International Journal of Theoretical Physics, Vol. 7, No. 2 (1973), pp. 129–143

Energy-Momentum Quanta in Fresnel's Evanescent Wave II

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Received: 3 *March* 1972

Abstract

Four generalisations of results appearing in a previous paper, referred to as I, are here produced. (1) Formulas for the field strengths of the evanescent wave generated inside a vacuum sandwiched between two identical refracting media propagating symmetrical incident plane waves; the classical exponential damping factor being then replaced by hyperbolic cosines or sines (according to the field components), an extremely close approximation to a plane tachyon wave is thus obtained; (2) Compact formulas for the case where the evanescent wave is generated by a superposition of plane incident waves with propagation vectors **k** parallel to a common incidence plane; (3) Compact formulas for the other typical case where the dispersion on **k** is parallel to the reflecting plane; (4) Formulas for refraction and total reflection of a photon with a non-zero rest mass.

We take the opportunity of this paper to review briefly various articles that had escaped us, where a transverse energy flux inside Fresnel's evanescent wave was discussed, and also some recent papers dealing with quantisation of the evanescent wave or related topics.

1. *Introduction*

Before introducing our present subject matter we shall review briefly some articles that had escaped us, where the question of a transverse energy flux inside Fresnel's evanescent wave is discussed, and also some recent papers dealing with quantisation of the Fresnel evanescent wave or related topics.

It seems that Boguslawski (1912) was the first to mention the existence of a transverse energy flux inside an absorbing medium supporting two interfering inhomogeneous plane waves. Soon after Wiegrefe (1914, 1916) discussed the much more interesting case of the Fresnel evanescent wave excited by a plane incident wave with a linear polarisation oblique on the incidence plane. However the fact that the transverse energy flux is very much stronger when the polarisation of the incident plane wave is elliptical, and is indeed maximal (given the incidence angle) when the evanescent wave is circularly polarised, escaped him. Rose & Wiegrefe (1916) have attempted to prove experimentally the existence of the transverse energy flux but, in the light of subsequent work on this subject, it is clear that their approach was not the best one.

Fedorov (1955) drew attention to Wiegrefe's paper, and produced compact formulas for the case of a general elliptic polarisation of both the incident plane wave and the evanescent wave. He mentioned that the existence of the transverse energy flux should entail a transverse shift of the reflected beam, just as the existence of the classical longitudinal energy flux (Kristoffel, 1956; Renard, 1964) entails the longitudinal Goos–Hänchen shift (Goos & Hänchen, 1947, 1949); however he produced no formula for the transverse shift.

None of these authors had related the existence of the transverse energy flux to the absence of a component of the photon momentum normal to the incidence plane; this seems queer, as the momentum of the photon is known since 1905. In 1964–5, not aware of the previous writings, we (de Beauregard, 1964a, 1965b) called attention to this phenomenon as a striking instance of non-collinearity of velocity and momentum of spinning particles. Calculations were produced for both the spinning electron (de Beauregard, 1964b, 1965a) and the spinning photon (de Beauregard, 1964a, 1965b), the latter case being retained as more interesting for experimental tests, due to the much larger values attainable for the wavelength.

Schilling (1965), using a stationary phase argumentation similar in spirit to Noether's (1931) explanation of the Goos–Hänchen shift, produced formulas for both the longitudinal and the transverse shifts in total reflection of an elliptically polarised incident wave. In the light of subsequent work, it turns out that Schilling's formula for the transverse shift should be corrected by a factor $\cos^2 i$, where i denotes the incidence angle.

Imbert (1968), unaware of Schilling's paper, calculated the transverse shift by using an energy flux conservation argument analogous to Kristoffel's (1956) and Renard's (1964) calculation of the Goos–Hänchen shift. His value is $\cos^2 i$ times larger than Schilling's, and is quite unambiguously supported by the experimental measurements he performed in 1969–70 (Imbert, 1969, 1970a, b).

So much for the energy flux inside Fresnel's evanescent wave.

A complementary facet of our problem is the study of the energy-momentum quanta inside Fresnel's evanescent wave. Let us first recall in this respect that absorption of the energy quanta inside Fresnel's evanescent wave is experimentally known since long ago (Hall, 1902).

Due to the imaginary character of the component of the propagation vector normal to the reflecting plane (Hall, 1902), the component of this vector parallel to the reflecting plane is (in units such that $c = 1$) larger than the angular frequency ω. This should entail remarkable 'tachyon properties' for the energy-momentum quanta inside Fresnel's evanescent wave, which would be testable in appropriate absorption or stimulated emission experiments. Unfortunately, for practical reasons, the most striking of these experiments, a *first order* photoelectric effect on electrons shot parallel to both the incidence and the reflecting planes with just the right velocity, is only a thought experiment. However Glass & Mendlowitz (1968) have drawn our attention to the fact that their theory of the Smith-Purcell (1953) effect uses a quite similar concept for electrons travelling parallel to the surface of a metallic light diffracting grating.

Finally Carniglia & Mandel (1971), simultaneously to our writings on tachyon photons, have quantised the Fresnel evanescent wave in an authoritative paper. They insist that 'evanescent photons' do not exist *per se*, but only as quanta of a complex comprising an incident and a reflected plane wave together with the associated evanescent wave. Unquestionable as it stands, this assertion certainly does *not* preclude the experimental possibility of joined absorption or stimulated emission of the energy ω *and* the momentum k_x quanta inside the evanescent wave; all the more so that Carniglia & Mandel's commutation formulas for the emission and absorption operators in the k representation are exactly those compatible with our own energy-momentum conservation formulas.

So much for energy momentum quanta inside Fresnel's evanescent wave.

As for the content of the present paper, it is exactly characterised by the content of the abstract. Therefore nothing is added here except reference to a previous paper (de Beauregard *et al.*, 1971), hereafter labelled I, to which this one is a natural sequel.

For simplicity units such that $c = 1$ and $\hbar = 1$ will be used throughout.

2. *Quasi Plane Tachyon-Photon Wave Generated by Double Total Reflection*†

(1) The idea underlying the following calculation (de Beauregard & Ricard, 1970) and the device it defines is that, by replacing the exponential $\exp(-by)$ which damps the field strengths in the direction orthogonal to the reflecting surface either by $\cosh(by)$ and $\sinh(by)$ (depending on the field components), an extremely close approximation to a plane tachyon wave is obtained at the height $y = 0$ where $\cosh(by) = 1$ and $\sinh(by) = 0$. Obviously such a wave is generated between the parallel plane interfaces $y = \pm a$ separating a vacuum from two identical isotropic media of index n, by two incoming symmetrical plane waves (Fig. 1).

At this point a close parallelism between the formulas that will be produced and those for the wave guide consisting of two parallel plane conductors $y = \pm a$ enclosing a vacuum is expected, because, as is well known, the field strengths are then functions of $\cos(N\pi y/a)$ and $\sin(N\pi y/a)$. The formulas corresponding to the two cases will now be displayed.

† The content of this section results from work in collaboration with J. Ricard (de Beauregard & Ricard, 1970).

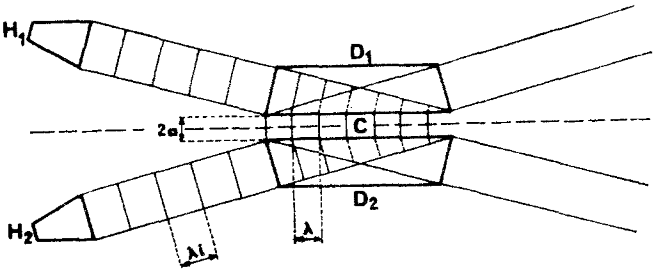

Figure 1.

(2) *Evanescent wave.* Denoting $x0y$ the incidence plane common to the two symmetrical incoming plane waves (Fig. 1), and setting

$$P \equiv \exp\{j\omega(t - \alpha x)\}, \qquad \alpha \equiv n a_0 > 1 \tag{2.1}$$

where α_0 denotes the incidence angle inside the two media of index n, and

$$\bar{\beta} \equiv (\alpha^2 - 1)^{1/2} \tag{2.2}$$

we write the general evanescent wave subject to our symmetry prescriptions as a superposition of a transverse electric

$$\mathscr{E}_z = EP\cosh(k\bar{\beta}y), \qquad \mathscr{H}_x = jEP\bar{\beta}\sinh(k\bar{\beta}y), \qquad \mathscr{H}_y = -EP\alpha\cosh(k\bar{\beta}y) \tag{2.3}$$

and a transverse magnetic wave

$$\mathscr{H}_z = HP\cosh(k\bar{\beta}y), \qquad \mathscr{E}_x = -jHP\bar{\beta}\sinh(k\bar{\beta}y), \qquad \mathscr{E}_y = +HP\alpha\cosh(k\bar{\beta}y) \tag{2.4}$$

E and H denote complex constants, and all non-written field strengths are zero. The Fresnel-like formulas for the transmission and reflection coefficients pertaining to this case have been explicitly calculated (de Beauregard & Ricard, 1970).

(3) *z independent guided wave.* The phase factor corresponding to (2.1) is now

$$P = \exp\{j\omega(t - \alpha x)\}, \qquad \alpha < 1 \tag{2.5}$$

and the transverse electric wave must assume the form $\mathscr{E}_z = EP{}^{\sin}_{\cos}|k\beta y|$ with

$$\beta \equiv (1 - \alpha^2)^{1/2} \tag{2.6}$$

and be zero for $y = \pm a$. Thus, N denoting an integer, the condition $k\beta = N\pi/a$ must be fulfilled which, in the case $N = 1$, yields for the sine type wave

$$\mathscr{E}_z = EP\sin(k\beta y), \qquad \mathscr{H}_x = jEP\beta\cos(k\beta y), \qquad \mathscr{H}_y = -EP\alpha\sin(k\beta y) \tag{2.7}$$

Similarly, the transverse magnetic wave must be such that $\mathscr{E}_x = 0$ for $y = \pm a$ so that, for $N = 1$, it assumes the form

$$\mathscr{H}_z = HP\cos(k\beta y), \qquad \mathscr{E}_x = jHP\beta\sin(k\beta y), \qquad \mathscr{E}_y = HP\alpha\cos(k\beta y) \quad (2.8)$$

As was expected, formulas (2.4) and (2.8) are isomorphic, while formulas (2.3) and (2.7) are less similar.

(4) *Group velocity in the x direction.* $\mathbf{k}$ denoting the propagation vector inside the sandwiched vacuum $-a < y < +a$, and assuming in both cases that there is no dispersion of the k_y component, the Rayleigh formula yields

$$v_x = dk/dk_x = k_x/k = \alpha \tag{2.9}$$

with $\alpha < 1$ for the guided wave and $\alpha > 1$ for the evanescent wave. The latter result is of course completely consistent with the fact that for $y \simeq 0$, formulas (2.1), (2.2), (2.3), (2.4) approximate very closely those of a plane tachyon wave with $k_x/k > 1$.

(5) *Energy flux, energy and momentum densities.* We will satisfy ourselves with a calculation of the components $M^{\alpha 4} = M^{4\alpha}$ and M^{44} of the (symmetric) Maxwell–Minkowski tensor, that is, of the Poynting vector and the classical energy density, and of the components $T^{4\alpha}$, $T^{\alpha 4}$ and T^{44} of the de Broglie (1949) asymmetric energy-momentum tensor $\frac{1}{8}A_l{}^*[\partial^i]H^{jl} + \text{c.c.}$

One easily obtains for the sandwiched evanescent wave

$$\begin{aligned}
w &= M^{44} = \tfrac{1}{4}\alpha^2\cosh 2(k\bar{\beta}y)(E^*E + H^*H)\\
S^x &= iM^{4x} = iT^{4x} = \tfrac{1}{2}\alpha\cosh^2(k\bar{\beta}y)(E^*E + H^*H)\\
S^y &= iM^{4y} = iT^{4y} = 0\\
S^z &= iM^{4z} = iT^{4z} = \tfrac{1}{4}j\alpha\bar{\beta}(E^*H - EH^*)\sinh 2(k\bar{\beta}y)\\
\tilde{w} &= T^{44} = \tfrac{1}{2}\mathscr{E}^*\mathscr{E} = \tfrac{1}{4}\{E^*E + H^*H +\\
&\qquad + [E^*E + H^*H(2\alpha^2 - 1)]\cosh 2K\bar{\beta}y)\}\\
\mathscr{P}^x &= iT^{x4} = \alpha\tilde{w} \quad \text{with} \quad \alpha > 1\\
\mathscr{P}^y &= iT^{y4} = 0\\
\mathscr{P}^z &= iT^{z4} = 0
\end{aligned}$$

and for the guided wave

$$\begin{aligned}
w &= M^{44} = \tfrac{1}{2}\{\alpha_0{}^2\cos 2(k\beta y)(H^*H - E^*E) + E^*E + H^*H\}\\
S^x &= iM^{4x} = iT^{4x} = \tfrac{1}{2}\alpha_0\{E^*E\sin^2(k\beta y) + H^*H\cos^2(k\beta y)\}\\
S^y &= iM^{4y} = iT^{4y} = 0\\
S^z &= iM^{4z} = iT^{4z} = \tfrac{1}{4}j\alpha_0\beta(E^*H - EH^*)\cos 2(k\beta y)\\
\tilde{w} &= T^{44} = \tfrac{1}{2}\mathscr{E}^*\mathscr{E} = \tfrac{1}{4}\{E^*E + H^*H +\\
&\qquad + [H^*H(2\alpha^2 - 1) + E^*E]\cos 2(k\beta y)\}\\
\mathscr{P}^x &= iT^{x4} = \alpha\tilde{w} \quad \text{with} \quad \alpha < 1\\
\mathscr{P}^y &= iT^{y4} = 0\\
\mathscr{P}^z &= iT^{z4} = 0
\end{aligned}$$

It should be noted that the (asymmetric) Géhéniau–de Broglie energy momentum tensor yields *locally* the same ratios $\mathscr{P}^x/w = \alpha$, $\mathscr{P}^y/w = \mathscr{P}^z/w = 0$, between the momentum densities and the energy density, as exist between the momentum quanta and the energy quanta in the wave: $k^x/w = \alpha$, $k^y/w = k^z/w = 0$. Moreover, $\alpha > 1$ for the tachyon-photons inside the evanescent wave, while $\alpha < 1$ inside the guided wave.

(6) *Concluding this section.* The sandwiched evanescent wave we have defined approximates very closely a guided tachyon wave having (in units such that $c = 1$) a group velocity $\alpha > 1$ and a ratio between momentum and energy quanta $k^x/w = \alpha > 1$.

3. *Evanescent Wave with a Dispersion of the Propagation Vector Either Parallel to the Incidence Plane or to the Reflecting Plane*

(1) It is certainly desirable to produce formulas describing more realistic situations than the highly idealised case of the evanescent wave generated by a pure plane incident wave. We display in this section compact formulas for the two cases where a dispersion of the propagation vector either parallel to the incidence plane or to the reflecting plane exists.

(2) *Dispersion parallel to the incidence plane* (de Beauregard, 1971b). We consider an electromagnetic field *in vacuo* such that the 10 wave amplitudes $\mathscr{V} \equiv (\mathbf{E}, \mathbf{H}, \mathbf{A}, V)$ are z independent, so that the photon has no momentum in the z direction, the time dependence being through a common factor $\exp(j\omega t)$.

The 10 functions

$$\mathscr{V}_0(x, y) = \mathscr{V} \exp(-j\omega t) \tag{3.1}$$

are thus by hypothesis solutions of the Helmholtz equation

$$(\partial_x{}^2 + \partial_y{}^2 + \omega^2)\mathscr{V}_0(x, y) = 0 \tag{3.2}$$

We then superpose a transverse electric

$$E_z \equiv E_z(x, y), \qquad H_x = j\omega^{-1}\,\partial_y E_z, \qquad H_y = -j\omega^{-1}\,\partial_x E_z \tag{3.3}$$

and a transverse magnetic

$$H_z \equiv H_z(x, y), \qquad E_x = -j\omega^{-1}\,\partial_y H_z, \qquad E_y = j\omega^{-1}\,\partial_x H_z \tag{3.4}$$

solution with the same angular frequency ω, and calculate the Poynting vector

$$\mathbf{S} \equiv \tfrac{1}{4}(\mathbf{E}^* \times \mathbf{H} + \mathbf{E} \times \mathbf{H}^*) \tag{3.5}$$

We obtain, $[\partial] \equiv \underset{\rightarrow}{\partial} - \underset{\leftarrow}{\partial}$ denoting the Schrödinger or Gordon operator,

$$S_{x,y} = \frac{j}{4\omega}(E_z{}^*[\partial_{x,y}]E_z + H_z{}^*[\partial_{x,y}]H_z) \tag{3.6}$$

that is, a Schrödinger-like energy flux inside the x, y plane, with no interference terms between E_z and H_z, and

$$S_z = \frac{1}{4\omega^2}(\partial_x E_z{}^* \partial_y H_z - \partial_x H_z{}^* \partial_y E_z) + \text{c.c.} \tag{3.7}$$

Thus, although the z component of the photon momentum is identically zero, there is a non-zero energy flux through the plane $z = 0$, the value of which depends critically on the phase relation between E_z and H_z.

One easily verifies the relation

$$\partial_x S_y - \partial_y S_x = \frac{j}{2\omega}(\partial_x E_z{}^* \partial_y E_z + \partial_x H_z{}^* \partial_y H_z) + \text{c.c.} \tag{3.8}$$

If the boundary conditions are such as to allow that

$$H_z(x, y) = jaE_z(x, y) \tag{3.9}$$

where a denotes a real constant, then

$$S_z = \frac{1}{(a + a^{-1})\,\omega}(\partial_x S_y - \partial_y S_x) \tag{3.10}$$

follows from (3.7) and (3.8). In integral form

$$\iint S_z\, dx\, dy = \frac{1}{(a + a^{-1})\,\omega}\oint (S_x\, dx + S_y\, dy) \tag{3.11}$$

By definition, left or right circular polarisation (that is, pure positive or negative helicity) correspond respectively to $a = \pm 1$.

Now, the Fresnel evanescent wave is indeed such that the condition (3.9) is fulfilled. Also, it is such that the double integral (3.11), extended either to the semi plane $z = 0$, $y < 0$, or to the whole plane $z = 0$, is non-zero (Imbert, 1969, 1970a, b).

The preceding formulas hold in the case where the evanescent wave is a Fourier superposition of pure Fresnel evanescent waves, with the dispersion on the complex propagation vector $\mathbf{k}$ lying inside the plane $z = 0$. This clearly allows a more general derivation of the new Imbert transverse shift (Imbert, 1969, 1970a, b) than had been initially given (Imbert, 1968).

(3) *Dispersion parallel to the reflecting plane* (de Beauregard, 1971a). Denoting $y = 0$ the reflecting plane separating the vacuum $y < 0$ from the medium of index $n, y > 0$, we consider the vacuum solutions of the Helmholtz equation

$$(\partial_z{}^2 + \partial_x{}^2 + n^2 \alpha^2 \omega^2)\,\mathscr{V}(x, z) = 0 \tag{3.12}$$

such that $n\alpha = \text{const.} > 1$; they represent a Fourier superposition of pure Fresnel evanescent waves, the dispersion on the complex propagation vector being now parallel to the reflecting plane. Setting

$$q \equiv (n^2 \alpha^2 - 1)^{1/2}, \qquad \tau \equiv 1/n^2 \alpha^2 \tag{3.13}$$

and denoting $E_y \equiv E_y(x, z)$, $H_y \equiv H_y(x,z)$ two solutions of (3.12), we write the solution of Maxwell's equations inside the vacuum $y < 0$ as

$$E_y \equiv E_y, \qquad E_z = \tau(q\partial_z E_y - j\,\partial_x H_y), \qquad E_x = \tau(q\,\partial_x E_y + j\,\partial_z H_y) \tag{3.14}$$

$$H_y \equiv H_y, \qquad H_z = \tau(q\,\partial_z H_y + j\,\partial_x E_y), \qquad H_x = \tau(q\,\partial_x H_y - j\,\partial_z E_y) \tag{3.15}$$

and calculate the Poynting vector (3.5) as

$$S_x = (2n\alpha)^{-2}\,\omega^{-1}\{j(E_y{}^*[\partial_x]\,E_y + H_y{}^*[\partial_x]\,H_y) + q(E_y{}^*[\partial_z]\,H_y - H_y{}^*[\partial_z]\,E_y)\}$$
$$(3.16)$$
$$S_z = (2n\alpha)^{-2}\,\omega^{-1}\{j(E_y{}^*[\partial_z]\,E_y + H_y{}^*[\partial_z]\,H_y) - q(E_y{}^*[\partial_x]\,H_y - H_y{}^*[\partial_x]\,E_y)\}$$

and

$$S_y = (2n\alpha\omega)^{-2}(\partial_z E_y{}^*\,\partial_x H_y - \partial_x E_y{}^*\,\partial_z H_y) + \text{c.c.} \tag{3.17}$$

The relation

$$\partial_z S_x - \partial_x S_z = 2j(2n\alpha)^{-2}\,\omega^{-1}(\partial_z E_y{}^*\,\partial_x E_y + \partial_z H_y{}^*\,\partial_x H_y) + \text{c.c.} \tag{3.18}$$

is easily verified, so that, if

$$H_y(x,z) = jaE_y(x,z) \tag{3.19}$$

with a denoting a constant, one has

$$S_y = \frac{1}{(a+a^{-1})\,\omega}(\partial_z S_x - \partial_x S_z) \tag{3.20}$$

or, in integral form,

$$\iint S_y\,dx\,dz = \frac{1}{(a+a^{-1})\,\omega}\oint (S_x\,dx + S_z\,dz) \tag{3.21}$$

Formulas (3.16) express the vector $(S_x, S_z, 0)$ as the sum of two vectors. The first one is an ordinary longitudinal Schrödinger-like current with no interference terms between E_y and H_y. The second one is in the form of a transverse Schrödinger current and depends critically on the phase relation between E_y and H_y; it is maximal when the condition (3.19) is fulfilled, with $a = \pm 1$, that is, left or right circular polarisation inside the evanescent wave.

Formula (3.21) yields the energy flux through an arbitrary element of the reflecting plane when the condition (3.19) holds.

4. *The Fresnel Evanescent Wave in the Case where the Photon has a Rest Mass*

(1) We have shown in I that, inside the vacuum of Fresnel's evanescent wave, the asymmetric de Broglie (1949) or canonical energy-momentum tensor, rather than the symmetric Maxwell–Minkowski one, rightly encodes the information pertaining to the energy fluxes (as measured in the longitudinal Goos–Hänchen (Goos & Hänchen, 1947, 1949) and in the

transverse Imbert (Imbert, 1969, 1970a, b) shifts in total reflection) and the momentum densities. Then, as the de Broglie energy-momentum tensor, which contains the 4-potential $A^i(i=1,2,3,4; A^4=iV)$ is *not* gauge invariant in this case, we have shown that it is only the gauge that is transverse in the rest frame of the refracting medium ($V\equiv 0, \mathbf{A}.\mathbf{k}=0$, with $\mathbf{k}$ complex) that yields the preceding result. We thus conclude that this gauge is selected as the only physically significant one in this case.

We will now prove that if we endow the photon with a non-zero (but exceedingly small) rest mass m, the Fresnel evanescent wave is necessarily a pure *spatially* transverse wave in the rest frame of the refracting medium.

As for the longitudinal waves which can exist if $m\neq 0$, we will prove that they are not refracted by the medium, and thus *cannot* contribute as a third degree of freedom of the evanescent wave.

(2) *Refracting indexes of the transverse and longitudinal waves inside a medium of electric and magnetic susceptibilities ε and μ.* We are not aware that the (generalised) Maxwell equations for the case $m\neq 0$, $\varepsilon\neq 1$, $\mu\neq 1$, have ever been written (not even in the well-known Bass & Schrödinger (1955) article).

We take (de Beauregard, 1972) the homogeneous set of equations in exactly the Maxwell form (for the case of a homogeneous, $\mathbf{k}$ real, or inhomogeneous, $\mathbf{k}$ complex plane wave):

$$\mathbf{k}\times\mathbf{A}=-j\mu\mathbf{H}, \qquad \omega\mathbf{A}-\mathbf{k}V=j\mathbf{E} \tag{4.1}$$

entailing

$$\mathbf{k}\times\mathbf{E}+\mu\omega\mathbf{H}\equiv 0, \qquad \mathbf{k}.\mathbf{H}\equiv 0 \tag{4.2}$$

and

$$\mathbf{E}.\mathbf{H}\equiv 0 \tag{4.3}$$

and the inhomogeneous set of equations in the Proca–de Broglie form (de Broglie, 1934a, b, c; Proca, 1936)

$$\mathbf{k}\times\mathbf{H}-\varepsilon\omega\mathbf{E}=jm^2\mathbf{A}, \qquad \varepsilon\mathbf{k}.\mathbf{E}=-jm^2V \tag{4.4}$$

entailing

$$\mathbf{k}.\mathbf{A}-\omega V\equiv 0 \tag{4.5}$$

In (4.4) the Proca–de Broglie vacuum current and charge densities stand in the place of the usual current and charge densities, but the situation is very different from the classical one in that *both* the (4.1)'s and the (4.4)'s are coupling the field strengths to the potentials. As the magnetic susceptibility μ is present only in the (4.1)'s and the electric susceptibility ε only in the (4.4)'s, a condition implying ε, μ, $\mathbf{k}$, ω, m is necessary for the compatibility of the (4.1)'s and the (4.4)'s.

In order to find it we first eliminate $\mathbf{A}$ between $(4.1)_1$ and $(4.4)_1$ by applying the operator $\mathbf{k}\times$ to $(4.4)_1$ and using the (4.2)'s, thus obtaining

$$(k^2-\varepsilon\mu\omega^2+\mu m^2)\mathbf{H}=0$$

whence the option

$$\mathbf{H}=0 \quad or \quad k^2=\varepsilon\mu\omega^2-\mu m^2 \tag{4.6}$$

Now, if we eliminate $\mathbf{E}$ between $(4.1)_2$ and $(4.4)_2$ by applying to the latter the operator $\mathbf{k}.$ and using (4.4), we obtain

$$(\omega^2 - k^2 - m^2/\varepsilon)\,V = 0$$

whence the option

$$V = 0 \quad or \quad k^2 = \omega^2 - m^2/\varepsilon \tag{4.7}$$

In the limit $\varepsilon \to 1$, $\mu \to 1$, both conditions $(4.6)_2$ and $(4.7)_2$ reduce to the vacuum propagation condition $k^2 = \omega^2 - m^2$. Here we find *two* different propagation modes for plane waves in the medium, one with index

$$n_t \equiv k_t/\omega = \{\varepsilon\mu(1 - m^2/\varepsilon\omega^2)\}^{1/2} \simeq (\varepsilon\mu)^{1/2} \tag{4.8}$$

and the other one with index

$$n_l = k_l/\omega = (1 - m^2/\varepsilon\omega^2)^{1/2} \simeq 1 \tag{4.9}$$

Moreover, as the formula $(4.7)_2$ does not contain μ and that μ appears in the (4.2)'s only as a factor of $\mathbf{H}$, we conclude that the condition $(4.7)_1$ entails the condition (4.6).

Now we show that the condition $(4.6)_1$ entails the condition $(4.7)_2$ and characterises longitudinal waves. $\mathbf{H} \equiv 0$ entails, *via* equations (4.2) to (4.5),

$$\mathbf{k} \times \mathbf{A} = 0, \qquad \mathbf{A} = a\mathbf{k}, \qquad V = ak^2/\omega \tag{4.10}$$

$$\mathbf{E} = ja(k^2/\omega)(k^2 - \omega^2)\,\mathbf{k} \tag{4.11}$$

with a denoting a (complex) constant. Applying the operator $\varepsilon\mathbf{k}.$ to (4.11) and using $(4.4)_2$ we obtain (as $a \neq 0$ if $\mathbf{A} \neq 0$) formula (4.9). Incidentally, we have proved that conditions $(4.6)_1$ and $(4.7)_2$ are equivalent, so that our dilemma can be written as

$$\mathbf{H} \equiv 0 \quad or \quad V \equiv 0 \tag{4.12}$$

Finally we verify that the condition $(4.7)_1$ entails the condition $(4.6)_2$ and characterises the transverse waves. $V \equiv 0$ entails, *via* equations (4.2) to (4.5), the formulas

$$V = 0, \qquad \mathbf{E} = -j\omega\mathbf{A}, \qquad \mathbf{H} = -j\mathbf{k} \times \mathbf{A} \tag{4.13}$$

and (4.3) characterising the transverse waves, and, through $(4.2)_1$ and $(4.4)_1$,

$$kE = \mu\omega H, \qquad kH = (\varepsilon\omega - m^2/\omega)\,E$$

whence (as $\mathbf{E} \neq 0$ and $\mathbf{H} \neq 0$ if $\mathbf{A} \neq 0$) formula $(4.7)_2$.

To conclude, if the photon has a non-zero (but exceedingly small) rest mass m, ordinary isotropic refracting media can support two different modes of plane propagating waves: the ordinary *transverse waves* characterised by

$$V \equiv 0 \quad and \quad n_t = \{\varepsilon\mu(1 - m^2/\varepsilon\omega^2)\}^{1/2} \simeq (\varepsilon\mu)^{1/2}$$

and the *longitudinal waves* characterised by

$$\mathbf{H} \equiv 0 \quad and \quad n_l = (1 - m^2/\varepsilon\omega^2)^{1/2} \simeq 1$$

(they are thus not appreciably refracted).

In other words, *ordinary isotropic refracting media must be thought of as polarisation state filters for the massive photon*: they will throw incoming photons either in the (twofold) locally transverse state $\mathbf{k}.\mathbf{A} = 0$ or in the (simple) locally longitudinal state $\mathbf{A} = a\mathbf{k}$, $V = ak^2/\omega$. If, for instance, the surface through which the photons enter the medium is a plane oblique on the incoming propagation vector, the two beams will be divergent (like in the case of ordinary birefringence).

Consider for instance the (highly arbitrary) assumption that the incoming beam is made of a 'natural' light where the three independent degrees of freedom have equivalent weights. Then, *in terms of numbers of photons per sec cm²*, the (locally) longitudinal and transverse beams filtered by the medium will have 'intensities' in the (very high) ratio 1/2. However, *in terms of electric field strengths* the corresponding intensity ratio will be the (exceedingly small) $m^2/2\omega^2$. The idea that, if the photon does have a non-zero rest mass, streams of longitudinal photons might be flowing undetected through the universe certainly sounds like good science fiction ....

Coming back to our problem of the (pure) Fresnel evanescent wave characterised by a well-defined complex propagation vector, it is clear from above that (1) it contains *rigorously* no contribution from the spin zero state, and (2) that the index n for building it is not quite exactly the classical $\sqrt{\varepsilon\mu}$ but rather the n defined by (4.8) (which makes practically no difference).

(3) *Fresnel evanescent wave for $m \neq 0$.* Using the same coordinates and conventions as in I together with the refracting index $n \equiv n_t$ defined by (4.8), we write the complex propagation vector inside the evanescent wave as

$$k_x = n\alpha\omega, \qquad k_y = -j\omega(n^2\alpha^2 - 1 + \omega^{-2}m^2)^{1/2}, \qquad k_z = 0 \qquad (4.14)$$

so that

$$k_x{}^2 + k_y{}^2 = \omega^2 - m^2 \qquad (4.15)$$

and k_x and k_z are continuous through the plane $y = 0$. The common phase factor is thus

$$P = \mathrm{Exp}\{j\omega(t - n\alpha x) - \omega(n^2\alpha^2 - 1 + \omega^{-2}m^2)^{1/2}\} \qquad (4.16)$$

As there is no admixture of spin 0 state, we must retain only solutions such that the (4.13)'s hold. However, when the vector potential enters the picture (as is necessarily the case if $m \neq 0$), the presence of the reflecting plane and the existence of the corresponding boundary condition entails some dissimilarity between the two linear polarisation states described as 'transverse electric' and 'transverse magnetic'.

Transverse electric wave. By hypothesis

$$A_x = 0, \qquad A_y = 0, \qquad A_z = j\omega^{-1}E_z, \qquad V = 0 \qquad (4.17)$$

entailing

$$\mathbf{k}.\mathbf{A}=0 \tag{4.18}$$

and, through (4.1),

$$E_x=0, \qquad E_y=0, \qquad E_z\equiv E_z \tag{4.19}$$

$$H_x=-j(n^2\alpha^2-1+\omega^{-2}m^2)^{1/2}E_z, \qquad H_y=-n\alpha E_z, \qquad H_z=0 \tag{4.20}$$

Transverse magnetic wave. By hypothesis

$$A_x=\frac{-jk_y}{\omega^2-m^2}H_z, \qquad A_y=\frac{jk_x}{\omega^2-m^2}H_z, \qquad A_z=0, \qquad V=0 \tag{4.21}$$

entailing (4.18) and, through (4.1),

$$E_x=\frac{j(n^2\alpha^2-1+\omega^{-2}m^2)^{1/2}}{1-\omega^{-2}m^2}H_z, \qquad E_y=\frac{n\alpha}{1-\omega^{-2}m^2}H_z, \qquad E_z=0 \tag{4.22}$$

$$H_x=0, \qquad H_y=0, \qquad H_z\equiv H_z \tag{4.23}$$

In both cases the homogeneous Maxwell and the inhomogeneous Proca–de Broglie equations are satisfied, as the (space-time) formulas ($i,j=1,2,3,4$; $x^4=it$)

$$jB^{ij}=k^iA^j-k^jA^i, \qquad k_ik^i+m^2=0, \qquad k_iA^i=0$$

entail

$$jk_jB^{ij}=m^2A^i, \qquad \sum_{\text{circ}}k^iB^{jk}=0$$

regardless of the real or complex character of the space vector k^α ($\alpha=1,2,3$).

Formulas (4.8) and (4.14) to (4.23) are either identical to, or differing only by corrective terms of order m^2/ω^2 from those given in I for the two polarisation states of the evanescent wave.

Finally we must derive the formulas for the continuity or discontinuity of the field strengths through the reflecting plane $y=0$. The de Broglie–Proca equations read, in the case of a harmonic and transverse solution,

$$\begin{aligned}\operatorname{curl}\mathbf{E}+j\mu\omega\mathbf{H}&=0, \qquad \operatorname{div}\mathbf{H}=0\\ \operatorname{curl}\mathbf{H}-j(\varepsilon\omega-m^2/\omega)\mathbf{E}&=0, \qquad \operatorname{div}\mathbf{E}=0\end{aligned} \tag{4.24}$$

They are isomorphic to the Maxwell equations in the case of a harmonic solution and an imaginary conduction coefficient

$$\sigma=-j\omega^{-1}m^2 \tag{4.25}$$

so that a simple transcription of the well-known continuity formulas for the field strengths through a discontinuity surface yields

$$H_1{}^T=H_2{}^T, \qquad E_1{}^T=E_2{}^T \tag{4.26}$$

$$\mu_1H_1{}^N=\mu_2H_2{}^N, \qquad (\varepsilon_1-\omega^{-2}m^2)E_1{}^N=(\varepsilon_2-\omega^{-2}m^2)E_2{}^N \tag{4.27}$$

where the suffixes T and N hold for the tangential and normal components respectively.

The field strengths inside the incident (i) and reflected (r) wave for $y=+0$, and inside the evanescent (e) wave for $y=-0$, must satisfy these conditions. First we notice that $(4.1)_1$ entails ($n=n_t$)

$$nE^{i,r}=\mu H^{i,r} \tag{4.28}$$

The transverse electric triplet. Setting

$$E_z^{\,e}=a, \qquad E_z^{\,i}=\tfrac{1}{2}(a+b), \qquad E_z^{\,r}=\tfrac{1}{2}(a-b) \tag{4.29}$$

we satisfy $(4.27)_2$. Then from (4.20)

$$H_y^{\,e}=-n\alpha a, \qquad H_x^{\,e}=-j(n^2\alpha^2-1+\omega^{-2}m^2)^{1/2}\,a \tag{4.30}$$

and from (4.28)

$$H_y^{\,i}=-n\alpha(a+b)/2\mu, \qquad H_x^{\,i}=-n\beta(a+b)/2\mu \tag{4.31}$$

$$H_y^{\,r}=-n\alpha(a-b)/2\mu, \qquad H_x^{\,r}=n\beta(a-b)/2\mu \tag{4.32}$$

From $(4.30)_1$, $(4.31)_1$ and $(4.32)_1$ we deduce, in accord with $(4.27)_1$,

$$H_y^{\,e}=\mu(H_y^{\,i}+H_y^{\,r}) \tag{4.33}$$

From $(4.30)_2$, $(4.31)_2$, $(4.32)_2$, $(4.27)_1$ and the obvious relation $\alpha^2+\beta^2=1$ we deduce

$$b=j\mu n^{-1}\left(\frac{n^2\alpha^2-1+\omega^{-2}m^2}{1-\alpha^2}\right)^{1/2} a \tag{4.34}$$

that is, the phase shifts in total reflection.

The transverse magnetic triplet. Setting

$$H_z^{\,e}=a', \qquad H_z^{\,i}=\tfrac{1}{2}(a'+b'), \qquad H_z^{\,r}=\tfrac{1}{2}(a'-b') \tag{4.35}$$

we satisfy (4.27). Then from (4.22)

$$E_y^{\,e}=\frac{n\alpha}{1-\omega^{-2}m^2}a', \qquad E_x^{\,e}=j\frac{(n^2\alpha^2-1+\omega^{-2}m^2)^{1/2}}{1-\omega^{-2}m^2}\,a' \tag{4.36}$$

and from (4.28)

$$E_y^{\,i}=\mu\alpha(a'+b')/2n, \qquad E_x^{\,i}=\mu\beta(a'+b')/2n \tag{4.37}$$

$$E_y^{\,r}=\mu\alpha(a'-b')/2n, \qquad E_x^{\,r}=\mu\beta(b'-a')/2n \tag{4.38}$$

From $(4.36)_1$, $(4.37)_1$, $(4.38)_1$ and (4.8) we deduce, in accord with $(4.27)_2$,

$$(1-\omega^{-2}m^2)\,E_y^{\,e}=(\varepsilon-\omega^{-2}m^2)(E_y^{\,i}+E_y^{\,r}) \tag{4.39}$$

From $(4.36)_2$, $(4.37)_2$, $(4.38)_2$ and $(4.26)_2$ we deduce the phase shift formula

$$b'=j\mu^{-1}n\left(\frac{n^2\alpha^2-1+\omega^{-2}m^2}{1-\alpha^2}\right)^{1/2}(1-\omega^{-2}m^2)^{-1}\,a' \tag{4.40}$$

Thus, contrary to Bass & Schrödinger's (1955) conclusion for the metallic reflection, we find that the total vitreous reflection of a massive photon (either in the transverse electric or the transverse magnetic mode) goes on without *any* interference with the longitudinal mode.

In the limit $m \to 0$ all the preceding formulas (4.26) to (4.40) reduce to the classical ones.

5. *Concluding Remarks*

We have given four very natural extensions to the contents of a previous paper (de Beauregard *et al.*, 1971) referred to as I:

(1) Formulas for a sandwiched evanescent wave generated by two symmetrical plane waves incident inside two identical refracting media; the Fresnel exponential damping factor being then replaced by hyperbolic cosines or sines (depending on the field strengths components) an extremely close approximation to a tachyon plane wave (with propagation vector $\mathbf{k}$ longer, in units such that $c = 1$, than the angular frequency ω) is thus obtained. This is not to say that the physical device thus defined would in fact be more convenient than the simple plane interface, for experimentally testing the tachyon-like quanta inside an evanescent wave. Our main motivation when producing these formulas has been to add physical credibility to the concept of the tachyon photons.

(2) We have also produced very similar compact formulas for the two cases where the evanescent wave can be thought of as a Fourier type superposition of elementary evanescent waves characterised by complex propagation vectors $\mathbf{k}$, the dispersion of which is either parallel to the incidence plane or to the reflecting plane. In both cases the existence of the transverse energy flux when the evanescent wave is elliptically polarised is quite conspicuous in the formulas.

(3) Finally we have written down the formulas for plane waves of massive photons inside a refracting medium, thus showing that the corrected index for the transverse waves is $\sqrt{[\varepsilon\mu(1 - m^2/\varepsilon\omega^2)]}$, where m denotes a photon (very small) rest mass, and that the index for the longitudinal waves is $\sqrt{[1 - m^2/\varepsilon\omega^2]}$. Thus an ordinary refracting medium should be birefringent with respect to the ordinary transverse photons on the one hand, and for the longitudinal photons on the other hand. Then (as no longitudinal evanescent waves are produced under such conditions) we have calculated the transverse electric and transverse magnetic types of evanescent waves for the massive photon, producing formulas which, in the limit $m \to 0$, transform into the well-known ones we have used in paper I. As was expected, the potential's gauge for the massive photons corresponds exactly to the transverse gauge we had selected in I for physical reasons.

Acknowledgements

It is a pleasure to thank Dr. C. Imbert and Dr. J. Ricard and (in connection with Section 4) Dr. J. P. Vigier for very stimulating discussions.

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