QUALITY AND RELIABILITY ENGINEERING INTERNATIONAL
*Qual. Reliab. Engng. Int.* 2005; **21**:387–411
Published online 27 January 2005 in Wiley InterScience (www.interscience.wiley.com). DOI: 10.1002/qre.668

**Research**

# *Engineering System Safety Analysis and Synthesis Using the Fuzzy Rule-based Evidential Reasoning Approach*

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***The main objective of this paper is to propose a framework for modelling, analysing and synthesizing system safety of engineering systems or projects on the basis of a generic rule-based inference methodology using the evidential reasoning (RIMER) approach. The framework is divided into two parts. The first one is for fuzzy rule-based safety estimation, referred to as a fuzzy rule-based evidential reasoning (FURBER) approach. The second one is for safety synthesis using the evidential reasoning approach. In the FURBER framework, parameters used to define the safety level, including failure rate, failure consequence severity and failure consequence probability are described using fuzzy linguistic variables; a fuzzy rule base designed on the basis of a belief structure is used to capture uncertainty and nonlinear relationships between these three parameters and the safety level; and the inference of the rule-based system is implemented using the evidential reasoning algorithm. Then the following steps involve synthesizing safety at higher levels of an engineering system to integrate all possible causes to a specific technical failure, or estimates made by a panel of experts. The synthesis is also based on the evidential reasoning approach. The final step describes the analysis and interpretation of the final synthesized safety of a system. The above framework has been applied to modelling system safety of an offshore and marine engineering system: the floating production storage offloading (FPSO) system. A series of case studies of collision risk between a FPSO and a shuttle tanker due to technical failure during a tandem offloading operation is used to illustrate the application of the proposed model. Copyright © 2005 John Wiley & Sons, Ltd.***

KEY WORDS: safety modelling; decision-making; fuzzy rule base; linguistic variable; evidential reasoning; design evaluation; maritime safety-based decision making

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Contract/grant sponsor: EPSRC; contract/grant numbers: GR/R30624 and GR/R32413; GR/S85498/01 and GR/S85504/01

*Received 29 October 2003*
*Revised 3 April 2004*

## 1. INTRODUCTION

The growing technical complexity of large engineering systems such as offshore platforms and support vessels, together with the intense public concern over their safety, has stimulated the research and development of novel safety analysis methods and safety assessment procedures.

Many typical safety assessment approaches such as the probabilistic risk assessment approach have been widely used, but may be difficult to use in situations where there is a lack of information and past experience, or in ill-defined situations in risk analysis[1]. In certain circumstances, probability theory can be a powerful tool. Under prevailing circumstances, only limited data are available on system failures, for which the statistical accuracy is poor, e.g. at initial design stages or for a system with a high level of innovation, and often data are inadequate or information imprecise when carrying out safety estimates for a novel system.

In engineering safety analysis, fuzziness is caused due to ill-defined concepts in observation, or the inaccuracy and poor reliability of instruments used to make observations. Incompleteness or ignorance is caused due to weak implication which occurs when an expert is unable to establish a strong correlation between premise and conclusion. This means that intrinsically vague information may coexist with conditions of 'lack of specificity' originating from evidence not strong enough to completely support a hypothesis but only with degrees of belief or credibility[2].

In addition, it is difficult to generate a mathematical model to represent and describe the safety discipline of an engineering system (e.g. a maritime engineering system), as safety is a multiple-level and multiple-variable optimization problem[3]. There are many instances where causes of an accident involve operational procedures, human errors and decisions taken by designers and management. In other words, the safety of a system is affected by various factors such as design, manufacturing, installation, commissioning, operations and maintenance. The safety of a structure is often determined by all the associated failure events of each individual component that makes up the structure. A component usually has several failure events. The problem may then arise as to how the uncertain evaluations of safety analyses for all the failure events of a component may be synthesized in a rational way so as to attain an evaluation of the safety of the component. The problem may be ultimately generalized to estimate the safety of a system with a hierarchy of components.

The aim of our work is to establish a framework that provides a basis for safety analysis and synthesis in engineering systems, in particular to deal with information that may be un-quantifiable due to its nature and that may be imprecise, ill-defined, and incomplete, for which traditional quantitative approaches (e.g. statistical approaches) cannot provide an adequate answer.

A more realistic approach to express fuzziness is to use linguistic assessments instead of numerical values. Such an approach allows the representation of information in a more natural and adequate form if it is difficult to express the information with precision. Fuzzy logic[4] provides a systematic way to represent linguistic variables in a natural decision-making procedure. It does not require an expert to provide a precise point at which a risk factor exists. So it can be used as a powerful tool complementary to traditional methods to deal with imprecise information, especially linguistic information[5,6] which is commonly used to represent risk factors in risk analysis[3,7–17].

Dempster–Shafer (DS) theory of evidence[18,19] based on the concept of *belief function* is well suited to modelling subjective credibility induced by partial evidence[20]. DS theory enlarges the scope of traditional probability theory, describes and handles uncertainties using the concept of the degrees of belief, which can model incompleteness and ignorance explicitly. It also provides appropriate methods for computing belief functions for combination of evidence[21]. The DS theory has been particularly useful in domains in which a hierarchical structure can be imposed on the hypotheses so that groups of hypotheses form classes in the hierarchy[22]. In addition, the DS theory also shows great potential in multiple attribute decision analysis (MADA) under uncertainty, where the rigorous yet pragmatic evidential reasoning (ER) approach for MADA under uncertainty has been developed on the basis of a distributed assessment framework and the evidence combination rule of the DS theory[23–28].

Fuzzy set theory is actually well suited to dealing with fuzziness and DS theory of evidence provides an ideal framework for handling incompleteness. It seems reasonable to extend the fuzzy logic framework to cover credibility uncertainty as well. The benefit of combining fuzzy logic and belief models may become substantial when a lack of specificity in data is prevalent[29]. Several researchers have investigated the relationships between fuzzy sets and DS theory and suggested different ways of integrating them[30–35]. A belief rule-base inference methodology using the evidential reasoning approach—RIMER has recently been proposed[36] on the ER approach, decision theory and fuzzy set theory.

In this paper, we propose a framework for modelling safety of engineering systems based on fuzzy logic and the ER approach. The framework is divided into two parts.

(1) Rule-based safety estimation, referred to as a fuzzy rule-based evidential reasoning (FURBER) approach, which is based on the RIMER approach. In this framework, safety-related parameters are described using fuzzy linguistic variables, and a fuzzy rule base with a belief structure, i.e. fuzzy rules with belief degrees for all possible safety output terms in the consequent, is used to capture uncertain causal relationships between these parameters and the safety level. Moreover, the antecedent of each IF–THEN rule is considered as an overall attribute, called a global attribute, which is assessed to an output term in the consequent of a rule with a degree of belief. Actual input can be transformed into a distributed representation for a linguistic term of an individual antecedent attribute. Finally, the inference of the rule base is implemented using the ER algorithm, where a global activation degree is used as the weight of a global attribute.
(2) Safety synthesis using the ER approach. Based on (1), in this framework, the modelling framework of multi-attributes or multi-experts or a combination of both based on the evidential reasoning approach is used to deal with problems having a hierarchy with uncertainty, i.e. safety synthesis, and the final step describes the calculation of the overall risk level ranking index. The identified potential causes are ranked on the basis of their ranking index values.

A case study of collision risk between a floating production storage and offloading (FPSO) system and a shuttle tanker due to technical failure during tandem offloading operations is used to illustrate the application of the proposed approach.

## 2. A FRAMEWORK FOR MODELLING SYSTEM SAFETY

A generic framework for a modelling system safety estimate using the FURBER approach and for safety synthesis using the ER approaches is depicted in Figure A1 in Appendix A. The issue of knowledge acquisition and representation for system safety modelling is outlined in Section 2.1. The framework for modelling system safety for risk analysis consists of five major steps, which include all the necessary steps required for safety analysis at the bottom level of a hierarchical system (i.e. each cause to technical failure) using the FURBER approach, which is based on RIMER proposed in Yang *et al.*[36]. The steps used are outlined in Section 2.2. In addition, an ER approach is used in the later stage of the framework to deal with safety synthesis at higher levels of the engineering system with complexity involving multi-experts, or multi-attributes, or a combination of both (this is to integrate all the possible causes to a specific technical failure, or estimates made by a panel of experts), which are outlined in Section 2.3; the ranking and interpretation of the final safety synthesis of a system are also given in Section 2.3.

### 2.1. *Knowledge acquisition and representation*

This component consists of the following steps for knowledge acquisition and representation.

#### 2.1.1. *Identification of causes/factors*

In this step, all anticipated causes/factors to technical failures of an engineering system are identified. This can be done by a panel of experts during a brainstorming session at the early conceptual design stages of the system.

 

*2.1.2. Identify and characterize fuzzy input and output variables*

The three fundamental parameters used to assess the safety level of an engineering system on a subjective basis are ***failure rate*** **(FR)**, ***consequence severity*** **(CS)** and ***failure consequence probability*** **(FCP)**. Subjective assessments (using linguistic variables instead of ultimate numbers in probabilistic terms) are more appropriate for analysis using these three parameters as they are always associated with uncertainty, especially for a novel system with a high level of innovation. These linguistic assessments can become the criteria for measuring attributes of objects, in this case, safety levels.

The typical linguistic variables used to describe **FR**, **CS** and **FCP** of a particular element may be defined and characterized as follows[37].

**FR** describes failure frequencies in a certain period, which directly represents the number of failures anticipated during the design life span of a particular system or an item. To estimate **FR**, one may choose to use such linguistic terms as '*very low* (VL)', '*low* (Lo)', '*reasonably low* (RLo)', '*average* (A)', '*reasonably frequent* (RF)', '*frequent* (F)', and '*highly frequent* (HF)'. Table AI in Appendix A describes the possible range of the frequencies of failure occurrence and defines the linguistic terms of **FR**. In that table, some numerical values for **FR** are given. Such values may vary with different engineering systems. If such numerical values are not available at all, then the modelling of **FR**s can be carried out using subjective judgements.

**CS** describes the magnitude of possible consequences, which is ranked according to the severity of failure effects. One may choose to use such linguistic terms as '*negligible* (N)', '*marginal* (Ma)', '*moderate* (Mo)', '*critical* (Cr)' and '*catastrophic* (Ca)'. Table AII in Appendix A shows the possible criteria used to define the linguistics terms for describing and ranking the **CS** of failure effects.

**FCP** defines the probability that consequences happened gives the occurrence of the event. One may choose to use such linguistic terms as '*highly unlikely* (HU)', '*unlikely* (U)', '*reasonably unlikely* (RU)', '*likely* (Li)', '*reasonably likely* (RLi)', and '*definite* (D)'. Table AIII in Appendix A describes the **FCP**.

The following step is to select the types of fuzzy membership functions used to delineate each input variable, and provide interpretation for each fuzzy set of each variable. Note that fuzzy membership functions are not compulsory to be constructed in our approach. Due to lack of information, it may be difficult to get fuzzy membership functions. The belief distribution assessment scheme proposed in Yang *et al.*[36] provides other alternative ways for inference while no membership function is available.

It is possible to have some flexibility in the definition of membership functions to suit different situations. Fuzzy membership functions are generated using linguistic categories identified in knowledge acquisition and consist of a set of overlapping curves. The application of categorical judgments has been quite positive in several practical situations[38]. It is also common and convenient for safety analysts to use categories to articulate safety information. Figure A2 in Appendix A shows the fuzzy **FR** set definition. The fuzzy **CS** set definition is shown in Figure A3 in Appendix A, and the fuzzy **FCP** set definition is shown in Figure A4 in Appendix A. They are the triangular membership function and trapezoidal membership function. Both of these memberships are commonly used to describe risk in safety assessment[14].

It is worth noting that qualitative parameters, e.g. **CS** and ***safety estimates***, are associated with concepts—a subjective (if not quite arbitrary) scale against which the range of the parameters is mapped. A subjective scale is called a *psychometric scale*, since it comes from the designer's mind[39]. The range of the psychometric scale is determined by the level of granularity and fine detail in the model. The fuzzy **CS** set definition shown in Figure A3 in Appendix A is just an example of this kind of psychometric scale (the domain runs from 0 to 10 indicating the degree to which the concept **CS** is *negligible*, *marginal*, ..., or *catastrophic*).

The above definition and identification are used in our case study in Section 3. It is noted that they can be modified according to different requirements in codes and standards (e.g. safety/risk guidelines, regulations, laws, etc.) and different aspects of engineering systems such as fire, explosions, structure, safety system, etc.

***Safety estimate*** is the only output fuzzy variable used in this study to produce safety evaluation for a particular cause to technical failure. This variable is also described linguistically, which is described and determined by the above parameters. In safety assessment, it is common to express a safety level by degrees to which it belongs to such linguistic variables as '*Poor*', '*Fair*', '*Average*', and '*Good*' which are referred to as safety expressions.

### 2.1.3. Construct a fuzzy rule base with the belief structure

Fuzzy logic systems are knowledge-based or rule-based systems constructed using human knowledge in the form of fuzzy IF–THEN rules[40]. An important contribution of fuzzy system theory is that it provides a systematic procedure for transforming a knowledge base into nonlinear mapping. For example, the following is a fuzzy IF–THEN rule for safety analysis:

*IF* **FR** of a hazard is *frequent* AND **CS** is *catastrophic* AND **FCP** is *likely*, THEN ***safety estimate*** is *Poor.*

In view of the increasing complexity of many knowledge-based systems, the knowledge representation power of fuzzy rule-based systems will be severely limited if only fuzziness is used to represent uncertain knowledge. As mentioned in the first section, there is another kind of uncertainty caused because an expert is unable to establish a strong correlation between premise and conclusion. In other words, evidence available is not sufficient or experts do not believe 100% in a hypothesis but only to a degree of belief. For example, we may only get the following rule with certain degrees of belief:

$R_1$: *IF* **FR** *is* <u>*frequent*</u> AND **CS** *is* <u>*critical*</u> AND **FCP** *is* <u>*unlikely*</u> THEN ***safety estimate*** *is* <u>*Fair*</u> with a belief degree of 0.7.

More generally, we may have fuzzy rules with belief degrees for multiple possible consequent terms, for example,

$R_k$: *IF* **FR** *is* <u>*frequent*</u> AND **CS** *is* <u>*critical*</u> AND **FCP** *is* <u>*unlikely*</u> THEN ***safety estimate*** *is* {(*Good*, 0), (*Average*, 0), (*Fair*, 0.7), (*Poor*, 0.3)}

where {(*Good*, 0), (*Average*, 0), (*Fair*, 0.7), (*Poor*, 0.3)} is a belief distribution representation for safety consequent, indicating that we are 70% sure that the safety level is *Fair*, and 30% sure that the safety level is *Poor*.

In order to model more general and complex decision-making problems under uncertainty, other important information such as weighting factors may also need to be considered, such as the relative weight of a rule (rule weight) used to represent the relative importance of the rule's contribution to reach the final conclusion, and the relative weight of an antecedent attribute (attribute weight).

Considering the above parameters, a normal fuzzy rule can be extended for safety estimates by assigning the rule a degree of belief, a rule weight, and an attribute weight.

In general, assume that the three antecedent attributes, $U_1 = \mathbf{FR}$, $U_2 = \mathbf{CS}$ and $U_3 = \mathbf{FCP}$ can be described by $J_i$ linguistic terms $\{A_{ij},\ j = 1, \ldots, J_i\}$, $i = 1, 2, 3$, respectively, and one consequent variable ***safety estimate*** can be described by $N$ linguistic terms, i.e. $D_1, D_2, \ldots, D_N$. Let $A_i^k$ be a linguistic term corresponding to the $i$th attribute in the $k$th rule, with $i = 1, 2, 3$. Thus the $k$th rule in a rule base can be written as follows:

$$\begin{aligned} R_k:\quad & \text{IF } \mathbf{FR} \text{ is } A_1^k \text{ AND } \mathbf{CS} \text{ is } A_2^k \text{ AND } \mathbf{FCP} \text{ is } A_3^k \text{ THEN } \textit{\textbf{safety estimate}} \text{ is} \\ & \{(D_1, \bar{\beta}_{1k}), (D_2, \bar{\beta}_{2k}), \ldots, (D_N, \bar{\beta}_{Nk})\}, \left(\sum_{i=1}^{N} \bar{\beta}_{ik} \le 1\right), \\ & \text{with the rule weight } \theta_k \text{, and the attribute weights } \delta_1, \delta_2, \delta_3 \end{aligned} \tag{1}$$

where $\bar{\beta}_{ik}$ ($i \in \{1, \ldots, N\}$; $k \in \{1, \ldots, L\}$, with $L$ being the total number of the rules in the rule base) is a belief degree measuring the subjective uncertainty of the consequent '***safety estimate*** is $D_i$' drawn due to the antecedent '**FR** is $A_1^k$ AND **CS** is $A_2^k$ AND **FCP** is $A_3^k$' in the $k$th rule. $\{A_1^k, A_2^k, A_3^k\}$ is called the *packet of antecedents* in the $k$th rule, for convenience, denoted as $A^k$, $k \in \{1, \ldots, L\}$. If $\sum_{i=1}^{N} \bar{\beta}_{ik} = 1$, the output assessment or the $k$th rule is said to be complete; if $\sum_{i=1}^{N} \bar{\beta}_{ik} = 1$ for all $k = 1, \ldots, L$, then the rule base is a complete rule base; otherwise, it is incomplete. Note that $\left(\sum_{i=1}^{N} \bar{\beta}_{ik} = 0\right)$ denotes total ignorance about the output given the input. The rule-base with the rules given in form (1) is referred to as *a fuzzy rule-base with the belief structure*. Note that a common fuzzy rule is the special cases of rule (1) with $\{\bar{\beta}_{1k}, \bar{\beta}_{2k}, \ldots, \bar{\beta}_{Nk}\}$ being given special values. In fact, if we assign $\bar{\beta}_{ik} = 1$, $\bar{\beta}_{jk} = 0$, $j \neq i$, $j = 1, \ldots, N$ in rule (1), we will get a normal fuzzy rule.

Note that parameters such as the belief degree distribution of a rule, rule weight, and attribute weight are usually assigned at the knowledge acquisition phase before a fuzzy rule base with the belief structure is established.

### 2.2. *Fuzzy rule-base inference mechanism based on the ER approach*

Once a rule base is established, the knowledge contained can be used to perform inference for given input. The inference procedure is basically composed of the following five steps.

#### 2.2.1. *Input transformation*

This is to transform the input into the distributed representation of linguistic values in antecedents using belief degrees. In general, we may consider a linguistic term in the antecedent as an evaluation grade, the input for an antecedent attribute $U_i$ can be assessed to a distribution representation of the linguistic terms using belief degrees[23] as follows:

$$S(U_i) = \{(A_{ij}, \alpha_{ij});\ j = 1, \ldots, J_i\}, \quad i = 1, 2, 3 \tag{2}$$

where $A_{ij}$ ($j \in \{1, \ldots, J_i\}$) is the $j$th linguistic term of the $i$th attribute, $\alpha_{ij}$ the likelihood to which the input for $U_i$ belongs to the linguistic term $A_{ij}$ with $\alpha_{ij} \geq 0$ and $\sum_j^{J_i} \alpha_{ij} \leq 1$ ($i = 1, 2, 3$), referred to as *the individual matching degree*. $\alpha_{ij}$ in Equation (2) could be generated using various ways depending on the nature of an antecedent attribute and the available data, as described in the following three cases:

(1) The matching function method while the input is in numerical form and the linguistic value is characterized using fuzzy membership functions (suitable for both quantitative and qualitative attributes).
(2) Rule-based or utility-based transformation methods while the input is in numerical forms but the fuzzy membership function is not available (only suitable for the quantitative attribute)[26,36]. The basic idea is that numerical data can be expressed as belief distributions using equivalence transformation techniques.
(3) The subjective assessment method (for quantitative and qualitative attributes). In this case the subjective judgments $\alpha_{ij}$ in Equation (2) can be assessed based on the historical data, statistical distributions or expert experience. This subjective assessment can be taken as an alternative solution due to lack of information, e.g. when neither the membership function of each linguistic term nor numerical forms of the input are available at all, and is especially useful for qualitative attribute assessment, which sometimes is totally subjective. In assessment of qualitative parameter **CS**, for example, an expert may provide the following assessment: 30% sure that **CS** is at a moderate level and 70% sure that it is critical.

For more details about the above three cases, we refer to Yang *et al.*[36]. Here we only consider case (1). For the purpose of safety modelling, it is assumed that each input parameter may be fed to the proposed safety model in any one of the following forms based on history data and expert experiences[37].

- A single deterministic value with 100% certainty.
- A closed interval defined by an equally likely range.
- A triangular distribution defined by a most likely value, with lower and upper least likely values.
- A trapezoidal distribution defined by a most likely range, with lower and upper least likely values.

Corresponding to the rule base (1), the general input form corresponding to the antecedent attribute in the $k$th rule is given as follows:

$$(A_1^*, \varepsilon_1) \text{ AND } (A_2^*, \varepsilon_2) \text{ AND } (A_3^*, \varepsilon_3) \tag{3}$$

where $\varepsilon_i$ ($\in [0, 1]$) expresses the degree of belief assigned by an expert to the association of $A_i^*$ ($i = 1, \ldots, 3$), which reflects the uncertainty of the input data.

Finally, $\alpha_{ij}$ in Equation (2) could be formulated in the following way:

$$\alpha_{ij} = \frac{\tau(A_i^*, A_{ij}) \cdot \varepsilon_i}{\sum_{j=1}^{J_i} [\tau(A_i^*, A_{ij})]}, \quad i = 1, 2, 3;\ j = 1, \ldots, J_i \tag{4}$$

where $(A_i^*, \varepsilon_i)$ is the actual input corresponding to the $i$th antecedent, $\tau$ is a matching function, $\tau(A_i^*, A_{ij}) = \tau_{ij}$ is a matching degree to which $A_i^*$ belongs to $A_{ij}$. One possible matching function $\tau$ is given as follows, as used in our case study in Section 3:

$$\tau(A_i^*, A_{ij}) = \max_x[\min(A_i^*(x), A_{ij}(x))] \tag{5}$$

$\tau$ could be generated using other matching functions[41].

*2.2.2. Activation weight for a packet antecedent*

Considering an input given by Equation (3) corresponding to the $k$th rule defined as in (1),

$$\textbf{FR} \text{ is } (A_1^k, \alpha_1^k) \text{ AND } \textbf{CS} \text{ is } (A_1^k, \alpha_2^k) \text{ AND } \textbf{FCP} \text{ is } (A_1^k, \alpha_3^k) \tag{6}$$

where $\alpha_i^k$ is the individual matching belief degree that the input belongs to $A_i^k$ of the individual antecedent $U_i$ appearing in the $k$th rule.

The global matching weight $w_k$ of the packet antecedent $A^k$ in the $k$th rule is generated by weighting and normalizing the $\alpha_k$ given by Equation (6) as follows:

$$w_k = (\theta_k \cdot \alpha_k) \Big/ \left( \sum_{j=1}^{L} \theta_j \alpha_j \right) \tag{7}$$

where $\theta_k$ $(\in \mathbf{R}^+,\ k = 1, \ldots, L)$ is the relative weight of the $k$th rule, $\alpha_k = \prod_{i=1}^{3} (\alpha_i^k)^{\bar{\delta}_i}$, $\bar{\delta}_i = \delta_i/(\max_{i=1,2,3}\{\delta_i\})$, so $\bar{\delta}_i \in [0, 1]$, here $\delta_i$ $(\in \mathbf{R}^+,\ i = 1, 2, 3)$ is the weight of the $i$th antecedent attribute, and $L$ is the number of rules in the rule base. Here $\theta_k$ and $\delta_i$ can be any value in $\mathbf{R}^+$ depending on the application context because finally $w_k \in [0, 1]$ by Equation (7). Moreover, note that 'AND' connective is used for three antecedents in a rule. In other words, the consequent of a rule is not believed to be true unless all the antecedents of the rule are activated. In such cases, the simple multiplicative aggregation function is used here to calculate $\alpha_k$. Note that $0 \le w_k \le 1$ $(k = 1, \ldots, L)$ and $\sum_{i=1}^{L} w_i = 1$.

*2.2.3. Update the degree of belief in the consequent of a rule based on the actual input*

An incomplete input for an attribute will lead to an incomplete output in each of the rules in which the attribute is used. In the inference procedure, such incompleteness should be considered. The original belief degree in the $i$th consequent term of the $k$th rule in (1) is updated based on the actual input information as follows:

$$\beta_{ik} = \bar{\beta}_{ik} * \left[ \sum_{t=1}^{3} \tau(t, k) * \sum_{j=1}^{J_t} \alpha t_{ij} \right] \Big/ \left[ \sum_{t=1}^{3} \tau(t, k) \right] \tag{8}$$

where

$$\tau(t, k) = \begin{cases} 1 & \text{if } U_t \text{ is used in defining } R_k \\ 0 & \text{otherwise} \end{cases} \qquad (t = 1, 2, 3)$$

$\alpha_{tj}$ is given in Equation (2) with $\alpha_{tj} \ge 0$ and $\sum_j^{J_t} \alpha_{tj} \le 1$. $\bar{\beta}_{ik}$ is given in (1) with $0 \le \sum_{i=1}^{N} \bar{\beta}_{ik} \le 1$. Note that $0 \le \sum_{i=1}^{N} \beta_{ik} \le 1$ for all $k$ and $1 - \sum_{i=1}^{N} \beta_{ik}$ denotes both the ignorance incurred in establishing $R_k$ and the incompleteness that may exist in the input information.

*2.2.4. Rule expression matrix for a fuzzy rule base with belief structure*

Suppose a fuzzy rule base with the belief structure is given by $R = \{R_1, \ldots, R_L\}$. The $k$th rule in (1) can be represented as follows:

$$R_k: \text{ IF } U \text{ is } A^k \text{ THEN } \textbf{\textit{safety estimate}} \text{ is } D \text{ with belief degree } \beta_k \tag{9}$$

Table I. Rule expression matrix for a fuzzy rule base with the belief structure

| Input | Output | | | | | |
|---|---|---|---|---|---|---|
| | $D_1$ | $D_2$ | $\dots$ | $D_i$ | $\dots$ | $D_N$ |
| $A^1\ (w_1)$ | $\beta_{11}$ | $\beta_{21}$ | $\dots$ | $\beta_{i1}$ | $\dots$ | $\beta_{N1}$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\dots$ | $\vdots$ | $\dots$ | $\vdots$ |
| $A^k\ (w_k)$ | $\beta_{1k}$ | $\beta_{2k}$ | $\dots$ | $\beta_{ik}$ | $\dots$ | $\beta_{Nk}$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\dots$ | $\vdots$ | $\dots$ | $\vdots$ |
| $A^L\ (w_L)$ | $\beta_{1L}$ | $\beta_{2L}$ | $\dots$ | $\beta_{iL}$ | $\dots$ | $\beta_{NL}$ |

where $U$ represents the antecedent attribute vector (**FR**, **CS**, **FCP**), $A^k$ the packet antecedents $\{A_1^k, A_2^k, A_3^k\}$, $D$ the consequent vector $(D_1, \dots, D_N)$, $\beta_k$ the vector of the belief degrees $(\beta_{1k}, \dots, \beta_{Nk})$ and $k \in \{1, \dots, L\}$. Each fuzzy rule with belief structure can be explained in the following way.

The packet antecedent $A^k$ of an IF–THEN rule can be considered as a global attribute, which is considered as being assessed to a linguistic term $D_i$ (the $i$th possible consequent term in the $k$th rule) with a belief degree of $\beta_{ik}$ ($i \in \{1, \dots, N\}$). This assessment can be represented by

$$S(A^k) = \{(D_i, \beta_{ik});\ i = 1, \dots, N\} \tag{10}$$

which is obviously a distributed assessment and is referred to as *a belief structure*, where $\beta_{ik}$ measures the degree to which $D_i$ is the consequent if the input activates the antecedent $A^k$ in the $k$th rule, which is given using (8) with $0 \le \sum_{i=1}^{N} \beta_{ik} \le 1$ for all $k$. Here $i = 1, \dots, N$, $k = 1, \dots, L$. $L$ is the number of rules in the rule base and $N$ is the number of the possible consequent terms in the $k$th rule.

A fuzzy rule base with the belief structure established using the rules given in Equation (10) can be summarized using the rule expression matrix shown in Table I. In the matrix, $w_k$ is the global actuation weight of $A^k$, which measures the degree to which the $k$th rule is weighted and activated.

In this rule-base representation framework, rules represent functional mappings for imprecise information with uncertainty. Belief degrees together with fuzzy linguistic variables provide a flexible approach to represent both uncertain evidence and uncertain knowledge. Using such uncertain rules, a more informative representation of knowledge becomes possible. The uncertainties are considered and handled in the following inference procedure, which is implemented to combine all rules to generate the final belief degrees of safety level for $D_1, \dots, D_N$ using the rule expression matrix.

### 2.2.5. *Rule combination using the ER approach*

Having represented each rule using the rule expression matrix, the ER approach[26–28] can be used to combine rules and generate final conclusions, which can be directly implemented as follows. First, transform the degrees of belief $\beta_{jk}$ for all $j = 1, \dots, N$, $k = 1, \dots, L$ into basic probability masses using the following equations[26–28]:

$$m_{j,k} = w_k \beta_{j,k}, \qquad j = 1, \dots, N$$

$$m_{D,k} = 1 - \sum_{j=1}^{N} m_{j,k} = 1 - w_k \sum_{j}^{N} \beta_{j,k}$$

$$\bar{m}_{D,k} = 1 - w_k \quad \text{and} \quad \tilde{m}_{D,k} = w_k \left(1 - \sum_{j=1}^{N} \beta_{j,k}\right)$$

For all $k = 1, \dots, L$, with $m_{D,k} = \bar{m}_{D,k} + \tilde{m}_{D,k}$ for all $k = 1, \dots, L$ and $\sum_{j}^{L} w_j = 1$. The probability mass assigned to the consequent $D$, which is unassigned to any individual output term $D_j$, is split into two parts, one caused by the relative importance of the $k$th packet antecedent $A^k$ or $\bar{m}_{D,k}$, the other by the incompleteness of the $k$th packet antecedent $A^k$ or $\tilde{m}_{D,k}$.

Then, aggregate all the packet antecedents of the $L$ rules to generate the combined degree of belief in each possible consequent term $D_j$ in $D$. Suppose $m_{j,I(k)}$ is the combined degree of belief in $D_j$ by aggregating the first $k$ packet antecedents $(A^1, \ldots, A^k)$ and $m_{D,I(k)}$ is the remaining degree of belief unassigned to any output term. Let $m_{j,I(1)} = m_{j,1}$ and $m_{D,I(1)} = m_{D,1}$. Then the overall combined degree of belief $\beta_j$ in $D_j$ is generated as follows:

$$
\begin{aligned}
\{D_j\}:\ & m_{j,I(k+1)} = K_{I(k+1)}[m_{j,I(k)}m_{j,k+1} + m_{j,I(k)}m_{D,k+1} + m_{D,I(k)}m_{j,k+1}] \\
& m_{D,I(k)} = \bar{m}_{D,I(k)} + \tilde{m}_{D,I(k)}, \quad k = 1, \ldots, L \\
\{D\}:\ & \tilde{m}_{D,I(k+1)} = K_{I(k+1)}[\tilde{m}_{D,I(k)}\tilde{m}_{D,k+1} + \tilde{m}_{D,I(k)}\bar{m}_{D,k+1} + \bar{m}_{D,I(k)}\tilde{m}_{D,k+1}] \\
\{D\}:\ & \bar{m}_{D,I(k+1)} = K_{I(k+1)}[\bar{m}_{D,I(k)}\bar{m}_{D,k+1}] \\
& K_{I(k+1)} = \left[1 - \sum_{j=1}^{N}\sum_{\substack{t=1\\t\neq j}}^{N} m_{j,I(k)}m_{t,k+1}\right]^{-1}, \quad k = 1, \ldots, L-1 \\
\{D_n\}:\ & \beta_j = (m_{j,I(L)})/(1 - \bar{m}_{D,I(L)}), \quad j = 1, \ldots, N \\
\{D\}:\ & \beta_D = (\tilde{m}_{D,I(L)})/(1 - \bar{m}_{D,I(L)})
\end{aligned}
$$

$\beta_D$ represents the remaining belief degrees unassigned to any $D_j$. It has been proved that $\sum_{j=1}^{N} \beta_j + \beta_D = 1$ (see Yang and Xu[27]).

The final conclusion generated by aggregating the $L$ rules, which are activated by the actual input $A^*$ for $U = (\mathbf{FR}, \mathbf{CS}, \mathbf{FCP})$, can be represented as follows:

$$S(A^*) = \{(D_j, \beta_j),\ j = 1, \ldots, N\} \tag{11}$$

The inference procedure is based on the FURBER approach. The logic behind the approach is that if the output in the $k$th rule is $D_i$, then the overall output must be $D_i$ to a certain degree. The degree is measured by both the degree to which the $k$th rule is important to the overall output and the degree to which the antecedents of the $k$th rule are activated by the actual input. The final result is still a belief distribution of safety expressions, which gives a panoramic view about the safety level for a given input.

### 2.3. *Safety synthesis framework using the ER approach*

#### 2.3.1. *Multi-attribute and multi-expert safety synthesis*

It is worth noting that in this section, in order to achieve a more effective and logical evaluation process, it is necessary to break down a complex system into simpler sub-systems. The hierarchical framework of attributes and experts is used to guide the overall evaluation of multi-attributes or multi-experts or a combination of multi-attribute–multi-expert decision problems. The first four components of the framework mainly focus on safety assessment of a single cause of a technical failure carried out by an expert. This component is concerned with safety synthesis of a system at various levels, such as:

(a) the synthesis of safety estimates of a specific cause of a technical failure done by a panel of experts; or
(b) the synthesis of safety estimates of various causes of a technical failure done by an expert; or
(c) a combination of the above two forms, i.e. a multi-attribute–multi-expert safety synthesis.

We consider two ways to generate final safety assessment, i.e. multi-expert–multi-attribute ((a) and (c)) and multi-attribute–multi-expert ((b) and (c)). In the former case, a panel of experts are the evaluators, not the final decision-makers; in the latter case, a panel of experts are the final decision-makers. We only discuss (a) and (c) in detail; (b) and (c) can be treated in a similar way. Normally the results generated for the two ways would be different.

 

We consider several particular causes $C = \{C_1, \ldots, C_d\}$ of a technical failure. For each particular cause $C_j$ ($j \in \{1, \ldots, d\}$), the description or the input of its antecedent attribute (**FR**, **CS**, **FCP**) in the safety rule can be derived from different sources or evaluated by different experts. Assume that there are several sources or experts $e_i$ ($i = 1, \ldots, K$). Without lost of generality, suppose input comes from different experts.

Note that it is likely for selected experts to be of different importance, so the weights of experts need to be taken into account. The assessment of weight for each expert is an important decision for the analyst to make in view of the safety of a system under scrutiny. Each expert is assigned a weight to indicate the relative importance of his or her judgment in contributing towards the overall safety evaluation. The analyst must decide which experts have higher authority and then assign weights accordingly. Weights also need to be taken into account for different sources, while the weights of a source reflect its reliability.

We assume that different experts/different sources have different reliability weights, $w_{Ei}$ ($i = 1, \ldots, K$). Suppose $A_{ei} = (A_{ei,1}, A_{ei,2}, A_{ei,3})$ is an input vector derived from $e_i$ for an antecedent attribute. For each input, we may get a corresponding safety estimate $D_{ei}$ using the above FURBER approach, which can be formulated as follows:

$$
\begin{aligned}
&\text{IF } U \text{ is } A_{e1} \text{ THEN } D_{e1} \text{ is } \{(D_1, \eta_{11}), (D_2, \eta_{21}), \ldots, (D_N, \eta_{N1})\} \\
&\text{IF } U \text{ is } A_{e2} \text{ THEN } D_{e2} \text{ is } \{(D_1, \eta_{12}), (D_2, \eta_{22}), \ldots, (D_N, \eta_{N2})\} \\
&\ldots \\
&\text{IF } U \text{ is } A_{et} \text{ THEN } D_{et} \text{ is } \{(D_1, \eta_{1t}), (D_2, \eta_{2t}), \ldots, (D_N, \eta_{Nt})\} \\
&\ldots \\
&\text{IF } U \text{ is } A_{eK} \text{ THEN } D_{eK} \text{ is } \{(D_1, \eta_{1K}), (D_2, \eta_{2K}), \ldots, (D_N, \eta_{NK})\}
\end{aligned}
$$

where $\{(D_1, \eta_{1i}), (D_2, \eta_{2i}), \ldots, (D_N, \eta_{Ni})\}$ resulting from Equation (11) are obtained using the FURBER approach. Then the actual safety estimates $D_c$ of a specific cause $C_i$ ($i = 1, \ldots, d$) can be generated by synthesizing multi-expert assessments, i.e. by aggregating $\{D_{e1}, D_{e2}, \ldots, D_{eK}\}$ using the ER algorithm, which is represented as

$$S(C_j) = \{(D_i, \eta_i^j);\ i = 1, \ldots, N\}, \quad j = 1, \ldots, d \tag{12}$$

When there are several particular causes of a technical failure, the final safety estimate of a technical failure $D$ is the synthesis of all the assessments $S(C_j)$ ($j = 1, \ldots, d$) for each particular cause using the ER algorithm again.

The ER algorithm is now used to perform safety synthesis at different levels of an engineering system with a structure that is capable of being decomposed into a hierarchy. The number of levels required in safety synthesis is solely decided by the degree of complexity of the system under scrutiny or by the number of experts taking part in assessment.

### 2.3.2. *Ranking and interpretation of results*

This section describes the calculation of the overall risk level ranking index associated with various causes of a technical failure. Then the identified potential causes are ranked on the basis of their ranking index values.

To calculate risk ranking index values associated with various causes of a technical failure, it is required to describe the safety expressions $\{D_1, D_2, \ldots, D_N\}$, for example, {*Good*, *Average*, *Fair*, *Poor*} using numerical values, i.e. the utility of each linguistic safety expression. The utility values associated with the defined safety expressions can be designated by experts. Suppose $u(D_i)$ ($i = 1, \ldots, N$) represents the utility of a safety expression $D_i$, and $C_j$ ($j = 1, \ldots, d$) is the anticipated cause of a technical failure. Then the risk ranking index value $R_j$ associated with the cause $C_j$ of the technical failure can be defined as follows:

$$R_j = \sum_{i=1}^{N} \eta_i^j \times u(D_i), \quad j = 1, \ldots, d \tag{13}$$

where $\eta_i^j$ ($i = 1, \ldots, N$) is obtained from Equation (12), and $d$ is the number of causes of the technical failure.

 

Obviously, the $R_i$ values obtained using the above expression can only show the relative risk level among all potential causes identified. The smallest $R_i$ is ranked first as it deserves more attention to reduce its potential risk to as low as reasonably practical (ALARP). The largest $R_i$ is ranked last to draw least attention and minimum effort for risk reduction measure consideration. A small $R_i$ means that cause $i$ has a relatively higher risk level and deserves more attention at the early design stages or the early stages of designing operational strategies.

Complementary to the distribution assessment (Equation (12)), a utility interval approach[26] can be used if the assessment is incomplete or imprecise, where maximum, minimum and average risk ranking index values are calculated as follows and used to rank the alternatives:

$$R_{\max}(C_j) = \sum_{i=1}^{N-1} \eta_i^j u(D_i) + (\eta_N^j + \eta_D^j) u(D_N) \tag{14}$$

$$R_{\min}(C_j) = (\eta_1^j + \eta_D^j) u(D_1) + \sum_{i=1}^{N} \eta_i^j u(D_i) \tag{15}$$

$$R_{\text{aver}}(C_j) = \frac{R_{\max}(C_j) + R_{\min}(C_j)}{2} \tag{16}$$

It is obvious that if all original assessments $S(C_j)$ in the distribution assessment are complete, then $\eta_D^j(C_j) = 0$ and $R_j = R_{\min}(C_j) = R_{\max}(C_j) = R_{\text{aver}}(C_j)$.

According to the maximum, minimum utilities and corresponding index value interval, the ranking of two causes may be made as follows. If $R_{\min}(C_l) \geq R_{\max}(C_k)$, $C_l$ is ranked first then $C_k$; if $R_{\min}(C_l) = R_{\min}(C_k)$ and $R_{\max}(C_l) = R_{\max}(C_k)$, $C_l$ is said to be independent of $C_k$. In other cases, *average* rank index may be used to generate an average ranking, but this kind of ranking may be inconclusive and unreliable. To produce a reliable ranking, the quality of original assessments must be improved by reducing imprecision and/or incompleteness present in the original information only associated with $C_l$ and $C_k$.

The ranking results for risk due to various potential causes may help designers understand the anticipated technical problem in question so that an improved risk reduction measure can be incorporated in the new design or a more innovative design can be carried out to reduce the potential estimated risk.

## 3. CASE STUDY IN MARINE AND OFFSHORE ENGINEERING

### 3.1. Problem description

FPSO vessels are by far the most popular platform for floating production systems in offshore oil and gas fields worldwide. According to a report by McCaul[42], almost 60% of the floating production systems now on order have ship-shape hulls. Subsequently, this is of growing importance in shuttle tankers, since they are often used to offload hydrocarbons directly from FPSOs. The tandem offloading concept is generally being adopted for offloading operations. Alongside offloading is a less-adopted concept.

FPSO systems combine traditional process technology with marine technology, and thus are quite dependent on technical design and operational safety control[42]. Since the tandem configuration is dominant in the North Sea, collision between a FPSO and a shuttle tanker during tandem offloading operation has caused a growing concern in the North Sea. Several recent contact incidents between FPSOs/FSUs (floating storage units) and shuttle tankers have clearly demonstrated a high possibility or likelihood of contact between vessels during tandem offloading. The large masses involved between a FPSO and a tanker significantly intensify the collision risk.

It is essential that anticipated hazards due to technical factors be identified, risk control options proposed, and risk reduction or control measures taken to reduce the risk to ALARP. Scenarios involving potential major hazards which might threaten an FPSO or loss of operational control are assessed at an early stage in the design of new facilities, in order to optimize technical and operational solutions[37]. A traditional ship/platform collision risk model may not be effective for tandem offloading operation.

 

This study concentrates on the risk evaluation of the major hazards threatening the FPSO overall rather than focusing on specific areas of design. The FPSO investigated is a turret-moored system connected through flexible risers to remote subsea wells. The safety assessment provides a means for screening the safety implications which would influence the development of the concept. This permits these safety and economic consequences of the concept to be considered in the early design processes and also highlights areas where little guidance or part experience is available, especially with these types of innovative developments.

In this section, safety assessment is carried out on the risk introduced by the collision of a FPSO and a shuttle tanker during a tandem offloading operation. Only technical failures causing risk are assessed here, though the operational failure has also been recognized as one of the major causes of collision.

According to the literature, the technical failure that might cause collisions between a FPSO and a shuttle tanker during tandem offloading operations is malfunction of propulsion systems[43]. The four major causes of technical failures are:

(1) controllable pitch propeller (CPP) failure;
(2) thruster failure;
(3) position reference system (PRS) failure;
(4) dynamics positioning system failure (DP).

For safety modelling purposes it is assumed that each input parameter (i.e. **FR**, **CS**, and **FCP**) will be fed to the proposed safety model in terms of any one of the four input forms described in Section 2.1.1 to address different levels of uncertainty.

In this case study, seven linguistic variables may be used for **FR**, five for **CS**, and seven for **FCP**. Definition of the linguistic variables and the corresponding membership functions are given in Tables AI–AIII and Figures A2–A4 in Appendix A.

A panel of five experts from different disciplines participated in risk analyses of the above four identified causes of technical failures, which may result in collision between a FPSO and a shuttle tanker.

The safety estimate of each technical failure is assessed by five experts separately. The assessment made by the five experts in terms of **FR**, **CS**, and **FCP** is depicted in Table II(a) for collision between a FPSO and a shuttle tanker during a tandem offloading operation due to a technical failure caused by CPP. The other three kinds of assessments are depicted in Table II(b)–(d), respectively. Note that the experts' assessments of the three input parameters in different input forms are used to address different levels of uncertainty.

### 3.2. *Safety estimate of each technical failure by each expert on collision risk*

In the following sections, the evaluations made by expert #1 on collision risk caused by the CPP failure are discussed in more detail to demonstrate the procedure involved in the proposed FURBER scheme for the safety model. For other cases we only provide the results generated using similar computation procedures.

#### 3.2.1. *Rule base with the belief structure*

According to the number of linguistic variables used for describing antecedents, a rule base having a total number of 245 fuzzy rules with the belief structure are used in the case study. The weights of the rules and the attributes are all assumed to be equal. This fuzzy rule base with the belief structure is constructed on the basis of the original rule base given by Sii and Wang[37]. For instance, three original fuzzy rules are listed as follows:

$R_{198}$: *IF* **FR** *is* $\underline{\textit{frequent}}$ AND **CS** *is* $\underline{\textit{critical}}$ AND **FCP** *is* $\underline{\textit{unlikely}}$ THEN ***safety estimate*** *is Fair*
$R_{199}$: *IF* **FR** *is* $\underline{\textit{frequent}}$ AND **CS** *is* $\underline{\textit{critical}}$ AND **FCP** *is* $\underline{\textit{reasonably unlikely}}$ THEN ***safety estimate*** *is Fair*
$R_{200}$: *IF* **FR** *is* $\underline{\textit{frequent}}$ AND **CS** *is* $\underline{\textit{critical}}$ AND **FCP** *is* $\underline{\textit{likely}}$ THEN ***safety estimate*** *is Fair*

These rules are reconstructed using the belief structure as follows:

$R^*_{198}$: *IF* **FR** *is* $\underline{\textit{frequent}}$ AND **CS** *is* $\underline{\textit{critical}}$ AND **FCP** *is* $\underline{\textit{unlikely}}$ THEN ***safety estimate*** *is* {(*Good*, 0), (*Average*, 0.2), (*Fair*, 0.7), (*Poor*, 0.1)}

 

Table II. Expert assessments of the three input parameters using different forms to address uncertainty for technical failure caused by malfunction of (a) the CPP, (b) the thruster, (c) the PRS and (d) the DP

| Expert | Shape of input form | **FR** | **CS** | **FCP** |
|---|---|---|---|---|
| (a) CPP | | | | |
| E # 1 | Triangular | (6.5, 8, 9.5) | (7.5, 8.5, 9.5) | (5.5, 7, 8.5) |
| E # 2 | Triangular | (5.5, 7.5, 9) | (7, 8.5, 10) | (5, 7.5, 9.5) |
| E # 3 | Closed interval | [6, 8] | [7, 9] | [6.5, 9] |
| E # 4 | Trapezoidal | {5.5, 6.5, 9, 10} | {5.5, 7, 8, 10} | {5, 7, 8, 8.5} |
| E # 5 | Single deterministic | 7.75 | 8.25 | 7.6 |
| (b) Thruster | | | | |
| E # 1 | Triangular | (6, 7, 7.5) | (6.5, 7, 8) | (4.5, 5.5, 6) |
| E # 2 | Triangular | (6, 6.5, 8) | (7, 8, 9) | (6, 7.5, 8) |
| E # 3 | Closed interval | [5.5, 7.5] | [6, 8] | [6, 8] |
| E # 4 | Trapezoidal | {5, 6, 7, 8} | {5, 7, 8, 9} | {5, 6, 7, 9} |
| E # 5 | Single deterministic | 7.15 | 7.95 | 7.25 |
| (c) PRS | | | | |
| E # 1 | Triangular | (6.5, 7, 7.5) | (8, 8.5, 9) | (5.5, 7, 8) |
| E # 2 | Triangular | (6, 7.5, 8) | (7.5, 8, 9.5) | (5, 6, 7) |
| E # 3 | Closed interval | [6.5, 8] | [7, 7.5] | [6.5, 7.5] |
| E # 4 | Trapezoidal | {6, 7, 8, 9} | {5, 7, 8, 8.5} | {6, 7, 8, 9} |
| E # 5 | Single deterministic | 7.5 | 7.2 | 7.1 |
| (d) DP | | | | |
| E # 1 | Triangular | (7, 7.5, 8) | (7.5, 8.5, 9) | (6, 7, 7.5) |
| E # 2 | Triangular | (6.5, 7, 8) | (6.5, 7, 8.5) | (5.5, 6, 7) |
| E # 3 | Closed interval | [7, 9] | [7.5, 9.5] | [7, 8] |
| E # 4 | Trapezoidal | {6.5, 7, 7.5, 8} | {6, 6.5, 7, 8} | {6.5, 7, 7.5, 9} |
| E # 5 | Single deterministic | 7.95 | 8.25 | 7.9 |

$R^*_{199}$: *IF* **FR** *is* <u>*frequent*</u> AND **CS** *is* <u>*critical*</u> AND **FCP** *is* <u>*reasonably unlikely*</u> THEN ***safety estimate*** *is* {(*Good*, 0), (*Average*, 0), (*Fair*, 0.8), (*Poor*, 0.2)}

$R^*_{200}$: *IF* **FR** *is* <u>*frequent*</u> AND **CS** *is* <u>*critical*</u> AND **FCP** *is* <u>*likely*</u> THEN ***safety estimate*** *is* {(*Good*, 0), (*Average*, 0), (*Fair*, 0.5), (*Poor*, 0.5)}

In the original rules, the different input linguistic terms lead to the same output term '*Fair*', which does not seem entirely convincing. In the reconstructed rules, one may see the difference of the rules from their belief variations. It is argued that rules with the belief structure provide a more flexible and rational way to construct fuzzy rule bases. Note that the belief degrees of the rules in the rule base given in Table AIV in Appendix A are provided for illustration purposes. The actual degrees of belief depend on the context of applications.

### 3.2.2. *Transformation of the input and the fuzzy rule expression matrix*

Expert #1 used the triangular distribution input form to address the inherent uncertainty associated with the data and information available while assessing the three input parameters. The **FR** is described triangularly as (6.5, 8.0, 9.5), the **CS** as (7.5, 8.5, 9.5), and the **FCP** as (5.5, 7.0, 8.5). As shown in Table III, these input values are transformed into the distributed representation on the linguistic terms in the antecedent using Equations (4) and (5), where $\varepsilon_i$ is assumed to be 1 in Equation (4), based on the membership function defined in Figures A2–A4 in Appendix A.

$\tau$ is calculated using Equation (5) and $\alpha_{ij}$ using Equation (4). We have not listed the corresponding linguistic term with $\tau = 0$ for each input parameter.

Similar transformation procedures are performed for the other four experts for CPP, and for the other three potential causes (the thruster, PRS, and DP) of technical failure, which are summarized in Table IV(a)–(d), where the numerical values associated with each linguistic term are the individual matching belief degrees, i.e. $\alpha_{ij}$ $(i = 1, 2, 3,\ \ j = 1, \ldots, J_i$ with $J_1 = 7$, $J_2 = 5$, and $J_3 = 7)$.

 

Table III. Transformation of the input for CPP assessed by expert #1 into the distributed representation of linguistic terms in the antecedent

| Input parameter | Linguistic term | $\tau$ | $\alpha_{ij}$ |
|---|---|---|---|
| Failure rate | Average | 0.2 | 0.108 |
| | Reasonably frequent | 0.70 | 0.378 |
| | Frequent | 0.70 | 0.378 |
| | Highly frequent | 0.25 | 0.135 |
| Failure consequence probability | Likely | 0.2 | 0.1 |
| | Reasonably likely | 1.0 | 0.5 |
| | Highly likely | 0.6 | 0.3 |
| | Definite | 0.2 | 0.1 |
| Consequence severity | Critical | 0.75 | 0.49 **(0.2)** |
| | Catastrophic | 0.78 | 0.51 **(0.6)** |

Table IV. Transformation of the input for (a) the CPP, (b) the thruster, (c) the PRS and (d) the DP assessed by five experts into the distributed representation of linguistic terms in the antecedent

| Expert | **FR** | **CS** | **FCP** |
|---|---|---|---|
| (a) CPP | | | |
| # 1 | {(A, 0.108), (RF, 0.378), (F, 0.378), (HF, 0.135)} | {(Cr, 0.49), (Ca, 0.51)} | {(L, 0.1), (RL, 0.5), (HL, 0.3), (D, 0.1)} |
| # 2 | {(RL, 0.081), (A, 0.238), (RF, 0.405), (F, 0.276)} | {(Cr, 0.5), (Ca, 0.5)} | {(L, 0.112), (RL, 0.345), (HL, 0.0.341), (D, 0.201)} |
| # 3 | {(A, 0.364), (RF, 0.364), (F, 0.273)} | {(Cr, 0.5), (Ca, 0.5)} | {(RL, 0.333), (HL, 0.333), (D, 0.333)} |
| # 4 | {(RL, 0.088), (A, 0.221), (RF, 0.294), (F, 0.294), (HF, 0.103)} | {(M, 0.263), (Cr, 0.439), (Ca, 0.298)} | {(L, 0.129), (RL, 0.379), (HL, 0.379), (D, 0.114)} |
| # 5 | {(RF, 0.626), (F, 0.374)} | {(Cr, 0.75), (C, 0.25)} | {(RL, 0.429), (HL, 0.571)} |
| (b) Thruster | | | |
| # 1 | {(A, 0.294), (RF, 0.588), (F, 0.118)} | {(M, 0.259), (Cr, 0.741)} | {(RL, 0.152), (HL, 0.455), (D, 0.394)} |
| # 2 | {(A, 0.4), (RF, 0.429), (F, 0.171)} | {(Cr, 0.667), (Ca, 0.333)} | {(RL, 0.533), (HL, 0.467)} |
| # 3 | {(RL, 0.182), (A, 0.364), (RF, 0.364), (F, 0.091)} | {(M, 0.5), (Cr, 0.5)} | {(RL, 0.5), (HL, 0.5)} |
| # 4 | {(RL, 0.175), (A, 0.351), (RF, 0.351), (F, 0.123)} | {(M, 0.0.302), (Cr, 0.465), (Ca, 0.233)} | {(L, 0.202), (RL, 0.405), (HL, 0.263), (D, 0.13)} |
| # 5 | {(RF, 0.95), (F, 0.05)} | {(Cr, 1)} | {(RL, 0.429), (HL, 0.571)} |
| (c) PRS | | | |
| # 1 | {(A, 0.226), (RF, 0.645), (F, 0.129)} | {(M, 0.259), (Cr, 0.741)} | {(L, 0.118) (RL, 0.588), (HL, 0.294)} |
| # 2 | {(A, 0.242), (RF, 0.515), (F, 0.242)} | {(Cr, 0.667), (Ca, 0.333)} | {(L, 0.333), (HL, 0.667)} |
| # 3 | {(A, 0.25), (RF, 0.5), (F, 0.25)} | {(Cr, 1.0)} | {(RL, 0.667), (HL, 0.333)} |
| # 4 | {(A, 0.233), (RF, 0.465), (F, 0.302)} | {(M, 0.302), (Cr, 0.465), (Ca, 0.233)} | {(RL, 0.4), (HL, 0.4), (D, 0.2)} |
| # 5 | {(RF, 0.75), (F, 0.25)} | {(Cr, 1)} | {(RL, 0.474), (HL, 0.526)} |
| (d) DP | | | |
| # 1 | {(RF, 0.667), (F, 0.333)} | {(Cr, 0.536), (Cr, 0.464)} | {(RL, 0.222), (HL, 0.778)} |
| # 2 | {(A, 0.182), (RF, 0.606), (F, 0.212)} | {(A, 0.226), (Cr, 0.645), (Ca, 0.129)} | {(L, 0.333), (HL, 0.667)} |
| # 3 | {(RF, 0.5), (F, 0.5)} | {(Cr, 0.5), (Ca, 0.5)} | {(RL, 0.5), (HL, 0.5)} |
| # 4 | {(A, 0.2), (RF, 571), (F, 0.229)} | {(M, 0.412), (Cr, 0.588)} | {(RL, 0.417), (HL, 0.417), (D, 0.167)} |
| # 5 | {(RF, 0.52), (F, 0.48)} | {(Cr, 0.75), (Ca, 0.25)} | {(RL, 0.15), (HL, 0.85)} |

In the rule base, 245 rules have been established, of which only 32 rules are fired due to the CPP failure in this particular case for expert #1, i.e. Rules #130–133, #137–140, #165–168, #172–175, #200–203, #207–210, #235–238, and #242–245. These rules are all listed in Table AV in Appendix A.

 

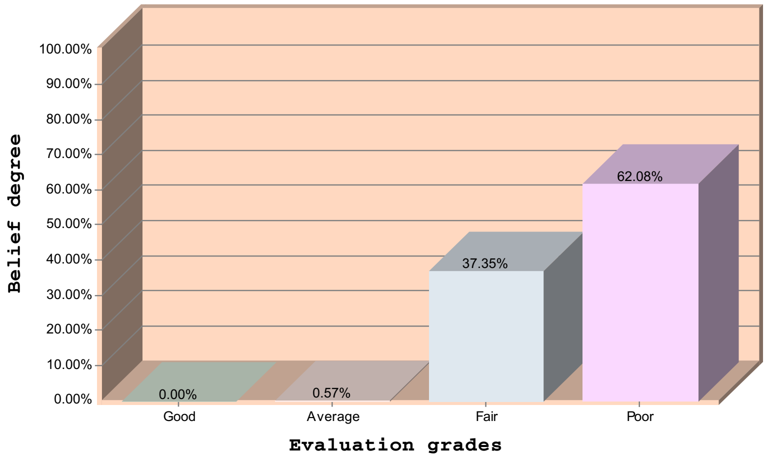


Figure 1. The safety estimate of CPP by expert #1

Based on the individual matching belief degrees, the activation weight $w_k$ ($k = 1, \ldots, 32$) of each rule in the fired sub-rule base is calculated using Equation (7), where we assume that the attribute weight $\delta_i = 1$ ($i = 1, 2, 3$) and the relative weight of the rules are all equal or $\theta_k = 1$ ($k = 1, \ldots, 32$). The fuzzy rule expression matrix for the sub-rule base with the fired 32 rules is shown in Table AV in Appendix A. Taking rule #132 for example, the assessment can be represented by

$$S(A^{132}) = \{(Good, 0), (Average, 0), (Fair, 0.2), (Poor, 0.7)\}$$

with the activation weight $w_{132} = 0.015$.

*3.2.3. Fired rule combination to get the safety estimate using the ER algorithm*

Based on Table AV in Appendix A, the Intelligent Decision System (IDS)[44] is used to implement the combination of the 32 rules and to generate safety estimates. The final assessment result for CPP by expert #1 is generated as follows and is shown in Figure 1.

$$\text{Generated result: } \{(Good, 0), (Average, 0.0057), (Fair, 0.3735), (Poor, 0.6208)\}$$

This result can be interpreted in such a way that the safety estimate of CPP to technical failure is '*Average*' with a belief degree of 0.0057, '*Fair*' with a belief degree of 0.3735 and '*Poor*' with a belief degree of 0.6208.

Similar computations are performed for the other four experts for CPP, and for the other three potential causes (the thruster, PRS, and DP) of technical failure. The safety estimates generated for the CCP, thruster, PRS and DP by five experts are summarized in Table V(a)–(d), respectively.

## 3.3. *Illustration of safety estimate based on the incomplete input*

To illustrate how the incompleteness of input can be reflected and dealt with in the FURBER inference engine, we use another example by modifying the input of expert #1 for CPP. We use the same input for **FR** and **FCP** as shown in Table II(a), but assume that the input for the **CS** is incomplete. Suppose the transformation of the input is {(*Critical*, 0.2), (*Catastrophic*, 0.6)}, as shown in Table III using bold fonts in the parentheses of the fourth column. It is possible in this case that no numerical value is available and only subjective judgements can be made with the incomplete belief assessment due to the lack of information.

Due to the change in the input, the activation weight of each rule is also changed using Equation (7), and the degrees of belief of the rules are updated using Equation (8). Because the input of **CS** is incomplete, i.e. $\sum_{j=1}^{5} \alpha_{2j} = 0.8 < 1$, together with $\sum_{j=1}^{7} \alpha_{1j} = \sum_{j=1}^{7} \alpha_{3j} = 1$, then the degree of belief $\bar{\beta}_{ik}$ for the fired

 

Table V. Safety estimate by each expert on the collision risk between a FPSO and a shuttle tanker due to technical failure caused by (a) the CPP, (b) the thruster, (c) the PRS and (d) the DP

| Expert | **FR** | **CS** | **FCP** | Safety estimate *Good* | *Average* | *Fair* | *Poor* |
|---|---|---|---|---|---|---|---|
| (a) CPP | | | | | | | |
| # 1 | (6.5, 8, 9.5) | (7.5, 8.5, 9.5) | (5.5, 7, 8.5) | 0 | 0.0057 | 0.3735 | 0.6208 |
| # 2 | (5.5, 7.5, 9.0) | (7, 8.5, 10) | (5, 7.5, 9.5) | 0 | 0.0075 | 0.3750 | 0.6175 |
| # 3 | [6, 8] | [7, 9] | [6.5, 9] | 0 | 0.0033 | 0.3090 | 0.6876 |
| # 4 | {5.5,6.5,9,10} | {5.5, 7, 8, 10} | {5, 7, 8, 8.5} | 0 | 0.0233 | 0.4751 | 0.5016 |
| # 5 | 7.75 | 8.25 | 7.6 | 0 | 0.0123 | 0.3641 | 0.6236 |
| (b) Thruster | | | | | | | |
| # 1 | (6, 7, 7.5) | (6.5, 7, 8) | (4.5, 5.5, 6) | 0 | 0.0373 | 0.7802 | 0.1825 |
| # 2 | (6, 6.5, 8) | (7, 8, 9) | (6, 7.5, 8) | 0 | 0.0640 | 0.4165 | 0.5195 |
| # 3 | [5.5, 5.5, 7.5, 7.5] | [6, 6, 8, 8] | [6, 6, 8, 8] | 0 | 0.0375 | 0.6503 | 0.3122 |
| # 4 | {5, 6, 7, 8} | {5, 7, 8, 9} | {5, 6, 7, 9} | 0 | 0.0274 | 0.5540 | 0.4186 |
| # 5 | 7.15 | 7.95 | 7.25 | 0 | 0.0013 | 0.4179 | 0.5808 |
| c) PRS | | | | | | | |
| # 1 | (6.5, 7, 7.5) | (8, 8.5, 9) | (5.5, 7, 8) | 0 | 0.0047 | 0.6151 | 0.3802 |
| # 2 | (6, 7.5, 8) | (7.5, 8, 9.5) | (5, 6, 7) | 0 | 0.0041 | 0.6142 | 0.3817 |
| # 3 | [6.5, 6.5, 8, 8] | [7, 7, 7.5, 7.5] | [6.5, 6.5, 7.5, 7.5] | 0 | 0.0055 | 0.3845 | 0.6100 |
| # 4 | {6, 7, 8, 9} | {5, 7, 8, 8.5} | {6, 7, 8, 9} | 0 | 0.0204 | 0.5111 | 0.4685 |
| # 5 | 7.5 | 7.2 | 7.1 | 0 | 0.0080 | 0.3694 | 0.6226 |
| (d) DP | | | | | | | |
| # 1 | (7, 7.5, 8) | (7.5, 8.5, 9) | (6, 7, 7.5) | 0 | 0.0102 | 0.3595 | 0.6303 |
| # 2 | (6.5, 7, 8) | (6.5, 7, 8.5) | (5.5, 6, 7) | 0 | 0.0097 | 0.6926 | 0.2977 |
| # 3 | [7, 7, 9, 9] | [7.5, 7.5, 9.5, 9.5] | [7, 7, 8, 8] | 0 | 0.0097 | 0.3930 | 0.5973 |
| # 4 | {6.5,7,7.5,8} | {6, 6.5, 7, 8} | {6.5,7,7.5,9} | 0 | 0.0200 | 0.5733 | 0.4067 |
| # 5 | 7.95 | 8.25 | 7.9 | 0 | 0.0256 | 0.2688 | 0.7056 |

32 rules ($i = 1, \ldots, 4$; $k = 1, \ldots, 32$) are updated into $\beta_{ik} = \bar{\beta}_{ik} * 2.8/3 = \bar{\beta}_{ik} * 0.9333$ using Equation (8). Hence, $0 < \sum_{i=1}^{4} \beta_{ik} < 1$ for all $k$. Table AVI in Appendix A is the fuzzy rule expression matrix for the sub-rule base, which is generated using the fired 32 rules based on this new input. The final assessment result is generated using IDS as follows and also shown in Figure 2:

$$\{(Good, 0),\ (Average, 0.0041),\ (Fair, 0.3847),\ (Poor, 0.5594),\ (\text{unknown}, 0.0518)\}$$

This result means that the output will also be incomplete due to the incompleteness of the input. The final output still gives a similar overall picture about the safety level with the possible incompleteness.

Similar computations may be performed for safety assessment using the proposed fuzzy-logic-based ER approach for the other three potential causes of technical failure.

### 3.4. *Safety synthesis*

The ER approach can be used not only to aggregate fuzzy rules for safety analysis in the FURBER framework, but also to assess the safety of the whole system. IDS is again used to synthesize safety estimates. Three examples are demonstrated. The first example is multi-cause synthesis, the second example is the multi-expert evaluation of a particular failure mode, and the last example is a multi-cause–multi-expert synthesis and evaluation.

#### 3.4.1. *Multi-expert safety synthesis*

Based on the results shown in Table V(a)–(d), Table VI(a) and (b) show the results of multi-expert safety synthesis on collision risk between a FPSO and a shuttle tanker due to CPP, thruster, PRS and DP caused

 

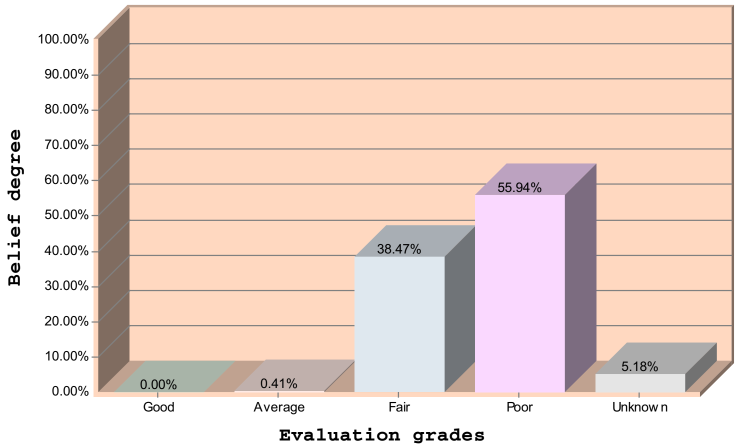


Figure 2. The final safety estimate of CPP by expert #1 based on the incomplete input

Table VI. Multi-expert safety synthesis on collision risk between a FPSO and a shuttle tanker due to CPP caused technical failure with (a) equal expert weights and (b) different expert weights $W_E^*$

| | Safety synthesis | | | |
|---|---|---|---|---|
| Technical failure cause | *Good* | *Average* | *Fair* | *Poor* |
| (a) Equal expert weights | | | | |
| CPP | 0 | 0.0063 | 0.3468 | 0.6469 |
| Thruster | 0 | 0.0336 | 0.6378 | 0.3286 |
| PRS | 0 | 0.0053 | 0.5538 | 0.4409 |
| DP | 0 | 0.0097 | 0.4563 | 0.5339 |
| (b) Different expert weights $W_E^*$ | | | | |
| CPP | 0 | 0.0086 | 0.3582 | 0.6333 |
| Thruster | 0 | 0.0305 | 0.6098 | 0.3597 |
| PRS | 0 | 0.0080 | 0.4903 | 0.5017 |
| DP | 0 | 0.0108 | 0.4993 | 0.4898 |

technical failure, respectively. The synthesis is carried out using both equal and different relative weights among experts.

Suppose the same relative weight is given to the panel of experts, i.e. $\mathrm{W_E} = \{w_{\text{expert\#1}};\ w_{\text{expert\#2}};\ w_{\text{expert\#3}};\ w_{\text{expert\#4}};\ w_{\text{expert\#5}}\} = \{1, 1, 1, 1, 1\}$. Based on the safety estimate of each expert on collision risk between a FPSO and a shuttle tanker due to four anticipated causes to technical failure, shown in Table V(a)–(d), the multi-expert safety synthesis is carried out using IDS and the results are summarized in Table VI(a) and (b).

Again IDS is used to carry out the multi-expert safety synthesis with different weights assigned to the experts, e.g. $W_E^* = \{w_{\text{expert\#1}};\ w_{\text{expert\#2}};\ w_{\text{expert\#3}};\ w_{\text{expert\#4}};\ w_{\text{expert\#5}}\} = \{0.3, 0.5, 0.9, 0.8, 0.1\}$, which is normalized as $\{0.12, 0.19, 0.35, 0.31, 0.04\}$ so that their total weights are summed to one. The synthesis result is depicted in Table VI(b).

### *3.4.2. Safety ranking*

To calculate risk ranking index values associated with various causes of technical failure, it is required to describe the four safety expressions, i.e. {*Good, Average, Fair, Poor*} using utility values. The utility values associated with the defined safety expressions can be assigned by experts. Suppose $u(Good)$, $u(Average)$, $u(Fair)$, $u(Poor)$ represent the utilities associated with '*Good*', '*Average*', '*Fair*', '*Poor*', respectively, and

$$\{u(Good),\ u(Average),\ u(Fair),\ u(Poor)\} = \{1, 0.8, 0.6, 0.2\}$$

 

Table VII. Risk ranking

| Rank items | Expert weights | Ranking | | | |
|---|---|---|---|---|---|
| | | CPP | Thruster | PRS | DP |
| Safety ranking index value | Equal weight | 0.346 34 | 0.450 88 | 0.404 28 | 0.387 24 |
| | Different weight $W_E^*$ | 0.348 4 | 0.462 2 | 0.400 9 | 0.406 2 |
| Safety ranking ordering | Equal weight | 1 | 4 | 3 | 2 |
| | Different weight $W_E^*$ | 1 | 4 | 2 | 3 |

Table VIII. Multi-cause safety synthesis for the technical failure by each expert

| Expert | Safety synthesis | | | |
|---|---|---|---|---|
| | *Good* | *Average* | *Fair* | *Poor* |
| Expert #1 | 0 | 0.0116 | 0.5438 | 0.4446 |
| Expert #2 | 0 | 0.0170 | 0.5352 | 0.4478 |
| Expert #3 | 0 | 0.0111 | 0.4217 | 0.5672 |
| Expert #4 | 0 | 0.0182 | 0.5402 | 0.4416 |
| Expert #5 | 0 | 0.0091 | 0.3258 | 0.6651 |

The risk ranking index value $R_{CPP}$ associated with the cause CPP is calculated based on the multi-expert safety synthesis shown in Table VI(a) with equal expert weight and using Equation (13) as follows:

$$R_{CPP} = \sum_{i=1}^{4} \eta_i^{CPP} \times u(D_i) = (0 \times 1) + (0.0063 \times 0.8) + (0.3468 \times 0.6) + (0.6469 \times 0.2) = 0.346\,43$$

Based on the results shown in Table VI(a) and (b), the risk ranking index values associated with the causes CPP, Thruster, PRS and DP of technical failure can be calculated and the results are summarized in Table VII.

From Table VII it can be noted that for the two sets of weights assigned to the experts, the potential risk caused by CPP is always the highest, so it deserves more attention to reduce its potential risk to ALARP. The thruster is always ranked to be the safest among the four options. When the relative weights of the panel experts are taken as equal, the potential risks caused by DP and PSR are ranked second and third, respectively. When different weights were given, the potential risks caused by PSR and DP are ranked second and third, respectively.

### *3.4.3. Multi-cause safety synthesis*

Table VIII shows the results of the multi-cause safety synthesis by each expert on the four anticipated causes of technical failure, which result in collision between a FPSO and a shuttle tanker. The result produced by expert #1 is as follows:

Multi-cause safety synthesis (Expert #1) = {(*Good*, 0), (*Average*, 0.0116), (*Fair*, 0.5438), (*Poor*, 0.4446)}.

### *3.4.4. Multi-cause–multi-expert safety synthesis*

Suppose equal relative weights are given to the panel of experts, i.e. $W_E = \{w_{expert\#1}; w_{expert\#2}; w_{expert\#3}; w_{expert\#4}; w_{expert\#5}\} = \{1, 1, 1, 1, 1\}$. Based on Table VIII, the multi-cause–multi-expert safety synthesis is given by {(*Good*, 0), (*Average*, 0.0106), (*Fair*, 0.4698), (*Poor*, 0.5196)}.

For the different weights, the multi-cause–multi-expert safety synthesis is given by {(*Good*, 0), (*Average*, 0.0115), (*Fair*, 0.4902), (*Poor*, 0.4983)}. The results of the multi-cause–multi-expert safety synthesis for the two sets of weights are summarized in Table IX.

The belief distribution {(*Good*, 0), (*Average*, 0.0106), (*Fair*, 0.4698), (*Poor*, 0.5196)} represents the final multi-cause–multi-expert safety synthesis of a technical factor, meaning that the panel of experts is 51.96% sure that the safety level is *Poor*, and 46.98% sure that the safety level is *Fair*, and 1.06% sure that the safety level is *Average*. This final result gives a panoramic view of the safety level for a technical factor.

 

Table IX. Multi-cause–multi-expert safety synthesis by experts for different weights

| Expert's weight | | | | | Safety synthesis | | | |
|---|---|---|---|---|---|---|---|---|
| Expert #1 | Expert #2 | Expert #3 | Expert #4 | Expert #5 | *Good* | *Average* | *Fair* | *Poor* |
| 1 | 1 | 1 | 1 | 1 | 0 | 0.0106 | 0.4698 | 0.5196 |
| 0.3 | 0.5 | 0.9 | 0.8 | 0.1 | 0 | 0.0115 | 0.4902 | 0.4983 |

## 4. CONCLUSION

A fuzzy rule-based hierarchical multi-expert safety analysis and synthesis framework using the ER approach was proposed. In this approach, information on different properties from various sources can be transformed and used in the rule base and in the inference process. In the framework, a normal 'IF–THEN' rule base can be extended to a rule base with a belief structure, so that subjective expert judgements with uncertainties of both a probabilistic and fuzzy nature can be taken into account. The FURBER approach can be used to accommodate domain-specific human experts' experiences and safety engineering knowledge with uncertainties. Multi-attribute and multi-expert decision-making can also be conducted on the basis of safety estimates using the ER algorithm. The new approach provides a flexible way to represent and a rigorous procedure to deal with hybrid uncertain safety assessment information to arrive at final conclusions.

The results generated from a series of case studies on collision risk between a FPSO and a shuttle tanker have demonstrated that such a framework can provide safety analysts and designers with a convenient tool that can be used at various stages of the design process of offshore engineering systems for risk analysis. In conclusion, the proposed framework offers great potential in safety assessment and deign decision support of engineering and management systems, especially in the initial conceptual design stages. Moreover, this new methodology provides scope and flexibility for rule training and self-learning/updating in a rule base, which will be investigated in future work.


### Acknowledgements

This work forms part of the projects supported by the UK Engineering and Physical Sciences Research Council (EPSRC) under grant Nos GR/R30624 and GR/R32413; GR/S85498/01 and GR/S85504/01.


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## APPENDIX A

Table AI. Failure rate[37]

| Rank | **FR** | Meaning (general interpretation) | Failure rate (1/year) |
|---|---|---|---|
| 1,2,3 | Very low | Failure is unlikely but possible during lifetime | $<10^{-6}$ |
| 4 | Low | Likely to happen once during lifetime | $0.25 \times 10^{-5}$ |
| 5 | Reasonably low | Between low and average | $0.25 \times 10^{-4}$ |
| 6 | Average | Occasional failure | $10^{-3}$ |
| 7 | Reasonably frequent | Likely to occur from time to time | $0.25 \times 10^{-2}$ |
| 8, 9 | Frequent | Repeated failure | $0.125 \times 10^{-1}$ |
| 9, 10 | Highly frequent | Failure is almost inevitable or likely to exist repeatedly | $>0.25 \times 10^{-1}$ |

Table AII. Consequence severity[37]

| Rank | **CS** | Meaning (generic marine and offshore structure/system interpretation) |
|---|---|---|
| 1 | Negligible | At most a single minor injury or unscheduled maintenance required (service and operations can continue) |
| 2, 3 | Marginal | Possible single or multiple minor injuries and/or minor system damage. Operations interrupted slightly, and resumed to their normal operational mode within a short period of time (say less than 2 h) |
| 4, 5, 6 | Moderate | Possible multiple minor injuries or a single severe injury, moderate system damage. Operations and production interrupted marginally, and resumed to their normal operational mode within, say, no more than 4 h |
| 7, 8 | Critical | Possible single death, probable multiple severe injuries or major system damage. Operations stopped, platform closed, shuttle tanker's failure to function. High degree of operational interruption due to the nature of the failure such as an inoperable platform (e.g. drilling engine fails to start, power system failure, turret mooring system failure) or an inoperable convenience subsystem (e.g. DP, PRS) |
| 9, 10 | Catastrophic | Possible multiple deaths, probable single death or total system loss. Very high severity ranking when a potential failure mode (e.g. collision between a FPSO and a shuttle tanker, blow-out, fire and explosion) affects safe platform operation and/or involves non-compliance with government regulations |

 

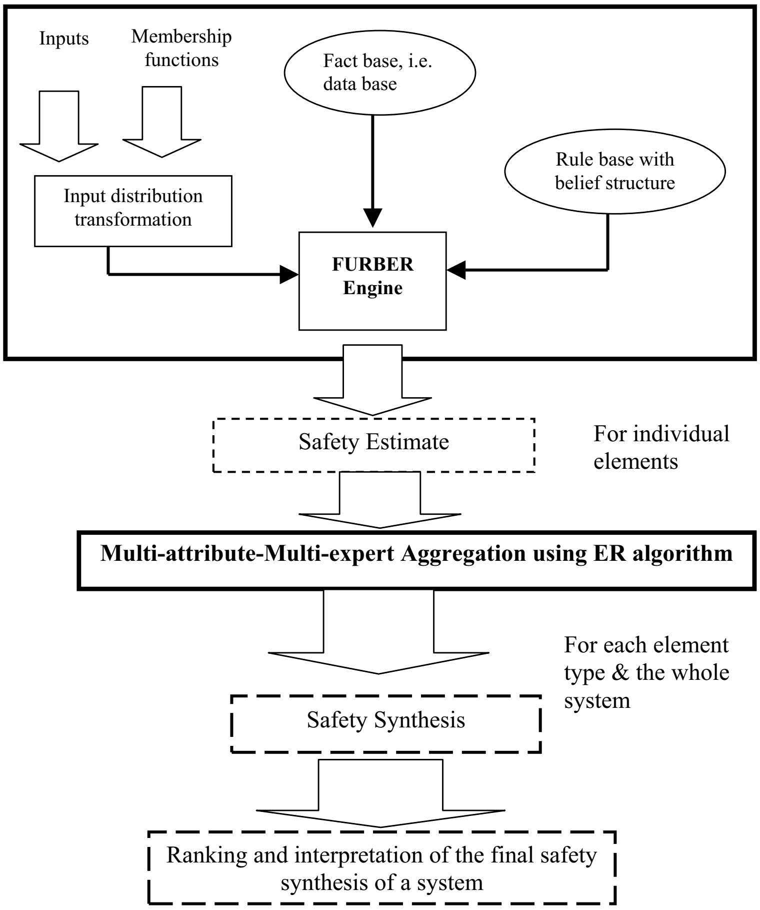


Figure A1. A generic safety assessment and synthesis framework

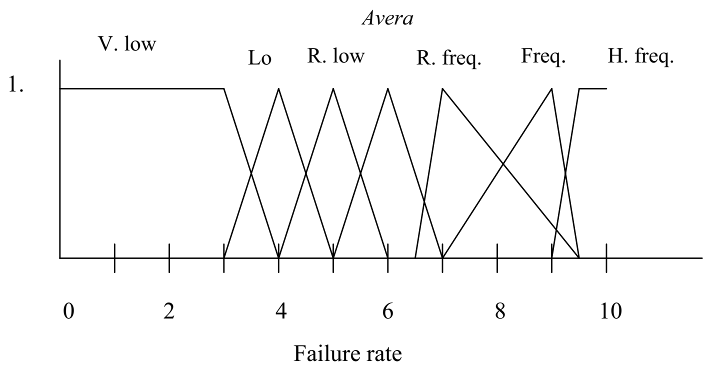


Figure A2. Fuzzy failure rate set definition[37]

 

Table AIII. Failure consequence probability[37]

| Rank | **FCP** | Meaning |
|---|---|---|
| 1 | Highly unlikely | The occurrence likelihood of possible consequence is highly unlikely given the occurrence of the failure event (extremely unlikely to exist on the system or during operations) |
| 2, 3 | Unlikely | The occurrence likelihood of possible consequences is unlikely but possible given that the failure event happens (improbable to exist even on rare occasions on the system or during operations) |
| 4 | Reasonably unlikely | The occurrence likelihood of possible consequences is reasonably unlikely given the occurrence of the failure event (likely to exist on rare occasions on the system or during operations) |
| 5 | Likely | It is likely that consequences happen given that the failure event occurs (a programme is not likely to detect a potential design or operations procedural weakness) |
| 6, 7 | Reasonably likely | It is reasonably likely that consequences occur given the occurrence of the failure event (i.e. exist from time to time on the system or during operations, possibly caused by a potential design or operations procedural weakness) |
| 8 | Highly likely | It is highly likely that consequences occur given the occurrence of the failure event (i.e. often exist somewhere on the system or during operations due to a highly likely potential hazardous situation or design and/or operation procedural drawback) |
| 9, 10 | Definite | Possible consequences happen given the occurrence of a failure event (i.e. likely to exist repeatedly during operations due to an anticipated potential design and operation procedural drawback) |

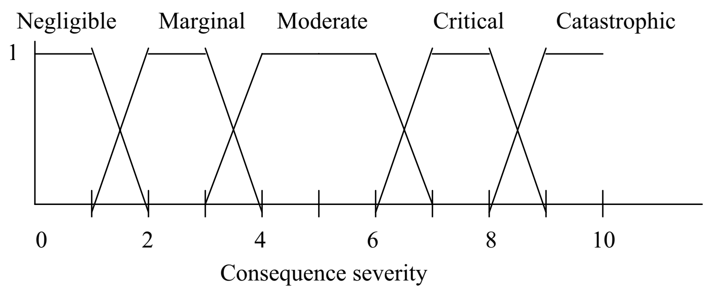


Figure A3. Fuzzy consequence severity set definition[37]

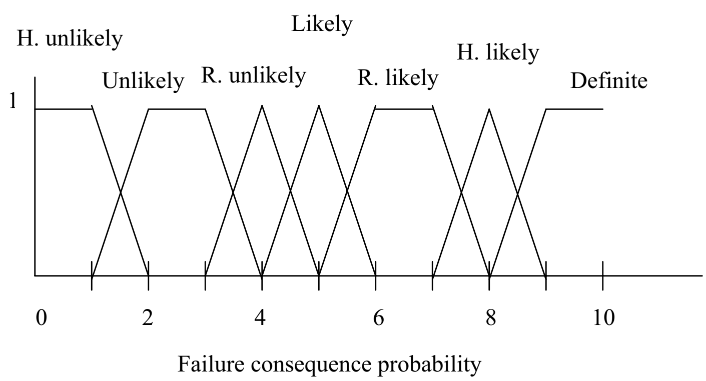


Figure A4. Fuzzy failure consequence probability set definition[37]

Table AIV. Illustration of safety rule base with belief structure

| Item | Antecedent attribute | Safety estimate | | | |
|---|---|---|---|---|---|
| | | *Good* | *Average* | *Fair* | *Poor* |
| R #1 | (*very low*, *negligible*, *highly unlikely*) | 1 | | | |
| R #2 | (*very low*, *negligible*, *unlikely*) | 0.8 | 0.2 | | |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| R #244 | (*highly frequent*, *catastrophic*, *highly likely*) | | | 0.05 | 0.95 |
| R #245 | (*highly frequent*, *catastrophic*, *definite*) | | | | 1 |

*Note:* (*very low*, *negligible*, *highly unlikely*) represents '**FR** is *very low* AND **CS** is *negligible* AND **FCP** is *reasonably unlikely*'. Other rules have the same meaning. Blank entries for the belief in the table mean '0'.

Table AV. Fuzzy rule expression matrix of the knowledge base with the fired 32 rules

| Input | Output | | | |
|---|---|---|---|---|
| | $D_1$ (*Good*) | $D_2$ (*Average*) | $D_3$ (*Fair*) | $D_4$ (*Poor*) |
| $A^{130}$ (0.005) | 0 | 0 | 1 | 0 |
| $A^{131}$ (0.024) | 0 | 0 | 0.5 | 0.5 |
| $A^{132}$ (0.015) | 0 | 0 | 0.3 | 0.7 |
| $A^{133}$ (0.010) | 0 | 0 | 0.2 | 0.8 |
| $A^{137}$ (0.005) | 0 | 0 | 0.5 | 0.5 |
| $A^{138}$ (0.028) | 0 | 0 | 0.4 | 0.6 |
| $A^{139}$ (0.015) | 0 | 0 | 0.2 | 0.8 |
| $A^{140}$ (0.010) | 0 | 0 | 0.15 | 0.85 |
| $A^{165}$ (0.017) | 0 | 0 | 0.8 | 0.2 |
| $A^{166}$ (0.084) | 0 | 0 | 0.5 | 0.5 |
| $A^{167}$ (0.050) | 0 | 0 | 0.4 | 0.6 |
| $A^{168}$ (0.034) | 0 | 0 | 0.3 | 0.7 |
| $A^{172}$ (0.017) | 0 | 0.1 | 0.8 | 0.1 |
| $A^{173}$ (0.087) | 0 | 0 | 0.9 | 0.1 |
| $A^{174}$ (0.052) | 0 | 0 | 0.4 | 0.6 |
| $A^{175}$ (0.035) | 0 | 0 | 0.3 | 0.7 |
| $A^{200}$ (0.017) | 0 | 0 | 0.5 | 0.5 |
| $A^{201}$ (0.084) | 0 | 0 | 0.3 | 0.7 |
| $A^{202}$ (0.050) | 0 | 0.1 | 0.1 | 0.8 |
| $A^{203}$ (0.034) | 0 | 0 | 0.2 | 0.8 |
| $A^{207}$ (0.017) | 0 | 0 | 0.5 | 0.5 |
| $A^{208}$ (0.087) | 0 | 0 | 0.4 | 0.6 |
| $A^{209}$ (0.052) | 0 | 0 | 0.3 | 0.7 |
| $A^{210}$ (0.035) | 0 | 0 | 0.2 | 0.8 |
| $A^{235}$ (0.006) | 0 | 0 | 0.4 | 0.6 |
| $A^{236}$ (0.031) | 0 | 0 | 0.2 | 0.8 |
| $A^{237}$ (0.019) | 0 | 0 | 0.1 | 0.9 |
| $A^{238}$ (0.012) | 0 | 0 | 0 | 1 |
| $A^{242}$ (0.006) | 0 | 0 | 0.2 | 0.8 |
| $A^{243}$ (0.032) | 0 | 0 | 0.15 | 0.85 |
| $A^{244}$ (0.019) | 0 | 0 | 0.05 | 0.95 |

*Note*: The values in the parentheses in the first column are the weights of the activation attributes of $A^k$ generated using Equation (7), where the weights of the rules and the weights of the antecedents are assumed to be equal. Moreover, because the inputs are complete, then the degrees of belief do not need to be updated.

Table AVI. Fuzzy rule expression matrix with the fired 32 rules for the incomplete input

| | Output | | | |
|---|---|---|---|---|
| Input | $D_1$ (*Good*) | $D_2$ (*Average*) | $D_3$ (*Fair*) | $D_4$ (*Poor*) |
| $A^{130}$ (0.002) | 0 | 0 | 0.933 333 | 0 |
| $A^{131}$ (0.011) | 0 | 0 | 0.466 667 | 0.466 667 |
| $A^{132}$ (0.006) | 0 | 0 | 0.28 | 0.653 333 |
| $A^{133}$ (0.002) | 0 | 0 | 0.186 667 | 0.746 667 |
| $A^{137}$ (0.006) | 0 | 0 | 0.466 667 | 0.466 667 |
| $A^{138}$ (0.032) | 0 | 0 | 0.373 333 | 0.56 |
| $A^{139}$ (0.019) | 0 | 0 | 0.186 667 | 0.746 667 |
| $A^{140}$ (0.006) | 0 | 0 | 0.14 | 0.793 333 |
| $A^{165}$ (0.008) | 0 | 0 | 0.746 667 | 0.186 667 |
| $A^{166}$ (0.038) | 0 | 0 | 0.466 667 | 0.466 667 |
| $A^{167}$ (0.023) | 0 | 0 | 0.373 333 | 0.56 |
| $A^{168}$ (0.008) | 0 | 0 | 0.28 | 0.653 333 |
| $A^{172}$ (0.023) | 0 | 0.093 33 | 0.746 67 | 0.093 33 |
| $A^{173}$ (0.113) | 0 | 0 | 0.84 | 0.093 333 |
| $A^{174}$ (0.068) | 0 | 0 | 0.373 333 | 0.56 |
| $A^{175}$ (0.023) | 0 | 0 | 0.28 | 0.653 333 |
| $A^{200}$ (0.008) | 0 | 0 | 0.466 667 | 0.466 667 |
| $A^{201}$ (0.038) | 0 | 0 | 0.28 | 0.653 333 |
| $A^{202}$ (0.023) | 0 | 0.093 333 | 0.093 333 | 0.746 667 |
| $A^{203}$ (0.008) | 0 | 0 | 0.186 667 | 0.746 667 |
| $A^{207}$ (0.023) | 0 | 0 | 0.466 667 | 0.466 667 |
| $A^{208}$ (0.113) | 0 | 0 | 0.373 333 | 0.56 |
| $A^{209}$ (0.068) | 0 | 0 | 0.28 | 0.653 333 |
| $A^{210}$ (0.023) | 0 | 0 | 0.186 667 | 0.746 667 |
| $A^{235}$ (0.003) | 0 | 0 | 0.373 333 | 0.56 |
| $A^{236}$ (0.014) | 0 | 0 | 0.186 667 | 0.746 667 |
| $A^{237}$ (0.008) | 0 | 0 | 0.093 333 | 0.84 |
| $A^{238}$ (0.003) | 0 | 0 | 0 | 0.933 333 |
| $A^{242}$ (0.008) | 0 | 0 | 0.186 667 | 0.746 667 |
| $A^{243}$ (0.041) | 0 | 0 | 14 | 0.793 333 |
| $A^{244}$ (0.024) | 0 | 0 | 0.046 667 | 0.886 667 |

 