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Enhanced Lepton Flavour Violation with Massless Neutrinos: A Study of Muon and Tau Decays

M. C. Gonzalez-Garcia[1]

and

J. W. F. Valle[2]

Instituto de Física Corpuscular - C.S.I.C.
Departament de Física Teòrica, Universitat de València
46100 Burjassot, València, SPAIN

Abstract

Lepton flavour violating rates can be experimentally measurable even if the observed neutrinos are strictly massless. We make a study of the attainable rates for anomalous leptonic muon and tau number violating decays such as $\mu \to e\gamma$, $\mu \to 3e$, $\tau \to \mu\gamma$, $\tau \to e\gamma$, $\tau \to \mu\mu^+\mu^-$, $\tau \to ee^+e^-$, $\tau \to e\mu^+\mu^-$, *etc.* as well as semileptonic lepton flavour violating tau decays such as $\tau \to \mu\pi^0$, $\tau \to e\pi^0$, $\tau \to e\eta$, *etc.* All muonic violating decays can be as large as the present limits from LAMPF, TRIUMF and PSI. The corresponding tau violating processes can all be at the limit of sensitivity of the upcoming τ factories.

[1] E-mail CONCHA at EVALUN11 or 16444::CONCHA
[2] E-mail VALLE at EVALUN11 or 16444::VALLE

1 Introduction

Although in the standard model all lepton flavours are exactly conserved, nothing is sacred about these conservation laws when one considers models beyond the standard model. Thus looking for such rare decays is a way to search for so far unknown interactions. The most conventional way to induce lepton flavour violation is through nonzero neutrino masses [1]. Flavour violation arises in this case from the impossibility of eliminating mixing from the leptonic charged current of the physical leptons, in a way analogous to the case of quarks. Although model dependent, the expected lepton flavour violating rates will generally tend to be very much restricted due to the smallness of neutrino masses that follows from laboratory, astrophysics and cosmology [1]. As a result, this is the most restrictive way to induce lepton flavour violating processes, given the tight observational limits on neutrino masses [2].

It has recently been noted [3] that neutrinos can be "protected" from acquiring a mass by some symmetry such as the conservation of the *total* lepton number, while leaving the door open to the possible violation of *partial* leptonic flavours [12] and CP [13, 14]. The possibility of having lepton flavour violation in models with massless neutrinos is specially interesting, not only conceptually, but also practically, since the stringent constraints on the resulting rates that follow from observational limits on neutrino masses are avoided altogether, thus allowing them to be large [2].

Either way, the observation of any lepton flavour violating process would imply physics beyond the standard model and thus such processes have been vigorously searched for experimentally. The prime examples of lepton flavour violation in purely leptonic processes are the decays $\mu \to e\gamma$ and $\mu \to 3e$. The most stringent limits on these processes arise from LAMPF, TRIUMF and PSI. The experimental limit on the $\mu \to e\gamma$ decay branching ratio is

$$B(\mu \to e\gamma) \leq 4.9 \times 10^{-11} \quad [4] \tag{1}$$

In the near future one hopes to reach sensitivities up to $B(\mu \to e\gamma) \sim 10^{-13}$. Another purely leptonic process is the decay $\mu \to 3e$. The corresponding experimental limit is

$$B(\mu \to 3e) \leq 1.0 \times 10^{-12} \quad [5] \tag{2}$$

There are also good limits on semileptonic muon violating processes such as μe conversion in atoms, which are likely to be improved with the construction of the SINDRUM II spectrometer at PSI [6]. Limits on flavour violating processes involving τ's are much poorer than the corresponding limits on muon violation, but one hopes to improve them [7] with the upcoming τ factories [8]. Some of these limits are:

$$\begin{array}{ll}
B(\tau \to \mu\gamma) \leq 5.5 \times 10^{-4} & [9] \\
B(\tau \to e\gamma) \leq 2.0 \times 10^{-4} & [10] \\
B(\tau \to \mu\mu^+\mu^-) \leq 2.9 \times 10^{-5} & [11] \\
B(\tau \to ee^+e^-) \leq 3.3 \times 10^{-5} & [11] \\
B(\tau \to e^+e^-) \leq 3.3 \times 10^{-5} & [11] \\
B(\tau \to e\mu^+\mu^-) \leq 3.8 \times 10^{-5} & [11] \\
B(\tau \to \mu\pi^0) \leq 8.2 \times 10^{-4} & [9] \\
B(\tau \to e\pi^0) \leq 1.4 \times 10^{-4} & [10] \\
B(\tau \to \mu K^0) \leq 1.0 \times 10^{-3} & [9] \\
B(\tau \to eK^0) \leq 1.3 \times 10^{-3} & [9] \\
B(\tau \to e\eta) \leq 2.4 \times 10^{-4} & [10]
\end{array} \tag{3}$$

As discussed above, when lepton flavour violation occurs *unrelated* to neutrino mass the corresponding branching ratios can be large. In ref. [18, 14] this enhancement was shown to take place in high energy processes such as $Z \to e + \bar{\tau}$ and $Z \to \mu + \bar{\tau}$ that might be observable at LEP with upgraded luminosity [18]. In this letter we will study the case of low energy processes. We will consider explicitly the attainable lepton flavour violation rates for the processes of eq. (1) and eq. (2) as well as for the tau decays in the third equation. We will demonstrate that all these processes may be well within reach of planned experimental facilities such as the proposed MEGA experiment at Los Alamos as well as a τ factory.

2 An Illustrative Model

To illustrate our point we consider the $SU(2) \otimes U(1)$ model suggested in ref [12, 13, 14]. The reason for our choice is twofold. First, the simplicity of the model allows for the study of flavour violating rates to be carried out explicitly and the implementation of the constraints on the rates for rare processes can also be performed in a simple way. Second, since neutrinos are massless in this model, the stringent constraints on these rates that follow from observational limits on neutrino masses are avoided. For completeness we recall the basics of the model. It contains the following set of (left-handed) leptons (repeated over generation index, i)

$$\begin{pmatrix} \nu_i \\ e_i \end{pmatrix}, {e^c}_i,\ {\nu^c}_i,\ S_i \tag{4}$$

where ν^c denotes a RH neutrino * and we have included in addition to right handed neutrinos, an equal number of gauge singlet leptons S_i. The quark sector is completely standard. In the presence of these extra singlets the masslessness of neutrinos is ensured by imposing the conservation of total lepton number, the same global symmetry that keeps neutrinos massless in the standard model. The difference is that here it is imposed while there it is automatic. This restriction leads to the following form for the neutral mass matrix (in the basis ν, ν^c, S)

$$\begin{pmatrix} 0 & D & 0 \\ D^T & 0 & M \\ 0 & M^T & 0 \end{pmatrix} \tag{5}$$

It is easy to see that the three light neutrinos are massless *Weyl* neutrinos while the other 6 neutral 2-component leptons combine exactly into 3 heavy *Dirac* isosinglet fermions. Individual leptonic flavour is however violated *despite physical neutrinos being strictly massless* as has been discussed in ref [12, 14].

The matrix given in eq. (5) has been suggested in *different* theoretical models [19], but specially and most naturally in a large class of superstring inspired models [20].

*We describe right-handed fermions in charge-conjugate notation

In this case not only the extra singlet fermions are present, but also one is often led to a mass matrix of the above form [3, 20], since the restricted set of Higgs fields available can not provide the usual Majorana mass terms needed in the simplest seesaw mechanism [20].

The isosinglet neutral heavy leptons (NHLS) couple to the standard $SU(2) \otimes U(1)$ weak bosons through mixing with ordinary isodoublet neutrinos. For example the charged leptonic weak interaction is given by

$$-\frac{g}{\sqrt{2}} W^\mu \sum_{\alpha\beta} \bar{e}_{L\alpha}\, \gamma_\mu\, K_{\alpha\beta}\, N_{L\beta} \tag{6}$$

It is completely described by a *rectangular* matrix K [21], which may be decomposed as

$$K = (K_L, K_H) \tag{7}$$

where the 3×3 K_L matrix couples the charged leptons to the massless neutrinos while K_H couples them to the NHLS. An explicit parametrization of the weak charged current mixing matrix K has been given in ref. [13, 14]. It involves in general n^2 mixing angles plus $(n-1)^2$ phases that could violate CP despite the fact that all neutrinos coupled in K_L are massless [13]. Here we neglect the possible effect of these CP violating phases.

The neutral current may be expressed in the following general form

$$-\frac{g'}{2\sin\theta_W} Z^\mu \sum_{\alpha\beta} \bar{N}_{L\alpha}\, \gamma_\mu\, P_{\alpha\beta}\, N_{L\beta} \tag{8}$$

where the matrix P is a projective hermitian matrix $P^2 = P = P^\dagger$, directly determined in terms of eq. (7) as

$$P = \begin{pmatrix} K_L^\dagger K_L & K_L^\dagger K_H \\ K_H^\dagger K_L & K_H^\dagger K_H \end{pmatrix} \tag{9}$$

The different entries in this matrix determine the neutral current couplings of the full set (denoted $N_{L\alpha}$) of mass-eigenstate neutral leptons. The admixture of NHLS in the weak currents leads to a violation of the GIM mechanism and a consequent enhancement in the rates for rare processes of interest to us here. It also leads in general to violations of weak universality. These limit the attainable values of the K_H matrix elements, as follows

$$
\begin{aligned}
(K_H K_H^\dagger)_{ee} &\lesssim 4.3 \times 10^{-2} \\
(K_H K_H^\dagger)_{\mu\mu} &\lesssim 0.8 \times 10^{-2} \\
(K_H K_H^\dagger)_{\tau\tau} &\lesssim 10 \times 10^{-2}
\end{aligned} \tag{10}
$$

These constraints were thoroughly discussed in ref [15]. They can be cleverly combined as follows [16]

$$
\mid \sum_a K_{Hia} F(x_a) K^*_{Hja} \mid^2 \leq \sum_a \mid K_{Hia} \mid^2 \sum_b \mid K_{Hjb} \mid^2 max(\mid F(x_a) \mid^2) \tag{11}
$$

This form of the writing the universality constraints is useful in order to determine accurately how large the rare μ, τ decay branching ratios can be. In any case, universality limits are still weak enough that a substantial improvement is possible at a high energy accelerator experiment, such as LEP [17].

3 Leptonic decays

Here we consider all decays of the type $l \to \gamma l_i$ as well as the rare 3-body decays $l \to e e^+ e^-$, $l \to \mu e^+ e^-$, $l \to e \mu^+ \mu^-$, $l \to \mu\mu^+\mu^-$, and $l \to e e^+ e^-$. We will denote them collectively by $l \to l_i l_j^+ l_j^-$, where l can be either a muon or a tau.

The partial decay width for the process $l \to \gamma l_i$ is [22, 23]

$$
\Gamma(l \to \gamma l_i) = \frac{\alpha G_F^2 m_l^5}{2^{11}\pi^4} \mid \tilde{F}_\gamma \mid^2 \tag{12}
$$

For the three body decays mentioned above, there are three different sets of 1-loop contributions coming from the diagrams with exchange of a Z^0 (Fig. 1.a), diagrams with photon exchange (Fig. 1.b), and box diagrams with W pair exchange (Fig. 1.c). With all that we get the following contributions to the amplitude of the process $l(p) \to l_i(p') l_j^+(q_1) l_j^-(q_2)$

$(q = p - p' = q_1 + q_2)$

$$
\begin{aligned}
M_b &= -i\frac{G_F^2 M_W^2}{8\pi^2} \sum K_{Hia} F_b(x_a) K^*_{Hla} \bar{u}_i(p')\gamma^\mu(1-\gamma^5)u(p)\bar{u}_j(q_2)\gamma_\mu(1-\gamma^5)v_j(q_1)\,, \\
M_z &= \frac{-iG_F^2 M_W^2}{8\pi^2} \sum F_Z(x_a) K^*_{Hla} K_{Hia} \bar{u}_i(p')\gamma^\mu(1-\gamma^5)u(p)\bar{u}_j(q_2)\gamma^\mu(1-4s_W^2-\gamma^5)v_j(q_1)\,, \\
M_\gamma &= \frac{-iG_F^2 s_W^2}{2\pi^2} \sum K^*_{Hla} K_{Hia} \bar{u}_i(p')\{F_1^\gamma(x_a)\gamma^\mu(1-\gamma^5) + \tfrac{m_l}{q^2} F_2^\gamma(x_a) \not{q}\gamma^\mu(1+\gamma^5)\} u(p)\bar{u}_j(q_2)\gamma_\mu v_j(q_1)\,.
\end{aligned} \tag{13}
$$

It is convenient to split up the sum of these amplitudes in a different way,

$$
M = M_b + M_z + M_\gamma = M_1 + M_2 + M_3\,, \tag{14}
$$

where

$$
\begin{aligned}
M &= M_1 + M_2 + M_3 \\
M_1 &= -i\frac{G_F^2 M_W^2}{2\pi^2}(\tilde{F}_{bil} + \tilde{F}_{Z^0 il})[\bar{l}_i\gamma^\mu L l][\bar{l}_j\gamma_\mu L l_j] \\
M_2 &= i\frac{G_F^2 M_W^2 \sin^2\theta_W}{\pi^2}(\tilde{F}_{Z^0 il} - \tilde{F}^\gamma_{1il})[\bar{l}_i\gamma^\mu L l][\bar{l}_j\gamma_\mu l_j] \\
M_3 &= -i\frac{G_F^2 M_W^2 \sin^2\theta_W m_l}{\pi^2 q^2}(\tilde{F}^\gamma_{2il})[\bar{l}_i q\gamma^\mu R l][\bar{l}_j\gamma_\mu l_j]
\end{aligned} \tag{15}
$$

where we have defined

$$
\tilde{F}_{ij} = \sum_a K_{Hia} F(x_a) K^*_{Hja} \tag{16}
$$

with $x_a = \frac{m_{N_a}^2}{M_W^2}$. The form factors have been calculated by Inami and Lim [24]

$$
\begin{aligned}
F_\gamma(x) &= -x\frac{1-5x-2x^2}{(1-x)^3} + \frac{6x^3 \ln x}{(1-x)^4}\,, \\
F_b(x) &= \frac{x}{1-x} + \frac{x\ln x}{(1-x)^2}\,, \\
F_{Z^0}(x) &= -\frac{1}{2}\left[\frac{5x}{1-x} + \frac{x(2+3x)}{(1-x)^2}\ln x\right], \\
F_1^\gamma(x) &= -\left[\frac{7}{12(x-1)} + \frac{13}{12(x-1)^2} - \frac{1}{2(x-1)^3}\right]x - \left[\frac{1}{6(x-1)} - \frac{7}{6(x-1)^2} - \frac{5}{6(x-1)^3} + \frac{1}{2(x-1)^4}\right]x\ln x\,, \\
F_2^\gamma(x) &= \left[\frac{1}{2(x-1)} + \frac{9}{4(x-1)^2} + \frac{3}{2(x-1)^3}\right]x - \frac{3x^3}{2(x-1)^4}\ln x = -\frac{1}{4}F_\gamma(x)\,.
\end{aligned} \tag{17}
$$

The decay rate for these processes is then

$$\Gamma(l \to l_i l_j^+ l_j^-) = \frac{G_F^4 M_W^4 m_l^5}{3x2^{11}\pi^7}\{|(\bar{F}_{bil} + \bar{F}_{Z^0il}) - 2\sin^2\theta_W(\bar{F}_{Z^0il} - \bar{F}_{1il}^\gamma)|^2 + 4\sin^4\theta_W\,|\bar{F}_{Z^0il} - \bar{F}_{1il}^\gamma|^2 - 8\sin^2\theta_W Re[(\bar{F}_{Z^0il} + \bar{F}_{bil})\bar{F}_{2il}^{\gamma*}] + 32\sin\theta_W^4 Re[(\bar{F}_{Z^0il} - \bar{F}_{1il}^\gamma)\bar{F}_{2il}^{\gamma*}] - |\bar{F}_{2il}^\gamma|^2\sin^4\theta_W[8\ln\frac{4m_l^2}{m_i^2} + \frac{44}{3}]\} \tag{18}$$

This result is valid for the case $i \neq j$. For the case $i = j$ we have two different possible assignments for the external momenta corresponding to

$$l(p) \to l_i^-(p')l_i^+(q_1)l_i^-(q_2) \quad q = p - p' = q_1 + q_2 \quad (a)\,,$$
$$l(p) \to l_i^-(q_2)l_i^+(q_1)l_i^-(p') \quad q = p - q_2 = q_1 + p' \quad (b)\,. \tag{19}$$

From the symmetrization rule for the case of identical particles we get

$$\Gamma = \frac{1}{2}\Gamma^{a+b}\,, \tag{20}$$

where Γ^{a+b} denotes the width obtained by adding the amplitudes corresponding to the two different contributions. After some algebra we get

$$\Gamma(l \to l_i l_i^+ l_i^-) = \frac{G_F^4 M_W^4 m_l^5}{3x2^{11}\pi^7}\{2\,|(\bar{F}_{bil} + \bar{F}_{Z^0il}) - 2\sin^2\theta_W(\bar{F}_{Z^0il} - \bar{F}_{1il}^\gamma)|^2 - 12\sin^2\theta_W Re[(\bar{F}_{Z^0il} + \bar{F}_{bil})\bar{F}_{2il}^{\gamma*}] - |\bar{F}_{2il}^\gamma|^2\sin^4\theta_W[8\ln\frac{4m_l^2}{m_i^2} + \frac{44}{3} + 12] + 64\sin\theta_W^4 Re[(\bar{F}_{Z^0il} - \bar{F}_{1il}^\gamma)\bar{F}_{2il}^{\gamma*}]\}\,. \tag{21}$$

The strength of these processes is constrained only by the laboratory limits on the size of the K_{Hij} matrix elements given in eq. (10) and not by the neutrino masses, that vanish identically [12]. This leads to observable rates. In order to estimate them we use the inequality in eq. (11).

In Fig. 2 we plot the attainable branching ratio for the process $\mu \to ee^+e^-$ after imposing also the constraint on the branching ratio of the decay $\mu \to e\gamma$ eq. (1) which is stronger than the universality limits for large NHL mass values. Our result for the attainable $\mu \to e\,\gamma$ branching ratio is clearly at the level of sensitivity of *present* experiments. The upcoming MEGA experiment could therefore be used to place important

constraints on the parameters of our model. For the case of τ decays the constraints on the K_H that follow from universality are weaker and as a result the branching ratios can be bigger, as we can see in Fig. 3. Here however, only with a τ factory can one really probe meaningfully these branching ratios.

4 Semileptonic Decays

Here we consider the decays $\tau \to S^0 l$, where S^0 denotes a pseudoscalar meson. This includes the decays $\tau \to \pi^0 l$, $\tau \to \eta l$, and $\tau \to K^0 l$. The first two have contributions coming from diagrams with interchange of the Z^0, and from the box diagrams. The last one only has a contribution from the box diagrams. The contributions to all of them come only from the axial part of the interaction, because of the pseudoscalar character of the mesons. Therefore the photon exchange does not contribute. The procedure is then to adapt the calculation for the leptonic decay to the process $l_i \to q_m \bar{q}_n l_j$ and later to sandwich it between the mesonic states of interest

$$<S^0 l_j \mid M \mid l_i>\,.$$

For the diagrams with photon and Z exchange this is trivially done by just modifying the corresponding vertex and one gets the amplitudes

$$M_\gamma = \frac{iG_F^2 s_W^2 Q_n}{2\pi^2}\sum_a K_{Hia}^* K_{Hja}\bar{u}_j\{F_1^\gamma(x_a)\gamma^\mu(1-\gamma^5) + \frac{m_i}{q^2}F_2^\gamma(x_a)\not{q}\gamma^\mu(1+\gamma^5)\}u_i\bar{u}_{q_n}\gamma_\mu v_{q_n}\,, \tag{22}$$

$$M_Z = \frac{iG_F^2 M_W^2}{4\pi^2}\sum_a K_{Hia}^* K_{Hja}F_Z(x_a)\bar{u}_j\gamma^\mu(1-\gamma^5)u_i\bar{u}_{q_n}\gamma_\mu(g_v^{q_n} - g_a^{q_n}\gamma^5)v_{q_n}\,. \tag{23}$$

Both are valid for either up or down-type quarks and are manifestly diagonal in quark flavour. For the case of the box diagrams we have to substitute two external legs and one of the internal fermion lines by the corresponding quark lines. The value of the corresponding amplitudes will be the same as for the leptonic decay after substituting $K_{ja}K_{ja}^* \to V_{hm}V_{hn}^*$ for the case of down quarks and $K_{ja}K_{ja}^* \to V_{nh}V_{mh}^*$ for the up quarks where V is the Cabbibo Kobayashi Maskawa (CKM) matrix. With all this we have

$$M_b = -i\frac{G_F^2 M_W^2}{8\pi^2}\sum_{a,h} K_{ja}K_{ia}^* I(x_a, x_h)\begin{bmatrix} V_{hm}V_{hn}^* \\ V_{nh}V_{mh}^* \end{bmatrix}[\bar{u}_j\gamma^\mu(1-\gamma^5)u_i]\begin{bmatrix} \bar{d}_m\gamma_\mu(1-\gamma^5)d_n \\ \bar{u}_m\gamma_\mu(1-\gamma^5)u_n \end{bmatrix}. \tag{24}$$

As a good approximation we set, for the case of d_h, $x_h = 0$ for $h = 1\ldots 3$ while, for u_h, we use $x_1 = x_2 = 0$ and $x_3 = x_{top} = \frac{m_{top}^2}{M_W^2}$. Using also the unitarity of the CKM matrix we get for the up quarks one finds

$$M_b^{u_n\bar{u}_m} = -i\frac{G_F^2 M_W^2}{8\pi^2}\delta_{mn}\bar{F}_{bij}\bar{u}_j\gamma^\mu(1-\gamma^5)u_i\bar{u}_m\gamma_\mu(1-\gamma^5)u_n\,, \tag{25}$$

whereas for the down quarks the amplitude is

$$M_b^{d_n\bar{d}_m} = -i\frac{G_F^2 M_W^2}{8\pi^2}[\delta_{mn}\bar{F}_{bij} - V_{tm}V_{tn}^*(\bar{F}_{bij} - \bar{F}_{bij}^t)]\bar{u}_j\gamma^\mu(1-\gamma^5)u_i\bar{d}_m\gamma_\mu(1-\gamma^5)d_n\,. \tag{26}$$

where

$$\begin{aligned} F_b^t(x) &= I(x, x_t) - I(0, x_t) \\ I(x,y) &= \tfrac{1}{x-y}\{(1+\tfrac{xy}{4})[\tfrac{1}{1-x} + \tfrac{x^2\ln x}{(1-x)^2} - (x\leftrightarrow y)] - 2xy[\tfrac{1}{1-x} + \tfrac{x\ln x}{(1-x)^2} - (x\leftrightarrow y)]\}\,. \end{aligned} \tag{27}$$

We now sandwich the amplitudes at the quark level between the vacuum and the final mesonic state. For the decay $l_i \to K^0 l_j$ the only contributing amplitude will be $M_b^{d\bar{s}}$, since the photon and Z exchange amplitudes are diagonal in quark flavour. Taking into account that

$$<0 \mid \bar{d}\gamma^\mu(1-\gamma^5)s \mid K^0> = -<0 \mid \bar{d}\gamma^\mu\gamma^5 s \mid K^0> =$$

$$= -\sqrt{2}f_\pi <0 \mid \partial^\mu \bar{K}^0 \mid K^0> = -\sqrt{2}f_\pi p_{K^0}^\mu\,,$$

we find that the decay width is given by

$$\Gamma(l_i \to K^0 l_j) = \frac{G_F^4 M_W^4 f_\pi^2}{2^6\pi^5} \mid V_{td}\mid^2\mid V_{ts}\mid^2\mid \bar{F}_{bij} - \bar{F}_{bij}^t \mid^2 m_i^3[1 - \frac{m_K^2}{m_i^2}]\,. \tag{28}$$

The decay $\tau \to K^0 l$ is therefore doubly suppressed due to the unitarity of the CKM matrix and it is exactly zero in the limit of light top. The branching ratio for this decay is always below 10^{-10}.

For the decays $l_i \to \pi^0 l_j$ and $l_i \to \eta l_j$ we know that the diagrams with photon exchange do not contribute since the electric current between the quarks is vector-like and it does not give any contribution to processes with one meson. Taking into account that

$$\begin{aligned} &<0 \mid \bar{u}\gamma^\mu\gamma^5 u \mid \pi^0> = f_\pi p_{\pi^0}\,, && <0 \mid \bar{u}\gamma^\mu\gamma^5 u \mid \eta> = \tfrac{f_\pi}{\sqrt{3}}p_\eta\,, \\ &<0 \mid \bar{d}\gamma^\mu\gamma^5 d \mid \pi^0> = -f_\pi p_{\pi^0}\,, && <0 \mid \bar{d}\gamma^\mu\gamma^5 d \mid \eta> = \tfrac{f_\pi}{\sqrt{3}}p_\eta\,, \\ &<0 \mid \bar{s}\gamma^\mu\gamma^5 s \mid \pi^0> = 0\,, && <0 \mid \bar{s}\gamma^\mu\gamma^5 s \mid \eta> = -\tfrac{2f_\pi}{\sqrt{3}}p_\eta\,, \end{aligned}$$

we have the following amplitudes

$$\begin{aligned} M_Z^{\pi^0} &= -\tfrac{iG_F^2 M_W^2 f_\pi}{4\pi^2}\bar{F}_{Zij}\bar{u}_j\gamma_\mu(1-\gamma^5)u_i p_{\pi^0}^\mu \pi^0\,, \\ M_Z^{\eta} &= -\tfrac{iG_F^2 M_W^2 f_\pi}{4\sqrt{3}\pi^2}\bar{F}_{Zij}\bar{u}_j\gamma_\mu(1-\gamma^5)u_i p_\eta^\mu \eta\,, \\ M_b^{\pi^0} &= -\tfrac{iG_F^2 M_W^2 f_\pi}{8\pi^2}\bar{F}_{bij}^{\pi^0}\bar{u}_j\gamma_\mu(1-\gamma^5)u_i p_{\pi^0}^\mu \pi^0\,, \\ M_b^{\eta} &= -\tfrac{iG_F^2 M_W^2 f_\pi}{8\pi^2\sqrt{3}}\bar{F}_{bij}^{\eta}\bar{u}_j\gamma_\mu(1-\gamma^5)u_i p_\eta^\mu \eta\,, \end{aligned}$$

with

$$\begin{aligned} \bar{F}_b^{\pi^0} &= -\mid V_{td}\mid^2(\bar{F}_b - \bar{F}_b^t)\,, \\ \bar{F}_b^{\eta} &= [\mid V_{td}\mid^2 - 2\mid V_{ts}\mid^2](\bar{F}_b - \bar{F}_b^t)\,. \end{aligned}$$

After adding the amplitudes and integrating over phase space we get the following decay widths

$$\Gamma(l_i \to l_j\pi^0) = \frac{G_F^4 M_W^4 f_\pi^2 m_i^3}{2^5\pi^5}\mid \bar{F}_{Zij} + \frac{\bar{F}_{bij}^{\pi^0}}{2}\mid^2[1 - \frac{m_\pi^2}{m_i^2}]^2\,, \tag{29}$$

and

$$\Gamma(l_i \to l_j\eta) = \frac{G_F^4 M_W^4 f_\pi^2 m_i^3}{2^5 3\pi^5}\mid \bar{F}_{Zij} + \frac{\bar{F}_{bij}^{\eta}}{2}\mid^2[1 - \frac{m_\eta^2}{m_i^2}]^2\,. \tag{30}$$

Using again the relation in eq. (11) in order to determine the allowed values of the rates, one can show that it is possible to get branching ratios which are within the sensitivities expected at a τ factory. In Fig. 3 we plot the ratios for all these processes.

5 Conclusions

In this letter we have analysed the possibility of having measurable lepton flavour violation in models with massless neutrinos based on the simplest $SU(2) \otimes U(1)$ gauge structure. A specific model was presented where the violation of lepton flavour can be potentially sizable due to the effects of the admixtures of new isosinglet fermions in the electroweak currents. The existence of these mixings is consistent with the masslessness of the observed neutrinos as a result of the imposition of an exact B-L symmetry. We have concentrated on the flavour-violating μ and τ decays and shown that most of muon decay processes can be larger than existing limits while tau decay processes could easily be within reach of future experimental facilities such as a τ factory. It is important to emphasize, in view of our results, the interest on more accurate tests for rare μ and τ decay branching ratios and we look forward with optimism to the prospects of having in the next few years very intense sources of muon and tau leptons. As we have illustrated, the physics case for such facilities is *complementary* to the physics of neutrino mass *per se*.

Figure Captions

Fig. 1.a:
Diagrams contributing to the effective vertex $Z\, l\, \bar{l}_i$.

Fig. 1.b:
Diagrams contributing to the effective vertex $\gamma\, l\, \bar{l}_i$.

Fig. 1.c:
Box diagrams contributing to the decay $l \to l_i l_j^+ l_j^-$ for $i \neq j$.

Fig. 2:
Attainable branching ratio for the decay $\mu \to 3e$ in the total lepton number conserving model (solid line). The dashed line corresponds to the decay $\mu \to e\gamma$ after imposing universality constraints. We see that for M bigger than 10 GeV, this branching ratio is bigger than the experimental limit, and therefore this will restrict the allowed branching ratio for $\mu \to 3e$.

Fig. 3:
Attainable branching ratios for the τ decays in the total lepton number conserving model. The solid line corresponds to the decay $l \to l_i l_j^+ l_j^-$, the dashed line corresponds to $\tau \to \pi^0 l_i$, the dotted line to $\tau \to \eta l_i$ and the dash-dotted line to $\tau \to l_i \gamma$. In all cases we have summed over all possible final-state leptons.

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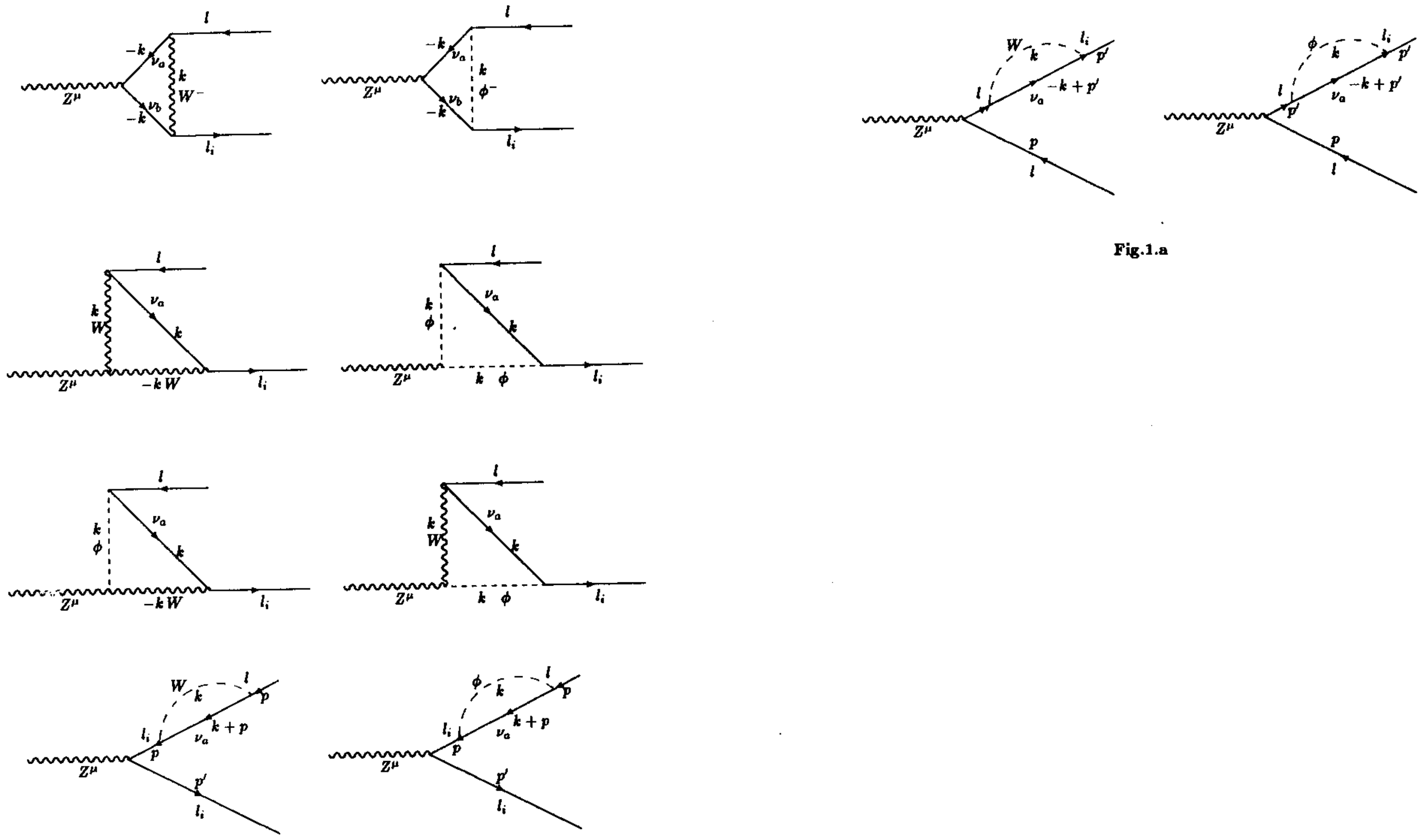

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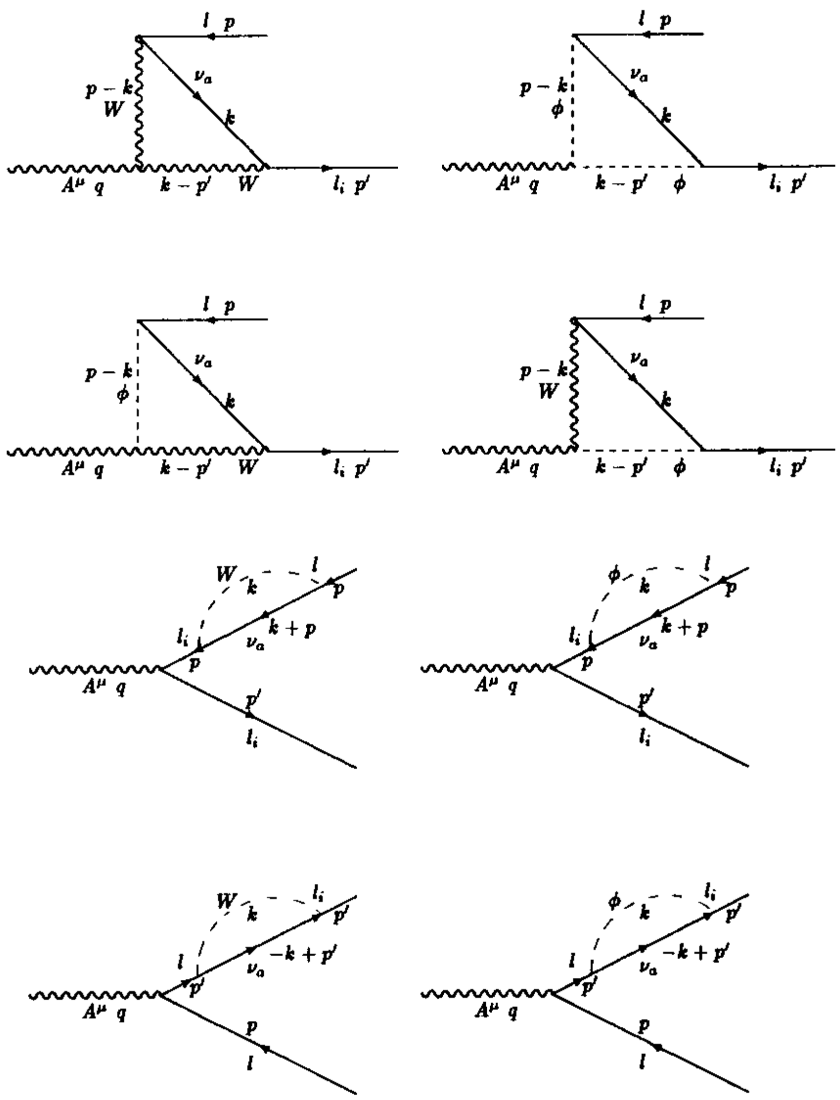

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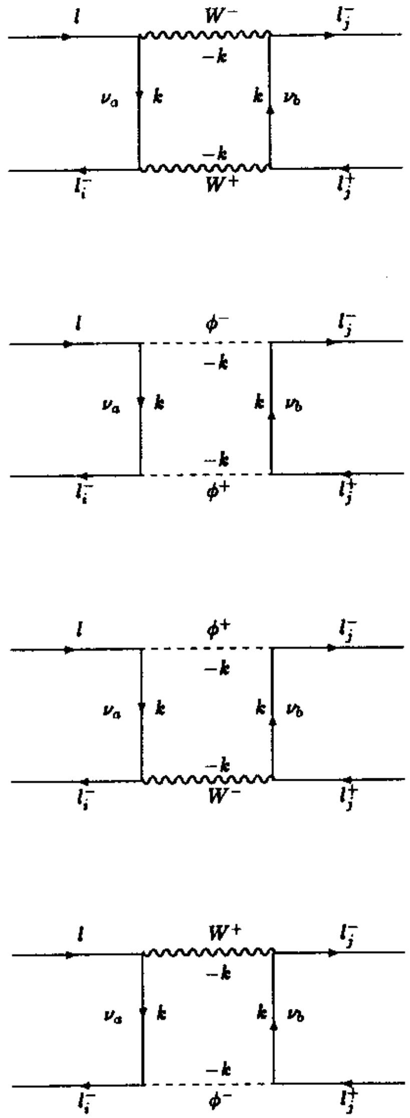

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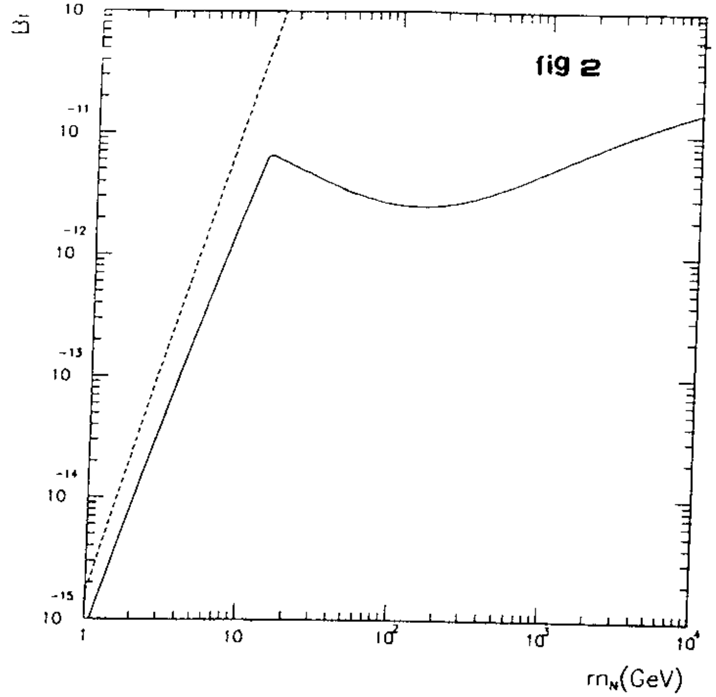
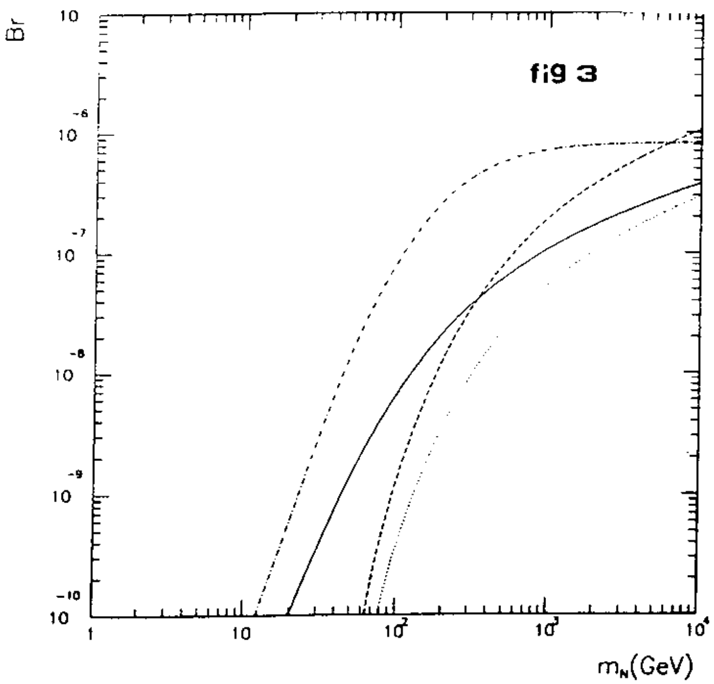


Br
10^{-11}
10^{-12}
10^{-13}
10^{-14}
10^{-15}
1
10
10^{2}
10^{3}
10^{4}
m_N(GeV)

fig 2



Br
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
1
10
10^{2}
10^{3}
10^{4}
m_N(GeV)

fig 3