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Energy-momentum tensor in the scalar diquark model

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Abstract

We compute all the gravitational form factors in the scalar diquark model at the one-loop level using two different regularization methods. We check explicitly that all the Poincaré sum rules are satisfied and we discuss in detail the results for the trace of the energy-momentum tensor. Finally we discuss the spatial distributions of energy and pressure in two and three dimensions.

Keywords: Scalar diquark model; energy-momentum tensor; gravitational form factors

1. Introduction

Recent years have seen a rich discussion on the energy-momentum tensor (EMT) in QCD. Specific focus has been put on the decomposition of the proton mass in QCD, see e.g. [1–9], and the relation with the trace of the EMT, which contains a well-known anomalous contribution. Beside the question of the proton spin decomposition, see e.g. [10–16], the EMT is also used in the literature to define the notions of pressure and shear distributions inside hadrons [17–23]. More generally, the EMT is a central object in the physics case of the future electron-ion collider in the US [24].

Our aim in this letter is to present a simple model computation for the proton respecting full Poincaré covariance. Specifically, we are going to present the one-loop results in the scalar diquark model. Some work based on the light-front wave function representation already exists for the EMT in some variant of this model [25], but lacks full covariance. We are going to present the complete one-loop results, including the counterterms for the proton field in the model, using two different types of regularization. The first is dimensional regularization, which has been used in similar works for QED and QCD. The second is Pauli-Villars regularization. In this second regularization scheme, the emergence of the anomalous contribution to the trace is conceptually different compared to dimensional regularization. We will highlights the key aspects of the comparison between the two regularization schemes. We will also derive the pressure and shear distributions in Fourier-conjugate space. However, they show distinct pathological features characteristic of perturbative computations.

2. Scalar diquark model

Different versions of the scalar diquark model exist. The main differences concern the inclusion of different flavors for the quark field, the inclusion of electromagnetic and color degrees of freedom for the quark and diquark fields. Since at one-loop level for external proton states all the differences amount to at most a global irrelevant factor, we choose to work with the simplest version of the model. The Lagrangian reads

$$\mathcal{L} = \bar{\Psi}\left(\frac{i}{2}\overleftrightarrow{\not\partial} - M\right)\Psi + \bar{q}\left(\frac{i}{2}\overleftrightarrow{\not\partial} - m\right)q + \frac{1}{2}\left(\partial^\mu\phi\partial_\mu\phi - m_s^2\phi^2\right) + g\phi\left(\bar{\Psi}q + \bar{q}\Psi\right), \tag{1}$$

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where Ψ is the proton field, q the quark field and ϕ the diquark field, and $\overleftrightarrow{\partial} = \overrightarrow{\partial} - \overleftarrow{\partial}$.

We will use two main regularization schemes. The first one is standard dimensional regularization (DR) with $D = 4 - 2\epsilon$, for which no modification of the Lagrangian is needed. The second one is Paulli-Villars (PV) regularization. Specifically, we introduce a ghost field c for the scalar diquark only, for which the Lagrangian reads

$$\mathcal{L}_{\mathrm{PV}} = -\frac{1}{2}\left(\partial^\mu c \partial_\mu c - M_{\mathrm{PV}}^2 c^2\right) + gc\left(\bar{\Psi} q + \bar{q}\Psi\right). \tag{2}$$

Notice that the kinetic term has opposite sign compared to the normal scalar, but the interaction term is identical.

For convenience, we report the equations of motion (EOM) for the theory:

$$i\partial\!\!\!/ S = \mathbb{M}S, \qquad \bar{S}i\overleftarrow{\partial\!\!\!/} = -\bar{S}\mathbb{M}, \qquad \left(\Box + m_s^2\right)\phi = g\bar{S}\sigma_1 S, \qquad \left(\Box + M_{\mathrm{PV}}^2\right)c = -g\bar{S}\sigma_1 S \tag{3}$$

with

$$S = \begin{pmatrix}\Psi \\ q\end{pmatrix}, \qquad \sigma_1 = \begin{pmatrix}0 & 1 \\ 1 & 0\end{pmatrix}, \qquad \mathbb{M} = \begin{pmatrix} M & -g(\phi + c) \\ -g(\phi + c) & m \end{pmatrix}. \tag{4}$$

We then derive the symmetric energy-momentum tensor (EMT) via the variation of the action $\int d^4x \sqrt{-g}\,(\mathcal{L} + \mathcal{L}_{\mathrm{PV}})$ with respect to a general metric, evaluated afterwards for the Minkowski metric $g_{\mu\nu} = \mathrm{diag}(+,-,-,-)$. After application of the EOM, we obtain

$$T^{\mu\nu} = \bar{S}\gamma^{\{\mu}\frac{i}{2}\overleftrightarrow{\partial}^{\nu\}}S + \partial^\mu\phi\partial^\nu\phi - \partial^\mu c\partial^\nu c - \frac{1}{2}g^{\mu\nu}\left(\partial_\alpha\phi\partial^\alpha\phi - m_s^2\phi^2\right) + \frac{1}{2}g^{\mu\nu}\left(\partial_\alpha c\partial^\alpha c - M_{\mathrm{PV}}^2 c^2\right), \tag{5}$$

where $a^{\{\mu}b^{\nu\}} = (a^\mu b^\nu + a^\nu b^\mu)/2$. Evidently, in the case of DR the ghost sector is not present.

We are going to investigate the proton matrix elements of Eq. (5) up to the one-loop level. For this, we will isolate the gravitational form factors, defined by the parametrization

$$\langle p', s'|T_i^{\mu\nu}|p, s\rangle = \bar{u}'\left[A_i(\Delta^2)\frac{P^\mu P^\nu}{M} + \left(A_i(\Delta^2) + B_i(\Delta^2)\right)\frac{P^{\{\mu}i\sigma^{\nu\}\rho}\Delta_\rho}{2M} + D_i(\Delta^2)\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{4M} + \bar{C}_i(\Delta^2)g^{\mu\nu}M\right]u, \tag{6}$$

where $u \equiv u(p, s)$ is the usual free Dirac spinor, $P = (p' + p)/2$ and $\Delta = p' - p$. Notice that the index i labels three different contributions, namely $i = \Psi$ for the proton operators, $i = q$ for the quark operators and $i = d$ for the diquark operators. In the case of PV regularization, we will merge the diquark and ghost contributions.

3. Loop results

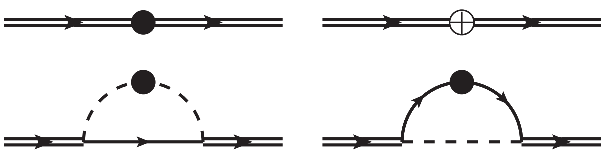

Figure 1: Relevant diagrams up to order g^2 for the proton matrix elements of the EMT operator. The black dot represents the EMT insertion into the Green's function, the white crossed dot represents the counterterm diagram. Solid, double solid and dashed lines represent the quark, proton and diquark fields, respectively.

In this section we report the main results of this work. First, we present the relevant Lagrangian counterterms in the two chosen regularization schemes. Then we proceed with the results for the EMT insertion in a quark line and in a diquark line, depicted in the second row of Fig. 1.

3.1. Lagrangian counterterms

The only necessary counterterms are the ones associated to the proton wave function $Z_\Psi = 1 + \delta Z_\Psi$ and the proton mass $Z_M = 1 + \delta Z_M$. We computed them in the on-shell scheme for the proton field. In general, the sum of the counterterm diagrams and the diagrams in which the loops are confined to one of the external legs is independent of the choice of renormalization scheme for the Lagrangian counterterms (individual diagrams though might naturally differ). Since here we are only interested in the total contribution from the proton operator, we can pick any scheme and the results will be unaffected.

For convenience, let us define

$$\bar{x} = 1 - x, \qquad a_s = \frac{g^2}{16\pi^2}, \qquad \Theta_0(x) = x m_s^2 + \bar{x} m^2 - x\bar{x} M^2. \tag{7}$$

In the on-shell scheme we find

$$\delta Z_\Psi^{\rm R} = -\frac{a_s}{2}\mathfrak{s}^{\rm R} + a_s \mathfrak{f}_\Psi^{\rm R} + a_s \int_0^1 dx\, x \left[\log\left(\frac{\Theta_0(x)}{\bar{\mu}^2}\right) - 2\bar{x}M\,\frac{m + xM}{\Theta_0(x)}\right], \tag{8}$$

$$\delta Z_M^{\rm R} = a_s\left(\frac{1}{2} + \frac{m}{M}\right)\mathfrak{s}^{\rm R} + a_s \mathfrak{f}_M^{\rm R} - a_s \int_0^1 dx \left(x + \frac{m}{M}\right)\log\left(\frac{\Theta_0(x)}{\bar{\mu}^2}\right), \tag{9}$$

where the singular and finite factors are given by

$$\mathfrak{s}^{\rm R} = \begin{cases} \frac{1}{\epsilon} & \text{for R=DR} \\ \log\left(\frac{M_{\rm PV}^2}{\bar{\mu}^2}\right) & \text{for R=PV} \end{cases}, \qquad \mathfrak{f}_\Psi^{\rm R} = \begin{cases} 0 & \text{for R=DR} \\ \frac{1}{4} & \text{for R=PV} \end{cases}, \qquad \mathfrak{f}_M^{\rm R} = \begin{cases} 0 & \text{for R=DR} \\ -\left(\frac{1}{4} + \frac{m}{M}\right) & \text{for R=PV} \end{cases}. \tag{10}$$

For DR we defined the typical $\overline{\rm MS}$ scale $\bar{\mu}^2 = 4\pi e^{-\gamma_E}\mu^2$. In the case of PV regularization, the scale $\bar{\mu}^2$ is a dummy scale for the sole purpose of having well-defined arguments for the logarithms and, at the same time, isolating the singularity.

At this point, we already have all the ingredients to deduce the gravitational form factors for the proton operator, namely

$$A_\Psi^{\rm R} = 1 + \delta Z_\Psi^{\rm R}, \qquad B_\Psi = 0, \qquad D_\Psi = 0, \qquad \bar{C}_\Psi = 0. \tag{11}$$

3.2. quark vertex

To express the results in a somewhat compact form, let us introduce the short-hand notation $\tau^2 = -\Delta^2/4$ and the following definitions

$$\Sigma_0(x,\tau) = \sqrt{\tau^2 + \Theta_0(x)}, \qquad \Lambda_0(x,\tau) = \log\left(\frac{\Sigma_0(x,\tau) + \tau}{\Sigma_0(x,\tau) - \tau}\right). \tag{12}$$

In this way the gravitational form factors for the quark operator simply read

$$\begin{aligned} A_q^{\rm R} &= \frac{a_s}{6}\mathfrak{s}^{\rm R} + a_s\mathfrak{f}_A^{\rm R} - a_s\int_0^1 dx\, x\bar{x}\left[\log\left(\frac{\Theta_0(x)}{\bar{\mu}^2}\right) + \frac{\Sigma_0(x;\bar{x}\tau)}{\bar{x}\tau}\Lambda_0(x,\bar{x}\tau)\right] \\ &\quad + \frac{a_s}{2\tau}\int_0^1 dx\, x\,\frac{\Theta_0(x) + (m + xM)^2}{\Sigma_0(x,\bar{x}\tau)}\Lambda_0(x,\bar{x}\tau), \end{aligned} \tag{13}$$

$$B_q = \frac{a_s M}{\tau}\int_0^1 dx\, x\bar{x}\,\frac{m + xM}{\Sigma_0(x,\bar{x}\tau)}\Lambda_0(x,\bar{x}\tau), \tag{14}$$

$$D_q = -\frac{a_s M^2}{\tau^2}\left(\frac{1}{3} + \frac{m}{M}\right) + \frac{a_s M^2}{\tau^3}\int_0^1 dx\left(x + \frac{m}{M}\right)\Sigma_0(x,\bar{x}\tau)\,\Lambda_0(x,\bar{x}\tau), \tag{15}$$

$$\bar{C}_q^{\rm R} = -\frac{a_s}{2}\left(\frac{1}{3} + \frac{m}{M}\right)\mathfrak{s}^{\rm R} + a_s\mathfrak{f}_{\bar{C}}^{\rm R} + a_s\int_0^1 dx\,\bar{x}\left(x + \frac{m}{M}\right)\log\left(\frac{\Theta_0(x)}{\bar{\mu}^2}\right), \tag{16}$$

where the singular factor $\mathfrak{s}^{\rm R}$ is the same as in Eq. (10) and the finite factors are here given by

$$\mathfrak{f}_A^{\rm R} = \begin{cases} 0 & \text{for R=DR} \\ \frac{1}{36} & \text{for R=PV} \end{cases}, \qquad \mathfrak{f}_{\bar{C}}^{\rm R} = \begin{cases} 0 & \text{for DR} \\ \frac{5}{36} + \frac{3m}{4M} & \text{for PV} \end{cases}. \tag{17}$$

We see that, depending on the regularization scheme, the finite part of the gravitational form factors may vary. It appears that the A_q gravitational form factor does not vanish when $\tau \to \infty$. In the case of $\bar{C}_q$, we observe that the whole gravitational form factor is actually τ-independent. Interestingly, the same observation has been made for the electron gravitational form factor $\bar{C}_e$ at one-loop in QED [26, 27].

3.3. *diquark vertex*

For the insertion of the EMT operator on the diquark line, everything proceeds in the same way. We obtain

$$A_d^{\rm R} = \frac{a_s}{3}\mathfrak{s}^{\rm R} + a_s\left(\frac{2}{3} - 10\mathfrak{f}_A^{\rm R}\right) - 2a_s\int_0^1 dx\, x\bar{x}\log\left(\frac{\Theta_0(\bar{x})}{\bar{\mu}^2}\right)$$
$$+ \frac{a_s}{\tau}\int_0^1 dx\, x\left[\frac{xM(m+\bar{x}M)}{\Sigma_0(\bar{x},\bar{x}\tau)} - 2\Sigma_0(\bar{x},\bar{x}\tau)\right]\Lambda_0(\bar{x},\bar{x}\tau), \tag{18}$$

$$B_d = -\frac{a_s M}{\tau}\int_0^1 dx\, x^2\,\frac{m+\bar{x}M}{\Sigma_0(\bar{x};\bar{x}\tau)}\Lambda_0(\bar{x},\bar{x}\tau), \tag{19}$$

$$D_d = -\frac{a_s M^2}{\tau^2}\left(\frac{2}{3} + \frac{m}{M}\right) + \frac{a_s M^2}{\tau^3}\int_0^1 dx\left(\bar{x} + \frac{m}{M}\right)\frac{\tau^2(\bar{x}^2-1) + \Theta_0(\bar{x})}{\Sigma_0(\bar{x},\bar{x}\tau)}\Lambda_0(\bar{x},\bar{x}\tau), \tag{20}$$

$$\bar{C}_d^{\rm R} = \frac{a_s}{2}\left(\frac{1}{3} + \frac{m}{M}\right)\mathfrak{s}^{\rm R} - a_s\mathfrak{f}_{\bar{C}}^{\rm R} - a_s\int_0^1 dx\, x\left(\bar{x} + \frac{m}{M}\right)\log\left(\frac{\Theta_0(\bar{x})}{\bar{\mu}^2}\right). \tag{21}$$

4. Renormalization

It is straightforward to see that all the gravitational form factors, once summed over the proton, quark and diquark contributions, are free from UV divergences. This means that the total symmetric EMT is finite and therefore does not require the introduction of additional counterterms beside the Lagrangian ones. The same observation has been made for an electron state in QED [28], and is consistent with the general arguments given in Ref. [29] in the context of non-abelian gauge theories.

In an MS scheme, limiting ourselves to external proton states, the subtraction of divergences can be performed by trivially removing the singular contribution from the individual proton, quark and diquark gravitational form factors. In Fig. 2 we show the contributions of order a_s to the A, B and D gravitational form factors based on the results of the previous section. For simplicity, we chose to illustrate the case $m = 0$ and $m_s = M$ in the $\overline{\rm MS}$ renormalization scheme.

5. Sum rules

A number of constraints on the gravitational form factors can be derived from Poincaré symmetry [11, 30–35]. In particular, four-momentum conservation implies

$$\sum_i A_i(0) = 1, \qquad \sum_i \bar{C}_i(\Delta^2) = 0, \tag{22}$$

and (generalized) angular momentum conservation implies in addition

$$\sum_i B_i(0) = 0. \tag{23}$$

These contraints should hold at any order in perturbation theory. Let us then write the gravitational form factors as

$$X_i = X_i^{(0)} + a_s X_i^{(1)} + \cdots, \tag{24}$$

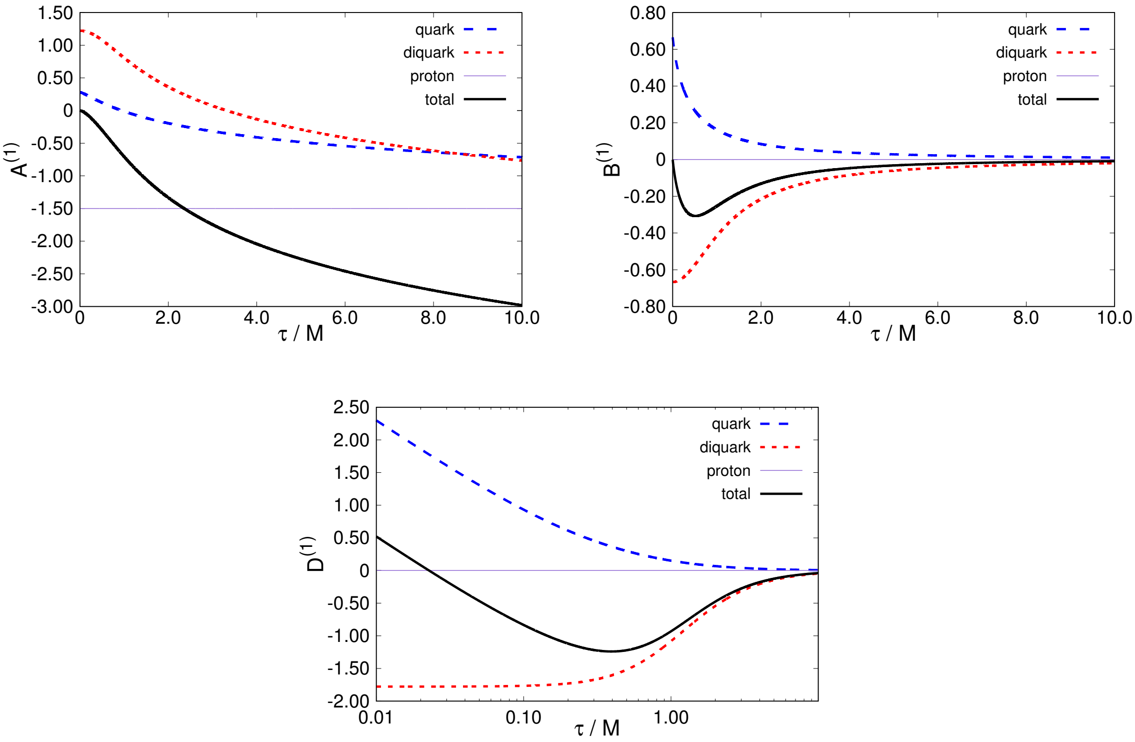

Figure 2: Gravitational form factors as functions of the dimensionless variable τ/M for the case $m = 0$, $m_s = M$. The thick black solid lines represent the total contributions. For all the panels, the thin solid lines, medium- and short-dashed lines represent the proton, quark and diquark contributions, respectively. All the results are shown in the $\overline{\text{MS}}$ renormalization scheme. Note that in the case of the D form factor, we used a log scale for the variable τ/M.

where the upper label indicates the order in a_s.

At tree level the Poincaré constraints are trivially satisfied since all the gravitational form factors vanish except $A_\Psi^{(0)}(\Delta^2) = 1$. Let us now check the $O(a_s)$ contributions. Comparing Eqs. (16) and (21), it is clear after a change of variable $x \mapsto \bar{x}$ in the integral that

$$\bar{C}_d^{(1)}(\Delta^2) = -\bar{C}_q^{(1)}(\Delta^2). \tag{25}$$

Combined with the result $\bar{C}_\Psi^{(1)}(\Delta^2) = 0$ from Eq. (11), we see that the second momentum sum rule in Eq. (22) is satisfied.

For the gravitational form factors A_i, we find in the limit of vanishing momentum transfer

$$\begin{aligned} A_\Psi^{(1)}(0) &= -\frac{1}{2}\,\mathfrak{s}^{\rm R} + \mathfrak{f}_\Psi^{\rm R} + \int_0^1 dx\, x \left[\log\left(\frac{\Theta_0(x)}{\bar{\mu}^2}\right) - 2\bar{x}M\,\frac{m + xM}{\Theta_0(x)}\right], \\ A_q^{(1)}(0) &= \frac{1}{6}\left(\mathfrak{s}^{\rm R} - 1\right) + \mathfrak{f}_A^{\rm R} - \int_0^1 dx\, x\bar{x}\left[\log\left(\frac{\Theta_0(x)}{\bar{\mu}^2}\right) - \frac{(m + xM)^2}{\Theta_0(x)}\right], \\ A_d^{(1)}(0) &= \frac{1}{3}\,\mathfrak{s}^{\rm R} - 10\mathfrak{f}_A^{\rm R} - 2\int_0^1 dx\, x\bar{x}\left[\log\left(\frac{\Theta_0(x)}{\bar{\mu}^2}\right) - \bar{x}M\,\frac{m + xM}{\Theta_0(x)}\right]. \end{aligned} \tag{26}$$

Owing to first momentum sum rule in Eq. (22), it is expected that the sum of these three contributions should vanish. We find indeed

$$\left[A_\Psi^{(1)} + A_q^{(1)} + A_d^{(1)}\right](0) = -\frac{1}{6} + \int_0^1 dx\,\left[(3x^2 - 2x)\log\left(\frac{\Theta_0(x)}{\bar{\mu}^2}\right) + x\bar{x}\,\frac{m^2 - x^2M^2}{\Theta_0(x)}\right] = 0, \tag{27}$$

where in the last step we integrated by parts and used the relation $\left(1 - x\,\frac{d}{dx}\right)\Theta_0(x) = m^2 - x^2 M^2$.

Finally, since we have

$$B_\Psi^{(1)}(0) = 0, \qquad B_q^{(1)}(0) = -B_d^{(1)}(0) = 2\int_0^1 dx\, x\bar{x}^2 M\, \frac{(m + xM)}{\Theta_0(x)}, \tag{28}$$

it follows automatically that the (generalized) angular momentum sum rule in Eq. (23) is also satisfied.

6. *D*-term

The last gravitational form factor is not constrained by Poincaré symmetry. It provides information about the spatial distribution of forces inside the system, and its value at vanishing momentum transfer is known as the D-term [17, 18, 36].

To order a_s in the scalar diquark model, only the quark and diquark sectors contribute to the D-term

$$D_q(0) = \frac{2a_s}{3}\int_0^1 dx\, \bar{x}^3 M\, \frac{(m + xM)}{\Theta_0(x)}, \qquad D_d(0) = \frac{2a_s}{3}\int_0^1 dx\, x(x^2 - 3)M\, \frac{(m + xM)}{\Theta_0(x)}. \tag{29}$$

They are both finite and non-zero. If we assume massless diquark and demand the validity of the stability condition $M < m + m_s$, we find that

$$D_d(0) \approx \frac{8Ma_s}{3(m - M)} \log(m_s). \tag{30}$$

Notice that, in line with the common expectation that $D(0)$ is negative for a stable bound state, we observe that $D(0) \to -\infty$ as $m_s \to 0$ with $M < m$. On the other hand, if we assume that $M = m$ and send the scalar diquark mass to zero, we find that the most singular behavior is of the form

$$D_d(0) \approx \frac{4Ma_s\pi}{3m_s}. \tag{31}$$

This is identical to the scaling behavior found for an electron state in QED with photon mass regularization [26]. We incidentally note that the situation is different for $D_q(0)$. In fact, in the limit of vanishing quark mass and for $M < m_s$, we find a finite value for $D_q(0)$. In the case $m \to 0$ and $m_s = M$ we find a divergence in D_q (but not in D_d, as illustrated by fig. 2). Specifically, we have

$$D_q(0) \approx -\frac{2a_s}{3} \log(m). \tag{32}$$

Let us stress that the two cases $m = M$, $m_s \to 0$ and $m_s = M$, $m \to 0$ present different scaling behaviours: the former is power-like $1/m_s$, the latter is logarithmic $\log(m)$.

The asymptotic behaviour in the large-τ limit is more cumbersome to extract. For this, let us work with $m_s = M$ and $m = 0$. We obtain

$$\begin{aligned} D_q &\underset{\tau \gg M}{\approx} -\frac{M^2}{6\tau^2}\left[2 - \log\left(\frac{4\tau^2}{M^2}\right)\right] + O\left(\frac{\log\tau^2}{\tau^4}\right), \\ D_d &\underset{\tau \gg M}{\approx} -\frac{2M^2}{3\tau^2}\left[1 + \log\left(\frac{4\tau^2}{M^2}\right)\right] + O\left(\frac{\log\tau^2}{\tau^4}\right). \end{aligned} \tag{33}$$

Changing the values of the masses leads to different numerical factors, but the overall structure remains always

$$D \sim \frac{c_1 + c_2 \log\tau^2}{\tau^2}. \tag{34}$$

This has implications for the pressure and shear distributions, which we will address in sec. 8.

7. Trace

From the definition of the EMT we can easily see that in DR (hence without ghost fields) the traces of the proton, quark and diquark EMTs are given by

$$\begin{aligned} T^{\mu}_{\Psi\,\mu} &= M\bar{\Psi}\Psi - \tfrac{1}{2}g\phi\left(\bar{\Psi}q + \bar{q}\Psi\right), \\ T^{\mu}_{q\,\mu} &= m\bar{q}q - \tfrac{1}{2}g\phi\left(\bar{\Psi}q + \bar{q}\Psi\right), \\ T^{\mu}_{d\,\mu} &= \partial_\mu\phi\partial^\mu\phi - (2-\epsilon)\left(\partial_\mu\phi\partial^\mu\phi - m_s^2\phi^2\right). \end{aligned} \tag{35}$$

So, as in QED, the anomaly emerges in DR from the bosonic sector

$$\text{Anomaly} = \epsilon\left(\partial_\mu\phi\partial^\mu\phi - m_s^2\phi^2\right). \tag{36}$$

The off-forward matrix elements of the relevant scalar operators are

$$\frac{\langle M\bar{\Psi}\Psi\rangle}{\bar{u}'u} = M\left(1 + \delta Z_M + \delta Z_\Psi\right), \tag{37}$$

$$\frac{\langle \frac{1}{2}g\phi\left(\bar{\Psi}q + \bar{q}\Psi\right)\rangle}{\bar{u}'u} = M\,\delta Z_M, \tag{38}$$

$$\begin{aligned} \frac{\langle m\bar{q}q\rangle}{\bar{u}'u} &= -a_s m\left(\frac{1}{\epsilon} + 1\right) + 2a_s m\int_0^1 dx\,\bar{x}\log\left(\frac{\Theta_0(x)}{\bar{\mu}^2}\right) \\ &\quad + a_s m\int_0^1 dx\left(\frac{(m+xM)^2 + \bar{x}^2\tau^2}{2\tau\Sigma_0(x,\bar{x}\tau)} + \frac{3\Sigma_0(x,\bar{x}\tau)}{2\tau}\right)\Lambda_0(x,\bar{x}\tau), \end{aligned} \tag{39}$$

$$\begin{aligned} \frac{\langle \partial_\mu\phi\partial^\mu\phi\rangle}{\bar{u}'u} &= -a_s\left(M + 2m\right)\left(\frac{1}{\epsilon} + 2\right) + 2a_s\int_0^1 dx\,\bar{x}\left[2(m+\bar{x}M) - xM\right]\log\left(\frac{\Theta_0(\bar{x})}{\bar{\mu}^2}\right) \\ &\quad + a_s\int_0^1 dx\left[\frac{(m+\bar{x}M)\left(M^2x^2 + \tau^2(x^2-1)\right)}{\tau\Sigma_0(\bar{x},\bar{x}\tau)} + \frac{\left[5(m+\bar{x}M) - 2xM\right]\Sigma_0(\bar{x},\bar{x}\tau)}{\tau}\right]\Lambda_0(\bar{x},\bar{x}\tau), \end{aligned} \tag{40}$$

$$\frac{\langle m_s^2\phi^2\rangle}{\bar{u}'u} = a_s m_s^2\int_0^1 dx\,\frac{m+\bar{x}M}{\tau\Sigma_0(\bar{x},\bar{x}\tau)}\,\Lambda_0(\bar{x},\bar{x}\tau). \tag{41}$$

These results are consistent with the expression for the trace of the general parametrization in Eq. (6)

$$\langle T^{\mu}_{i\,\mu}\rangle = M\,\bar{u}'u\left[A_i + 4\bar{C}_i + \frac{\Delta^2}{4M^2}\left(B_i - 3D_i\right) + 2\epsilon\left(\frac{\Delta^2}{4M^2}D_i - \bar{C}_i\right)\right]. \tag{42}$$

In particular, the matrix element of the trace anomaly reads

$$\langle p', s'|\epsilon\left(\partial_\mu\phi\partial^\mu\phi - m_s^2\phi^2\right)|p, s\rangle = -a_s\left(M + 2m\right)\bar{u}'u \tag{43}$$

and arises purely from the kinetic term. Since the tree-level matrix element of the EMT between proton states already gives the total proton mass, we expect in the forward limit $\tau \to 0$ that the anomaly is exactly compensated by the $\mathcal{O}(a_s)$ contribution to the classical EMT trace:

$$(T^{\mu}{}_{\mu})_{\text{class}} = \bar{S}\,\mathbb{M}S + \partial_\mu\phi\partial^\mu\phi - 2\left(\partial_\mu\phi\partial^\mu\phi - m_s^2\phi^2\right). \tag{44}$$

This is indeed what is found.

In PV regularization we find for the trace of the total EMT

$$T^{\mu}{}_{\mu} = \bar{S}\,\mathbb{M}S + \partial_\mu\phi\partial^\mu\phi - 2\left(\partial_\mu\phi\partial^\mu\phi - m_s^2\phi^2\right) - \partial_\mu c\partial^\mu c + 2\left(\partial_\mu c\partial^\mu c - M_{\text{PV}}^2 c^2\right). \tag{45}$$

In this case, the anomalous part comes from the ghost sector, even though it is not as transparent as DR. We cannot however isolate the anomalous operator in PV since the ghost sector is also needed to regulate the integrals in the physical sector.

8. Fourier transform

Fourier transforms of the gravitational form factors can be interpreted in terms of spatial distributions of energy, linear/angular momentum, and forces inside the target [14, 17–20]. Three- and two-dimensional Fourier transforms are respectively defined as

$$\hat{F}(r) = \int \frac{d^3\varDelta}{(2\pi)^3}\, e^{-i\boldsymbol{\varDelta}\cdot\boldsymbol{r}} F(-\boldsymbol{\varDelta}^2) = \int_0^\infty \frac{d\kappa}{2\pi^2}\, \kappa^2 F(-\kappa^2) j_0(\kappa r), \tag{46}$$

$$\tilde{F}(b_\perp) = \int \frac{d^2\varDelta_\perp}{(2\pi)^2}\, e^{-i\boldsymbol{\varDelta}_\perp\cdot\boldsymbol{b}_\perp} F(-\boldsymbol{\varDelta}_\perp^2) = \int_0^\infty \frac{d\kappa}{2\pi}\, \kappa F(-\kappa^2) J_0(\kappa b_\perp), \tag{47}$$

where j_0 and J_0 are the spherical and cylindrical Bessel functions of the first kind. The explicit expressions for the gravitational form factors are given in Appendix A. Any constant term in the gravitational form factors has singular Fourier transformation, contributing as $\delta^3(\boldsymbol{r})$ or $\delta^2(\boldsymbol{b}_\perp)$. We will discard any such contributions in the following discussion, since they emerge as pathological features of the perturbative nature of the presented results.

To study the physics in position space, we introduce the tangential and radial pressures $p_{t,r}$ in three dimensions and $\sigma_{t,r}$ in two dimensions. We also introduce the energy densities ε in three dimensions and ρ in two dimensions. The definitions in terms of the gravitational form factors are [19]

$$\begin{aligned}
p_t(r)/M &= \frac{1}{4M^2 r}\frac{d}{dr}\left(\rho\,\frac{d}{dr}\hat{D}(r)\right),\\
p_r(r)/M &= \frac{1}{2M^2 r}\frac{d}{dr}\hat{D}(r),\\
\varepsilon(r)/M &= \hat{A}(r) + \frac{1}{4M^2 r^2}\frac{d}{dr}\left[r^2\frac{d}{dr}\left(\hat{B}(r)-\hat{D}(r)\right)\right],\\
\sigma_t(b_\perp)/M &= \frac{1}{4M^2}\frac{d^2}{db_\perp^2}\tilde{D}(b_\perp),\\
\sigma_r(b_\perp)/M &= \frac{1}{4M^2 b_\perp}\frac{d}{db_\perp}\tilde{D}(b_\perp),\\
\rho(b_\perp)/M &= \tilde{A}(b_\perp) + \frac{1}{4M^2 b_\perp^2}\frac{d}{db_\perp}\left[b_\perp^2\frac{d}{db_\perp}\left(\tilde{B}(b_\perp)-\tilde{D}(b_\perp)\right)\right].
\end{aligned} \tag{48}$$

The $\bar{C}$ form factors being constant, their contributions have been discarded in the above expressions. We stress that the D form factors have non-singular Fourier transforms $\tilde{D}$ and $\hat{D}$, but their derivatives present singular behaviors. Since derivatives in the radial variable of order n are related to Fourier transforms of $\kappa^n D(-\kappa^2)$, it is trivial to conclude that the fall-off of $D(-\kappa^2)$ for large values of $\kappa = 2\tau$ given in Eq. (34) is not fast enough to guarantee the absence of singular contributions to pressure and shear in two and three dimensions. The singular contributions are however fundamental to ensure the von Laue condition for mechanical equilibrium [37]

$$\int_0^\infty dr\, r^2(p_r + 2p_t) = 0. \tag{49}$$

Interestingly, the combination of singular and regular contributions resemble the definition of +-distributions commonly used in QCD

$$\left[\frac{f(z)}{1-z}\right]_+ \equiv \frac{f(z)}{1-z} - \delta(1-z)\int_0^z dy\,\frac{f(y)}{1-y}. \tag{50}$$

Similar considerations apply to the derivatives of the B form factor.

While the perturbative approach ensures that Poincaré symmetry is preserved, which is the focus of this work, we cannot however consider that the one-loop results for the gravitational form factors provide a realistic picture of a bound state, and even less of the nucleon structure. For this reason, we refrain from interpreting in detail our results in position space.

9. Conclusions

In this work we studied in detail the symmetric energy-momentum tensor of the scalar diquark model to one-loop level in perturbation theory. Contrary to the light-front wave function overlap formalism, the perturbative approach allows us to maintain exact Poincaré symmetry throughout the calculations. We extracted the perturbative expressions of all the gravitational form factors using two different regularization methods, namely dimensional and Pauli-villars regularizations. We checked explicitly that including Lagrangian counterterms are sufficient to make the energy-momentum tensor finite, in agreement with general arguments given in the literature. We also showed that all the Poincaré constraints on the gravitational form factors are satisfied, and demonstrated the consistency of our results with expectations based on the trace anomaly. Finally, since Fourier transforms of the gravitational form factors can be used to define spatial distributions of energy, pressure and shear, we provided explicit expressions for the regular parts. Like in the QED case, it appeared however that some contributions to the gravitational form factors do not decrease sufficiently fast at large momentum transfer, which is a common pathological feature of the perturbative approach.

Appendix A. Explicit results for Fourier transforms

For the B form factors we find in two and three dimensions

$$\begin{pmatrix}\tilde{B}_q\\ \hat{B}_q\end{pmatrix} = \frac{2a_s M}{\pi}\int_0^1 dx\, x(m+xM)\begin{pmatrix}K_0^2(\zeta_q)\\ \frac{1}{r}K_0(2\zeta_q)\end{pmatrix}, \tag{A.1}$$

$$\begin{pmatrix}\tilde{B}_d\\ \hat{B}_d\end{pmatrix} = -\frac{2a_s M}{\pi}\int_0^1 dx\, \frac{x^2}{\bar{x}}(m+\bar{x}M)\begin{pmatrix}K_0^2(\zeta_d)\\ \frac{1}{r}K_0(2\zeta_d)\end{pmatrix}, \tag{A.2}$$

where we defined

$$\zeta_q = \frac{\sqrt{\Theta_0(x)}}{\bar{x}}\begin{pmatrix}b_\perp\\ r\end{pmatrix}, \qquad \zeta_d = \frac{\sqrt{\Theta_0(\bar{x})}}{\bar{x}}\begin{pmatrix}b_\perp\\ r\end{pmatrix}. \tag{A.3}$$

For the A form factors, we isolate and remove the constant contribution (in the momentum transfer) that leads to singular Fourier transforms

$$A_q^{\text{R,const}} = \frac{a_s}{6}\left(\mathfrak{s}^{\text{R}}-1\right) + a_s\mathfrak{f}_A^{\text{R}} - a_s\int_0^1 dx\, x\bar{x}\log\left(\frac{\Theta_0(x)}{\bar{\mu}^2}\right), \tag{A.4}$$

$$A_d^{\text{R,const}} = \frac{a_s}{3}\mathfrak{s}^{\text{R}} - 10\mathfrak{f}_A^{\text{R}} - 2a_s\int_0^1 dx\, x\bar{x}\log\left(\frac{\Theta_0(x)}{\bar{\mu}^2}\right). \tag{A.5}$$

Subtracting these constant terms, we find

$$\begin{pmatrix}\tilde{A}_q\\ \hat{A}_q\end{pmatrix} = \frac{a_s}{\pi}\int_0^1 dx\, \frac{x}{\bar{x}}\left[(m+xM)^2\begin{pmatrix}K_0^2(\zeta_q)\\ \frac{1}{r}K_0(2\zeta_q)\end{pmatrix} + \Theta_0(x)\begin{pmatrix}K_1^2(\zeta_q)\\ \frac{1}{r}K_2(2\zeta_q)\end{pmatrix}\right], \tag{A.6}$$

$$\begin{pmatrix}\tilde{A}_d\\ \hat{A}_d\end{pmatrix} = \frac{2a_s}{\pi}\int_0^1 dx\, \frac{x}{\bar{x}}\left[xM(m+\bar{x}M)\begin{pmatrix}K_0^2(\zeta_d)\\ \frac{1}{r}K_0(2\zeta_d)\end{pmatrix} + \Theta_0(\bar{x})\begin{pmatrix}K_1^2(\zeta_d)-K_0^2(\zeta_d)\\ \frac{1}{r}K_2(2\zeta_d)-\frac{1}{r}K_0(2\zeta_d)\end{pmatrix}\right]. \tag{A.7}$$

For the D form factors we find

$$\begin{pmatrix}\tilde{D}_q\\ \hat{D}_q\end{pmatrix} = \frac{4a_s M}{\pi}\int_0^1 dx\, \bar{x}(m+xM)\int_0^1 dz\, \frac{\sqrt{1-z^2}}{z}\begin{pmatrix}K_0\left(\frac{2}{z}\zeta_q\right)\\ \frac{1}{2r}\exp\left(-\frac{2}{z}\zeta_q\right)\end{pmatrix}, \tag{A.8}$$

$$\begin{pmatrix}\tilde{D}_d\\ \hat{D}_d\end{pmatrix} = -\frac{4a_s M}{\pi}\int_0^1 dx\, \frac{m+\bar{x}M}{\bar{x}}\int_0^1 dz\, \frac{1-\bar{x}^2(1-z^2)}{z\sqrt{1-z^2}}\begin{pmatrix}K_0\left(\frac{2}{z}\zeta_d\right)\\ \frac{1}{2r}\exp\left(-\frac{2}{z}\zeta_d\right)\end{pmatrix}. \tag{A.9}$$

The integral

$$F(y) = \int_0^1 dz\, \frac{\sqrt{1-z^2}}{z}\begin{pmatrix}K_0\left(\frac{y}{z}\right)\\ \exp\left(-\frac{y}{z}\right)\end{pmatrix} \tag{A.10}$$

is a solution of the differential equation

$$\left(1 - y\,\frac{d}{dy}\right)F(y) = \begin{pmatrix} \frac{1}{2}K_0^2\left(\frac{y}{2}\right) \\ K_0\left(y\right) \end{pmatrix}. \tag{A.11}$$

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