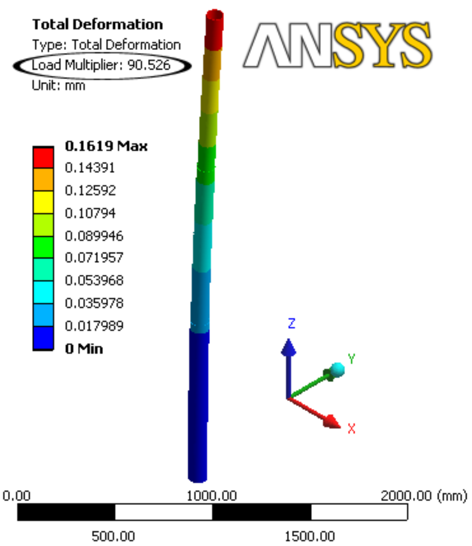


Figure 4. Elastic buckling critical force for D2=9.50[cm] and D3=9.00[cm] (bolded value in Table 3 - point R1)

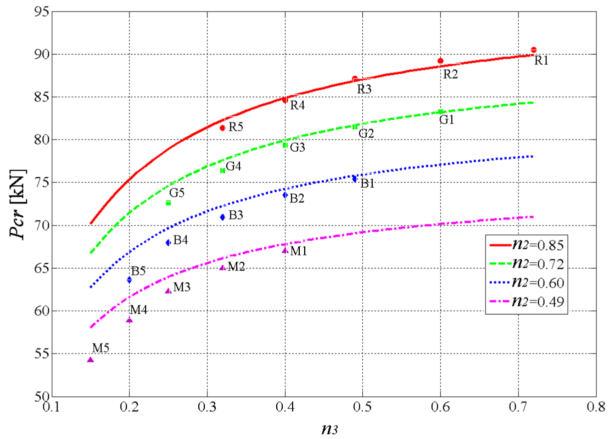


Figure 5. Critical force versus bending stiffness ratio for analytical and FEM prediction

3.2 Variation of segments lengths ratios within fixed pipe diameters (bending stiffness)

Lengths variation is done by changing the relative segments participation in overall column length for several combinations of pipe diameters. It is adopted that the first segment ratio takes values in the range 0.20÷0.60, while other two segments have equal lengths. Since it must be $s_1+s_2+s_3=1$, it follows that $s_2=s_3=0.5(1-s_1)$.

In the same manner as Table 3, Table 4 comparatively shows elastic buckling critical force values P_{cr} obtained from analytical and numerical FEM model for various combinations of segments lengths.

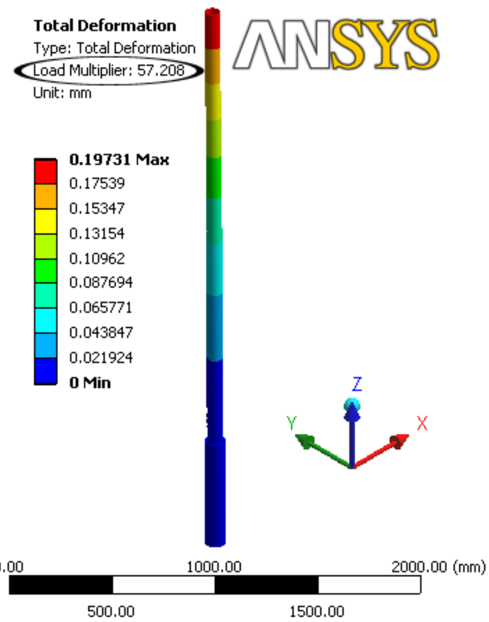


Figure 6. Elastic buckling critical force for D_2=8.00[cm], D_3=7.50[cm], s_1 =0.20, s_2=s_3=0.40 (bolded value in Table 4 - point M5)

Since the applied force is P=1[kN] (Fig.3), the load multiplier itself represents the critical force in [kN], Fig.6.

Fig. 7 represents the diagram of critical force (19) based on variable segments lengths ratios and fixed segments bending stiffness.

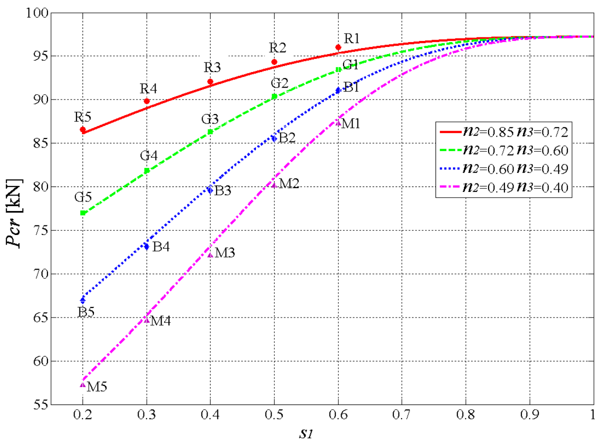


Figure 7. Critical force versus segments lengths ratio for analytical and FEM prediction

4. CONCLUSION

Considering the variation of segments bending stiffness, relative deviations of elastic buckling critical force are generally less than 3% (Table 3) except for a few cases with largest stiffness reduction (the largest relative error 7.06% is for the case with D_2=8.00[cm], D_3=5.50[cm]). Greater differences in pipes diameters size within segments transitions caused larger relative error of critical force value due to deviation of column real elastic line in comparison with assumed sine shape.

On the other hand, within the variation of segments lengths with fixed bending stiffness (fixed set of pipe diameters), relative error is less than 1.39% (Table 4).

For both analysis regimes, the influence of variable parameters on the elastic buckling critical force can be evaluated through general dimensionless influence parameter q, whose diagrams for analysis regimes, presented in sections 3.1 and 3.2, are given in Fig.8 and Fig. 9 respectively.

Diagram in Fig. 8 shows the expected increase of dimensionless influence parameter q, higher values of critical force versus increase of bending stiffness ratio of second and third segment n_2 and n_3 respectively. The same conclusion can be made upon the diagram in Fig.9, wherein the values of dimensionless influence parameter q converge to 1 when participation of first segment length in overall column length exceeds 80% ($s_1 > 0.8 \Rightarrow P_{cr} \approx P_{cr}^1$).

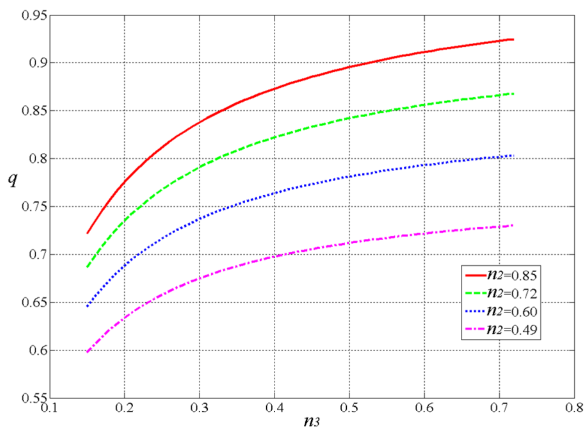


Figure 8. Diagram of dimensionless influence parameter *q* for bending stiffness ratios variation

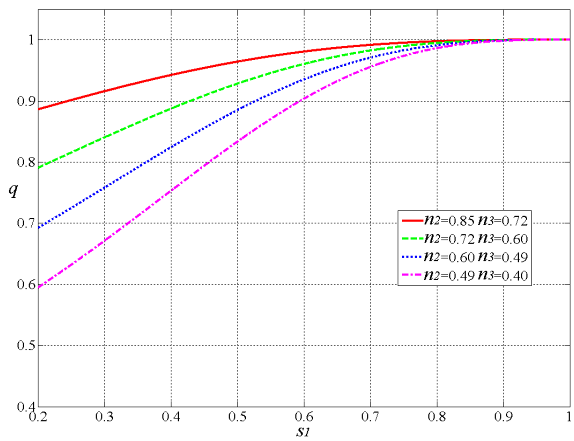


Figure 9. Diagram of dimensionless influence parameter *q* for lengths ratios variation

Comparing the results obtained from analytical and FEM model, it can be seen that relative deviations are acceptable for application purposes. Therefore, it can be pointed out that presented methodology can be used for calculation of elastic buckling critical load of axially loaded stepped column. Besides, derived analytical dependence between critical buckling load and stiffness and lengths ratios of three-segment stepped column is a valuable basis for parameters optimization. Further, introduced methodology can be used for managing more complex cases with more segments and eccentric axial loads. Finally, it generates a convenient platform for building the application software solution for calculating the elastic buckling critical load of axially loaded multi-segment stepped column.

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NOMENCLATURE

ΔV	strain energy
ΔT	deformation work
λ	lowering of column's free end
P	compressive force
E	Young's modulus
δ	free end deflection
l	overall column length
$l_1, l_{2,} l_3$	segments lengths
h_i , $i = 1,...,6$	sections heights
$M_i(x)$, $i = 1,...,6$	sections bending moments
$G_1, G_{2,} G_3$	segments self-weights
G_4	payload - compressive force
$I_1, I_{2,} I_3$	sectional moments of inertia
δ_i , $i = 1,3,5$	deflections at acting points
$m_1, m_{2,} m_3$	loads ratios
$n_{2,} n_3$	bending stiffness ratios
$s_1, s_{2,} s_3$	lengths ratios
q	elastic buckling dimensionless influence parameter
$D_1, D_{2,} D_3$	sections diameters
w	pipe wall thickness

ЕНЕРГЕТСКИ МЕТОД У ЕФИКАСНОМ ОДРЕЂИВАЊУ КРИТИЧНОГ ОПТЕРЕЋЕЊА ИЗВИЈАЊА АКСИЈАЛНО ОПТЕРЕЋЕНОГ ТРОСЕГМЕНТНОГ СТУБА

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У раду је разматрана еластична стабилност тросегментног аксијално оптерећеног стуба помоћу енергетске методе. Спроведена је методологија са циљем формирања прорачунског модела који је погодан за брзо и истовремено довољно тачно одређивање критичног оптерећења које доводи до извијања. Уводећи у анализу односе оптерећења, крутости савијања и дужина сегмената, дефинисан је општи бездимензиони утицајни коефицијент за било коју комбинацију параметара. Дати су дијаграми промене критичног оптерећења у односу на варијацију геометријских параметара. Резултати аналитичког модела су дати упоредо са резултатима добијеним из нумеричке анализе методом коначних елемената. Кроз два режима испитивања, засебно су анализирани утицаји од промене крутости савијања и дужина сегмената.

Table 3. Comparative results from analytical model and FEM for the regime of variable pipe diameters (bending stiffness) with fixed and equal segment lengths

Segment I diameter D1 [cm]	I1[cm4]	Pcr for n2=n3=1	Segment II diameter D2 [cm]	I2[cm4]	n2=I2/I1	Segment III diameter D3 [cm]	I3[cm4]	n3=I3/I1	Pcr theory [kN]	Pcr FEM [kN]	Point designation in diagram	Relative error [%]
10.00	168.81	97.19	9.50	143.58	0.85	9.00	121.00	0.72	89.89	90.53	R1	-0.71%
						8.50	100.92	0.60	88.57	89.21	R2	-0.72%
						8.00	83.20	0.49	86.87	87.12	R3	-0.29%
						7.50	67.69	0.40	84.86	84.62	R4	0.28%
						7.00	54.24	0.32	82.27	81.36	R5	1.12%
			9.00	121.00	0.72	8.50	100.92	0.60	83.21	83.33	G1	-0.14%
						8.00	83.20	0.49	81.70	81.47	G2	0.28%
						7.50	67.69	0.40	79.92	79.35	G3	0.72%
						7.00	54.24	0.32	77.62	76.37	G4	1.64%
						6.50	42.71	0.25	74.61	72.63	G5	2.73%
			8.50	100.92	0.60	8.00	83.20	0.49	75.80	75.42	B1	0.50%
						7.50	67.69	0.40	74.26	73.56	B2	0.95%
						7.00	54.24	0.32	72.27	70.96	B3	1.85%
						6.50	42.71	0.25	69.66	67.95	B4	2.52%
						6.00	32.94	0.20	66.89	63.66	B5	5.07%
			8.00	83.20	0.49	7.50	67.69	0.40	67.80	67.00	M1	1.19%
						7.00	54.24	0.32	66.14	64.97	M2	1.80%
						6.50	42.71	0.25	63.94	62.27	M3	2.68%
						6.00	32.94	0.20	61.61	58.91	M4	4.58%
						5.50	24.79	0.15	58.07	54.24	M5	7.06%

Table 4. Comparative results from analytical model and FEM for the regime of variable segment lengths for some combinations of pipe diameters (fixed bending stiffness)

Segment I diameter D_1 [cm]	I_1 [cm^4]	*Pcr* for $n_2=n_3=1$	Segment II diameter D_2 [cm]	I_2 [cm$_4$]	$n_2=I_2/I_1$	Segment III diameter D_3 [cm]	I_3 [cm^4]	$n_3=I_3/I_1$	s_1	s_2	s_3	*Pcr* theory [kN]	*Pcr* FEM [kN]	Point designation in diagram	Relative error [%]
10.00	168.81	97.19	9.50	143.58	0.85	9.00	121.00	0.72	0.60	0.20	0.20	95.29	96.00	R1	-0.74%
									0.50	0.25	0.25	93.69	94.36	R2	-0.71%
									0.40	0.30	0.30	91.56	92.10	R3	-0.59%
									0.30	0.35	0.35	88.99	89.81	R4	-0.91%
									0.20	0.40	0.40	86.10	86.56	R5	-0.53%
			9.00	121.00	0.72	8.50	100.92	0.60	0.60	0.20	0.20	93.33	93.49	G1	-0.17%
									0.50	0.25	0.25	90.19	90.41	G2	-0.24%
									0.40	0.30	0.30	86.21	86.32	G3	-0.13%
									0.30	0.35	0.35	81.64	81.88	G4	-0.29%
									0.20	0.40	0.40	76.81	76.99	G5	-0.23%
			8.50	100.92	0.60	8.00	83.20	0.49	0.60	0.20	0.20	90.86	91.11	B1	-0.27%
									0.50	0.25	0.25	85.97	85.49	B2	0.56%
									0.40	0.30	0.30	80.05	79.58	B3	0.59%
									0.30	0.35	0.35	73.64	73.14	B4	0.68%
									0.20	0.40	0.40	67.26	66.91	B5	0.52%
			8.00	83.20	0.49	7.50	67.69	0.40	0.60	0.20	0.20	87.78	87.26	M1	0.60%
									0.50	0.25	0.25	80.95	80.12	M2	1.04%
									0.40	0.30	0.30	73.14	72.14	M3	1.39%
									0.30	0.35	0.35	65.18	64.60	M4	0.90%
									0.20	0.40	0.40	57.76	57.21	M5	0.96%