Enhanced Lattice-Based Signatures on Reconfigurable Hardware

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Lattice Based Cryptography

Many theoretical advantages over ECC/RSA:

- Strong theoretical guarentee of hardness
- Resist known quantum algorithms
- Very versatile: PKE, Signatures, IBE, FHE . . .
- Asymptotically efficient $O(n^2)$ or even $O(n \log n)$.

In practice ?
Simple and very fast PKE (NTRU-Encrypt, LWE Encryption).
Efficient signatures have been more problematic.

BLISS: An optimized Signature Scheme

Signature without Trapdoors (Fiat-Shamir transform).

[Lyu09] Fiat-Shamir with aborts ($\approx 50 Kbits$)

[Lyu12] Abort Rate improved using Gaussians ($\approx 12 Kbits$)

[DDLL13] Abort Rate improved using Bimodal Gaussians ($\approx 5 Kbits$)

BLISS vs. ECDSA vs. RSA on Software

Scheme.	Security	Sign Size	Sign./s	Ver./s
BLISS-I	**128 bits**	**5.5kbits**	**8k**	**33k**
RSA 2048	112 bits	2kbits	0.8k	27k
RSA 4096	$\geqslant$ **128 bits**	**4kbits**	**0.1k**	**7.5k**
ECDSA 256	**128 bits**	**512 bits**	**9.5k**	**2.5k**

BLISS [DDLL13] compared to OpenSSL implem. of RSA and ECDSA on x86-64.

Fiat-Shamir with aborts [Lyu09, Lyu12, DDLL13]

sk : $\mathbf{S} \in \mathbb{Z}_{2q}^{m \times k}$, short

pk : $\mathbf{A} \in \mathbb{Z}_{2q}^{n \times m}$, random

$\mathbf{T} = \mathbf{AS} = q\,\mathbf{Id}$

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Sample $\mathbf{y} \leftarrow D_{\mathbb{Z},\sigma}^{m}$, short

$$\xrightarrow{\mathbf{w} = \mathbf{A}\mathbf{y} \in \mathbb{Z}^n}$$

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Sample $\mathbf{c} \in \mathbb{Z}^k$, short

$$\xleftarrow{\mathbf{c}}$$

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$\mathbf{z} = \mathbf{y} \pm \mathbf{Sc}$

Abort with proba.
$D_{\mathbb{Z},\sigma}^m(\mathbf{z})/M \cdot D_{\mathbb{Z},\sigma}^m(\mathbf{z} \pm \mathbf{Sc})$

$$\xrightarrow{\mathbf{z}}$$

Rejection probability is such that $\mathbf{z} \sim D_{\mathbb{Z},\sigma}^m$ is independent from $\mathbf{S}$.

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sk : $\mathbf{S} \in \mathbb{Z}_{2q}^{m \times k}$, short

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$\mathbf{T} = \mathbf{AS} = q\,\mathbf{Id}$

Sample $\mathbf{y} \leftarrow D^m_{\mathbb{Z},\sigma}$, short

$\xrightarrow{\mathbf{w} = \mathbf{Ay} \in \mathbb{Z}^n}$

Sample $\mathbf{c} \in \mathbb{Z}^k$, short

$\xleftarrow{\mathbf{c}}$

$\mathbf{z} = \mathbf{y} \pm \mathbf{Sc}$

Abort with proba.

$D^m_{\mathbb{Z},\sigma}(\mathbf{z})/M \cdot D^m_{\mathbb{Z},\sigma}(\mathbf{z} \pm \mathbf{Sc})$

$\xrightarrow{\mathbf{z}}$

Check that

$\|\mathbf{z}\|$ is short, and

$\mathbf{Az} = \mathbf{Tc} + \mathbf{w} \bmod q$

Rejection probability is such that $\mathbf{z} \sim D^m_{\mathbb{Z},\sigma}$ is independent from $\mathbf{S}$.

Hardware Implementation of BLISS

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- Polynomial multiplications $\mathbf{A}\mathbf{y}$ in $\mathbb{Z}_q[X]/(\Phi_{2n}(X))$
  Already addressed in [PG13] and [RVM$^+$14] (next talk)

Hardware Implementation of BLISS

The two costly steps are:

- Polynomial multiplications $\mathbf{Ay}$ in $\mathbb{Z}_q[X]/(\Phi_{2n}(X))$
  Already addressed in [PG13] and [RVM$^+$14] (next talk)
- Discrete Gaussian Sampling $\mathbf{y} \leftarrow D^m_{\mathbb{Z},\sigma}$ for large σ
  Needs high precision sampling (learning attacks)
  - Long Floating Points Arith. [GPV08]
  - Slow algorithms [DDLL13]
  - Large tables/trees [DG14]

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Most of our contributions deal with hardware implementation of **Discrete Gaussian Sampling**.

Algorithmic contributions to Gaussian samping

We focus on the fastest method to sample Discrete Gaussian using **Cumulative Distribution Tables (CDT)**.

Operation: Binary search accelerated by guide tables

Problem: Naïve implementation would require $42KB$

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We introduce two new techniques:

- Gaussian sampling by **convolution**
- **Kullback-Leibler-divergence** based security argument

Our algorithm requires a table of $2.1KB$, and is almost as fast

1 Introduction

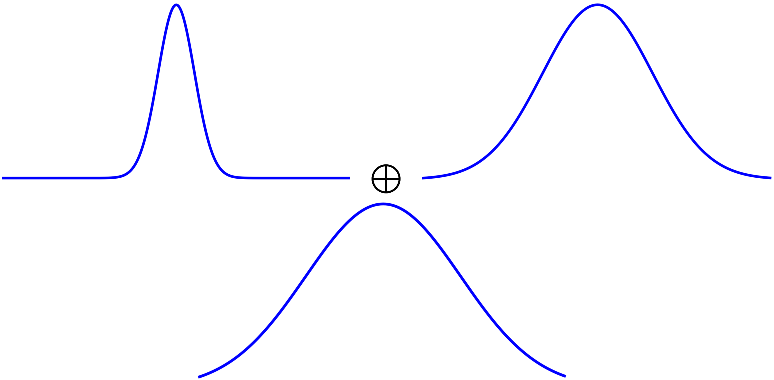
2 Algorithmic contributions

3 Implementation and Performances

4 Conclusion

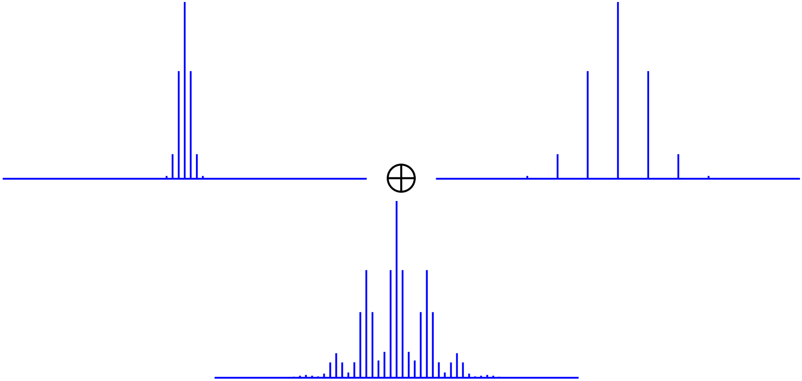
(Discrete) Gaussians convolutions

A convolution of gaussians is a gaussian.



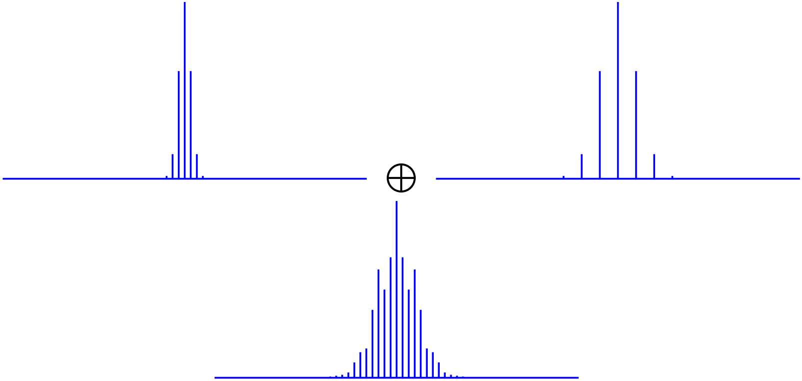
(Discrete) Gaussians convolutions

And what about discrete Gaussians ?



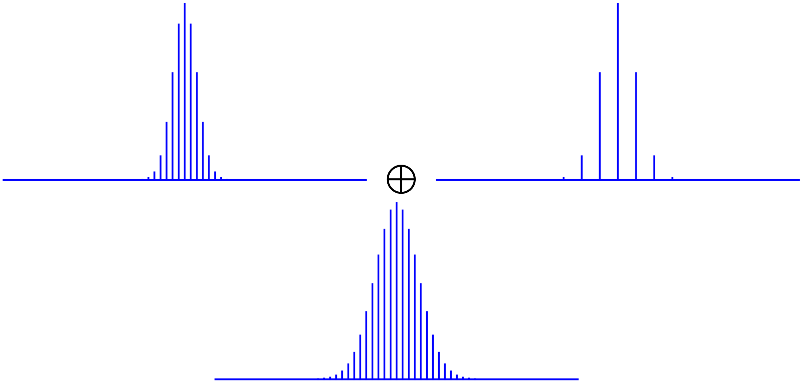
(Discrete) Gaussians convolutions

Well, depending on the parameter...



(Discrete) Gaussians convolutions

It may seem quite Gaussian.



Peikert's Convolution Theorem

Lemma (Adapted from [Pei10])

Let $x_1 \leftarrow D_{\mathbb{Z},\sigma_1}$, $x_2 \leftarrow \cdot D_{\mathbb{Z},\sigma_2}$ *and set* $\sigma_3^{-2} = \sigma_1^{-2} + \sigma_2^{-2}$, *and* $\sigma^2 = \sigma_1^2 + \sigma_2^2$. *If* $\sigma_1 \geqslant \omega(\sqrt{\log n})$ *and* $\sigma_3 \geqslant k \cdot \omega(\sqrt{\log n})$, *then:*

$$x_1 + kx_2 \simeq D_{\mathbb{Z},\sigma}.$$

Application to BLISS-I: We can sample two variables x_1, x_2 of deviation $\sigma' = 19.5$ to obtain $x = x_1 + 11x_2$, of deviation $\sigma = 215$.

Impact: Size of table is reduced from 42KB to 4.5KB.
Running time is less than doubled (binary search is faster for σ').

Kullback-Leibler divergence

Definition (Kullback-Leibler Divergence)

Let $\mathcal{P}$ and $\mathcal{Q}$ be distribution over S. The KL divergence, of $\mathcal{Q}$ from $\mathcal{P}$ is defined as:

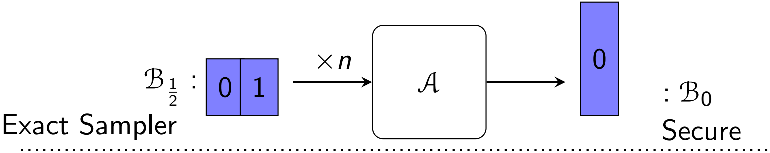
$$D_{\mathbf{KL}}(\mathcal{P}\|\mathcal{Q}) = \sum_{i\in S} \ln\left(\frac{\mathcal{P}(i)}{\mathcal{Q}(i)}\right)\mathcal{P}(i).$$

KL-divergence allows the same arguments as Statistical Distance:

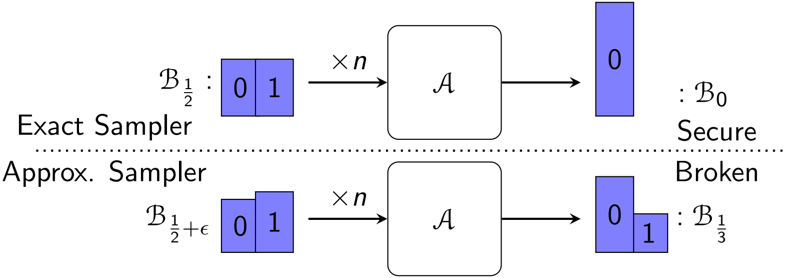
Fact (Addivity and Datta Processing inequality)

- $D_{\mathbf{KL}}(\mathcal{P}_0 \times \mathcal{P}_1 \| \mathcal{Q}_0 \times \mathcal{Q}_1) = D_{\mathbf{KL}}(\mathcal{P}_0\|\mathcal{Q}_0) + D_{\mathbf{KL}}(\mathcal{P}_1\|\mathcal{Q}_1)$
- *for any function* f: $D_{\mathbf{KL}}(f(\mathcal{P})\|f(\mathcal{Q})) \leqslant D_{\mathbf{KL}}(\mathcal{P}\|\mathcal{Q})$

Kullback-Leibler vs. Statistical distance: Example



Kullback-Leibler vs. Statistical distance: Example



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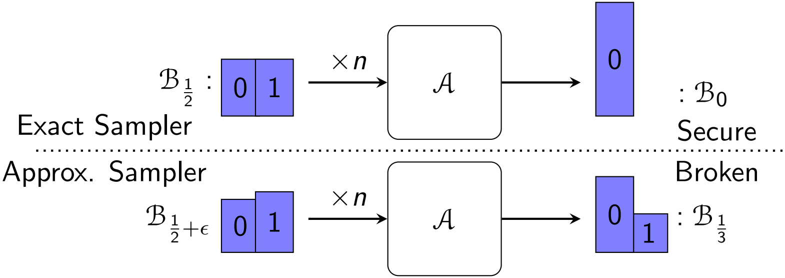
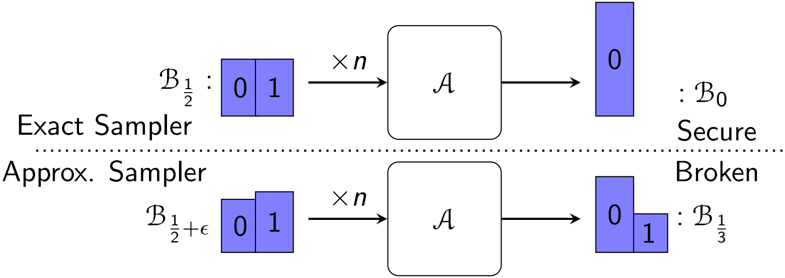


Statistical distance argument

$$\Delta(\mathcal{B}_{\frac{1}{2}}, \mathcal{B}_{\frac{1}{2}+\epsilon}) = \Theta(\epsilon), \qquad \Delta(\mathcal{B}_0, \mathcal{B}_{\frac{1}{3}}) = \Theta(1)$$

$$n \geqslant \Theta(1/\epsilon)$$

Kullback-Leibler vs. Statistical distance: Example



KL-Divergence argument

$$D_{\mathbf{KL}}(\mathcal{B}_{\frac{1}{2}} \| \mathcal{B}_{\frac{1}{2}+\epsilon}) = \Theta(\epsilon^2), \qquad D_{\mathbf{KL}}(\mathcal{B}_0 \| \mathcal{B}_{\frac{1}{3}}) = \Theta(1)$$

$$n \geqslant \Theta(1/\epsilon^2)$$

Kullback-Leibler vs. Statistical distance: Rule of Thumb

Δ Averaged **absolute error**

$$\Delta(\mathcal{P}, \mathcal{Q}) = \frac{1}{2}\sum_i |\mathcal{P}(i) - \mathcal{Q}(i)|$$

$D_{\mathbf{KL}}$ Averaged squared **relative error**

$$D_{\mathbf{KL}}(\mathcal{P}\|\mathcal{Q}) \leqslant 2\sum_i \left|\frac{\mathcal{P}(i) - \mathcal{Q}(i)}{\mathcal{P}(i)}\right|^2 \mathcal{P}(i).$$

Limits of KL-divergence

- $D_{\mathbf{KL}}$ is not symmetric
- Can be worse than Δ (e.g. Tailcutting)
- Improvements only for reduction to **Search Problems**

Truncating Cumulative Distribution Table

Full CDT

```
11111100001010111001011011000010110110100011001010100001110011001110111010110000
10110001100110100001010001110011010010101110001010101101010100101111110000011001
01110001001010001100101100010001011100100100101111111101000001111011101011001101
01000000101100011101100111000110101110111110011100010101000111100010101110111011
00100000111110011111110100000001001010011101000011010101101011101111001010101111
00001110111010011000100111111100111010111000000101000100011010001001111001000011
00000101111101011010011011000010000000100010110001000100010101110100111110101010
00000010000110010011101100010101011110000001100100011001101100110100011101000101
00000000101001100100101111001101010110100101100011111001000000000100100100110011
00000000010110100101010100011110000000001001000000100110001100011011000011010000 0
00000000000010101011111110110011101010101110110000001110110110010010110011111011 1
00000000000000100011110011101111011010001010101000100001001111011001111000000001
00000000000000000110100001001100111010111100000000010110101011110100110101001000
00000000000000000001000010010110000110100110111111001011000111000011001011101001
00000000000000000000001001001101011101001100100011111100000101010011001010010100
00000000000000000000000001000111011001010010111101010100110110111111111101101011
00000000000000000000000000001111000101010001101101000001100110110010000111110111
00000000000000000000000000000001011000111000010111001011000011001000000000111010
00000000000000000000000000000000001110010000110111101010111101000101111001011 1
00000000000000000000000000000000000011111110100000010101101111111011100110 1
00000000000000000000000000000000000000011111000101001101110100001110011111 0
000000000000000000000000000000000000000000110100101110101110011011101000 01
00000000000000000000000000000000000000000000001001101111000011100000011101 1
0000000000000000000000000000000000000000000000000001100100001000010010010 1
000000000000000000000000000000000000000000000000000000111000000001010000 0
0000000000000000000000000000000000000000000000000000000000110110100011 0
000000000000000000000000000000000000000000000000000000000000000010111000
```

Truncating Cumulative Distribution Table

Truncating left-most zeros

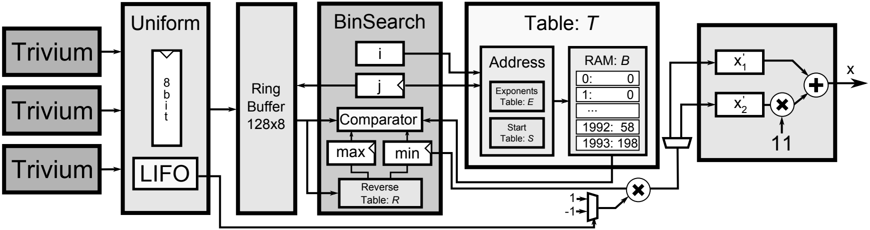
```
1111110000101011100101101100001011011010001100101010000111001100111011101011 0000
1011000110011010000101000111001101001010111000101010110101010010111111000001 1001
0111000100101000110010110001000101110010010010111111110100000111101110101100 1101
0100000010110001110110011100011010111011111001110001010100011110001010111011 1011
0010000011111001111111010000000100101001110100001101010110101110111100101010 1111
0000111011101001100010011111110011101011100000010100010001101000100111100100 0011
0000010111111010110100110110000100000001000101100010001000101011101001111010 1010
0000001000011001001110110001010101111000000110010001100110110011010001110100 0101
0000000010100110010010111100110101011010010110001111100100000000010010010011 0011
0000000000101101001010101000111100000000100100000100110001100011011000011010 0000
0000000000001010101111110110011101010101110110000001110110110010010110011111 0111
0000000000000010001111001110111101101000101010100010000100111101100111100000 0001
0000000000000000011010000100110011101011110000000000101101010111101001101010 1000
0000000000000000000100001001011000011010011011111100101100011100001100101110 1001
0000000000000000000000100100110101110100110010001111110000010101001100101001 0100
0000000000000000000000000100011101100101001011110101010011011011111111110110 1011
0000000000000000000000000000011110001010100011011010000011001101100100001111 0111
0000000000000000000000000000000010110001110000101110010110000110010000000011 1010
0000000000000000000000000000000000001110010000110111101010111101000101111001 0111
0000000000000000000000000000000000000000111111110100000010101101111111011100 1101
0000000000000000000000000000000000000000000011111000101001101110100001110011 1110
0000000000000000000000000000000000000000000000001101001011101011100110111010 0001
0000000000000000000000000000000000000000000000000000100110111110000111000001 1011
0000000000000000000000000000000000000000000000000000000001100100001000010010 0101
0000000000000000000000000000000000000000000000000000000000000111000000000101 0000
0000000000000000000000000000000000000000000000000000000000000000011011010001 10
000000000000000000000000000000000000000000000000000000000000000000000000101110 00
```

Truncating Cumulative Distribution Table

Truncating right-most bits using KL-divergence

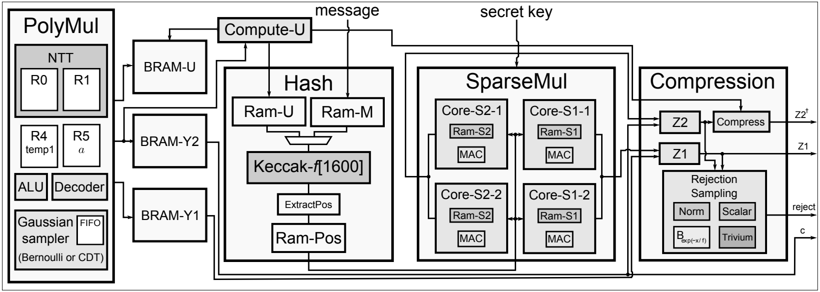
```
1111110000101011100101101100001011011010001100101010000111001100111011101011 0000
10110001100110100001010001110011010010101110001010101101010100101111110000011001
01110001001010001100101100010001011100100100101111111101000001111011101011001101
01000000101100011101100111000110101110111110011100010101000111100010101110111011
00100000111110011111110100000001001010011101000011010101101011101111001010101111
00001110111010011000100111111100111010111000000101000100011010001001111001000011
00000101111110101101001101100001000000010001011000100010001010111010011111010101 0
00000010000110010011101100010101011110000001100100011001101100110100011101000101
00000000101001100100101111001101010110100101100011111001000000000100100100110011
00000000001011010010101010001111000000000100100000100110001100011011000011010000 0
00000000000010101011111101100111010101011101100000011101101100100101100111110111
00000000000000100011110011101111011010001010101000100001001111011001111000000001
00000000000000000110100001001100111010111100000000010110101011110100110101001000
00000000000000000010000100101100001101001101111110010110001110000110010111010 01
00000000000000000000001001001101011101001100100011111100000101010011001010010100
00000000000000000000000010001110110010100101111010101001101101111111110110 1011
00000000000000000000000000001111000101010001101101000011001101100100001111 0111
00000000000000000000000000000001011000111000010111001011000011001000000000111010
00000000000000000000000000000000000111001000011011110101011110100010111100101 11
00000000000000000000000000000000000001111111101000000101011011111110111001101
00000000000000000000000000000000000000001111100010100110111010000111001111 10
00000000000000000000000000000000000000000001101001011101011100110111010000 1
00000000000000000000000000000000000000000000000100110111100001110000011101 1
00000000000000000000000000000000000000000000000000110010000100001001001 01
00000000000000000000000000000000000000000000000000000011000000001010000
00000000000000000000000000000000000000000000000000000000010110 1000110
00000000000000000000000000000000000000000000000000000000000010111000
```

FPGA Implementation of BLISS-I : CDT Sampler



- Ring-buffer to store random numbers generated by Trivium instantiation
- Binary-search component operates on block RAM B
- **Biggest challenge:** Critical path of binary search

FPGA Implementation of BLISS-I : Signing



- Number theoretic transform (NTT) multiplier ($\mathbf{a}\mathbf{y}_1$)
- Keccak-1600 hash function
- Fast sparse multiplier ($\mathbf{z}_1 = \mathbf{s}_1\mathbf{c} + \mathbf{y}_1, \mathbf{z}_2 = \mathbf{s}_2\mathbf{c} + \mathbf{y}_2$)

FPGA Implementation of BLISS-I : Results

Algorithm	LUT	FF	BRAM	DSP	OPs/s
BLISS-1[Sign]	7,491	7,033	7.5	6	7.9k signs/s
BLISS-1[Ver]	5,275	4,488	4.5	3	14,4k verifs/s
CDT-Sampler	928	1,121	1	0	17,4 M samp./s
Bernoulli Sampler	1,178	1,183	0	1	7,4 M samp./s

- Results are given for a 1024-bit message on Spartan6-LX25-3
- High-speed signing and verification
- DT sampler is twice as fast as Bernoulli sampler for similar resource consumption

FPGA Implementation of BLISS-I : Comparision

Algorithm	LUT	FF	BRAM	DSP	OPs/s
BLISS-1[Sign]	7,491	7,033	7.5	6	7,958 signs/sec
BLISS-1[Ver]	5,275	4,488	4.5	3	14,438 signs/sec
GLP-1[Sign/Ver]	6,088	6,804	19.5	4	1,627/7,438
RSA-2048 [Sign]	4190 slices		7	17	79
Curve25519	2783	3592	2	20	2518
ECDSA-256 [Sign/Ver]	32,299 LUT/FF pairs		0	0	139/110

- Implementation faster that RSA and prime curve ECC/ECDSA
- Faster, shorter and more secure than GLP lattice-signature
- Reasonable area consumption

Conclusion

Lattice-based Crypto is ready.

- BLISS compares to standardized signature schemes, in both Software and Hardware
- New geometric analysis will make it **even faster** [D14, To appear, see Rump Session]

Time has come for **standardization** of lattice-based cryptography.

- Provide alternative/fallbacks to ECC/RSA
- Mostly **unpatended**
- Motivate more work:
  Comprehensive Cryptanalysis, Improved Algorithms,
  Lightweight Implementation, Side Channel Attacks, ...

Open Access, Open-Sources

Full and updated paper:

`http://eprint.iacr.org/2014/254`

Software implementation:

`http://bliss.di.ens.fr/`

Hardware implementation:

`http://www.sha.rub.de/research/projects/lattice/`

Thanks !

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