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AEC-NASA TECH BRIEF

Energy-Storage of a Prescribed Impedance

A mathematical method for deriving the minimum storage of impedance networks is presented (*1*). The problem of inferring the total average energy-storage of a passive, linear, electrical impedance, from its observed or specified terminal behavior alone, is discussed. This electromagnetic energy-storage is determined at different excitation frequencies through analysis of the network's terminal and reactance characteristics. In only a few cases is the energy-storage (for sinusoidal external excitation) uniquely determinable.

A general expression for the energy-storage is derived that involves, in addition to terminal properties, properties of a set of functions describing the separate dissipative processes. This expression is used for finding a new minimum-energy storage, for a lumped-element network, that all realizations of the impedance must equal or exceed.

Results show that the average energy-storage of a sinusoidally excited, generally linear, passive network is not determined by terminal behavior alone; that is, equivalent circuits in the usual terminology do not store the same energy for the same excitation. The class of networks related by Cauer's equivalence transformation do, however.

The energy-storage is uniquely determined by the terminal behavior whenever the system dissipates no energy, when it is an RL or RC network, or when the dependence of the terminal behavior on the individual dissipative processes is known. For reciprocal-element networks, a simple lower bound exists for the energy-storage at each frequency. This bound is attained at all frequencies by RL or RC networks.

The general expression found for energy-storage shows that for any linear, passive network there is a minimum possible energy-storage corresponding to a prescribed impedance. Furthermore this bound is attainable at all frequencies as a minimum-energy synthesis. The minimum-energy synthesis is a minimum-phase-shift Darlington synthesis, and normally nonreciprocal elements (gyrators) are required for its realization.

The methods used in the development are not intrinsically restricted to lumped-elements systems, so that, with perhaps some qualifications, the results could extend to distributed systems with nonrational $Z_0(p)$. Also, the results are not restricted to electrical impedances; for example, the same arguments apply for an acoustic impedance. An extension to multiport systems also may be possible. That nonreciprocal elements may sometimes be necessary, for the general case to attain the minimum energy, can be expected intuitively from relations between group time-delay and energy-storage.

Reference:

1. W. E. Smith, "The energy storage of a prescribed impedance" (Argonne National Laboratory, Nov. 1967).

Notes:

1. This information may interest designers or manufacturers of microwave-antenna systems.
2. Inquiries concerning this innovation may be directed to:

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(continued overleaf)

Patent status:

Inquiries concerning rights for commercial use of this innovation may be made to:

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